Logique & Analyse **198** (2007), 99–111

A THREE-VALUED TEMPORAL LOGIC FOR FUTURE CONTINGENTS

SEIKI AKAMA, YASUNORI NAGATA AND CHIKATOSHI YAMADA

Abstract
It is now recognized that Lukasiewicz's three-valued logic cannot supply a satisfactory treatment of future contingents. The defense of Lukasiewicz's three-valued logic can be found in the work of Prior. Inspired by Prior's suggestions to future contingents, we propose a new three-valued temporal logic FCP, in which the interpretation of negation differs from Lukasiewicz's negation. It is shown in the logic that contingent propositions have the indeterminate truth-value while logically true propositions like the law of excluded middle have the determinate truth-values. We discuss some features of the proposed logic in comparison with the other approaches to future contingents in the literature.

1. *Introduction*

By the problem of *future contingents*, we mean whether future contingent events already have the determined truth-values, i.e. true or false. It seems perfectly coherent to maintain that there is already now a fact of the matter whether a future contingent F is true, but that nevertheless our world could have been such that F had a different truth-value, in the sense that there are possible worlds in which F has another truth-value. The problem has a long history since the age of Aristotle. It is well known that Lukaisiewicz [8] proposed his three-valued logic to deal with future contingents.

It is now recognized that Lukasiewicz's three-valued logic cannot give a satisfactory treatment of future contingents. A defense of Lukasiewicz's three-valued logic can be found in the work of Prior [9]. We believe that existing three-valued logics are not suited to the formalization of future contingents. The desired solution is to develop a new three-valued temporal logic.

In this paper, we proposed a new three-valued temporal logic FCP, in which the interpretation of negation differs from Lukasiewicz's negation. It is shown in the logic that contingent propositions have the indeterminate truth-value while logically true propositions like the law of excluded middle have the determinate truth-values.

The structure of this paper is as follows. In section 2, we discuss the problem of future contingents of Aristotle and gives the exposition of Lukasiewicz's solution by means of three-valued logic. In section 3, a three-valued temporal logic FCP is introduced. It is described as an alternative to Lukasiewicz's three-valued logic. We note some features of FCP. The completeness argument is given using a Kripke type semantics. Some philosophical discussions on future contingents are also presented. Section 4 concludes this paper with directions of future research.

2. *Future Contingents*

There are many propositions to refer to future contingents. As is well known, future contingents raise many philosophical problems. Aristotle [3] already noted in *De Interpretatione IX*, that only propositions about the future which are either necessarily true, or necessarily false, or something determined have a determinate truth-value. In other words, Aristotle accepts the law of excluded middle, but rejects the principle of law of bivalence for future contingents. There are, however, in fact different interpretations of Aristotle's *De Interpretatione*; see Anscombe [2].

Lukasiewicz [8] attempted to formalize Aristotle's idea by developing a *three-valued logic* in which the third truth-value reads "indeterminate". We know that Lukasiewicz presented the truth-value tables for negation, conjunction, disjunction and implication.

For example, consider (1):

(1) I will go to Warsaw.

It seems correct to say that my going to Warsaw remains open. It is thus plausible to give (1) the third truth-value. Namely, we may be able to claim that future contingent propositions have no determinate truth-value. We thus think that the use of three-valued logic is promising.

Unfortunately, many philosophers (or logicians) believe that Lukasiewicz's three-valued logic is not successful as a formalization of future contingents. It produces some counterintuitive results.

Consider the following disjunctive future contingent proposition (2):

(2) Either I will go to Warsaw or I will play Chopin's first concerto.

Since both disjuncts are contingent, (2) receives an indeterminate truth-value according to Lukasiewicz's truth-value table. But the interpretation does not work when we deal with the disjunctive proposition in which one disjunct is the negation of the other disjunct like (3).

(3) Either I will go to Warsaw or I will not go to Warsaw.

How should we interpret (3)? Obviously, we should claim that (3) is definitely true.

If we rely on Lukasiewicz's three-valued logic, the truth-value of (3) is indeterminate, and the result is not intuitively justified. (3) is an instance of the so-called *law of excluded middle* of the form $A \vee \neg A$. A similar defect can be recognized in the treatment of the *law of non-contradiction* of the form $\neg(A \wedge \neg A)$. These two principles have the truth-value "indeterminate" according to Lukasiewicz's three-valued logic. The difficulty lies in the interpretation of disjunction (conjunction) with indeterminate disjuncts (conjuncts). In addition, Lukasiewicz's three-valued logic cannot formalize the above mentioned Aristotle's idea because Aristotle wants necessarily true propositions like $A \vee \neg A, \neg(A \wedge \neg A)$ to have the determinate truth-value, namely "true". On these grounds, we are dissatisfied with Lukasiewicz's three-valued logic as the formal system for future contingents. The criticism of Lukasiewicz's three-valued logic is also found in Haack [7].

3. *Three-Valued Temporal Logic FCP*

To overcome the difficulties in Lukasiewicz's three-valued logical solution to the problem of future contingents consonant to Aristotle's writing, we need to develop a version of three-valued temporal logic. The desired feature to be considered here is to allow *truth-value gap* for future contingent propositions, but confirming necessarily true propositions. Technically speaking, there are two approaches to carry out the feature. One approach is to use three-valued semantics for temporal logic. This can be done by the so-called *supervaluation* of van Fraassen [6]. The dual of supervaluation, namely *subvaluation* is also a candidate for the required three-valued semantics.

The other approach is to revise the underlying three-valued logic. In other words, Lukasiewicz's logic is replaced by other logic. And the new three-valued logic can be properly extended to temporal logic. This paper explores the second approach.

What is a desired three-valued logic? Is it a new one? Or is it an old one? To sketch such a three-valued logic, we expect that the truth-value table agrees with the classical one when no truth-value gap arises. In addition, "indeterminate" truth-value can be intuitively interpreted by logical connectives.

There are surely contingent propositions which receive the indeterminate truth-value. However, we have to accommodate logically necessary propositions which are true. Recall that Lukasiewicz's three-valued logic has the following truth-value tables:

$\sim$	
t	f
u	u
f	t

$\wedge$	t	u	f
t	t	u	f
u	u	u	f
f	f	f	f

$\vee$	t	u	f
t	t	t	t
u	t	u	u
f	t	u	f

$\rightarrow$	t	u	f
t	t	u	f
u	t	t	u
f	t	t	t

Here, t reads "true", f "false", and u "indeterminate", respectively.

For the entries of determinate truth-values, the above agree with the classical truth-value tables. We may have no objections to the truth-value tables for conjunction and disjunction. The gist of our revision is to change the truth-value table for negation, which causes several difficulties mentioned above. There seem no reasons to justify Lukasiewicz's interpretation of negation. The alternative interpretation is:

$\neg$	
t	f
u	t
f	t

Here, we use $\neg$ for the new negation connective. This new interpretation is inspired by Prior's passage ([9], p. 136):

> Perhaps 'neither true nor false' is simply a possible way of describing the kind of falsehood which 'It will be that p' has, in Peircean logic, when the matter is undecided.

According to Prior's remark, we could identify "neither true nor false" as a subcase of "false". From this, the negation of "indeterminate" evaluates as "true". The proposed interpretation can also be justified as follows. If A is indeterminate, then it is not true that A. This gives rise to a possibility that something related to A is true. Thus, one can equate something related to A to the negation of A, supporting the new truth-value table.

Those who are familiar with many-valued logic can recognize that the new negation $\neg$ is in fact the *external* negation due to Bochvar [4]. This does not amount to adopting other external connectives in our system. This is because the interpretation of other external connectives is hardly defended for our purposes.

By our interpretation of "indeterminate", we should define the implication $A \supset B$ as $\neg(A \wedge \neg B)$, which has the truth-value table different from Lukasiewicz's:

$\supset$	t	u	f
t	t	f	f
u	t	t	t
f	t	t	t

Now, we are ready to formulate a three-valued temporal logic FCP. The language of FCP contains logical symbols: negation, conjunction, disjunction and implication, and temporal operators: F (it will be the case) and P (it has been the case). We can introduce other two temporal operators by definition.

G $=_{\text{def}}$ $\neg$F$\neg$ (it will always be the case)

H $=_{\text{def}}$ $\neg$P$\neg$ (it has always been the case)

We can also define the equivalence $\equiv$ as:

$A \equiv B =_{\text{def}} (A \supset B) \wedge (B \supset A)$.

The axiomatization of FCP, similar to that of the minimal temporal logic K_t, consists of axioms and rules of inference:

Three-Valued Temporal Logic FCP

Axioms

(A1) Axioms of Positive Classical Logic

(A2) $(\neg B \supset \neg A) \supset (A \supset B)$

(A3) $\mathrm{G}(A \supset B) \supset (\mathrm{G}A \supset \mathrm{G}B)$

(A4) $\mathrm{H}(A \supset B) \supset (\mathrm{H}A \supset \mathrm{H}B)$

(A5) $A \supset \mathrm{HF}A$

(A6) $A \supset \mathrm{GP}A$

Rules of Inference

(R1) $\vdash A, \vdash A \supset B \Rightarrow \vdash B$

(R2) $\vdash A \Rightarrow \vdash \mathrm{G}A$

(R3) $\vdash A \Rightarrow \vdash \mathrm{H}A$

Here, $\vdash A$ reads "A is provable in FCP". (R1) is *modus ponens*, and (R2) and (R3) are a temporal version of *necessitation*. The notion of a proof is defined as usual.

Surprisingly, FCP has classical flavors. For example, $A \supset A$ is true. We also note that $A \supset B$ is equivalent to $\neg B \supset \neg A$ or $\neg A \vee B$. This means that $A \supset B$ is interpreted as material implication. De Morgan and double negation laws hold. These facts can be well-understood by the interpretation of "indeterminate" sentences as a possible description of false sentences. This enables us to improve Lukasiewicz's approach. In FCP. we express (3) for (4).

(4) $\mathrm{F}A \vee \mathrm{F}\neg A$

which is equivalent to:

(5) $\mathrm{F}(A \vee \neg A)$

Even if A is indeterminate, we wish (5) to be true because $A \vee \neg A$ is a logical truth. In fact, this is possible in FCP as the semantics below shows.

We are now in a position to describe a Kripke type semantics for FCP. A *FCP-model* is a triple $\langle T, \leq, V\rangle$, where T is a set of time points, $\leq$ is a binary relation on T, and V is a partial valuation function: $FOR \times T \to \{t, f\}$. Here, FOR is a set of formulas. $V(A, w) = t$ reads "A is true at $w \in T$" and $V(A, w) = f$ "A is false at $w \in T$", respectively. If neither $V(A, w) = t$ nor $V(A, w) = f$, $V(A, w) = u$, where u is "indeterminate". The valuation function V is extended for other connectives as follows:

$V(\neg A, w) = t$ iff $V(A, w) \neq t$
$V(\neg A, w) = f$ iff $V(A, w) = t$
$V(A \wedge B, w) = t$ iff $V(A, w) = V(B, w) = t$
$V(A \wedge B, w) = f$ iff $V(A, w) = f$ or $V(B, w) = f$
$V(A \vee B, w) = t$ iff $V(A, w) = t$ or $V(B, w) = t$
$V(A \vee B, w) = f$ iff $V(A, w) = V(B, w) = f$
$V(\mathrm{F}A, w) = t$ iff $\exists v(w \leq v$ and $V(A, v) = t)$
$V(\mathrm{F}A, w) = f$ iff $\forall v(w \leq v$ imply $V(A, v) = f)$
$V(\mathrm{F}A, w) = u$ iff $\forall v(w \leq v$ imply $V(A, v) = u)$
$V(\mathrm{P}A, w) = t$ iff $\exists v(v \leq w$ and $V(A, v) = t)$
$V(\mathrm{P}A, w) = f$ iff $\forall v(v \leq w$ imply $V(A, v) = f)$
$V(\mathrm{P}A, w) = u$ iff $\forall v(v \leq w$ imply $V(A, v) = u)$

A formula A is *valid*, written $\models A$, iff for every FCP-model and for every time point $w \in T, V(A, w) = t$.

By the tandem presentation of truth and falsity, conjunction and disjunction can be appropriately interpreted. For negation, the specification is trivial to simulate the truth-value table. Namely, negated sentences do not allow a truth-value gap. For temporal operators, we need to give three separate descriptions capable of interpreting future contingents. The proposed semantics can be viewed as a partial version of Kripke semantics for the classical minimal temporal logic K_t.

Below we give some technical results about FCP. First, the interpretations of G and H can be described by the duality of F and P.

Theorem 1: $V(\mathrm{G}A, w) = t$ iff $\forall v(w \leq v$ imply $V(A, v) = t)$
$V(\mathrm{G}A, w) = f$ iff $\exists v(w \leq v$ and $V(A, v) \neq t)$
$V(\mathrm{H}A, w) = t$ iff $\forall v(v \leq w$ imply $V(A, v) = t)$
$V(\mathrm{H}A, w) = f$ iff $\forall v(w \leq v$ and $V(A, v) \neq t)$

Proof. $\mathrm{G}A$ is equivalent to $\neg\mathrm{F}\neg A$, but $\neg A$ is not indeterminate in any time point. Then, we have the following proofs by using the truth-value table for $\neg$:

$$
\begin{array}{lll}
V(\neg\mathrm{F}\neg A, w) = t & \text{iff} & V(\mathrm{F}\neg A, w) = f \\
 & \text{iff} & \forall v(w \leq v \text{ imply } V(\neg A, v) = f) \\
 & \text{iff} & \forall v(w \leq v \text{ imply } V(A, v) = t) \\
V(\neg\mathrm{F}\neg A, w) = f & \text{iff} & V(\mathrm{F}\neg A, w) = t \\
 & \text{iff} & \exists v(w \leq v \text{ and } V(\neg A, v) = t) \\
 & \text{iff} & \exists v(w \leq v \text{ and } V(A, v) \neq t)
\end{array}
$$

□

A similar justification is also available for H.

Theorem 1 reveals that indeterminacy can be found only for tensed sentences with H and P. Next, we show the soundness.

Theorem 2: $\vdash A \;\Rightarrow\; \models A$.

Proof. This can be established by checking that all the axioms of FCP are valid and the rules of inference can preserve validity. It is easy to show that the theorems of classical propositional logic are also theorems of FCP. It thus suffices to consider the axioms (A3)–(A6). Because (A3) and (A4) ((A5) and (A6)) are similarly proved to be valid. We take up the proofs of (A5) and (A6).

Assume that $\mathrm{G}(A \supset B) \supset (\mathrm{G}A \supset \mathrm{G}B)$ is not valid. Then, there is a time point w such that $V(\mathrm{G}(A \supset B) \supset (\mathrm{G}A \supset \mathrm{G}B), w) \neq t$. This is equivalent to the following:

$$
\begin{array}{ll}
\text{iff} & V(\neg(\mathrm{G}(A \supset B) \wedge \neg(\mathrm{G}A \supset \mathrm{G}B)), w) \neq t \\
\text{iff} & V(\mathrm{G}(A \supset B) \wedge \neg(\mathrm{G}A \supset \mathrm{G}B), w) = t \\
\text{iff} & V(\mathrm{G}(A \supset B), w) = t \text{ and } V(\neg(\mathrm{G}A \supset \mathrm{G}B), w) = t
\end{array}
$$

From the first conjunct of the last clause, we have:

$$
\begin{array}{lll}
V(\mathrm{G}(A \supset B), w) = t & \text{iff} & \forall v(w \leq v \text{ imply } V(A \supset B, v) = t) \\
 & \text{iff} & \forall v(w \leq v \text{ imply } V(\neg(A \wedge \neg B), v) = t) \\
 & \text{iff} & \forall v(w \leq v \text{ imply } V(A \wedge \neg B, v) \neq t) \\
 & \text{iff} & \forall v(w \leq v \text{ imply } (V(A, v) \neq t \\
 & & \quad \text{or } V(\neg B, v) \neq t)) \\
 & \text{iff} & \forall v(w \leq v \text{ imply } (V(A, v) \neq t \\
 & & \quad \text{or } V(B, v) = t))
\end{array}
$$

From the second conjunct of the last clause, we have:

$$\begin{array}{lll} V(\neg(\mathrm{G}A \supset \mathrm{G}B), w) = t & \text{iff} & V(\mathrm{G}A \supset \mathrm{G}B, w) \neq t \\ & \text{iff} & V(\neg(\mathrm{G}A \wedge \neg\mathrm{G}B), w) \neq t \\ & \text{iff} & V(\mathrm{G}A \wedge \neg\mathrm{G}B, w) = t \\ & \text{iff} & V(\mathrm{G}A, w) = t \text{ and } V(\neg\mathrm{G}B, w) = t) \\ & \text{iff} & \forall v(w \leq v \text{ imply } V(A, v) = t) \\ & & \quad \text{and } V(\mathrm{G}B, w) \neq t \\ & \text{iff} & \forall v(w \leq v \text{ imply } V(A, v) = t) \\ & & \quad \text{and } \exists v(w \leq v \text{ and } V(B, v) \neq t) \end{array}$$

From these, it is shown that we have no V and w such that $V(\mathrm{G}(A \supset B) \supset (\mathrm{G}A \supset \mathrm{G}B), w) \neq t$. This means that (A3) is valid.

Next, we deal with (A5). Assume that $A \supset \mathrm{HF}A$ is not valid. Then, there is a time point such that $V(A \supset \mathrm{HF}A, w) \neq t$. This is equivalent to the following:

$$\begin{array}{lll} V(A \supset \mathrm{HF}A, w) \neq t & \text{iff} & V(\neg(A \wedge \neg\mathrm{HF}A), w) \neq t \\ & \text{iff} & V(A \wedge \neg\mathrm{HF}A, w) = t \\ & \text{iff} & V(A, w) = t \text{ and } V(\neg\mathrm{HF}A, w) = t \\ & \text{iff} & V(A, w) = t \text{ and } V(\mathrm{HF}A, w) \neq t \\ & \text{iff} & V(A, w) = t \text{ and } \exists v(v \leq w \text{ and} \\ & & \quad V(\mathrm{F}A, v) \neq t) \\ & \text{iff} & V(A, w) = t \text{ and } \exists v(v \leq w \text{ and} \\ & & \quad \forall u(v \leq u \text{ imply } V(A, u) \neq t)) \end{array}$$

Here, assuming $u = w$ implies contradiction. This means that (A5) is valid. □

The next theorem shows that FCP is a three-valued extension of K_t.

Theorem 3: For future non-contingent (determinate) sentences A,
$\models_{K_t} A$ iff $\models A$.

Here, $\models_{K_t}$ denotes standard K_t validity.

Proof. Since non-contingent sentences are interpreted as two-valued, FCP-model is shown to be equivalent to a Kripke model for K_t. Then, the stated proposition follows.

Now, we turn to a completeness proof of FCP. Here, we need some notions. We say that a set of formulas Δ is *inconsistent* iff there are $A_1, ..., A_n$ in Δ such that $\vdash \neg(A_1 \wedge ... \wedge A_n)$. Δ is *consistent* iff it is not inconsistent. Γ is *FCP-maximal* iff $A \in \Gamma$ or $\neg A \in \Gamma$ for some A and Γ is a subset of maximal set Δ. FCP-maximal consistent set Γ is like standard maximal consistent set, in which neither $A \in \Gamma$ nor $\neg A \in \Gamma$, but maximality can be emulated by possible extensions. We know that a consistent set of formulas can be expanded to a maximal consistent set of formulas. This means that we can construct a FCP-maximal consistent set of formulas from a maximal consistent set of formulas. □

Lemma 4: Let Γ be a FCP-maximal consistent set of formulas and Δ be a maximal consistent set extending Γ. Then, we have:

(1) if $\vdash A$, then $A \in \Gamma$.
(2) $\neg A \in \Gamma$ iff $A \notin \Delta$.
(3) $A \wedge B \in \Gamma$ iff $A \in \Gamma$ and $B \in \Gamma$.
(4) $A \vee B \in \Gamma$ iff $A \in \Gamma$ or $B \in \Gamma$.
(5) if $A \in \Gamma$ and $A \supset B \in \Gamma$, then $B \in \Gamma$.

Proof. (1), (3), (4), (5) are trivial. For (2): Suppose $\neg A \in \Gamma$. This implies $\neg A \in \Delta$. Since Δ is FCP-maximal, we have $A \notin \Delta$. Next, assume $A \notin \Delta$. Because $A \in \Delta$ or $\neg A \in \Delta$, we have $\neg A \in \Delta$. This means that there is a $\Gamma \subseteq \Delta$ such that $\neg A \in \Delta$.

Now, we define a *canonical FCP-model* $\mathcal{M}^c = (T^c, \leq^c, V^c)$ as follows:

(1) T^c is a FCP-maximal consistent set.
(2) $w \leq^c v$ iff $\{A \mid \mathrm{F}A \in w\} \subseteq v$ iff $\{A \mid \mathrm{P}A \in v\} \subseteq w$.
(3) $V^c(A, w) = t$ iff $A \in w$,
$V^c(A, w) = f$ iff $\neg A \in w$,
$V^c(A, w) = u$ otherwise.

Here, w is a member of T. It is shown that $\mathcal{M}$ is a FCP-model. We are now ready to present the fundamental theorem to prove completeness. □

Theorem 5: For any formula A and FCP-maximal consistent set w,
$V^c(A, w) = t$ iff $A \in w$.
$V^c(A, w) = f$ iff $\neg A \in w$.

Proof. The theorem is proved by induction on A.

$(\neg)$:	$V^c(\neg A, w) = t$	iff	$V^c(A, w) \neq t$
		iff	$A \notin w$ (IH)
		iff	$\neg A \in w$ (Lemma 4(2))
	$V^c(\neg A, w) = f$	iff	$V^c(A, w) = t$
		iff	$A \in w$ (IH)
		iff	$\neg\neg A \in w$ (double negation)

$$
\begin{array}{lll}
(\wedge): & V^c(A \wedge B, w) = t & \text{iff } V^c(A, w) = V^c(B, w) = t \\
& & \text{iff } A \in w \text{ and } B \in w \text{ (IH)} \\
& & \text{iff } A \wedge B \in w \text{ (Lemma 4(3))} \\
& V^c(A \wedge B, w) = f & \text{iff } V^c(A, w) = f \text{ or } V^c(B, w) = f \\
& & \text{iff } \neg A \in w \text{ or } \neg B \in w \text{ (IH)} \\
& & \text{iff } \neg A \vee \neg B \in w \text{ (double negation)} \\
& & \text{iff } \neg(A \wedge B) \in w \text{ (de Morgan)} \\
(\mathrm{F}): & V^c(\mathrm{F}A, w) = t & \text{iff } \exists v(\{A \mid \mathrm{F}A \in w\} \subseteq v \text{ and } V^c(A, v) = t) \\
& & \text{iff } \exists v(\{A \mid \mathrm{F}A \in w\} \subseteq v \text{ and } A \in v) \text{ (IH)} \\
& & \text{iff } \mathrm{F}A \in w \\
& V^c(\mathrm{F}A, w) = f & \text{iff } \forall v(\{A \mid \mathrm{F}A \in w\} \subseteq v \text{ imply } V^c(A, v) = f) \\
& & \text{iff } \forall v(\{A \mid \mathrm{F}A \in w\} \subseteq v \text{ imply } \neg A \in v) \text{ (IH)} \\
& & \text{iff } \forall v(\{A \mid \mathrm{F}A \in w\} \subseteq v \text{ imply } A \notin v) \\
& & \quad \text{(Lemma 4(2))} \\
& & \text{iff } \mathrm{F}A \notin w \\
& & \text{iff } \neg \mathrm{F}A \in w \text{ (Lemma 4(2))} \\
(\mathrm{P}): & V^c(\mathrm{P}A, w) = t & \text{iff } \exists v(\{A \mid \mathrm{F}A \in w\} \subseteq v \text{ and } V^c(A, v) = t) \\
& & \text{iff } \exists v(\{A \mid \mathrm{P}A \in w\} \subseteq v \text{ and } A \in v) \text{ (IH)} \\
& & \text{iff } \mathrm{P}A \in w \\
& V^c(\mathrm{P}A, w) = f & \text{iff } \forall v(\{A \mid \mathrm{P}A \in w\} \subseteq v \text{ imply } V^c(A, v) = f) \\
& & \text{iff } \forall v(\{A \mid \mathrm{P}A \in w\} \subseteq v \text{ imply } \neg A \in v) \text{ (IH)} \\
& & \text{iff } \forall v(\{A \mid \mathrm{P}A \in w\} \subseteq v \text{ imply } A \notin v) \\
& & \quad \text{(Lemma 4(2))} \\
& & \text{iff } \mathrm{P}A \notin w \\
& & \text{iff } \neg \mathrm{P}A \in w
\end{array}
$$

As a consequence, we can reach the completeness. □

Theorem 6: (completeness) $\vdash A$ iff $\models A$.

Some people assume that temporal ordering be transitive in K_t. Such an extension of FCP, denoted $FCP4$ requires additional two axioms.

(A7) $\mathrm{G}A \supset \mathrm{GG}A$
(A8) $\mathrm{H}A \supset \mathrm{HH}A$

A Kripke semantics needs the restriction that $\leq$ be transitive. And the completeness proof of $FCP4$ presents no difficulty.

4. *Conclusions*

We proposed a new three-valued temporal logic FCP to solve the problem of future contingents. The gist of the development of our system is to allow a truth-value gap for future contingents, while necessarily true propositions can be interpreted to be true. To achieve the requirement, we mix Lukasiewicz’s three-valued logic with Bochvar’s external negation. In this sense, our three-valued temporal logic is novel. As far as we know, this kind of temporal (also three-valued) logic was not investigated in the literature. The axiomatization and semantics of FCP is also given with a completeness argument.

A similar, but slightly different, three-valued modal system Q was also proposed by Prior [9]; also see Bull [5] and Akama and Nagata [1]. The starting point of Q is to develop a “correct” modal logic and its temporal variant Q_t can also be formalized. However, Q_t seems inadequate as a logic for future contingents for not supporting the law of excluded middle.

Now, we must give philosophical remarks on Aristotle’s interpretations of future contingents. There are at least two different interpretations. The “standard” interpretation says that Aristotle rejects “necessarily (A or $\neg A$)”. Then, future contingents need not have a truth-value, namely they allow the truth-value gap. Thus, the so-called *determinism* does not follow.

The standard interpretation is dominant and was formalized by Lukasiewicz. However, even for the standard interpretation, there are several options. In other words, we can reject the law of bivalence or the law of the excluded middle, or both.

On the other hand, the “non-standard” interpretation states that Aristotle denies “necessarily A or necessarily $\neg A$” (cf. [2]). This focuses on necessity of a proposition rather than its truth-value. The formalization of future contingents based on the non-standard interpretation has some interest.

Finally, we list further problems to be worked out. First, we must address the following important philosophical problem. According to the truth-value table for implication given in section 3, the sentence (6) will have to receive the value t.

(6) If I will go to Warsaw, then I will eat spaghetti.

This is because both its antecedent and its consequent receive the value u (since they are “atomic” future contingents). And of course this phenomenon generalizes. But this seems counterintuitive, and hard to reconcile with Aristotle’s theory of future contingents. Note that the supervaluation theory does not have this problem: this appears to be an advantage of the supervaluational approach.

However, we still regard our theory of future contingents philosophically interesting in the light of this problem. In other words, we are not disappointed with the problem. There are two reasons. One reason is that we rely on Prior’s interpretation of the truth-value u. Motivated by Prior’s passage quoted above, u can be interpreted as a possible way to describe the kind of the truth-value f. In addition, the use of new negation $\neg$ enables us to claim that the negation of indeterminate proposition receives the truth-value t. Because our implication $A \supset B$ is defined as $\neg(A \wedge \neg B)$, it follows that $\mathrm{F}A \supset \mathrm{F}B$ for future contingents A, B, which is a logical form of (6), receives the truth-value t. The feature in our system is technical, and the difficulty could be logically overcome by incorporating the supervaluational method.

The other reason is philosophical. In fact, the truth-value of the future contingent $\mathrm{F}A$ seems to rely on some adequate metaphysical grounds for the truth condition of A. So we believe that (6) cannot be simply expressed as $\mathrm{F}A \supset \mathrm{F}B$. It is here necessary to consider some philosophical grounds like *causality* holding between A and B. Based on these two reasons, the philosophical problem above does not seem to decrease the philosophical significance of our proposal. However, at the present time, we do not know how to fully solve the problem in our system.

Now to other further topics. From philosophical and formal point of view, a quantificational extension of FCP should be considered. However, the presence of quantifiers appears to raise serious defects with completeness.

Our approach should be compared with other approaches like supervaluational and subvaluational ones to motivate FCP; see van Fraassen [6]. These approaches can revise not a logic but a semantics to accommodate truth-value gaps (or truth-value gluts). It is, however, true that such semantical techniques enable us to non-bivalent logic while classical laws are preserved. We also notice that *Peircean branching time temporal logic* has some connections with our logic; see Prior [9]. According to Peirce, future contingents have no definite truth-values. Based on his idea, Prior developed a branching time temporal logic with "will necessarily eventually be" temporal operator allowing future contingents. But the logic must assume the particular time structure, i.e. *branching time*.

ACKNOWLEDGEMENTS

We would like to thank the anonymous referee for suggesting some improvements of the previous drafts. In particular, he pointed out the important philosophical problem of our approach discussed in section 4.

Akama:
1-20-1 Higashi-Yurigaoka
Asao-ku
Kawasaki-shi 215-0012, Japan.
E-mail: sub-akama@jcom.home.ne.jp

Nagata:
Department of Electrical and Electronics Engineering
Faculty of Engineering
University of the Ryukyus
1 Senbaru Nishihara
Okinawa 903-0213, Japan.
E-mail: ngt@eee.u-ryukyu.ac.jp

Yamada:
Takushoku University Hokkaido College
4558 Memu
Fukagawa
Hokkaido 074-8585, Japan.
E-mail: cyamada@takushoku-hc.ac.jp

REFERENCES

[1] Akama, S. and Nagata, Y. (2005): On Prior's three-valued modal logic Q, *Proc. of ISMVL'2005*, 14–19.
[2] Anscombe, G. (1956): Aristotle and the sea-battle, De Interpretationes Chapter IX, *Mind*, LXV, 257, 1–15.
[3] Aristotle (1963): *De Interpretatione*, translated by E.M. Edghill, W.D. Ross (ed.), *The Works of Aristotle*, Oxford University Press, Oxford.
[4] Bochvar, D. (1939): On three-valued logical calculus and its application to the analysis of contradictions (Russian), *Mathematiceskij Sbornik*, 4, 353–369.
[5] Bull, R.A. (1964): An axiomatization of Prior's modal calculus Q, *Notre Dame Journal of Formal Logic*, 5, 211–214.
[6] van Fraassen, B. (1966): Singular terms, truth-value gaps, and free logic, *Journal of Philosophy*, 63, 481–495.
[7] Haack, S. (1978): *Philosophy of Logics*, Cambridge University Press, Cambridge.
[8] Lukasiewicz, J. (1920): On 3-valued logic, S. McCall (ed.), *Polish Logic*, 16–18, Oxford University Press, Oxford, 1967.
[9] Prior, A.N. (1967): *Past, Present and Future*, Clarendon Press, Oxford.