

Logique & Analyse 199 (2007), 317–334

PRIOR ON THE LOGIC AND THE METAPHYSICS OF TIME

ROBERTA BALLARIN

Abstract

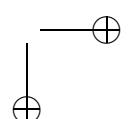
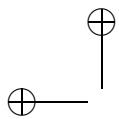
In this paper I explore three related topics emerging from Prior's work on the logic of time. First, what is the proper province of logic, if any? Is temporal (modal) logic just logic, on a par with the paradigmatic case of first-order quantification theory or even simple propositional logic? Second, what counts as an interpretation of a formal system? In particular, can formal semantics provide an interpretation? Third, what is the proper role of the model-theoretic meta-theory? In connection with this last question we will see how Prior's attitude towards instants of time may teach us something about the analogous case of possible worlds.

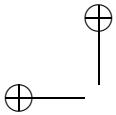
1. *Introduction*

The preface to *Time and Modality* starts with the words: “These lectures are the expression of a conviction that formal logic and general philosophy have more to bring to one another than is sometimes supposed.” And it is soon added: “Certainly we have a duty to notice that facts X and Y do not fit into such-and-such a formal logician’s straight-jacket..., but we should not neglect either to hunt for some better-fitting clothing for them, especially since the formal logician’s shop is now so much more variously stocked.”¹

So, from the very introduction to his 1956 lectures on time and modality, Prior expresses forcefully his fundamental attitude towards logic and metaphysics: No rigid presuppositions on the proper province of logic are assumed, and formal innovations are open-mindedly pursued. New formal systems are devised to better symbolize and systematize our ordinary tensed and modal talk. Thanks to the formalization, the logical connections (independence, derivability, etc.) between (the formal analogues of) our ordinary tensed/modal sentences are more easily explored. Nonetheless, the job of the logician is never assumed to take over and replace the need for philosophical

¹ A.N. Prior, *Time and Modality*, p. vii.





analysis. The basic observations on the nature of time and modality are to be supplied by the metaphysician, for the logician to systematize.

Taking inspiration from the above remarks, in this paper I explore three different related topics emerging from Prior's work on the logic of time. The unifying theme is the general question: How far can logic reach? I will explore three different aspects of this question. First, what is the proper province of logic, if any? Is temporal (modal) logic just logic, on a par with the paradigmatic case of first-order quantification theory or even simple propositional logic? Second, where does an interpretation for our formal systems come from? In particular, can formal semantics replace old fashioned metaphysics? Third, what is the proper role of metalogic? In connection with this last question we will see how Prior's attitude towards instants of time may teach us something about the analogous case of possible worlds.

2. *The Arbitrary Province of Logic*

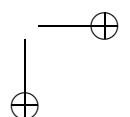
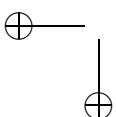
It is no accident that when listing contemporary formal logicians whose shops offer straight-jackets in lieu of comfortable clothes Prior mentions Quine.² Quine's insistence on delimiting the province of logic proper to first-order logic is bound to feel tight to the temporal/modal logician. Surely idiosyncratic ontological prejudices and metaphysical qualms have contributed to Quine's rejection of second-order and modal logics.³ Such anti-metaphysical sentiments were no doubt no part of Prior's philosophical temperament. Nonetheless a larger theme looms behind such disputes. Ontological parsimony aside, wherein lies the perceived divide?

Traditionally, formal systems are divided into two separate camps: logics on the one hand and theories on the other. Perhaps the availability of some formal results, especially proofs of completeness, has contributed to an initial partition of the formal landscape. However, a deeper theme in partitioning the field has consisted in connecting logic, pure logic — as opposed to the formal rendition of a theory — to universality. For example, when grouping identity theory with logic, rather than mathematics, Quine points out that:

Another respect in which identity theory seems more like *logic* than mathematics is universality: it treats of all objects impartially. Any

² The logicians mentioned are Aristotle, Russell and Quine. Cf. Prior, *ibid.*, p. vii.

³ Cf. W.V. Quine, *Philosophy of Logic*, chapter 5, for the case of second order logic; Quine's stand on modal logic is more complex, but see, among others, “The Problem of Interpreting Modal Logic”.



theory can likewise be formulated with general variables, ranging over everything, but still the only values of the variables that matter to number theory, for instance, or set theory, are the numbers and the sets; whereas identity theory knows no preference.⁴

From universality to lack of subject matter the step may seem short. So Russell, the other contemporary logician whose formal strait-jackets Prior mentions, in one of his moods emphasizes the lack of a (specific) subject matter for logic:

Thus the absence of all mention of particular things or properties in logic or pure mathematics is a necessary result of the fact that this study is, as we say, “purely formal.”⁵

And soon after:

[L]ogic (or mathematics) is concerned only with forms, and is concerned with them only in the way of stating that they are always or sometimes true...⁶

In Russell's view, the pure formality of logic is witnessed by the lack of particular constituents for logical propositions. It is of such displays of pure structure that logical truths state the truth.

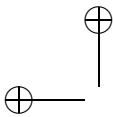
I believe we may reconstruct a progression of views making logic, so to speak, more and more formal. First, logic is supposed to be *universal*, i.e., logical truths are true of all individuals (and properties/relations). To this view Prior subscribes. Nonetheless, from the universal applicability of logical truths it does not follow, as Prior points out, that logical truths cannot be about specific individuals. So, it is a logical truth about John that he is sick if he is sick.⁷ The subject matter is specific, John and sickness, i.e., this

⁴ W.V. Quine, *Philosophy of Logic*, p. 62, emphasis added. A bit earlier Quine appealed to completeness results as the dividing factor.

⁵ B. Russell, *Introduction to Mathematical Philosophy*, p. 198. In this passage Russell sides mathematics with logic, whereas Quine had contrasted them, but this is irrelevant to the main point about the universality and formality of logic.

⁶ Russell, *ibid.*, pp. 199–200. See also *The Philosophy of Logical Atomism*, pp. 85–99, especially p. 96.

⁷ Cf. Prior, “A Statement of Temporal Realism”, p. 45, in *Logic and Reality*, pp. 45–6.



truth is about John even if it is true of everything that if it is sick it is sick (similarly for sickness).

Second, we find in Russell, but not in Quine and Prior, a *generalistic* conception of pure logic, according to which logical truths are about no individuals whatsoever. According to such a view it is logically true that if anything is F, then it is F. But the particular instance concerning John and sickness is not a pure logical truth, lacking as it does in generality.⁸

Third, possibly moving beyond Russell's original intentions, we proceed to abstract even from whatever purely general content had remained for logic. Left with no content whatsoever, logical truths are thought of not only as universal and general, but as *purely structural or formal*. At this point the question arises of what logic owes its formality to, and two main alternatives emerge, differing in what they take the formality of logic to reside in. According to the first kind of view, which I will label *linguistic*, the key to the formality of logic lies in its freedom from real (empirical/factual) content. The task of logic is not to inform us about the real world, rather to express ultimately linguistic (or conceptual) connections. The second view instead, which I will label *realist*, places the formality of logic not so much in a lack of factual content, rather in its concern with the most structural/formal features of reality itself. In this second sense, logic is formal in virtue of having real forms themselves as its specific subject matter. Logic takes as its proper subject matter some very general and abstract (structural) features of reality.

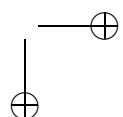
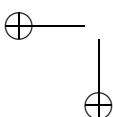
It is safe to say that Prior never fell prey to the above fascination with purely structural truths — be the structure in virtue of which they are true worldly or linguistic. To tackle the linguistic strand first: No statement is purely formal in the sense of being true only in virtue of how we use words (except for the trivial case of statements whose subject matter is our own use of words).⁹ In this sense, Prior connects himself to a logical tradition typically linked to Frege,¹⁰ who in comparing his concept-script to what he calls “Boole’s logical calculus” eloquently says: “Right from the start I had in mind the *expression of a content*.¹¹ All truths, including logical truths,

⁸ See Russell, *Our Knowledge of the External World*, p. 66.

⁹ See Prior, *ibid.*, p. 45, and also “What is Logic?”, p. 123, in *Papers in Logic and Ethics*, pp. 122–9.

¹⁰ See Heijenoort “Logic as Calculus and Logic as Language”; Sluga “Frege against the Booleanians”; and Hintikka “On the Development of the Model-Theoretic Viewpoint in Logical Theory”.

¹¹ G. Frege, *Posthumous Writings*, p. 12.



have a subject matter (be it particular or general) and as such may be labeled *not* purely structural in the linguistic sense.

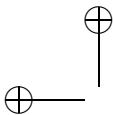
In defense of the linguistic conception of logic, it may perhaps be suggested that insofar as they are necessary, logical truths are independent of the accidental features of the world, hence in some important sense free of a world-dependent subject matter. Without entering into the complex topic of the necessity of logical truths (Is necessity a defining mark of logical truths?), we can at least say that the appeal to necessity to explain the world-free nature of logic displaces the problem without really solving it. The supposed world-independence of necessary truths remains to be explained — both in the sense that we still have to explain (i) to what necessary truths owe their necessity and (ii) in what sense independence from accidental features of the world amounts to independence from the world tout-court — and Prior’s suggestion is *at least* that appealing to a notion of truth simply in virtue of how we use words does not provide a full explanation.¹²

This of course is not to deny that some subjects are more formal than others. And here enters the worldly strand of the structural tradition, according to which the formality of logic is due not so much to a lack of subject matter/content for logic, but rather to the peculiar formal nature of such subject matter/content. Prior considers two possible non-linguistic ways of cashing in the idea of the generality and formality (truth in virtue of form) of logic. In a first sense — what he calls ‘a strictest sense’ — logic may be understood as having as its own subject matter the properties of implication and universality. In a second ‘looser’ sense, logic is concerned with implication *in any sort of field*. In this sense too logic is universal, insofar as it is concerned with the most general principles of inference (and truths) *in any field whatsoever*.¹³ This is the sense in which we sometimes speak of the logic of necessity, the logic of time, or even the logic of life. Here arbitrariness enters the scene in two forms. First, some subjects have more structure than others, for example life is more unruly than time. This will make a logic of time easier to formulate than a logic of life, but there is no sharp, principled divide between more and less general principles of inference, to make some fields but not others logical. Second, inside a particular field, be it time or life, there will be different truths, some more general than others. Here again there is no principled divide to separate the logical from the merely metaphysical truths of the specific matter.¹⁴

¹² I owe the main point raised in this paragraph to an anonymous referee.

¹³ Prior, “What is Logic?”, pp. 128–9. See also, *Past, Present and Future*, p. 51.

¹⁴ Cf. *Past, Present and Future*, p. 51, where speaking of tense logic in particular Prior says: “[T]he line between logic and other subjects seems to me in any case not an easy one



We may indeed say that Prior's robustly realist view of logic ("Logic is ... about the real world")¹⁵ places him firmly in the realist camp, however Prior does not deeply endorse a structuralist outlook. Though he briefly suggests that in a somewhat idealized sense logic has its own proper well-delimited subject matter (the properties of implication and universality), Prior refuses to draw clear boundaries for logic. The universality of logic has in Prior's hands interesting egalitarian consequences. Logic does not encompass an elite of universal, general, formal truths and modes of inference; in its most fruitful applications, logic is concerned with all fields and all modes of inference ("even the truth that all feathered animals breathe air *can* be used as a principle of inference"¹⁶). Not only is logic as concerned with the real world as any other discipline ('What else is there to be concerned with?' one might legitimately ask), but no sharp boundaries can be legitimately drawn between logic and metaphysics, let alone Russell's mathematics. If this is the case, the above mentioned traditional divide between logics and theories can be seen as ill conceived. We may conclude that the importance of the question of what counts as logic is downplayed by Prior.

3. Whence Interpretations?

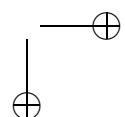
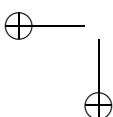
In the previous section we have considered Prior's stance on the philosophical thesis that logic is a purely formal discipline and on the resulting philosophical question of what logic owes its formality to. In a more technical vein, a formalistic view of logic regards formal systems in general as uninterpreted calculi. At this technical level of abstraction, the logician may be seen as abstracting even from the intended interpretation of the logical particles, thus opening the way to *formal reinterpretations* of such symbols. In this section, we will see what Prior has to say concerning this more technical level of abstraction.

When a formal system is taken as a pure uninterpreted calculus, the question arises of what suffices to provide an interpretation of the logical symbols of the system. In my reading, Prior seems to suggest that nothing short of the attribution of an 'intended' meaning by means of a translation into an

to draw *except arbitrarily*", and then considers better and worse ways of drawing the line between logical and non-logical tense-truths.

¹⁵ Prior, "A Statement of Temporal Realism," p. 45. Naturally, such a realism connects Prior to another Russellian line of thought which emphasizes the robustness and worldliness of logic. Cf. Russell, *Introduction to Mathematical Philosophy*, especially p. 169.

¹⁶ "What is Logic?", p. 128.



already interpreted language will suffice for the task, and this even in the paradigmatic simple case of propositional logic.

In his “Conjunction and Contraction Revisited” Prior considers both the case of whether interpretations of the propositional connectives are achieved by means of inferential definitions and the case of whether such interpretations are achieved by means of truth-tables. In both cases the answer is negative.

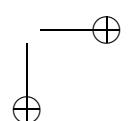
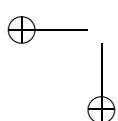
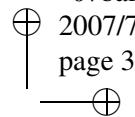
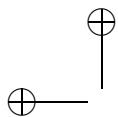
Let us first consider the case of an inferential definition of a logical sign by means of spelling out rules of inference for its introduction and elimination. One might think that at least in the paradigmatic simple case of propositional logic, no gap is there to be filled between the proof theory on the one hand and the interpretation of the logical signs on the other. Not so for Prior, who, with the characteristic incisiveness, tells us that “to believe that anything of this sort can take us beyond the symbols to their meaning, is to believe in magic.”¹⁷ Naturally, the rules of inference may put us on the right track to assign the *intended* meanings, but they are not by themselves sufficient for such an assignment.

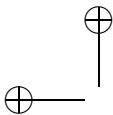
On a purely formalistic view of logic, according to which a logic is just a calculus, there is no intended interpretation to capture, hence no place for the thesis that the intended interpretation of the connectives depends on reflecting some substantial truths about that subject. In this case, no gap can be envisioned between the formal system — the calculus — on the one hand, and the intended interpretation of the logical signs, on the other. When there is no antecedent interpretation to capture, there is no possibility for the formal system to fail.

However, we have seen that according to Prior a gap between the formal system and the intended interpretation arises *already* in the very basic case of propositional logic. We might put it in the following way: the propositional calculus is no calculus. We should rather speak of the propositional theory. Such a theory aims to capture an antecedently given natural subject matter, the truth-functional truths (the so called tautologies). To capture the truth-functional truths and inferences, we need to reflect essential features of the truth-functional connectives.

Of course, it may be the case that one is not interested in discovering and formalizing the truths about conjunction, disjunction, negation, and so on (i.e., one’s interest may not reside in what I called ‘propositional theory’); rather one may be interested in the propositional *calculus* as an uninterpreted game. Then in the meta-theory of such a calculus one is free to *define* the uninterpreted logical signs in terms of the introduction and elimination rules. But, from Prior’s point of view, this would not be a direct interest in the

¹⁷ Prior, “Conjunction and Contraction Revisited,” pp. 159–60.





logical truths concerning conjunction, negation, etc., rather a meta-logical inquiry:

In the meta-theory of such a game we can indeed define terms like ‘conjunction-forming sign’, either directly in terms of the design of the symbols..., or in terms of the permissions to transform.¹⁸

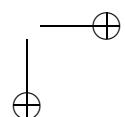
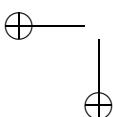
Secondly, Prior considers the case of whether the truth-tables can be taken to provide the interpretation that the pure calculus stands in need of. In this case too, perhaps more surprisingly, Prior’s answer is negative, thus revealing a skeptical attitude towards formal semantics. Once again, at least in the core case of propositional logic, if in no other case, it is natural to think that formal semantics is sufficient to provide the intended interpretation of the logical symbols. All that there is to the meanings of the logical connectives is given by the clauses in the truth definition, graphically represented by the truth-tables. So, for example, the meaning of the negation sign is given by the truth-clause according to which ‘ $\sim A$ ’ is true if and only if ‘ A ’ is not true, and the meaning of the conjunction connective is completely captured by the corresponding clause in the truth definition. We may think of the truth definitions as spelling out recipes for value assignments. So ‘ $P \& Q$ ’ is assigned True when P and Q are both True. In all other cases it is assigned False. This is thought to specify the meaning of the conjunction sign — at least if the values are taken to be indeed the True and the False.

However, Prior insists that he sees in principle no difference between inferential definitions and truth-tables. The availability of unintended interpretations that meet the truth-tabular (as well as the inferential) conditions is taken to show that the truth-tables are not apt by themselves to provide the intended meaning of the connectives. At most, given an intended meaning, they may be used to display its logical structure.¹⁹

Prior does not elaborate much on this matter, but his comments on truth-tables reveal a deeper dissatisfaction with formal semantics than the mere metaphysical concerns about some of the entities that formal semantics makes use of (possible worlds and instants of time). We may distinguish two separate questions:

¹⁸ Prior, *ibid.*, p. 159.

¹⁹ One of Prior’s examples is that the truth conditions for ‘ P and Q ’ are satisfied by ‘Either P and Q , or Oxford is the capital of Scotland’ (abbreviated as ‘ P ett Q ’), *ibid.*, p. 162. An anonymous referee has suggested that the truth-tables for ‘and’ and for ‘ett’ will diverge once possible situations are considered in which Oxford is the capital of Scotland. This move however is not available for unintended interpretations making use of necessary falsehoods, for example ‘Either P and Q , or $2+2=5$ ’.



1. Are formal models, of which truth-table rows are elementary exemplars, capable in principle of providing the semantics of modal, tense and even propositional logic?
2. Are the formal models philosophically viable, i.e. metaphysically unobjectionable?

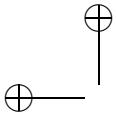
In the next section, we will focus on the second question and consider Prior's more metaphysical concerns. Our present concern however is with the first question. Given Prior's comments on truth-tables, and the lack of metaphysical concerns in this simple case, it seems to me that Prior's answer to the first question is no. Formal models are not capable of providing interpretations, and this independently of and prior to their philosophical viability. Naturally a formal model may represent (model) an interpretation otherwise supplied, but the model in and of itself does not provide the interpretation. Why so? One concern here might be that in order to provide a complete interpretation we need to interpret also the non-logical signs of our language. Hence, truth-tables are inadequate simply in the sense that they are insufficient to the task. They provide only a partial interpretation. To complete the formal interpretation, we have also to single out a particular model, in this case a row of the truth-table, as the intended one.

However, Prior's semantic concerns — as they emerge from his comments regarding unintended interpretations — seem to go beyond such questions. If the problem is that truth-tabular conditions are insufficient to narrow down the intended meaning of *the logical connectives*, a specific assignment of truth values to the non-logical signs can offer no help. Moreover, the additional question arises of whether a truth value assignment to the non-logical symbols constitutes an interpretation of such symbols, and here too one might be skeptical. In this sense, I connect Prior once again to a broadly Fregean perspective which has seen more to interpretations than the provision of models. I have in mind here a view such as Carnap's:

An interpretation should not be identified with a model, as is sometimes done. It is true that an interpretation can be given by the specification of a model. But there is no one-one correspondence between interpretations and models; two different (i.e., non logically equivalent) descriptions of the *same* model represent two *different* interpretations.²⁰

It seems then that all logical systems — no matter how elementary — are viewed by Prior as pursuing an antecedent subject matter, and so an *intended*

²⁰ R. Carnap, “My Conception of Semantics”, p. 902. Carnap then proceeds to provide an example of the gap between a model and an interpretation.



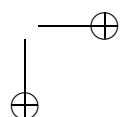
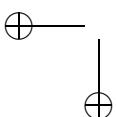
interpretation, reflecting fundamental truths about such subject matter, has to be given. Formal ‘interpretations’ (definitions) however, either in proof-theoretic or model-theoretic terms, are not semantically adequate.

The obvious consequence of such a view is that no semantic/interpretive gap separates tense and modal logics in general from the pure core of logic. Quine’s province of logic proper, encompassing propositional and first-order logic, is artificially delimited. One may interpret Quine’s qualms about modal logics as qualms about the logicality of such logics. Modal logic, according to such an interpretation, is a theory in disguise, and so is second-order logic (where standard incompleteness results conveniently attest to ontological excesses). Hence they are to be viewed as fundamentally different from propositional and first-order logic where the model-theoretic formal semantics is perfectly adequate to the interpretational task and no residual interpretational concerns remain.

Not so for Prior. To legitimize modal logics however there is no need to brand them pure by providing interpretationally adequate formal definitions. Such an enterprise is fundamentally misguided. Moreover, no completeness or other metatheoretical technical result can ever change the fact that the formal semantics of modal logic is interpretationally inadequate. Interpretationally, what needs to be done is not so much to prove modal logic model theoretically on a par with first-order logic, though it may well be so, nor is it sufficient to legitimize the entities that the modal models make use of. We only need to point out that *in no case* are formal interpretations adequate. Semantically, propositional theory stands in need of a non-model-theoretic interpretation as much as temporal logic, modal logic and second-order logic for that matter — though the metaphysical load might well be less controversial. From this perspective it is not so much the provision of an extensional metatheory for a modal, tense, or second-order calculus to legitimize it as logical, even when accompanied by completeness results. It is rather the fact that even the traditional first-order extensional logic does not receive its interpretation from the extensional metatheory. A non-formal interpretation is needed in any case. If the formal semantics is seen as inadequate to provide the intended interpretation of our logic, though it may well represent (model) such independently provided interpretation, the question naturally arises of the proper role of the model theory and of what philosophical significance can be attributed to formal completeness results.

4. Metaphysics and Metalogic

As noticed, Prior’s considerations about propositional logic place him in the camp of a broadly Fregean tradition according to which the objects of truth and falsity are *interpreted* sentences, and this holds for all kinds of truths,



logical truths included. Logical truths are first and foremost truths. We might summarize this view with the slogan: Interpretation before truth and truth before logical truth. This concerns the original informal notions of logical truth and inference. According to Prior, they are fundamentally semantic notions, holding of interpreted sentences. Formal systems are then devised to help systematize such truths and inferences.

Opposed to this view is the Boole/Schroeder algebraic tradition that sees logic as independent from and prior to content, and according to which the same logical calculus can legitimately receive different interpretations. In this pre-content logical tradition, logical truths need not be truths. If logical truths are characterized in terms of their role in a system of inference, i.e. their provability, then truth is no intrinsic part of the notion of logical truth.

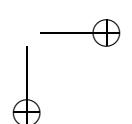
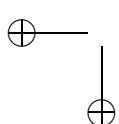
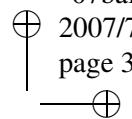
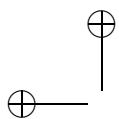
We must keep two questions separate. The first is about English sentences. Are the pre-formal notions of entailment and logical truth for English sentences essentially semantic, or do they apply to pre-interpreted sentences, let's say in virtue of their syntactical form? The second concerns what counts as an interpretation of (the signs of) a formal system. Does such an interpretation consist in an assignment of English meanings, i.e., what we might call an ‘interpretation by translation’, according to which for example ‘&’ means ‘and’? Or is it possible to interpret a formal language mathematically by means of a formal valuation, be it a model-theoretic assignment of values or a proof-theoretic definition of its symbols?

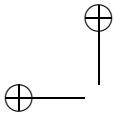
Though separate, the two questions are connected. It is only insofar as one conceives of the pre-formal notions of entailment and logical truth for English sentences as essentially semantic, and thinks of formal systems as renditions of their natural counterparts and not just calculi, that one may, though perhaps need not, find space for a semantic criticism of an otherwise technically adequate model-theoretic interpretation of such systems.

Prior seems to conceive of the job of the formal logician as follows: We start with some English sentences, ordinarily interpreted. With such an intended interpretation in mind, we devise a formal language apt to represent the internal logical structure of the original English sentences. We then build a formal calculus aimed at capturing the original inferential relations that obtain between the English counterparts of the formal sentences.²¹ Next we construct a formal semantics, by means of truth-clauses, of model-theoretic constructions, or both.

Once a formal calculus has been constructed and a formal semantics has been provided, questions of *external* adequacy will arise. First and foremost:

²¹ Of course, the logician as such has no metaphysical biases, and will offer adequate symbolizations both of sentences that, qua philosopher, he takes to be literally true and of sentences that he regards as literally false; similarly, he will build systems to whose axioms and rules of inference, under the intended interpretation, he may or may not subscribe.





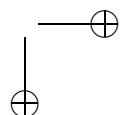
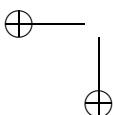
Are the original inferential relations adequately represented by the formal notion of provability in the system? And is the original notion of logical truth adequately represented by the formal notion of truth across all models? Moreover, purely formal, and in this sense *internal*, questions of soundness and completeness will also arise: Which class of models corresponds to the calculus?²² These internal questions concern the relation between the formal calculus and the formal model-theoretic semantics. In such a sense, a model-theoretic semantics may be proved *formally* adequate by a formal completeness result. Yet such a result has no immediate philosophical relevance, unless the model-theoretic semantics is taken to be semantically and philosophically adequate. We have already considered Prior's stand on the semantic (in)adequacy of the model theory. But, semantic concerns aside, what else is required for the model theory to be philosophically adequate? First, the formal notion of validity as truth across all models must capture (at least extensionally) the informal notion of validity; second, the model theory must make no use of metaphysically dubious entities. In this second sense, Prior raises a sharp metaphysical criticism to the model theory.

As the founding father of temporal logic, Prior has worked on the construction of both tense systems, which belong to the general class of modal systems, and U-calculi which build temporal logic as an extension of first-order logic. In the tense systems tense operators corresponding to the English phrases ‘It has always been the case that’, ‘It will always be the case that’, etc. are introduced to operate on formal sentences. In the U-calculi quantification over a special class of variables, ranging over instants of time, and an ordering relation (U) between such instants are introduced. Both kinds of calculi are of interest to the logician, and much of Prior's logical work consists in showing how such systems are related and how to ‘translate’ from one kind of system into the other.²³

However, the question arises of which kind of system, interpreted in the intended way, formulates the right ontological commitments. Is the structure of reality fundamentally tensed, or is time rather like a tapestry of instants (to use Prior's fortunate phrase)? Naturally, English itself is a tensed language, but this does not in and of itself answer the metaphysical question. The grammar of English sentences might itself be ontologically misleading and not a good guide to metaphysics (though it is hard to see how it would be possible to completely disregard linguistic data when facing ontological

²² See Kreisel, “Informal Rigour and Completeness Proofs”.

²³ I here use ‘system’ and ‘calculus’ interchangeably, but, interestingly enough, Prior talked of tense systems and U-calculi.



questions). According to Prior, “In doing metaphysics there is still no substitute for ‘the choice of the soul’; or, if you like, prejudice.”²⁴ And Prior made no secret of the choices of his soul. He endorsed the tensed fabric of reality, and forcefully rejected instants — and, for that matter, time too — as genuine entities:

But ‘instants’ as literal objects, or a cross-section of a literal object, go along with the picture of ‘time’ as a literal object, a sort of snake which either eats its tail or doesn’t, either has ends or doesn’t, either is made of separate segments or isn’t; and this picture I think we must drop.²⁵

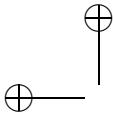
It is well known that Prior viewed instants as logical constructions out of tensed facts or propositions. Ontologically, he aimed at individuating the metaphysically basic elements of reality and then constructed other ‘things’ out of such basic and ultimately only real individuals. Prior regarded ordinary individuals, like me, you, chairs and tables as metaphysically basic, and viewed facts, events and propositions as constructions out of these basic items of his ontology. Finally, instants were placed at an even higher level of logical complexity, as constructions out of events or propositions:

I cannot *understand* ‘instants’, and the earlier-later relation that is supposed to hold between them, except as logical constructions out of tensed facts. Tense logic is for me, if I may use the phrase, *metaphysically fundamental*, and not just an artificially torn-off fragment of the first-order theory of the earlier-later relation.²⁶

²⁴ Prior, *Worlds, Times and Selves*, p. 93.

²⁵ Prior, *Past, Present and Future*, p. 189. The focus of this paper is not on Prior’s extremely interesting metaphysical views. Let me just say that there is (much) more to Prior’s metaphysics than what emerges from the above considerations concerning his endorsement of tenses and rejection of instants. First, there is the very difficult question of how to understand Prior’s view according to which reality flows, it is dynamic and not static, hence fundamentally tensed, yet there is, in some important sense, no such thing as time (time itself is not a genuine *thing*). Also, how are we to make sense of the tensed nature of reality if only the present time and ultimately present things are real? And in what sense is the future open given that there literally is no future? Personally, I have a lot of sympathy for Prior’s metaphysical views, but I am not sure I can really grasp them. Though I am inclined to think that the complexities reside in the subject matter itself, which Prior simply refused to simplify.

²⁶ Prior, *Worlds, Times and Selves*, p. 37.



Considerations of this kind touch upon the purely metaphysical question of what entities are metaphysically fundamental, and of which degree of reality belongs to logically constructed items — not much according to Prior. Because of such metaphysical qualms, Prior viewed the formal semantics of tense logic as philosophically unacceptable. Instants are meta-theoretical fictions. They play an important role in the metatheory of tense logic, but metatheory is no substitute for metaphysics. Not only, as we have seen, are meta-theoretical constructions not adequate semantic interpretations — they are also not metaphysically viable. Instants are just mathematically useful fictions:

It is true that in our technical work, when we are deciding which formulae express discreteness, finitude, etc., we always turn to ‘U-calculi’ in which the terminology is decidedly more abstract, and time appears as something like a class of classes of propositions ordered by a certain relation. This in itself, however, doesn’t make U-calculi more than handy diagrams; they need not be taken with any great metaphysical seriousness.²⁷

These mathematical fictions however are enormously useful from a technical point of view. They make it possible to provide an extensional metatheory for tense and modal logics in general, which extends the meta-theory of first-order logic. Technical questions of independence and completeness can now be handled in a familiar framework:

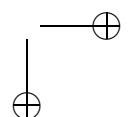
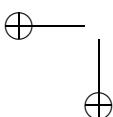
The ‘metalogical utility’ of associating tense-logical systems with systems developed within predicate logic and the theory of ordering relations is in fact not only ‘considerable’ but enormous, and something like it ... is now standard procedure in handling questions of independence and completeness not only in tense logic, but also, even especially, in modal logic.²⁸

In this last passage Prior is latching on to a familiar Quinean theme, according to which possible world semantics is a technical enterprise that makes it possible to prove some technical results, but affords no explication of the modal vocabulary.²⁹

²⁷ Prior, *Past, Present and Future*, p. 75.

²⁸ Prior, *ibid.*, p. 42.

²⁹ See Quine: “The notion of possible world did indeed contribute to the semantics of modal logic, and it behooves us to recognize the nature of its contribution: it led to Kripke’s



When ‘interpreting’ the modal system S5 as a fragment of ordinary predicate logic (‘the uniform monadic predicate calculus’), whose variable quantifies over possible worlds when bound, and stands for a privileged individual when free, Prior explicitly refers to Quine (“To use a distinction I once heard Quine insisting upon, what we have in th[is] calculus ... may be a *model* for modal logic, but it is not an *interpretation* of the modal words.”³⁰). This particular way of ‘interpreting’ modal logic, as the uniform monadic predicate calculus, is to Prior particularly non-illuminating, since it serves no good philosophical purpose. Its appeal to possible worlds is not ontologically secure. Possible worlds, even more clearly than instants, are mere meta-logical constructions. Prior once called a metaphysics of possible worlds ‘a tall story’, and this *independently* of whether possible worlds are understood David Lewis’s way, or actualistically as made up of actual individuals.³¹ Not to mention the additional questions raised by the appeal to a privileged world. Logically, the metatheory of the uniform monadic fragment of first-order logic may shed some additional light on the metatheory of modal logic, given the decidability of this fragment of first-order logic, but at the price of philosophical obscurantism.

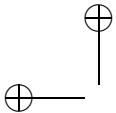
We noticed earlier that when a formal system is constructed and a model-theoretic semantics is provided both questions concerning the *external* adequacy of the model theory and questions *internal* to the model theory proper will arise. One such internal question which has captured much attention from philosophers is how to identify individuals across worlds (or instants). The problem of the cross-world identification of individuals has seemed at times to be the main obstacle to the adoption of a possible world metaphysics. I find Prior’s attitude towards the corresponding tense problem very illuminating.

We may think of quantified tense logic as an intermediate case between non-modal quantification theory and quantified modal logic, ‘intermediate’ in terms of resistance to straightforward interpretation. Prior believed that there is no identity problem across time; his basic items were enduring individuals whose trans-temporal existence and identity is primitive. Nonetheless Prior went on to argue that instants are not genuine individuals and do

precocious and significant theory of models of modal logic. Models afford consistency proofs; also they have heuristic value; but they do not constitute explication. Models, however clear in themselves, may leave us still at a loss for the primary, intended interpretation” (“Review of *Identity and Individuation*,” p. 492).

³⁰ Prior, *Worlds, Times and Selves*, p. 244.

³¹ Prior, “Modal Logic and the Logic of Applicability” (p. 283), in *Papers on Time and Tense*, pp. 275–92.



not provide an analysis of our natural tense languages. The model theory — of which he was the founder — was just this for Prior: a model theory.

Prior simply separated two independent issues: (i) the *internal* question within a model theory for tense logic — does identity across moments of time raise any special problems? And (ii) the *external* question of interpretation — does the moments-of-time model theory provide the intended interpretation of natural language tenses? Solving the first internal problem does not automatically legitimize a metaphysics of instants. The provision of an unproblematic metatheory leaves tense logic uninterpreted.

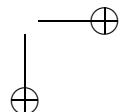
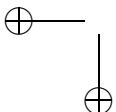
When it comes to a metaphysically viable interpretation, we find in Prior a non-ambivalent attitude: nowhere does Prior try to legitimize the model theory's instants by arguing they are *bona fide* items after all, albeit abstract entities (“Point-instants ... seem as mythical to me as matter did to Berkeley”³²). Hence, the *external* question of whether the model theory can provide an analysis of interpretational matters stands despite the resolution of the *identity* question *internal* to the model theory.³³

In conclusion, given his skepticism on the possibility (and desirability) of drawing clear boundaries for logic, Prior never deeply engaged with the question of which truths about time are to be regarded as logically pure.³⁴ This factor, combined with his emphasis on the indispensability of philosophical labor both for providing an interpretation of the logical/temporal/modal signs and for discovering the truths about time, contributed to a philosophical disregard of the model-theoretic apparatus. Consequently, Prior

³² Prior, *Past, Present and Future*, p. 200.

³³ This is not the right place to focus on purely metaphysical questions. However, I would like to suggest that according to Prior the inter-definability of instants as classes of propositions and propositions as classes of instants offers some kind of evidence of the thin metaphysical status of instants. In the model-theoretic setting, it is natural to take the individuals of the domain to be point-like bare particulars that support any logically consistent qualities whatsoever, or alternatively, to identify them with the bundles of the qualities proper. Similarly for the model theory's points of evaluation, be they ‘worlds’ or ‘instants’. Again, each such point may be seen in two ways: first, as a bare point of evaluation at which formulae are assigned truth values; second, as a bundle of qualities, this time with propositions (facts, true sentences) making up the bundle. However, when it comes to real individuals, Prior argued both against bare individuals (*haecceitism*) and against bundles of properties. See “Identifiable Individuals”, in *Papers on Time and Tense*, pp. 81–92.

³⁴ One exception is *Past, Present and Future*, pp. 50–1, where Prior criticizes Cocchiarella's choice of an axiomatic base for temporal logic which includes an axiom expressing time's linearity, but excludes, among others, axioms expressing either the discreteness or the density of time. According to Prior, it is questionable to regard some truths about time as logical, while rejecting others which are expressible in the same technical vocabulary. If one has to insist on demarcating the pure truths about time, a better choice would be to exclude *all* special assumptions concerning the earlier-later relation.



never fell into the temptation of substituting the logician’s work of building models for the metaphysician’s labor of providing ontologically secure interpretations. He never thought that the (in his eyes) misguided philosophical quest for the pure (logical) truths about time could receive some legitimacy rephrased as a technical questions of validity — truth across a bunch of set theoretic structures — for formal sentences. Nor did he ever engage in the search for the ‘intended model’. In so doing Prior placed himself securely in a Fregean and Quinean (at least for what concerns modal logic) tradition which sharply separates semantic and metaphysical questions from technical considerations. The model theory is insufficient to provide an interpretation exactly because we are after the intended interpretation of the logical, temporal and modal signs and the real truth of the matter.³⁵

ACKNOWLEDGEMENTS

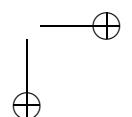
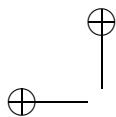
I wish to thank John Woods and an anonymous referee of this journal for some very helpful suggestions. The anonymous referee’s comments in particular have been very useful in structuring the third section of the paper.

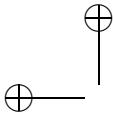
Department of Philosophy
University of British Columbia
Canada
E-mail: rballari@interchange.ubc.ca

REFERENCES

- [1] Carnap, R., 1963: My Conception of Semantics, in P.A. Schilpp (ed.), *The Philosophy of Rudolf Carnap*, Open Court, La Salle, Illinois.
- [2] Copeland, B.J. (ed.), 1996: *Logic and Reality*, Clarendon Press, Oxford.
- [3] Frege, G., 1979: *Posthumous Writings*, H. Hermes, F. Kambartel and F. Kaulbach (eds.), P. Long and R. White (trans.), University of Chicago Press, Chicago.
- [4] van Heijenoort, J., 1967: Logic as Calculus and Logic as Language, *Synthese* 17: 324–30.

³⁵ Interesting in this respect are Prior’s remarks about Lemmon’s algebraic ‘interpretation’ of modal logic, and the possibility of similarly ‘interpreting’ tense logic. In this perspective the U-calculus is simply a Boolean algebra and the derivability of theses about U from theses about tense operators amounts to an algebraic proof of completeness. See “Tensed Propositions as Predicates”, in *Papers on Time and Tense*, pp. 195–211, especially pp. 209–11.





- [5] Hintikka, H., 1988: On the Development of the Model-Theoretic Viewpoint in Logical Theory, *Synthese* 77: 1–36.
- [6] Kreisel, G., 1967: Informal Rigour and Completeness Proofs, in I. Lakatos (ed.), *Problems in the Philosophy of Mathematics*, North-Holland, Amsterdam: 138–71.
- [7] Lemmon, E.J., 1966a: Algebraic Semantics for Modal Logics I, *Journal of Symbolic Logic* 31: 46–65.
- [8] Lemmon, E.J., 1966b: Algebraic Semantics for Modal Logics II, *Journal of Symbolic Logic* 31: 191–218.
- [9] Prior, A.N., 1957: *Time and Modality*, Clarendon Press, Oxford.
- [10] Prior, A.N., 1964: Conjunction and Contonktion Revisited, *Analysis* 24: 191–5. Reprinted in *Papers in Logic and Ethics*: 159–64.
- [11] Prior, A.N., 1967: *Past, Present and Future*, Clarendon Press, Oxford.
- [12] Prior, A.N., 1976: *Papers in Logic and Ethics*, P.T. Geach and A.J.P. Kenny (eds.), Duckworth, London.
- [13] Prior, A.N. and Fine K., 1977: *Worlds, Times and Selves*, Duckworth, London.
- [14] Prior, A.N., 2003: *Papers on Time and Tense*, P. Hasle, P. Ohrstrom, T. Brauner and J. Copeland (eds.), Oxford University Press, Oxford.
- [15] Quine, W.V., 1947: The Problem of Interpreting Modal Logic, *Journal of Symbolic Logic* 12: 43–8.
- [16] Quine, W.V., 1970: *Philosophy of Logic*, Harvard University Press.
- [17] Quine, W.V., 1972: Review of *Identity and Individuation*, *Journal of Philosophy* 69: 488–97.
- [18] Russell, B., 1918: The Philosophy of Logical Atomism, in D. Pears (ed.), *Russell's Logical Atomism*, Fontana, 1972.
- [19] Russell, B., 1919: *Introduction to Mathematical Philosophy*, Allen and Unwin, London.
- [20] Russell, B., 1926: *Our Knowledge of the External World*, Revised ed., Allen and Unwin, London.
- [21] Sluga, H., 1987: Frege against the Booleans, *Notre Dame Journal of Formal Logic* 28: 80–98.

