Logique & Analyse **214** (2011), 249–285

WHITEHEAD'S MEREOTOPOLOGY AND THE PROJECT OF FORMAL ONTOLOGY

SÉBASTIEN RICHARD

Rationalism is an adventure in the clarification of thought, progressive and never final. But it is an adventure in which even partial success has importance.
(WHITEHEAD, A.N., *Process and Reality*, p. 9)

Mereology is the theory of wholes and parts. The first formal mereology was developed by Husserl in his third *Logical Investigation* at the beginning of the twentieth century. In 1916 Stanisław Leśniewski gave the first axiomatization of a classical extensional formal mereology. That same year, Alfred North Whitehead also gave a sketch of a mereology in "La théorie relationniste de l'espace". It was developed in the perspective of a theory of space in which the concept of point is no longer considered as primitive, but is built in terms of the relations between objects. This project was then taken up and amplified in the wider perspective of the method of extensive abstraction presented in *An Enquiry Concerning the Principles of Natural Knowledge* and *The Concept of Nature*. Afterwards, Whitehead added to what was first a theory of the part-whole relation some definitions of topological notions such as junction. This would allow a first analysis of the concept of boundary. These topological reflections were then only reduced to mereological ones and it is only in *Proces and Reality* that Whitehead developed a directly topological theory in which the mereological concepts can be defined. Our purpose in this paper is to study these three mereo-topological theories and to set them out in a formalized way, in order to finally question about their possible axiomatization.

Today the project of formal ontology is closely related to mereotopology which constitutes its largest part. This is why, in the final section, we will study the links between Whitehead's project and the projet of formal ontology. Our approach will be to investigate the meaning that one can attribute to such an expression, in particular in Husserl's philosophy.

1. *The mereology of "La théorie relationniste de l'espace"*

In "La théorie relationniste de l'espace", published in French in 1916,[1] Whitehead develops a spatial theory in which the notion of "point" is not considered as simple, but as a "complex entity", i.e. a logical function of the relations between objects that constitute space.[2] By not proceeding this way, one would immediately arrive at "an absolute theory of space"[3] while Whitehead wants to build "a relational theory of space".

He begins by distinguishing between three main meanings of the word "space":[4]

a) the "apparent space" is "the place of the objects as they appear to us", or as Whitehead says, the space in which the green trees, the sounds, the smells, and so on, are perceived. The objects of the apparent space are the "perceived objects", or the "apparent objects". Consequently, the apparent space of a perceiving subject will be different from the apparent space of another perceiving subject. It consists of certain relations between the objects perceived by a certain perceiving subject. The apparent space is further subdivided into two different spaces:

   1) the "immediate apparent space" is the space of what appears immediately, in a fragmentary and limited way, to the subject who perceives, such as, for example, the part of a room or the side of a mountain;

   2) the "complete apparent space" is the "idea of the total space of a complete world of apparent objects in which we do not refer

[1] WHITEHEAD, A.N., 1916, "La théorie relationniste de l'espace", *Revue de métaphysique et de morale*, 23 (3), pp. 423–454. In fact, the text had already been presented in April 1914 at the Congress of mathematical logic in Paris.

[2] *Ibid.*, p. 429. According to Jules Vuillemin, the choice of a relational conception of space appears to be related to the non-published fourth volume of *Principia Mathematica*. It would be through an analogy with the concept of number that Whitehead would have conceived the concept of point: it should be conceived as a class of some entities (VUILLEMIN, J., 1971, *La logique et le monde sensible. Études sur les théories contemporaines de l'abstraction*, Flammarion, coll. Nouvelle bibliothèque scientifique, Paris, pp. 62–63).

[3] WHITEHEAD, A.N., 1916, *op. cit.*, p. 430.

[4] *Ibid.*, pp. 423–426.

exclusively to a perceiving subject". This completion has the characteristic, on the one hand, of the adjustment of immediate spaces of different individuals and, on the other hand, of the adjunction of the idea of all the perceptions which could appear to hypothetical subjects according to laws and the state of the physical world;

b) the "physical space" is a unique and universal hypothetical space of objects, whose relations would correspond exactly to our sensations. In this space what appears immediately to a subject is related to "a complex of relations between physical objects". This is the space in which the electrons and the molecules move and interact with each other;[5]

c) the "abstract space" is the space of abstract geometry.

While there are a lot of immediate apparent spaces and geometrical spaces, Whitehead notices that it is common to suppose that there exists only one complete apparent space and only one physical space. While this supposition seems to be justified for the complete apparent space, this is less obvious as far as the physical space is concerned. Indeed, as this space is constituted by a complex of relations between physical objects, it seems possible to build different complexes "which will give rise to different definitions of points, lines, planes, and so on, with the logical type of the properties which are defined as spatial".[6] However Whitehead adds that there is certainly "a dominant interest" for a particular complex of relations which constitutes the only physical space from a practical point of view.

What the paper of 1916 tries to show is that geometry needs not be conceived abstractedly, but may be conceived in such a way that its properties and the abstract space are articulated either to the apparent space or to the physical space. We must then define the geometrical objects "in function of the relations between objects",[7] either perceived or apparent. Here Whitehead is opposed to "the absolute theory of space" which considers the notion

[5] There is a "parallelism" between the apparent space and the physical space, but Whitehead put his study off until later. As noted by Durand, we find here the idea of what will be called "the bifurcation of nature" in *The Concept of Nature* and that Whitehead will criticize (DURAND, G., 2006, *Des événements aux objets. La méthode d'abstraction extensive chez A.N. Whitehead*, Ontos Verlag, coll. Chromatiques Whiteheadiennes, Frankfurt, p. 78).

[6] WHITEHEAD, A.N., 1916, *op. cit.*, p. 426.

[7] *Ibid.*, p. 430.

of point as simple and takes the relation of being at a point as primitive. The ultimate facts of a geometry based on such a conception of space are then "the primitive relations of the objects to their absolute positions".[8] On the other hand, the relational theory of space that Whitehead develops here states as essential that the points be considered as complex entities. These are logical functions of the relations between the objects that constitute space, wether it be physical or apparent. Consequently, the relations of the objets to their absolute positions are no longuer ultimate, contrary to the relations between objects. This is not the space which comes first but "a world of things in relation". In a way, the implementation of such a theory will then consist of inverting the traditional link of definition between points and volumes: thanks to a mereological relation of inclusion between objects, Whitehead is able to define the notion of point as a certain class of volumes. Consequently points will be derived entities that we do not need anymore. This method of definition, which will later be called "the method of extensive abstraction", is then extended to the definitions of the concepts of line and surface. It is not our purpose here to examine them. We want only to study the formal theory of the (mereological) relation of inclusion which it requires as premise.

The relational theory of space starts with the fundamental idea of a particular class 'σ' of relations. The possible definitions of the spatial concepts and the properties of 'σ' must then be specified in such a way that the common propositions may be true of the concepts thus defined. The world we obtain from 'σ' is called the "σ-world".[9]

Whitehead begins with the application of the class 'σ' to the apparent space. In this space we suppose that the class 'σ' represents "the class of relations between a possible perceiving subject and the extended and perceived apparent object".[10] Consequently, some member 'R' of 'σ' is "a mode of perception of an object by the senses", such that '$p\ R\ x$' means that the possible subject 'p' perceives the extended apparent objects 'x' according to the mode of perception 'R'. The class of converse domains of the relations belonging to 'σ', what Whitehead calls the class of "σ-objects" or the "σ-domain", is then composed of all the extended apparent objects which are perceived and the class of domains of the relations belonging to 'σ' is composed of all the possible perceiving subjects.

If we consider now the physical world, 'σ' is "the class of direct relations between physical objects".[11] So '$x\ R\ y$' means that the physical object 'x'

[8] *Idem.*

[9] *Ibid.*, p. 431.

[10] *Idem.*

[11] *Idem.*

has the direct relation 'R' to the physical object 'y'. The σ-domain and its converse are in that case identical and composed of all the physical objects.

How can we define the geometrical concept of point for the σ-worlds we have just specified? Whitehead suggests we define a derived relation 'E_σ', called 'σ-inclusion', "which is analogous, in his formal properties, to the relation of whole to parts".[12] But what is the meaning of this relation? Whitehead distinguishes between three meanings:[13]

a) the notions of 'whole' and 'part' mean the 'all' and 'some' that we find in logic. The whole and its parts are then classes, the latter being subclasses of the former;[14]

b) 'parts' mean the "heterogenous" parts of a more complex whole. In other words, the parts are different "by essence" from the complex objects, as it is the case, for example, of the sugar in a pudding or of a note in a chord. Whitehead calls a heterogenous part a "component", but this meaning will not be used later on;

c) the notions of 'whole' and 'part' are understood in a spatial sense. For example, the head of a horse is a spatial part of the body of a horse, a province is a spatial part of a territory, or a meter is a spatial part of a kilometer. We then have "homogenous" parts, i.e. parts which are of the same kind as the whole of which they are parts.

Strangely, Whitehead does not characterize as homogenous the third meaning of the word "part". Yet it seems obvious that the parts in this sense are also homogenous. Indeed, in this case, the whole and one of its parts are two classes of the same type, the second being included in the first. On the other hand, if he had considered the membership between a class and one of its elements as also being like a part-whole relation, this would have been heterogenous, since, according to the theory of types, a class must always be of a different type than that of its members.

Nonetheless, Whitehead asks himself if the third meaning is not a particular case of the first.[15] If this were to be the case then a spatial body would

[12] *Ibid.*, p. 432.

[13] *Ibid.*, p. 433.

[14] In this sense, Whitehead stresses that a part cannot be the class of which it is a part. So we can claim that the part-whole relation must be understood as the proper part-whole relation.

[15] WHITEHEAD, A.N., 1916, *op. cit.*, p. 434.

be a part of another spatial body because the first body would be a class of things, or a collection, included in the class of things constituted by the second body. But what are the things that constitute a spatial body? They cannot be smaller bodies that would be spatial homogenous parts of the given body, for we would then use the notion of "spatial homogenous part" to explain the notion of a body conceived as a class of things. In this case there would be a circle in the explanation.

However, if we want to maintain the identification between spatial homogenous parts and the parts as proper subclasses, "we are led to consider an extended section of space as a collection of points and an extended body as a collection of things occupying points".[16] These points-things are the "components" of the extended bodies that are the classes of points-things. According to Whitehead, this conception does not involve any contradiction, either from the point of view of a physical world or from the point of view of the apparent world. But it can be questioned from another point of view.

Let us first consider the perspective of the apparent world. The problem we have here is that we perceive the apparent objects. These objects are always perceived as "units", and not as classes of points.[17] Furthermore, an extended perceived object is always perceived in spatial relation with other objects and with its parts. It is now precisely what the conception in terms of logical classes cannot justify. On the one hand, when we fix our attention on a part of a whole, the immediate perception of the whole is lost in the explanation in terms of logical parts. On the other hand, when we perceive a whole, it is, according to the reductionist point of view, only a mental reconstruction as a logical whole of its perceived parts, these coming first. Then, the perception of a big size whole cannot be perceived as an immediately perceived unit. If we are to understand Whitehead correctly, his objection to reductionism regarding the perception of apparent objects is that it describes the perception of objects as a perception isolated from the parts, which are conceived as classes of points. Consequently, the relation to the whole or the perception of the whole is lost from the point of view of the immediate perception: they can only be reconstructed by another mental operation.

Let us proceed here with the perspective of the physical world. The σ-domain, or the class of σ-objects, must "necessarily" be constituted of "ultimate units".[18] If this were not the case, the relations of the class σ would not be "ultimate relations". Whitehead states that, as in geometry where we

[16] *Idem*.

[17] *Idem*.

[18] *Ibid.*, p. 435.

have to start with points "without parts or size", in the σ-world we must also begin with unbreakable units.

Having established the meaning of the part-whole relation that he wants to use, Whitehead now comes to the definition of the relation of inclusion.[19] If for any pair of σ-objects 'a' and 'b', '$a\ E_\sigma\ b$' means that 'a includes b', the relation of inclusion 'E_σ' can be defined in the following way:[20]

$$(a\ E_\sigma\ b) \equiv ((\exists x)(\exists R)^\ulcorner(R \in \sigma) \wedge (x\ R\ b)^\urcorner \wedge (\forall x)(\forall R)^\ulcorner((R \in \sigma) \wedge (x\ R\ b)) \supset (x\ R\ a)^\urcorner).$$

As we can easily see here, two conditions pertain to this definition:

a) '$(\exists x)(\exists R)^\ulcorner(R \in \sigma) \wedge (x\ R\ b\ )^\urcorner$' which means that '$b$ is a σ-object', i.e. a member of the converse σ-domain. This condition ensures that the entities 'a' and 'b' are σ-objects;

b) '$(\forall x)(\forall R)^\ulcorner((R \in \sigma) \wedge (x\ R\ b)) \supset (x\ R\ a)^\urcorner$' which means that 'if R is a member of σ and x is any entity having the relation R to b, then x has always the relation R to a'. For example, from the point of view of the apparent space, this condition maintains that 'every subject who perceives an object b, also perceives an apparent object a'. From the point of view of the physical space, it means that 'any physical object in direct relation with the physical object b, is in direct relation with the physical object a'.

When both conditions are satisfied, we can say that 'a includes b regarding σ', or 'b is a σ-part of a'. We understand then that the second condition asserts that to be in a direct relation or in a relation of perception with a part is to be in a direct relation or in a relation of perception with the whole. For example, to touch the head of a horse is to touch the horse.

[19] Whitehead avoids calling this relation "the part-whole relation" and considers as uninteresting the question of knowing if it would deserve this label (*idem*).

[20] We modify somewhat Whitehead's notation (cf. SIMONS, P., 2007, "Whitehead and Mereology", *in* DURAND, G. and WEBER, M., 2007, Les principes de la connaissance naturelle *d'Alfred North Whitehead*, Ontos Verlag, coll. Chromatiques Whiteheadiennes, Frankfurt, p. 218), which is the notation of *Principia Mathematica*. Furthermore we use '$^\ulcorner$' and '$^\urcorner$' to indicate the scope of quantifiers. This last notation is taken from Leśniewski.

Let us now consider some "hypotheses"[21] on the relation 'E_σ' derived from 'σ'. They will ensure us that this relation has the "formal" properties characteristic of the spatial part-whole relation in the apparent space:[22]

a) the domain of 'E_σ', i.e. the class of objects which are subject to this relation, is the total set of σ-objets. It follows from this that if 'x' is some σ-object, then 'x' is σ-included by some σ-object which includes it or is included by it;

b) the part-whole relation is either "reflexive" or "envelops a diversity".[23] It is "only a matter of words or of technical commodity" that a part be always a part of itself or not. But we cannot have some objects which would be part of themselves and others which would not. Here the relation is considered reflexive:

$$(\forall x)^\ulcorner(\exists y)^\ulcorner(x\ E_\sigma\ y) \vee (y\ E_\sigma\ x)^\urcorner \supset (x\ E_\sigma\ x)^\urcorner;$$

c) the part-whole relation is transitive:

$$(\forall x)(\forall y)(\forall z)^\ulcorner((x\ E_\sigma\ y) \wedge (y\ E_\sigma\ z)) \supset (x\ E_\sigma\ z)^\urcorner;$$

d) the part-whole relation is asymmetrical:

$$(\forall x)(\forall y)^\ulcorner((x\ E_\sigma\ y) \wedge (x \neq y)) \supset \sim (y\ E_\sigma\ x)^\urcorner;$$

e) every spatial object has other spatial parts than itself:

$$(\forall x)(\forall y)^\ulcorner(x\ E_\sigma\ y) \supset (\exists z)^\ulcorner(z \neq x) \wedge (z\ E_\sigma\ x)^{\urcorner\urcorner}.$$

So the whiteheadian mereology is non-atomistic. Here we have the "hypothesis of infinite divisibility" which is the basis of the "continuity" of space. Whitehead sets it out as an hypothesis on the class 'σ', but does not give any real justification for it.

[21] As it is the case in Whitehead's other mereo-topological theories, the axiomatic status of the properties attributed the part-whole relation is not clear.

[22] WHITEHEAD, A.N., 1916, *op. cit.*, pp. 435–437.

[23] Whitehead does not explain what he means by "envelops a diversity". According to the afterward text, we can suppose that it means 'to be irreflexive'.

The theory which is set out in "La théorie relationniste de l'espace" is only a sketch and for the moment it contains no topological element. It is what we could call a formal non-atomistic ground mereology. It is only in *An Enquiry Concerning the Principles of Natural Knowledge* that Whitehead will develop a mereology worthy of this name, containing the first outline of a formal topology.

2. *The mereo(-topology) of* An Enquiry Concerning the Principles of Natural Knowledge

In *An Enquiry Concerning the Principles of Natural Knowledge*, and also in *The Concept of Nature*, Whitehead develops a mereology more accomplished than in "La théorie relationniste de l'espace". Furthermore, he tries to define topological notions, such as "junction", in mereological terms. These notions constitute the first analysis of the concept of "boundary". Thus, we can say that Whitehead sets out here a mereo(-topology). The perspective in which this theory is developed is similar to the one of the 1916 paper, but the ambition here is wider. Indeed, Whitehead does not consider only geometry and space but also modern physics and nature from the point of view of his "process", or "passage".[24]

The spatial point or the temporal instant are no longer data of experience. Consequently, Whitehead will show how the scientific concepts of space and time can be derived with the help of an abstraction from experience. These concepts are complex in their essence and can be built from mutual relations between events which are the ultimate data of experience. These spatio-temporal relations between events constitute their spatio-temporal structure and are expressed by "the relation of extension". The principle of this "method of extensive abstraction" consists in a progressive limitation of the spatial and temporal extension specific to the phenomenon considered. The ideal event at which we arrive so by a progressive approximation is such that it is "without extension". Thus, abstracting from concrete events, we obtain some "events-particules", "point flash of instantaneous duration",[25] which are "the exactly determined concepts on which the whole fabric of science rests".[26] The method of extensive abstraction allows

[24] WHITEHEAD, A.N., 1964 (1920), *The Concept of Nature. The Tarner Lectures Delivered in Trinity College November 1919*, Cambridge University Press, Cambridge, p. 54.

[25] *Ibid.*, p. 172.

[26] WHITEHEAD, A.N., 1919, *An Enquiry Concerning the Principles of Natural Knowledge*, Cambridge University Press, Cambridge, 18.3, p. 76.

Whitehead then to show how the scientific concepts of time, space and matter issue "from fundamental relations between events, and from recognition of the characters of events".[27]

We do not wish here to outline the method of extensive abstraction, but the theory which it presupposes, that is the theory of the spatio-temporal structure of events, i.e. the theory of the relation of extension. This theory is, as in "La théorie relationniste de l'espace", a mereology because the relation of extension, also called "the second constant of the externality",[28] is the converse of the part-whole relation. It is symbolized 'K' and can be read, when applied to two events 'x' and 'y', ('$x\ K\ y$'), 'x' covers 'y', or according to the converse reading: 'y is a (proper) part of x',[29] what we will symbolize '$y \ll x$'.

As we have already stated, the relation of extension has events for *relata*. But what does Whitehead mean with this notion? He gives us the following example:

> *Thus the event which is the passage of the car is part of the whole life of the event which is the passage of the car. Similarly the event which is the continued existence of the house extends over the event which is the continued existence of a brick of the house, and the existence of the house during one day extends over its existence during one specified second of that day.*[30]

So, the relation of extension is one of "the simplest characteristics without which no datum of knowledge would be recognized as an event belonging to the order of nature".[31] It is spatio-temporal; the concepts of space and time and their differentiation being largely derived from the properties of this relation.

[27] *Ibid.*, 2.5, p. 8.

[28] *Ibid.*, 18.1, p. 101. The "constants of the externality" are "those characteristics of a perceptual experience which it possesses when we assign the property of being an observation of the passage of external nature to it, namely when we apprehend it". An event is what possesses those characteristics (*ibid.*, 17.1, pp. 71–72).

[29] Whitehead specifies further that "the term "part" means here "proper part"" (*ibid.*, 27.2, p. 101).

[30] *Ibid.*, 18.1, p. 75.

[31] *Idem.*

An event being extended possesses parts and every part of an event is itself an event. Whitehead maintains the non-atomicity of the notion of event, i.e. there is no minimal event (an event without parts). The divisibility of an event is infinite and an event is, consequently, never the sum of its parts (these being infinitely many). This does not proscribe the unity of the event which is qualified by Whitehead as "substantial".[32] This unity is quite simply not "an abstract derivative from logical construction".

For Whitehead nature is a continuous passage of events in other events. We see then the fundamental importance of the relation of extension for the comprehension of the "process of nature", "his creative advance":

> *Events never change. Nature develops, in the sense that an event* e *becomes part of an event* e′ *which includes (i.e. extends over)* e *and also extends into the futurity beyond* e*. Thus in a sense the event* e *does change, namely, in its relations to the events which were not and which become actual in the creative advance of nature. The change of an event* e *in this meaning of the term "change", will be called the "passage" of* e*; and the word "change" will not be used in this sense. Thus we say that events pass but do not change. The passage of an event is its passing into some other event which is not it.*[33]

Thus, an event "passes" in becoming part of another event, that is an event which covers it.

Jules Vuillemin and Peter Simons have each proposed a formalization of Whitehead's mereo(-topology).[34] The first expresses it with the relation of extension 'K', the second with the relation of proper part '$\ll$'. We set out here a formalization inspired by Simons.[35] In the same way as in "La théorie relationniste de l'espace", Whitehead does not formulate his mereo (-topology) in an axiomatic fashion, but with properties and definitions. We first formalize this theory following Whitehead's text and then consider its possible axiomatization.

[32] *Ibid.*, 18.4, p. 77.

[33] *Ibid.*, 14.3, p. 62.

[34] Cf. VUILLEMIN, J., 1971, *op. cit.*, pp. 67–68; and SIMONS, P., 2007, *op. cit.*, pp. 219–223.

[35] We indicate in footnotes Vuillemin's corresponding formalization.

The first property is the following:[36]

P1 $(a \ll b) \supset (a \neq b)$,

which means that 'nothing is a part of itself'[37] and expresses the property of irreflexivity of the part-whole relation.[38]

The second property:[39]

P2 $(\forall x)^\ulcorner(\exists y)^\ulcorner x \ll y^\urcorner \wedge (\exists y)^\ulcorner y \ll x^{\urcorner\urcorner}$,

which means that 'every event is a part of another event and has another event as part'. Simons calls this property the "principle of openness above and below".[40] The first part denies the existence of a maximal event and the second denies the existence of an atomic event.

The third property:[41]

P3 $((\forall x)^\ulcorner(x \ll a) \supset (x \ll b)^\urcorner \wedge (a \neq b)) \supset (a \ll b)$,

which means that 'if every part of a is a part of b and a and b are different,

[36] P1′ $(aKb) \supset (a \neq b)$,
which means that 'if a covers b, then a is distinct from b'.

[37] As Simons suggests, these formulas must be interpreted as universally quantified propositions:
P1″ $(\forall x)(\forall y)^\ulcorner(x \ll y) \supset (x \neq y)^\urcorner$.
However, we keep the formalization in terms of individual constants which is closer to Whitehead's text.

[38] Simons speaks of the property of "asymmetry" (SIMONS, P., 2007, *op. cit.*, p. 219). We can express it the following way:
$(a \ll b) \supset \sim (b \ll a)$.
This property is the result of the irreflexivity (P1) and the transitivity (P4) of '$\ll$'.

[39] P2′ $(\forall x)^\ulcorner(\exists y)^\ulcorner xKy^\urcorner \wedge (\exists y)^\ulcorner yKx^{\urcorner\urcorner}$,
which means that 'each event covers other events and is covered by other events'.

[40] SIMONS, P., 2007, *op. cit.*, p. 219.

[41] P3′ $((\forall x)^\ulcorner(bKx) \supset (aKx)^\urcorner \wedge (a \neq b)) \supset (aKb)$,
which means that 'if every event covered by b is covered by a and if a is distinct from b, then a covers b'.

then a is a part of b'. Simons calls this property the "proper parts principle".[42]

The fourth property:[43]

P4 $((a \ll b) \wedge (b \ll c)) \supset (a \ll c)$,

which means that 'if a first event is a part of a second event and this second event is a part of a third event, then the first is a part of the third'. It is the property of transitivity.

The fifth property:[44]

P5 $(a \ll b) \supset (\exists x)\ulcorner(a \ll x) \wedge (x \ll b)\urcorner$,

which means that 'if a is a part of b, then there exists an event of which a is a part and which is a part of b'. It is the property of density.

The sixth property:[45]

P6 $(\forall x)(\forall y)(\exists z)\ulcorner(x \ll z) \wedge (y \ll z)\urcorner$,

which means that 'for any two events, there exists an event of which they are both part'. It is what Simons calls the property of "upper bound".[46]

Whitehead then defines several notions from the notion of proper part. He starts with the notion of "intersection" that we symbolize '$\circ$':

D1 $(a \circ b) \equiv (\exists x)\ulcorner(x \ll a) \wedge (x \ll b)\urcorner$,

which means that 'two events intersect when they have parts in common'.

[42] SIMONS, P., 2007, *op. cit.*, p. 219. On this principle and his consequences, particularly the extensionality, cf. SIMONS, P., 1987, *Parts. A Study in Ontology*, Clarendon Press, Oxford, pp. 28 and 112–117.

[43] P4′ $((aKb) \wedge (bKc)) \supset (aKc)$,

which means that 'if a first event covers a second and this second event covers a third, then the first covers the third'.

[44] P5′ $(aKb) \supset (\exists x)\ulcorner(aKx) \wedge (xKb)\urcorner$,

which means that 'if a covers b, then there exists an event covered by a and which covers b'.

[45] P6′ $(\forall x)(\forall y)(\exists z)\ulcorner(zKx) \wedge (zKy)\urcorner$,

which means that 'for any two events, there exists an event which covers them both'.

[46] SIMONS, P., 2007, *op. cit.*, p. 220.

Let us notice that this notion is often called the relation of "overlapping".[47] Following this first definition, Whitehead sets out a proposition whose status is not clear and that can be formalized in the following way:

P7 $(\forall x)\ulcorner(x \circ a) \supset (x \circ b)\urcorner \supset ((a \ll b) \vee (a = b))$,

which means that 'if anything that intersects a intersects b, then either a is a part of b or a is identical to b'. Simons calls this the "strong supplementation principle". As we can easily see, this principle is closely linked to (P3) and differs from it only by the stronger condition according to which anything that intersects 'a' must also intersect 'b'. In fact, we can demonstrate that (P3) is deductible from (P7) and (D1).[48]

Whitehead then defines the notion of 'separation' that we symbolize '$\urcorner$':

D2 $(a \urcorner b) \equiv \sim (a \circ b)$,

which means that 'two events are separated if and only if they do not intersect'. Whitehead extends this definition to sets, so that a "separated set" can be defined in the following way:

D3 $sep(\alpha) \equiv (\forall x)(\forall y)\ulcorner((x \in \alpha) \wedge (y \in \alpha) \wedge (x \neq y)) \supset (x \urcorner y)\urcorner$,

which means that 'a set is separated if and only if every pair of distinct events of this set are separated'. Simons adds the condition of distinction on pair of events '$x \neq y$' which is not present in Whitehead's formulation.[49] This condition is necessary because two identical events can obviously not be separated.

Whitehead defines the notion of "the dissection of an event":

D4 $(\alpha\ diss\ a) \equiv (sep(\alpha) \wedge (\forall x)\ulcorner(x \circ a) \equiv (\exists y)\ulcorner(y \in \alpha) \wedge (x \circ y)\urcorner\urcorner)$,

which means that 'a set α is a dissection of an event a if and only if α is a separated set and anything that intersects a also intersects a member of α, and conversely'. So, according to Whitehead, "a dissection is a non-overlapping exhaustive analysis of an event into a set of parts, and conversely the dissected event is the one and only event of which this set is a dissection".[50] In

[47] Cf., for example, SIMONS, P., 1987, *op. cit.*, p. 28.

[48] For this demonstration, cf. *ibid.*, p. 29.

[49] Cf. SIMONS, P., 2007, *op. cit.*, p. 220.

[50] WHITEHEAD, A.N., 1919, *op. cit.*, p. 102.

other words, the dissection of an event is a set whose members constitute a complete division in disjoined parts of this event. Furthermore, a dissection is the dissection of only one event, what can be formulated as follows:

P8 $((\alpha \; diss \; a) \wedge (\alpha \; diss \; b)) \supset (a = b)$.

According to Whitehead, there is always an "indefinite number" of dissections of a given event:

P9 $(\exists\alpha)(\exists\beta)\ulcorner(\alpha \; diss \; a) \wedge (\beta \; diss \; a) \wedge (\alpha \neq \beta)\urcorner$.

Simons interprets this affirmation as saying that there is always two different dissections of an event, however one could also interpret this as saying that there are always infinitely many dissections of an event, what is "plausible" with density.[51]

Whitehead adds yet:

P10 $(a \ll b) \supset (\exists\alpha)\ulcorner(\alpha \; diss \; b) \wedge (a \in \alpha)\urcorner$,

which means that 'if a is a part of b, then there exists a dissection of b, whose a is a member'. According to Whitehead, it follows from this:[52]

P11 $(a \ll b) \supset (\exists x)\ulcorner(x \;\rceil\; a) \wedge (x \ll b)\urcorner$,

which means that 'if a is a part of b, then there exists at least one event separated from a which is also a part of b'. Simons calls this property the "weak supplementation principle".[53]

Whitehead adds other definitions which are no longer mereological, but topological.[54] He begins with the definition of the relation of 'junction':

D5 $(a \; junction \; b) \equiv (\exists x)\ulcorner(x \circ a) \wedge (x \circ b) \wedge (\exists\alpha)\ulcorner(\alpha \; diss \; x) \wedge (\forall z)\ulcorner(z \in \alpha) \supset ((z \ll a) \vee (z \ll b))\urcorner\urcorner\urcorner$,

[51] SIMONS, P., 2007, *op. cit.*, p. 220.

[52] Strangely, Simons does not consider this consequence in his presentation of Whitehead's mereo(-topology). This may be due to the fact that he considers that it can easily be deduced from (P7) (cf. SIMONS, P., 1987, *op. cit.*, p. 29).

[53] *Ibid.*, p. 28.

[54] WHITEHEAD, A.N., 1919, *op. cit.*, pp. 102–103.

which means that 'two events a and b are joined if and only if there exists a third event x such that x intersects both a and b and there exists a dissection of x of which each member is a part of a, or of b, or of both'. In *The Concept of Nature*, Whitehead gives a somewhat different formulation of this definition: two events are joined if and only if there is a third event such that they are both part of it and no part of it is separated from both of the two given events.[55] We can formalize this in the following way:

D5′ $(a\ junction'\ b) \equiv (\exists x)^{\ulcorner}(a \ll x) \wedge (b \ll x) \wedge \sim (\exists y)^{\ulcorner}(y \ll x) \wedge (y \upharpoonright a) \wedge (y \upharpoonright b)^{\urcorner\urcorner}$.

Whitehead seems to consider these two definitions as independent when he says that if one of the two is adopted, the other appears as an axiom "respecting the character of junction as we know it in nature".[56]

According to Whitehead, this relation of junction expresses "entirely" the concept of "the continuity of nature, that is the fact that "two joined events are continuous with each other"[57] and they form "exactly one event which is their sum".[58] So, two events which intersect are always joined:

P12 $(a \circ b) \supset (a\ junction\ b)$,

which is a consequence of the definition of junction. However this last notion is broader than the notion of intersection, for two events can be joined and yet separated:

P13 $(\exists x)(\exists y)^{\ulcorner}(x\ junction\ y) \wedge (x \upharpoonright y)^{\urcorner}$.

Such events will be said "adjoined":[59]

[55] WHITEHEAD, A.N., 1964 (1920), *op. cit.*, p. 76.

[56] *Idem*. Palter argues that this is not the case and that (D5) is a consequence of (D5'). However although he offers good arguments for the difference between the first condition of the two definitions, he completely neglects their second part (cf. PALTER, R.M., 1960, *Whitehead's Philosophy of Science*, University of Chicago Press, Chicago, p. 46). Simons, for his part, neglects in his formalization of (D5') its difference with the first condition of (D5') when he expresses it with '$\circ$' and not with '$\ll$' as we have (cf. WD5' *in* SIMONS, P., 1987, *op. cit.*, p. 85).

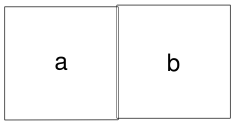
[57] WHITEHEAD, A.N., 1919, *op. cit.*, p. 102.

[58] WHITEHEAD, A.N., 1964 (1920), *op. cit.*, p. 76.

[59] We can notice the analogy of this definition with the definition of two intervals which "abut" given by Needham: "two intervals abut if they are separated and there is an interval

D6 $(a\ adjunction\ b) \equiv ((a\ junction\ b) \wedge (a \rceil b))$.

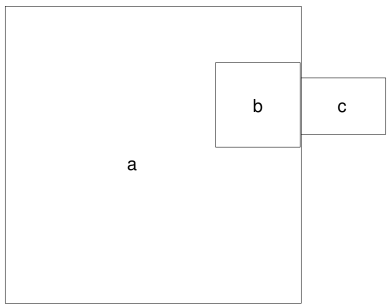
For example, in the following figure 'a' and 'b' are adjoined:



If two events 'a' and 'b' cover each other and there is a third event separated from 'a' and 'b', then 'a' and 'b' are said to be "injoined":

D7 $(a\ injunction\ b) \equiv ((b \ll a) \wedge (\exists x)\ulcorner(x\ sep\ a) \wedge (a\ adjunction\ b)\urcorner)$.

In the following figure 'a' and 'b' are injoined:



Indeed, 'a' covers 'b', and 'c' is separated from 'a' and adjoins 'b'. Whitehead claims yet that:

overlapping both but not any other separated from both" (NEEDHAM, P., 1981, "Temporal Intervals and Temporal Order", *Logique et analyse*, 29, definition 4, p. 53). We could formalize this definition in the following way:

DN4 $(a\ abut\ b) \equiv ((a \rceil b) \wedge (\exists x)\ulcorner(x \circ a) \wedge (x \circ b) \wedge \sim (\exists y)\ulcorner(x \circ y) \wedge (y \rceil a) \wedge (y \rceil b)\urcorner\urcorner)$.

Needham tries to develop a theory of the notion of temporal interval underlying the analysis of natural languages. To do this he formulates a mereology having separation '$\rceil$' as primitive notion. This mereology draws his inspiration from Leonard and Goodman's calculus of individuals (cf. LEONARD, H. and GOODMAN, N., 1940, "The Calculus of Individuals and its Uses", *The Journal of Symbolic Logic*, 5 (2), pp. 45–55), but with a weaker summation condition on temporal intervals (the sum operation only occurs between connected intervals).

P14 $((b \ll a) \wedge (c \upharpoonright a) \wedge (c\, adjunction\, b)) \supset (c\, adjunction\, a)$,

which means that 'if b is a part of a, and c is separated from a and adjoins b, then c adjoins a'. The situation is still illustrated by the last figure above.

With the definitions of the notions of junction, injunction and adjunction, Whitehead gives us the first analysis of the notion of "boundary".[60] The injunction and the adjunction are so types of "boundary unions": the injunction being a boundary union of an event and one of its parts and adjunction being a boundary union of two separated events.

In *On the Foundations of Mathematics*, Stanisław Leśniewski criticized Whitehead's mereo(-topological) system for not being presented in an axiomatic and deductive form.[61] Although it was not Whitehead's intention to present his theory in an axiomatic form,[62] and that, to some extend, Leśniewski's criticism is unjustified, however it is an interesting question to know precisely which properties are independent of the others. Leśniewski focuses his attention only on the mereological properties which have a close link with his own thesis. So he does not consider the definitions of the topological notions of junction, adjunction and injunction.

Using "the interpretative method",[63] Leśniewski demonstrates the independence of (P7) and (P11) in relation to the system (P1)–(P6). Unfortunately, he does not consider the independence of the other properties. Simons, on the other hand, demonstrates that the system composed of (P1),

[60] WHITEHEAD, A.N., 1919, *op. cit.*, p. 103.

[61] LEŚNIEWSKI, S., 1992 (1927–1931), *On the Foundation of Mathematics*, trans. D.I. Barnett, *in* SURMA, S.J., SZREDNICKI, J.T. et BARNETT, D.I. (eds.), S., 1992, *Stanisław Leśniewski: Collected Works*, vol. I, Polish Scientific Publishers/Kluwer, coll. Nijhoff International Philosophy Series, Dordrecht, pp. 259–263. It is Tarski who attracted Leśniewski's attention in 1926 on the relationship of his own mereological system and Whitehead's theory of extension. It is still he who put forward the hypothesis that the system of properties (P1)–(P6) does not constitute an axiomatic basis for the theory of extension, because it is impossible to deduce the properties (P7) and (P11) in it.

[62] Cf. WHITEHEAD, A.N., 1919, *op. cit.*, p. 76.

[63] As Kotarbiński says, this methods "consists in seeking for the primitive terms of the system meanings for which all axioms become true propositions" (KOTARBIŃSKI, T., 1964, *Leçons sur l'histoire de la logique*, trans. A. Posner, Presses Universitaires de France, Paris, p. 296–296; on this method, see also TARSKI, A., 1995, *Introduction to Logic and to the Methodology of Deductive Sciences*, trans. O. Helmer, Dover, New York, pp. 120–125). If it is the case, the consistency of the axiomatic system is demonstrated. Leśniewski chooses as a model the system of rational numbers and '$\ll$' is interpreted by the relation '(is) strictly smaller than'. In this interpretation the properties (P1)–(P6) are true, but (P7) and (P11) are false.

(P4), (P5), (P6), (P7) and (P10) constitutes a possible system of independent axioms for Whitehead's mereology.[64]

3. *The (mereo-)topology of* Process and Reality

Soon after the publication of *An Enquiry Concerning the Principles of Natural Knowledge* and *The Concept of Nature*, Theodore de Laguna criticized Whitehead's mereo(-topology) on several points.[65] He suggests first to simplify and to reinforce it by choosing the relation '(is) contained in' instead of the relation of extension, or of part to whole. This relation is understood along the same lines as 'including as part and completely enveloping'. Thus a solid contains another solid when the second is a part of the first and no solid external to the first can touch the second. This primitive relation allows him then to define 'a extends over b' as meaning 'there is no event which is contained in b and not contained in a, and there is an event which is contained in a and not contained in b'. Thus, if we symbolize '(is) contained in' with 'C', we have the following definition:

$$(a\ K\ b) \equiv \sim (\exists x)\ulcorner(x\ C\ b) \wedge \sim (x\ C\ a)\urcorner \wedge (\exists x)\ulcorner(x\ C\ a) \wedge \sim (x\ C\ b)\urcorner.$$

Secondly, referring to a definition he had given before, Laguna criticizes the "indirect" character of the notion of "event-particule" given by Whitehead.[66] According to him, we can remedy this defect by taking the relation '(is) contained in' as primitive.

Following Laguna's criticisms, Whitehead reformulated his theory in *Process and Reality*. The primitive relation is no longer the mereological part-whole relation, or, more precisely, its converse, i.e. the relation of extension, but the topological relation of 'extensive connection'.[67] We are now immediately in a (mereo-)topology, because the theory is first topological and permits then to define the mereological notions. On the other hand, the preceding theory was first mereological and allowed us then to define the

[64] SIMONS, P., 2007, *op. cit.*, pp. 221–222.

[65] LAGUNA, Th. de, 1921, "Extensive Abstraction : A Suggestion", *Philosophical Review*, 30 (2), pp. 216–218.

[66] Cf. WHITEHEAD, A.N., 1919, *op. cit.*, p. 121.

[67] We can also see this change as a result of Whitehead's philosophical development. Indeed, at least from 1925, Whitehead considers the notion of process as fundamental and the notion of extension as derived (cf. the second note added by Whitehead in 1924 to *An Enquiry Concerning the Principles of Natural Knowledge*, p. 202).

topological notions. Whitehead calls this new theory "the theory of extension", although the relation of extension is no more primitive and is not even defined.[68]

The new relation of extensive connection no longer applies to events, but to what Whitehead calls "regions" which are simply defined as the *relata* of the extensive connection.[69] In an informal way, two regions 'a' and 'b' will be said to be (extensively) connected, which we symbolize '$a \odot b$', when they have at least one point in common.[70] The notions of "region" and extensive connection" constitute the two primitive notions of the theory of extension that we describe here.

Again, Whitehead's (mereo-)topology is not presented in an axiomatic and symbolic fashion. It is formulated with "definitions" and "assumptions". We formalize it once more in drawing our inspiration from Simons.[71] Whitehead begins with the definition of the relation of "mediate connection",[72] that we symbolize '$\boxdot$':

D1 $(a \boxdot b) \equiv (\exists x)\ulcorner(x \odot a) \wedge (x \odot b)\urcorner$,

which means that 'two regions a and b are mediately connected if and only if there is a third region with which they are connected'

The first assumption asserts the "symmetry" of the relations of connection and mediate connection:

H1a $(a \odot b) \supset (b \odot a)$;

H1b $(a \boxdot b) \supset (b \boxdot a)$.

[68] Whitehead defines a relation of inclusion analogous to the relation of extension (cf., *infra*, (D2)).

[69] WHITEHEAD, A.N., 1978 (1929), *Process and Reality. An Essay in Cosmology*, ed. D.R. Griffin and D.W. Sherburne, The Free Press, New York, p. 294. Palter considers that the notion of region is almost formally equivalent to the notion of event. The sole formal difference between the two notions is the fact that regions are limited in extend, whereas events may be unbounded (PALTER, R.M., 1960, *op. cit.*, p. 110).

[70] This can be deduced from the figures given by Whitehead to illustrate the different cases of connection (cf. figure I *in* WHITEHEAD, A.N., 1978 (1929), *op. cit.*, p. 294).

[71] SIMONS, P., 2007, *op. cit.*, pp. 224–226. We should mention here the pioneering work of Bowman Clarke who proposed a mereotopology drawing his inspiration from Whitehead's one: CLARKE, B.L., 1981, A Calculus of Individuals Based on "Connection", *Notre Dame Journal of Formal Logic*, 22 (3), pp. 204–218. The system developed by Clarke is axiomatized and expressed in a symbolic fashion, contrary to Whitehead's one.

[72] WHITEHEAD, A.N., 1978 (1929), *op. cit.*, p. 294.

The second part of this assumption immediately ensues from the first and (D1), as Whitehead notices it himself.[73]

The second assumption claims, on the one hand, that:

H2a $\sim (\exists x)(\forall y)\ulcorner x \odot y \urcorner$,

which means that 'no region is connected with all the other regions', and, on the other hand, that:

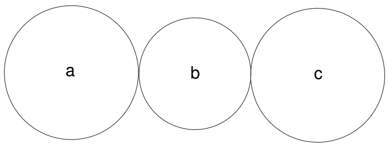
H2b $(\forall x)(\forall y)\ulcorner x \boxdot y \urcorner$,

which means that 'any two regions are always mediately connected'.

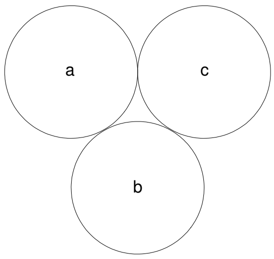
The third assumption asserts that connection is not transitive:

H3 $\sim (\forall x)(\forall y)(\forall z)\ulcorner((x \odot y) \wedge (y \odot z)) \supset (x \odot z)\urcorner$,

which means that 'there exists regions such that if the first is connected with the second and the second is connected with the third, then it does not follow that the first is connected with the third' Such a case is illustrated by the figure below:



This does not mean that the relation of connection is antitransitive in all the cases, as the figure below illustrates it:



[73] Cf. *ibid.*, p. 295.

The fourth assumption:

H4a $\sim (\exists x)\ulcorner x \odot x \urcorner$,

which means that 'no region is connected with itself'. It follows from this that 'no region is mediately connected with itself':

H4b $\sim (\exists x)\ulcorner x \boxdot x \urcorner$.

In other words, the relation of connection and mediate connection are both irreflexive.

Whitehead defines now the relation of "inclusion"[74] that he owes to Laguna and that constitutes an important addition to the theory of extension.[75] We use the converse relation that we symbolize '$\leq$'[76] and such that '$a \leq b$' can be read 'a is a (proper) part of b',[77] or 'b includes a':

D2 $(a \leq b) \equiv (\forall x)\ulcorner (x \odot a) \supset (x \odot b)\urcorner$,

which means that 'the region a is a part of b if and only if every region connected with a is connected with b'. It follows immediately from this that when a region is a part of another, the two are connected:

H5 $(a \leq b) \supset (a \odot b)$.

The relation of part to whole, or of inclusion, is transitive, irreflexive and asymmetrical. These three properties are expressed in the three following assumptions:

H6 $((a \leq b) \wedge (b \leq c)) \supset (a \leq c)$;

H7 $\sim (\exists x)\ulcorner x \leq x \urcorner$;

H8 $(a \leq b) \supset \sim (b \leq a)$.

[74] It is obviously the relation analogous to the relation of extension 'K' that we find in *An Enquiry Concerning the Principles of Natural Knowledge*.

[75] WHITEHEAD, A.N., 1978 (1929), *op. cit.*, p. 295.

[76] It is the relation analogous to the part-whole relation '$\ll$' that we find in *An Enquiry Concerning the Principles of Natural Knowledge*.

[77] Whitehead gives himself this reading (cf. WHITEHEAD, A.N., 1978 (1929), *op. cit.*, p. 295). The fact that the relation '(is) a part of' must be understood in the proper meaning is explicitly asserted with (H7).

This relation is thus a relation of partial strict order and defines what we call a "ground mereology".[78]

The first part of the ninth assumption asserts the non-atomicity of the notion of region. In other words, there is no region not including other region as part:

H9 $(\forall x)(\exists y)\ulcorner y \leq x \urcorner$.

The second part of the ninth assumption:

H9b $(\forall x)(\exists y)(\exists z)\ulcorner (y \leq x) \wedge (z \leq x) \wedge \sim (y \odot z)\urcorner$,

which means that 'every given region has parts which are not connected'.

Whitehead continues with the definition of the mereological notion of 'overlapping':[79]

D3 $(a \circ b) \equiv (\exists x)\ulcorner (x \leq a) \wedge (x \leq b)\urcorner$,

which means that 'two regions overlap when there exists a third which they both include'. This relation is symmetrical:

H10 $(a \circ b) \supset (b \circ a)$,

what ensues immediately from the definition of '$\circ$'. It is also the case of the following assumption:

H11 $(a \leq b) \supset (a \circ b)$,

which means that 'if a region is a part of another region, then they overlap'.

The twelfth assumption asserts that the overlapping of two regions entails that they are connected:

H12 $(a \circ b) \supset (a \odot b)$.

The notions of inclusion and overlapping being defined, Whitehead can now define the notion of "dissection". Nonetheless, this definition is not

[78] Cf. CASATI, R. and VARZI, A.C., 1999, *Parts and Places: The Structures of Spatial Representation*, MIT Press, Cambridge (Mass.), p. 36.

[79] It is the same notion as the notion of intersection defined in *An Enquiry Concerning the Principles of Natural Knowledge*, except that it is defined with '$\leq$', and not with '$\ll$'.

identical to the one defined in *An Enquiry Concerning the Principles of Natural Knowledge*:

D4 $(\alpha\ diss\ a) \equiv (\forall x)^{\ulcorner}(x \in \alpha) \supset (x \leq a)^{\urcorner} \wedge (\forall x)(\forall y)^{\ulcorner}((x \in \alpha) \wedge (y \in \alpha) \wedge (x \neq y)) \supset \sim (x \circ y)^{\urcorner} \wedge (\forall x)^{\ulcorner}((x \leq \alpha) \supset (x \notin \alpha)) \supset ((\exists y)^{\ulcorner}(y \in \alpha) \wedge (x \leq y)^{\urcorner} \vee (\exists y)(\exists z)^{\ulcorner}(y \in \alpha) \wedge (z \in \alpha) \wedge (y \neq z) \wedge (x \circ y) \wedge (x \circ z)^{\urcorner}),$

which means that 'a set α is a dissection of a region a if and only if (1) all the members of α are included in a and (2) no two members of α overlap and (3) any region included in a without being a member of α is either included in one member of α or overlaps more than one member of α'.

The thirteenth assumption:

H13 $(\forall x)(\exists \alpha)(\exists \beta)^{\ulcorner}(\alpha\ diss\ x) \wedge (\beta\ diss\ x) \wedge (\alpha \neq \beta)^{\urcorner},$

which means that 'there is more than one dissection for every given region'.

The fourteenth assumption:

H14 $(\alpha\ diss\ a) \supset \sim (\exists x)^{\ulcorner}(\alpha\ diss\ x) \wedge (x \neq a)^{\urcorner},$

which means that 'the dissection of a region is not the dissection of another one'.

Whitehead gives then the definition of the relation of a region that "intersect" two regions which overlap:

D5 $(a\ int\ bc) \equiv ((b \circ c) \wedge ((a \leq b) \wedge (a \leq c)) \vee ((a = b) \wedge (a \leq c)) \vee ((a = c) \wedge (a \leq b)) \wedge (\forall x)^{\ulcorner}((x \leq b) \wedge (x \leq c)) \supset \sim ((x \circ a) \wedge \sim (x \leq a))^{\urcorner}),$

which means that 'a region is the intersect of two regions b and c which do not overlap if and only if either (1) it is included in b and c, or (2) it is one of the two regions and is included in the other, and (3) no other region included in both b and c can overlap it without being included in it' There can be one or several intersects of two given regions.[80]

The fifteenth assumption:

H15 $(\forall x)^{\ulcorner}((x \leq a) \wedge (x \leq b) \wedge (a \circ b) \wedge \sim (x\ int\ ab)) \supset (\exists y)^{\ulcorner}(y\ int\ bc) \wedge (x \leq y) \wedge (\forall z)^{\ulcorner}((z\ int\ bc) \wedge (x \leq z)) \supset (z = y)^{\urcorner\urcorner},$

[80] SIMONS, P., 2007, *op. cit.*, p. 225.

which means that 'every region included in two regions which overlap and which is not itself an intersect is included in an intersect, and only one'.

The sixteenth assumption:

H16 $(a \leq b) \supset ((a\ int\ ab) \wedge (\forall x)\ulcorner(x\ int\ ab) \supset (x = a)\urcorner)$,

which means that 'if a is included in b, then a is the only intersect of a and b'.

The seventeenth assumption:

H17 $((a\ int\ bc) \wedge (a \neq b) \wedge (a \neq c)) \supset ((a \leq b) \wedge (a \leq c))$,

which means that 'an intersect of two regions, which is not one of the two regions, is included in both regions'.

The eighteenth assumption:

H18 $(a \circ b) \supset (\exists x)\ulcorner x\ int\ ab\urcorner$,

which means that 'every pair of regions which overlap has at least one intersect'.

After these several mereological definitions and assumptions, Whitehead defines the topological relation of "external connection", that we symbolize '$\bowtie$':

D7 $(a \bowtie b) \equiv ((a \odot b) \wedge \sim (a \circ b))$,

which means that 'two regions are externally connected when they are connected and do not overlap'.[81] Whitehead sees in this definition one of the advantages of his approach in terms of "extensive connection", in comparison with the approach in terms of "extensive whole and extensive part" used in *An Enquiry Concerning the Principles of Natural Knowledge*.[82] According to him, it is an important step towards the elaboration of the notion of "surface".[83] Two examples of externally connected regions are given by the two figures below:

[81] We can notice that it is the definition of what Laguna calls "to be in contact" (cf. LAGUNA, Th., 1922, "Point, Line, and Surface, as Sets of Solids", *The Journal of Philosophy*, 19 (17), definition VI, p. 452).

[82] WHITEHEAD, A.N., 1978 (1929), *op. cit.*, p. 297.

[83] *Idem*.

Whitehead defines the relation of "tangential inclusion", that we symbolize '$\ltimes$':

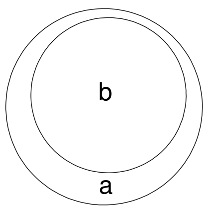
D8 $(a \ltimes b) \equiv ((a \leq b) \wedge (\exists x)^{\ulcorner}(x \bowtie a) \wedge (x \bowtie b)^{\urcorner})$,

which means that 'a region is tangentially included in another region if and only if the first is included in the second and they are both externally connected with a third'. This relation can be illustrated with the two examples below:

Whitehead finally defines the relation of "non-tangential inclusion", that we symbolize '$\lessdot$':

D9 $(a \lessdot b) \equiv ((a \leq b) \wedge \sim (\exists x)^{\ulcorner}(x \bowtie a) \wedge (x \bowtie b)^{\urcorner})$,

which means that 'a region is non-tangentially included in another region if and only if the first is included in the second and there is no region which is externally connected with them both'. This relation can be illustrated by the figure below:



Generally speaking we can say that two regions are connected if one of the two is included in the other, be it tangentially or non-tangentially, or if they overlap, or if they are externally connected.

Palter noticed that this system is inconsistent. Indeed, from the definition (D2) of '$\leq$' and (H5), we can deduce that'$\odot$' is reflexive:[84]

$$(\forall x)\ulcorner x \odot x \urcorner,$$

what contradicts (H4a). We can deduce with (D1) that the immediate connection is also reflexive:

$$(\forall x)\ulcorner x \boxdot x \urcorner,$$

what contradicts (H4b). But this is not a real problem, since it suffices to drop the property of non-reflexivity for the mediate connection. Besides Whitehead claims that this assumption is merely a "convenient arrangement of nomenclature".[85] In fact, we must admit that the non-reflexivity of the relation of connection is counterintuitive. In any case if we interpret connection along the same lines as having at least one point in common, we do not see why two regions which have all their points in common would not be connected according to this sense.

If we want to keep the non-reflexivity of the relation of connection, Gerla and Tortora gives us a consistant system of seven independent axioms:[86]

A1 $(\forall x)\ulcorner \sim (x \boxdot x)\urcorner$;

A2 $(\forall x)(\forall y)\ulcorner (x \odot y) \supset (y \odot x)\urcorner$;

A3 $(\forall x)(\exists y)\ulcorner (x \neq y) \wedge \sim (x \odot y)\urcorner$;

A4 $(\forall x)(\forall y)(\exists z)\ulcorner (x \odot z) \wedge (z \odot y)$;

A5 $(\forall x)(\forall y)(\forall z)\ulcorner ((x \leq y) \wedge (y \leq z)) \supset (x \leq z)\urcorner$;

A6 $(\forall x)(\exists y)(\exists z)\ulcorner (y \leq x) \wedge (z \leq x) \wedge (y \neq z) \wedge \sim (y \odot z)\urcorner$;

A7 $(\forall x)(\forall y)\ulcorner (x \leq y) \supset (x \odot y)\urcorner$.

[84] PALTER, R.M., 1960, *op. cit.*, note 8, p. 108. See also SIMONS, P., 2007, *op. cit.*, p. 226.

[85] WHITEHEAD, A.N., 1978 (1929), *op. cit.*, p. 295.

[86] Cf. the system (B1)–(B7) *in* GERLA, G. and TORTORA, R., 1996, "La relazione di connessione in A.N. Whitehead: aspetti matematici", *Epistemologia*, 15, pp. 356. Cf., also, RIDDER, L., 2002, *Mereologie. Ein Beitrag zur Ontologie und Erkenntnistheorie*, Vittorio Klostermann, Frankfurt, pp. 240–244; and GERLA, G. and TORTORA, R., 1996, "Dissezioni e intersezioni di regioni in A.N. Whitehead", *Epistemologia*, 19, pp. 289–308.

(A1) corresponds to (H4b), (A2) corresponds to (H1a), (A3) corresponds to (H2a) to which we have added a condition of difference between two entities, (A4) corresponds to (H2b), (A5) corresponds to (H6), (A6) corresponds to (H9b) to which we have also added a condition of difference between two entities, and (A7) corresponds to (H5). In this system, the definitions are somewhat modified in comparison with the ones given by Whitehead. For example (D1) becomes:

$$(\forall x)(\forall y)\ulcorner(x \boxdot y) \equiv ((x \neq y) \wedge (\exists z)\ulcorner(z \odot x) \wedge (z \odot y)\urcorner)\urcorner,$$

The condition of difference '$x \neq y$' prevents us from deducing '$(\forall x)\ulcorner x \boxdot x\urcorner$' from this definition. (D2) becomes:

$$(\forall x)(\forall y)\ulcorner(x \leq y) \equiv ((x \neq y) \wedge (\forall z)\ulcorner((z \neq y) \wedge (z \odot x)) \supset (z \odot y)\urcorner)\urcorner,$$

The first condition of difference '$x \neq y$' ensures us that the relation of inclusion is non-reflexive, whereas the second '$z \neq y$' ensures us that we cannot deduce '$(\forall x)\ulcorner x \odot x\urcorner$' from this definition. So, the difficulties suggested by Simons are avoided and the system is consistent.

4. *Whitehead's mereotopology and formal ontology*

It is common to divide Whitehead's philosophy into three different periods:[87]

a) the first logical mathematical period stretches from the beginning of Whitehead's teaching in mathematics in 1885 to the publication of the third volume of *Principia Mathematica* in 1913, that is, for the essential, the period of Cambridge;

b) a second period marked by the philosophy of sciences which covers the period 1914 to 1923, that is the period of London;

c) a third period devoted to metaphysics and lasting from 1924 to 1947 when Whitehead was at Harvard.

[87] PARMENTIER, A., 1968, *La philosophie de Whitehead et le problème de Dieu*, Beauchesne, coll. Bibliothèque des archives de philosophie, Paris, p. 21. Cf., also, MAYS, W., 1959, *The Philosophy of Whitehead*, George Allen & Unwin LTD, coll. Muirhead Library of Philosophy, London, p. 17; and LOWE, V., 1962, *Understanding Whitehead*, The John Hopkins University Press, Baltimore, pp. 117–296.

According to Michel Weber, Whitehead's conceptual development can be described as "a creative advance from formal to existential ontology, an advance that has always featured, *mutatis mutandis*, the same focus: the axiomatization of the uniform extensiveness structuring our world".[88] Thus in the development of Whitehead's philosophy there would be first a formal ontology whose point of view would have been then "transcended" by a proper existential ontology standpoint during the Harvard period.

As for us, we think that there is simply no formal ontology in Whitehead's philosophy, unless this expression is understood in a naive way. We claim instead that Whitehead developed an important theory that could pertain to formal ontology, namely a particular mereotopology. However this link between formal ontology and mereotopology is not necessary. Yet it is precisely this link that Whitehead did not envisage thematically: the mereotopology that he developed has never been considered as pertaining to a formal ontological project and, what is more, Whitehead's philosophical project is irrelevant to all formal ontology.

The closest conception of the relations between logic, mathematics and ontology with formal ontology that we can find in Whitehead's philosophy seems to be in the first period of his philosophical development.[89] In *A Treatise on Universal Algebra* of 1898, Whitehead, who gets his inspiration from Grassmann's two *Ausdehnungslehren* of 1844 and 1862, formulates the "ideal of mathematics" which

> [...] *should be to erect a calculus to facilitate reasoning in connection with every province of thought, or of external experience, in which the succession of thought, or of events can be definitely ascertained and precisely stated. So that all serious thought which is no philosophy, or inductive reasoning, or imaginative literature, shall be mathematics developed by means of a calculus*.[90]

Unfortunately Whitehead does not elaborate on what he understands with this "serious thought" that should be developed as a calculus. On the other hand, he expresses an idea of mathematics conceived as the development of "all kinds of formal necessary and deductive reasonings" and what interests

[88] WEBER, M., 2007, "PNK's Creative Advance from Formal to Existential Ontology", *in* DURAND, G. and WEBER, M., 2007, *op. cit.*, p. 259.

[89] On this point we agree with Weber's thesis

[90] WHITEHEAD, A.N., 1898, *A Treatise on Universal Algebra, with Application*, Cambridge University Press, Cambridge, p. vii.

him in Boole's algebra or Grassman's one is that they reach "beyond the traditional domain of pure quantity".

In "On Mathematical Concepts of the Material World" (1906), Whitehead outlines his project that aims at expressing the ground entities of euclidian geometry with the help of only one kind of ultimate entitiy, namely those of the material world:

> *The object of this memoir is to initiate the mathematical investigation of various possible ways of conceiving the nature of the material world. In so far as its results are worked out in precise mathematical details, the memoir is concerned with the possible relations to space of the ultimate entities which (in ordinary language) constitute the "stuff" in space (particles of matter).*[91]

Thus Whitehead seems to want to develop an ontology of the material world, formulated with symbolic propositions which are organized systematically as an axiomatic and deductive system. The theory of extension that we presented in the former section is, in our opinion, in continuity with such a project, although it was neither formalized nor axiomatized, which, as we saw, it could be. Logic, and formalism generally speaking, is then considered a "tool" that enables us to analyse reality with this important requirement that the theory must be anchored in the concreteness of nature.

At first sight, this project can seem very close to the project of formal ontology formulated at the beginning of the twentieth century by Husserl in his *Logical Investigations*.[92] A similar one can also be found later in Stanisław Leśniewski's philosophy and in the brentanian tradition.[93] After falling into relative oblivion, this project was taken up thirty years ago by anglo-saxon philosophy, in particular by the School of Manchester, namely Barry Smith, Kevin Mulligan and Peter Simons. Currently other philosophers such as Nino Cocchiarella, Roberto Poli or Frédéric Nef claim to belong to the tradition of formal ontology.

[91] WHITEHEAD, A.N., 1906, "On Mathematical Concepts of the Material World", *Philosophical Transactions, Royal Society of London, Series A*, 25, p. 465.

[92] Cf. particulary the third *Investigation* and the first part of *Formal Logic and Transcendental Logic*.

[93] We think here particulary of Meinong, but Husserl himself has forcefully distinguished his own formal ontological project from the menongian theory of objects (cf. HUSSERL, E., 1939 (1913), "Entwurf einer "Vorrede" zu den "Logischen Untersuchungen" (1913)", *Tijdschrift voor Philosophie*, 1, §7, pp. 319–323.

But what is formal ontology really? In our opinion, we must distinguish between two types of formal ontology, one that we would qualify as naive and the other as substantial. For Roberto Poli it consists in:

> *[...] attempts to use formal methods to solve classical philosophical problems relating to the notions of being, object, state of affairs, existence, property, relation, universal, particular, substance, accident, part, boundary, measure, causality, and so on.*[94]

If we simplify somewhat this definition, we obtain what we previously called the naive conception of formal ontology, i.e. an attempt to solve traditional metaphysical problems with formal tools. Such a conception, defended notably by Nino Cocchiarella, eliminates completely the question of the ontological formal categories which are the state of affairs, part, boundary, and so on. Furthermore, it puts in the foreground the use of logical formal tools. According to this meaning, the philosophical project defended by Whitehead in the first period of his philosophical development can be associated with formal ontology. However a project such as this one, which tries to solve philosophical problems, more particulary those concerning the world, with formal tools is absolutely not new: it has inherited a great deal from the traditional aristotelian ontology and runs across the whole scholastic tradition. Formal ontology would then only be a label for a project already quite old but which would now use the latest tools of modern formal logic.

We want to contrast this conception of formal ontology with the more substantial one defended by Husserl in his *Logical Investigations*. To a large extent, in our opinion, the project of formal ontology can already be found broadly outlined in the *Schullmetahysik* of the eighteenth century, but it is only with Husserl that it has received a precise formulation. This is notably due to the new formal conception of mathematics developed during the nineteenth century. Husserl took this revolution into account to conceive an ontology as *mathesis universalis*.

In his *Logical Investigations*, Husserl develops a two-level ontological device: one formal and the other material. But there also exists for Husserl another conception of ontology which is pejorative to him: it is an ontology that deals with the general characters of "effective reality", a science that can entirely be identified with metaphysics.[95] Yet the phenomenology as it

[94] POLI, R. and SIMONS, P., (éds.), 1996, *Formal Ontology*, Kluwer, Dordrecht, Foreword, p. vii.

[95] Cf. HUSSERL, E., 1984 (1906–1907), *Einleitung in die Logik und Erkenntnistheorie Vorlesungen (1906–1907)*, *Husserliana* XXIV, Martinus Nijhoff, Dordrecht, p. 99; and BÉGOUT, Br., 2003, "L'ontologie dans les limites de la phénoménologie", *in*, FISETTE, D. and LAPOINTE, S., 2003, *Aux origines de la phénoménologie. Husserl et le contexte des*

is presented in the *Logical Investigations* is exempt from any metaphysical presupposition. It disqualifies the ontology of the *Wirklichkeit*. The formal ontology developed by Husserl is in continuity with this project and is then not a traditional ontology in the sense where it does not treat of what is effective.

Formal ontology as conceived by Husserl is thus exempt of any ontic presupposition. This is one of the main characteristics of the substantial conception of the project of formal ontology that we also find in Leśniewski's "ontologically neutral" ontology.[96] The husserlian formal ontology endeavors to "describe all the forms of objectivity and the *a priori* laws which link them". It is a purely rational science of objects which, contrary to traditional ontology, is "derealized", which is only possible because it solely considers the object as object, what Husserl calls "the object in general (*Gegenstand überhaupt*)" or "the something in general",[97] and the other formal ontological categories that are related to it. Then it does not focus on such and such object, but on any object as belonging to a certain formal ontological domain. The husserlian formal ontology is formal not because it uses the tools of formal logic but because it formulates exact laws independent of material content. These laws are "analytic" in that they are "propositions absolutely general (and consequently exempt from all position of existence, explicit or implicit, of the individual)".[98] This is because the empty domain of the object in general is governed by "laws of essence" that can be formulated *a priori* and independently from their subject matter. For example, the law of transitivity of the part-whole relation can be formulated independently of all material content, because it is valid for every object in general, these being events, material things, and so on. But what are the different domains of husserlian formal ontology? They are the different formal ontological categories which are "correlated" with the logical formal categories. It is the object, the relation, the quality, the unity, the number, the part, the whole, and

Recherches logiques, Vrin and Les Presses de l'Université de Laval, coll. Zêtêsis, Paris and Quebec, pp. 151–153.

[96] MIÉVILLE, D., 1984, *Un développement des systèmes logiques de S. Leśniewski. Protothétique-Ontologie-Méréologie*, Peter Lang Verlag, Berne, p. 269.

[97] HUSSERL, E., 1993 (1900/1913), *Logische Untersuchungen. Zweiter Band. Untersuchungen zur Phänomenologie und Theorie der Erkenntnis. Erste Teil*, Max Niemeyer, Tübingen, *LU III*, §11, p. 252.

[98] *Ibid.*, §12, p. 254.

so on.[99] Thus for Husserl, the part-whole relation is a formal ontological notion and every theory which formulates his *a priori* eidetic laws belongs to formal ontology.

The husserlian formal ontology must also be distinguished from the "material ontologies", or "regional ontologies". The essential difference between these two types of ontologies lies in the kind of laws that govern them. While the laws of formal ontology are analytic, those of the several regional ontologies are synthetic *a priori*.[100] The synthetic *a priori* laws are not formal, i.e. they depend on the kind of objects they govern. For example, an analytic law will be 'there is no part without whole'. Such a law can be particularized to obtain what Husserl calls an "analytic necessity", as for exemple 'the existence of this house entails the existence of its walls, its roof, and so on'. Such a proposition can be formalized and it is by formalizing it, by making it independent of all material content, that we obtain an analytic law belonging to formal ontology for the ontological formal categories of part and whole. On the other hand, a proposition such as 'there is no color without some extent covered by it' is not a law pertaining to formal ontology, for it is a synthetic *a priori* law belonging to a material ontology. The particularisation of a proposition of this kind, as for example 'if there is a color red, then it must be on some surface', cannot be formalized. It is impossible to eliminate the material content of this proposition without losing the law. The synthetic laws do not pertain to the formal ontological categories, but to the more general kinds. These cannot be obtained by formalization, but by abstraction.

Finally, regarding symbolic formal logic, Husserl does not see the symbolic tool as an essential characteristic of formal ontology. Furthermore, his pure theory of the forms of wholes and parts is not even symbolized, the important thing being its formal character. That does not mean however that the symbolic mathematical form is undesirable, quite the contrary in fact. It allows us to give to formal ontology its systematic character and the true form of a science. From this point of view, we could say that Whitehead is close to Husserl, because he has never given a symbolic and axiomatized form to his different mereotopological theories, except in "La théorie relationniste de l'espace". This form is not what is most important even if it is possible and desirable.[101]

[99] *Ibid.*, §11, p. 252; cf., also, HUSSERL, E., 1928 (1913), *Ideen zur einer reinen phaenomenologischen Philosophie*, Max Niemeyer, Halle, §10, p. 21.

[100] Cf. *ibid.*, §10, pp. 38–43 [20–23]; cf. also HUSSERL, E., 1993 (1900/1913), *op. cit.*, *LU III*, §§11–12, pp. 251–256.

[101] In *Process and Reality*, Whitehead defines the "speculative philosophy" as "the attempt to form a system of general ideas that is necessary, logical, coherent and in function of which

If we go back now to Whitehead and the links between his different mereotopologies and formal ontology, we must first emphasize that mereology and topology constitute today the most important parts of the contemporary project of formal ontology inspired by Husserl.[102] From this point of view, we must acknowledge that Whitehead's contribution is important: numerous properties and assumptions developed by him in his different theories are today part of the formal mereotopology that constitute the bulk of formal ontology. Nevertheless, it is incorrect to consider that Whitehead has developed a formal ontology in the substantial sense, and this for several reasons. First, the different mereotopological theories set out by Whitehead are never developed for their own sake, that is as being able to constitute an ontology, even regional, but always for something else, namely, *mutatis mutandis*, the method of extensive abstraction, and thus a particular interpretation of experience. It is here an instrumental conception which, as we saw previously, characterizes a naive conception of formal ontology, but by no means the substantial conception, the only truly original one. Secondly, if whiteheadian mereotopology is, in certain respects, formal, it is not so in his entirety. Indeed it is always developed for a particular ontological domain, be it the domain of space, of events, of regions, and so on. It does not care about the formal domain of the something in general. It is not ontologically neutral. In this sense, it is still an ontology of the *Wirklichkeit*, of the effective reality. The anchoring in nature and in concreteness is one of the main characteristics of Whitehead's metaphysics. Finally, the laws formulated by Whitehead are not conceived as being obtained by formalization, but by abstraction from concrete data of experience. Consequently the ontology that we could try to find in the whiteheadian mereotopology is closer

the elements of our experience can be interpreted" (WHITEHEAD, A.N., 1978 (1929), *op. cit.*, p. 3). If he insists here on the rational constraints of "necessity", "logicality", "coherence" and "applicability", we must not forget that what prevails is the "interpretation" of immediate experience.

[102] On the importance of mereology and topology for the contemporary project of formal ontology, cf., among others, SIMONS, P., 1987, *op. cit.*; SMITH, B., 1996, "Mereotopology: A Theory of Parts and Boundaries", *Data and Knowledge Engineering*, 20, pp. 287–303; and VARZI, A., 1996, "Parts, Wholes, and Part-Whole Relations: The Prospects of Mereotopology", *Data and Knowledge Engineering*, 20, pp. 259–286.

to Husserl's material ontologies, because it tries to formulate the laws concerning the most general kinds of beings, namely those of events, regions, or others kinds of entities.[103]

Université Libre de Bruxelles (U.L.B.)
E-mail: sebastien.richard@ulb.ac.be

REFERENCES

[1] CASATI, R. and VARZI, A.C., 1999, *Parts and Places: The Structures of Spatial Representation*, MIT Press, Cambridge (Mass.).

[2] CLARKE, B.L., 1981, A Calculus of Individuals Based on "Connection", *Notre Dame Journal of Formal Logic*, 22 (3), pp. 204–218.

[3] DURAND, G., 2006, *Des événements aux objets. La méthode d'abstraction extensive chez A.N. Whitehead*, Ontos Verlag, coll. Chromatiques Whiteheadiennes, Frankfurt.

[4] DURAND, G. and WEBER, M., 2007, Les principes de la connaissance naturelle *d'Alfred North Whitehead*, Ontos Verlag, coll. Chromatiques Whiteheadiennes, Frankfurt.

[5] FISETTE, D. and LAPOINTE, S., 2003, *Aux origines de la phénoménologie. Husserl et le contexte des* Recherches logiques, Vrin and Les Presses de l'Université de Laval, coll. Zêtêsis, Paris and Quebec.

[6] GERLA, G. and TORTORA, R., 1996, "La relazione di connessione in A.N. Whitehead: aspetti matematici", *Epistemologia*, 15, pp. 351–364.

[7] GERLA, G. and TORTORA, R., 1996, "Dissezioni e intersezioni di regioni in A.N. Whitehead", *Epistemologia*, 19, pp. 289–308.

[8] HUSSERL, E., 1928 (1913), *Ideen zur einer reinen phaenomenologischen Philosophie*, Max Niemeyer, Halle.

[9] HUSSERL, E., 1939 (1913), "Entwurf einer "Vorrede" zu den "Logischen Untersuchungen" (1913)", *Tijdschrift voor Philosophie*, 1, pp. 106–133 and 319–323.

[10] HUSSERL, E., 1984 (1906–1907), *Einleitung in die Logik und Erkenntnistheorie Vorlesungen (1906–1907)*, *Husserliana* XXIV, Martinus Nijhoff, Dordrecht.

[11] HUSSERL, E., 1993 (1900/1913), *Logische Untersuchungen. Zweiter Band. Untersuchungen zur Phänomenologie und Theorie der Erkenntnis. Erste Teil*, Max Niemeyer, Tübingen.

[103] A more complete discussion of the empirical status of Whitehead's work should at least also include a discussion of his theory of prehension. However this is impossible for us to achieve in a single paper.

[12] KOTARBIŃSKI, T., 1964, *Leçons sur l'histoire de la logique*, trans. A. Posner, Presses Universitaires de France, Paris.
[13] LAGUNA, Th. de, 1921, "Extensive Abstraction: A Suggestion", *Philosophical Review*, 30 (2), pp. 216–218.
[14] LAGUNA, Th., 1922, "Point, Line, and Surface, as Sets of Solids", *The Journal of Philosophy*, 19 (17), pp. 449–461.
[15] LEONARD, H. and GOODMAN, N., 1940, "The Calculus of Individuals and its Uses", *The Journal of Symbolic Logic*, 5 (2), pp. 45–55.
[16] SURMA, S.J., SZREDNICKI, J.T. et BARNETT, D.I. (eds.), S., 1992, *Stanisław Leśniewski: Collected Works*, vol. I, Polish Scientific Publishers/Kluwer, coll. Nijhoff International Philosophy Series, Dordrecht.
[17] LOWE, V., 1962, *Understanding Whitehead*, The John Hopkins University Press, Baltimore.
[18] MAYS, W., 1959, *The Philosophy of Whitehead*, George Allen & Unwin LTD, coll. Muirhead Library of Philosophy, London.
[19] MIÉVILLE, D., 1984, *Un développement des systèmes logiques de S. Leśniewski. Protothétique-Ontologie-Méréologie*, Peter Lang Verlag, Bern.
[20] NEEDHAM, P., 1981, "Temporal Intervals and Temporal Order", *Logique et analyse*, 29, pp. 49–64.
[21] PALTER, R.M., 1960, *Whitehead's Philosophy of Science*, University of Chicago Press, Chicago.
[22] PARMENTIER, A., 1968, *La philosophie de Whitehead et le problème de Dieu*, Beauchesne, coll. Bibliothèque des archives de philosophie, Paris.
[23] POLI, R. and SIMONS, P., (eds.), 1996, *Formal Ontology*, Kluwer, Dordrecht.
[24] RIDDER, L., 2002, *Mereologie. Ein Beitrag zur Ontologie und Erkenntnistheorie*, Vittorio Klostermann, Frankfurt.
[25] SIMONS, P., 1987, *Parts. A Study in Ontology*, Clarendon Press, Oxford.
[26] SIMONS, P., 2007, "Whitehead and Mereology", *in* DURAND, G. and WEBER, M., 2007, Les principes de la connaissance naturelle *d'Alfred North Whitehead*, Ontos Verlag, coll. Chromatiques Whiteheadiennes, Frankfurt, pp. 215–233.
[27] SMITH, B., 1996, "Mereotopology: A Theory of Parts and Boundaries", *Data and Knowledge Engineering*, 20, pp. 287–303.
[28] TARSKI, A., 1995, *Introduction to Logic and to the Methodology of Deductive Sciences*, trans. O. Helmer, Dover, New York.
[29] VARZI, A., 1996, "Parts, Wholes, and Part-Whole Relations: The Prospects of Mereotopology", *Data and Knowledge Engineering*, 20, pp. 259–286.

[30] VUILLEMIN, J., 1971, *La logique et le monde sensible. Études sur les théories contemporaines de l'abstraction*, Flammarion, coll. Nouvelle bibliothèque scientifique, Paris.

[31] WEBER, M., 2007, "PNK's Creative Advance from Formal to Existential Ontology", *in* DURAND, G. and WEBER, M., 2007, Les principes de la connaissance naturelle *d'Alfred North Whitehead*, Ontos Verlag, coll. Chromatiques Whiteheadiennes, Frankfurt, pp. 259–273.

[32] WHITEHEAD, A.N., 1898, *A Treatise on Universal Algebra, with Application*, Cambridge University Press, Cambridge.

[33] WHITEHEAD, A.N., 1906, "On Mathematical Concepts of the Material World", *Philosophical Transactions, Royal Society of London, Series A*, 25, pp. 465–525.

[34] WHITEHEAD, A.N., 1916, "La théorie relationniste de l'espace", *Revue de métaphysique et de morale*, 23 (3), pp. 423–454.

[35] WHITEHEAD, A.N., 1919, *An Enquiry Concerning the Principles of Natural Knowledge*, Cambridge University Press, Cambridge.

[36] WHITEHEAD, A.N., 1964 (1920), *The Concept of Nature. The Tarner Lectures Delivered in Trinity College November 1919*, Cambridge University Press, Cambridge.

[37] WHITEHEAD, A.N., 1978 (1929), *Process and Reality. An Essay in Cosmology*, D.R. Griffin and D.W. Sherburne (eds.), The Free Press, New York.