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INFORMATION FLOW AND IMPOSSIBLE SITUATIONS*

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Abstract
Classical semantic information (*CSI*) assigns the same *body* of semantic information (Cont) as well as the same *amount* of semantic information (cont or inf) to logically equivalent sentences. A corollary of this fact is that all logically true sentences will receive a null assignment of semantic information via Cont as well as a zero measure of semantic information via cont or inf. The originators of *CSI* allowed that there be (but did not develop) a sensible notion of *psychological information (PI)* whereby its measure on logically true sentences would be greater than zero. By extension they allowed that this notion of *PI* would be such that non-identical measures could be assigned to logically equivalent sentences in general. The task undertaken in the article is to specify the basis of a theory of *PI* that satisfies these constraints. This project utilizes the frame semantics of substructural logics. The threat of a perceived *ad hoc* association is countered by an independent argument for the explication of the incomplete and inconsistent worlds (*impossible situations*) constituting frame semantics in terms of confused doxastic states. Combined with existing interpretations of frame semantics in terms of information flow, the result is a specification of the basis of a novel theory of the information flow between the doxastic states of agents, i.e. the specification of the basis of a novel theory of *PI*. A substructural logic no stronger than the linear logic DMALL is proposed as providing the appropriate semantics.

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1. *Introduction*

The theory of *classical semantic information* (*CSI*) (Bar-Hillel and Carnap, 1952) is built upon the classical modal space that Carnap (1955, 1956) used to define his notion of *intension* and which is commonly used to explicate metaphysical necessity.[1] The intension of a declarative sentence is taken to be the set of possible worlds that make the sentence true (equivalently, those worlds included by the sentence).[2] The notion of intension is co-definable with Bar-Hillel and Carnap's notion of *semantic information* as comprised by *CSI*. Semantic information is also referred to as *content* and denoted by 'Cont'. The content of a declarative sentence is taken to be the set of possible worlds that make the sentence false (equivalently, those worlds excluded by the sentence). Letting W be the set of all possible worlds, and X be the set of possible worlds identified with the intension of a declarative sentence s, and Y be the set of possible worlds identified with the content of s, we have (1.1):

(1.1) $W \setminus X = Y$ iff $W \setminus Y = X$

Hence X and Y will always be mutually exclusive and jointly exhaustive on W (i.e., they are a partition on W). Explicitly, the content of s will be identical to the set of possible worlds included by the negation of s. This is just to say that content of s is identified with the set of possible worlds included by $\neg s$. Where $X \subseteq W$ we have (1.2):[3]

(1.2) $\text{Cont}(s) =_{df} \{x \in W : x \models \neg s\}$

[1] Bar-Hillel and Carnap built *CSI* around a monadic predicate language. Possible worlds are modeled as state descriptions. The number of possible worlds is calculated accordingly. Where there are n individual constants (standing for n individuals) and m primitive monadic predicates, the number of atomic sentences will be nm, the number of possible worlds 2^{nm}, and the number of "Q-predicators" 2^m (Q-predicators are individuations of possible types of objects given a conjunction of predicates whereby each primitive predicate occurs either negated or un-negated (but not both)). A full sentence of a Q-predicator is a Q-sentence where a predicate is attached to a term. Hence a possible world is a conjunction of n Q-sentences as each Q-sentence describes a possibly existing individual. Since this article deals with nothing more fine-grained than the propositional calculus, these details will be ignored.

[2] We speak of the intension associated with a sentence as opposed to the intension associated with a *proposition* because, on the possible worlds understanding of propositions, a proposition *just is* an intension.

[3] We require '$\subseteq$' instead of the stronger '$\subset$' here because of the possibility that $\not\models s$, in which case $X = W$.

For any logically true sentence t, $\neg t$ will include no possible world. Via (1.2) we have (1.3):

(1.3) $\text{Cont}(t) = \oslash$

(1.3) is a special case of a more general result involving any pair (or more) of logically equivalent sentences s and s'. Given that logically equivalent sentences pick out the same sets of possible worlds, and given the definition stipulated by (1.2) above, we have (1.4):

(1.4) $\text{Cont}(s) = \text{Cont}(s') = Z$

If s (and s') $\neq t$, then $Z \neq \oslash$.

CSI is concerned not only with the individuation of semantic information (Cont) but also with its *measure*. The guiding intuition is that the informativeness of a sentence s is inversely proportionate to the probability of the state of affairs it describes being the case. *CSI* involves two distinct methods for obtaining measures of semantic information, a *content measure* (cont) and an *information measure* (inf).

Beginning with cont, Bar-Hillel and Carnap denote the logical (*a priori*) probability of a sentence s by $m(s)$, where m designates 'measure' (*op. cit.*, 302). m is acquired via an *a priori* probability distribution onto the set of all possible worlds. The distributed values sum to 1. For simplicities sake, we may assume that the distribution pattern is equiprobable.[4] *CSI* defines the cont of a sentence s as the measure of the complement of s, (1.5):

(1.5) $\text{cont}(s) =_{df} 1 - m(s)$

A logically true sentence t is true in every possible world, hence (1.6):

(1.6) $m(t) = 1$

A logically true sentence will return a minimal content measure. From (1.5) and (1.6) we have (1.7):

(1.7) $\text{cont}(t) = 1 - 1 = 0$

Similarly to (1.3), (1.7) is a special case of a more general result concerning any pair (or more) of logically equivalent sentences s and s'. Since logically

[4] As we are only considering logically equivalent sentences, this assumption is (strictly speaking) irrelevant for present purposes.

equivalent sentences are true across identical sets of possible worlds, we know that they will possess identical logical probabilities, hence (1.8):

(1.8) $m(s) = m(s')$

Hence logically equivalent sentences will return identical content measures. From (1.5) and (1.8) we have (1.9):

(1.9) $\text{cont}(s) = 1 - m(s) = \text{cont}(s') = 1 - m(s')$

Bar-Hillel and Carnap introduced the notion of an *information measure* (inf) to capture additivity on *inductive independence*. Two sentences are said to be inductively independent when the conditional probability of each sentence given the other is identical to its initial probability. Additivity on inductive independence fails for cont. For any two arbitrary sentences s and s', we cannot guarantee that $\text{cont}(s \wedge s') = \text{cont}(s) + \text{cont}(s')$ because it may be the case that $m(s)$ and $m(s)'$ have worlds in common. s and s' may have shared *content*. For additivity to hold on cont, it is *content independence* (not inductive independence) that is required.

The definition of inf may proceed via either cont (1.10) or m (1.11):

(1.10) $\text{inf}(s) =_{df} \log_2 \dfrac{1}{1 - \text{cont}(s)}$

(1.11) $\text{inf}(s) =_{df} \log_2 \dfrac{1}{m(s)} = -\log_2 m(s)$

(1.10) and (1.11) are equivalent, hence we consider only (1.10). Similarly to cont, any logically true sentence t will return a minimal information measure. From (1.10) and (1.7) we have (1.12):

(1.12) $\text{inf}(t) = \log_2 \dfrac{1}{1 - 0} = 0$

Again similarly to cont, any pair of logically equivalent sentences s and s' will return identical information measures. If $1 - \text{cont}(s) = x$ and $1/x = y$, then from (1.10) and (1.9) we have (1.13):

(1.13) $\text{inf}(s) = \log_2 \dfrac{1}{1 - \text{cont}(s)} = \log_2 \dfrac{1}{x} = \log_2 y = \text{inf}(s')$

With respect to logically true sentences returning a zero value, the authors of *CSI* comment that:

> This, however, is by no means to be understood as implying that there is no good sense of 'amount of information' in which the amount of information of these sentences will not be zero at all, and for some people, might even be rather high. To avoid ambiguities, we shall use the adjective 'semantic' to differentiate both the presystematic sense of 'information' in which we are interested at the moment and their systematic explicata from other senses (such as "amount of psychological information for the person P") and their explicata (*ibid.*, 223).

We may extend this into the more general result concerning pairs (or more) of logically equivalent sentences in general. We shouldn't understand (1.13) (or (1.9)) as implying that there is no good sense of 'amount of information' (or its individuation) such that logically equivalent sentences will return distinct values.[5]

The central argument of this essay is that said *psychological information* (*PI*) is best modeled via frame semantics.[6]

2. *Frame Semantics*

The formal constraints underlying *CSI* that force the results concerning logically true and logically equivalent sentences are transparent. Classical possible worlds semantics dictates that logically true sentences hold across all

[5] Strictly speaking, there is a sense in which we are restricting Bar-Hillel and Carnap's comments as well as extending them. When they write 'these sentences' they mean *any* necessary *a priori* sentence (such as mathematical and analytical truths etc.), whereas we are concerned with a restricted subset of such sentences.

[6] The quest for a realistic agent-based notion of information(s) (in some sense or other) is becoming increasingly urgent, as the contents of this volume attest. Mark Jago develops a psychological (i.e. non-idealized) type of epistemic possibility in order to formalize the notion of 'being informed'. Luciano Floridi and Patrick Allo (independently) develop a logic of 'being informed' via an informational interpretation of the modal logic KTB. María J. Frápolli and Francescc Camós (together) expound several non-formal approaches via which logical truths may be understood as being informative. Edwin D. Mares uses partial (i.e. non-classical) information and relevant semantics to develop a realistic position on indicative conditionals. Although several of the specific concerns, approaches, and conclusions differ, the motivation is generally identical: There is a dissatisfaction with the idealization explicit in *CSI* (and the classical semantics underlying it) when it comes to modelling informativeness.

possible worlds, and that logically equivalent sentences in general hold across identical sets of possible worlds. However, it is well known that there are non-classical logics with semantics such that logically true sentences do not hold across all worlds, and logically equivalent sentences do not hold across identical sets of possible worlds.

The semantics for substructural logics, *frame semantics*, is built upon *Routley-Meyer Frames* (Routley and Meyer, 1973).[7] We refer to such semantic frames simply as *frames*. Frames are similar to Kripke-style frame semantics for modal logic. A relevance frame for the class of substructural logics that are relevance logics is a 4-tuple $\langle 0, S, R, \sqsubseteq \rangle$. S is a set of *situations*. Situations serve the same function/purpose as do worlds, with the augmentation that they may be incomplete and/or inconsistent. On account of this inconsistency, S is therefore said to contain *impossible situations*.[8] $\sqsubseteq$ is a partial order on S. R is a ternary accessibility relation on S. Where x, y, z, ... are elements of S, R is defined via (2.1):

(2.1) If $Rxyz$ and if $x' \sqsubseteq x$, $y' \sqsubseteq y$, and $z \sqsubseteq z'$, then $Rx'y'z'$.

In (2.1) R is downward closed in its first two argument places and upward closed in its third argument place. That is, x and y are downward closed whilst z is upward closed. (2.1) ensures that R and $\sqsubseteq$ relate to each other in the desired manner. $0 \in S$ and is the *logic situation* (or logic *point*). 0 records logical consequence. Relevance semantics does not understand satisfaction in a model as truth in all situations.

A *relevance model* is a relevance frame with the addition of a binary relation $\vDash$ between the elements of S and all sentences of the language. An evaluation $\vDash$ on a relevance frame is a relation between situations x, y, ... and formulae ϕ, ψ, ... that respects the following four conditions:

(2.2) For all atomic propositions ϕ, if $x \sqsubseteq y$ and $x \vDash \phi$, then $y \vDash \phi$.

(2.3) $x \vDash \phi \wedge \psi$ iff $x \vDash \phi$ and $x \vDash \psi$.

(2.4) $x \vDash \phi \vee \psi$ iff $x \vDash \phi$ or $x \vDash \psi$.

(2.5) $x \vDash \phi \rightarrow \psi$ iff for all y and z such that $Rxyz$, if $y \vDash \phi$ then $z \vDash \psi$.

[7] We are ignoring multi-valued approaches.

[8] A situation may be impossible in more than one respect, as this is contingent upon the type of possibility under consideration (metaphysical, epistemic etc., see §2.3 of (Barwise, 1997). Throughout this article, we are only concerned with *classical logical impossibility*, hence 'impossible situation' is shorthand for 'logically impossible situation'.

By definition, *nothing* will be true in all situations in all models. Hence relevance semantics requires the specification of a point in S that makes a record of logical consequence. Validity is defined as truth at 0 in all *evaluations* in all models. $R0yz$ if and only if $y \sqsubseteq z$. $Rxyz$ may be rewritten as $y \sqsubseteq_x z$ to reemphasise an understanding of x as a connection (of what sort it is the purpose of §3 below to explicate) between y and z (see Gregory, 2001, 8). With $R0yz$ the subscript is omitted from $\sqsubseteq$ since the situation playing the role of the connection is the logic situation. 0 is the semantic analogue to the syntactic notion of the *empty premise structure*. Anything that follows from the empty premise structure (or its semantic analogue, the logic situation) follows without it.

(2.5) stipulates the evaluation conditions for $\rightarrow$, or *relevant implication*. $\rightarrow$ is evaluated using distinct situations for the composing formula from that used for the formula as a whole. Such *intensional connectives* are contrasted with extensional connectives such as those specified by (2.3) and (2.4). We cannot assume that $\phi \rightarrow \phi$ will be true at all points in the model, or at all situations. A situation x will fail to support $\phi \rightarrow \phi$ just in case $Rxyz$ for some situations y and z such that $y \vDash \phi$ and $z \nvDash \phi$. Similarly, $\phi \leftrightarrow \psi$ is no guarantee that ϕ and ψ will hold at identical situations (it may be the case that x supports $\phi \rightarrow \psi$ but not $\psi \rightarrow \phi$). Given the structure of relevance frames, there is no implication from contradictions to arbitrary formulae. This can be ensured by R being such that the impossible situation in the second argument place supports the contradiction, whilst the situation in the third argument place fails to support the consequent. Arbitrary situations will fail to support logical truths just when one of the situations accessible by the arbitrary situation is such that logical truths fail hold at that situation.

It might appear that constructing a theory of *PI* is now relatively academic; we proceed in a manner analogous to *CSI*, only on the back of relevance semantics as opposed to classical modal semantics. Although we may do so, the resulting theory would remain philosophically opaque. There are many unanalysed concepts in frame semantics. What are *situations?* What are *impossible situations?* What could 0, the *logic situation* conceivably be? What concepts underpin the ternary relation R? How are we supposed to understand the partial order relation $\sqsubseteq$? Such issues have caused some to claim that they simply do not *understand* relevance semantics (for example, Copeland, 1979). Recent work in *situation semantics* has attempted to analyse the concepts underpinning relevance semantics in terms of *information flow*. This analysis is cashed out in concepts that might not be any more transparent than those they were intended to explicate. Rendering them transparent delivers us a non *ad hoc* basis upon which to build our theory of psychological information.

3. *Information Flow*

The analysis of relevance semantics under consideration takes relevance frames to model *information flow*. $\sqsubseteq$ is understood as *informational development* or *informational inclusion*. $y \sqsubseteq z$ may be read as *the information in z is a development of the information contained in y*, or *the information specified by the situation y is included in the information specified by the situation z*. Anything supported by y, will also be supported by z (see Restall, 2000b, 853). In this case, a subscript on $\sqsubseteq$ indicates the linkage, or *information channel* via which the informational development was accomplished. $y \sqsubseteq_x z$ is read as: *the information contained in x, when applied to the information in y, gives the information in z*. We know from §2 above that $y \sqsubseteq_x z$ is a way of rewriting $Rxyz$. Hence the ternary relation of frame semantics may be understood in terms of information flow. Importantly, information channels *themselves* must also be taken to be situations.

The origin of the theory of information flow is located within *situation semantics* (Barwise and Perry, 1981, 1983).[9] The theory of information flow involves the notion of an *information structure*. We ignore its components with the exception of those directly relevant to further clarifying information channels. Information structures contain a set of *situations* S, a set of *types* T, a set of *channels* C, and a binary relation $\vDash$ that relates pairs of situations and types, and pairs of information channels and constraints. The relevant *condition* on information structures is the following (given in Restall, 1996, 466):

(3.1)
For each $\phi, \psi \in T$, $\phi \to \psi$ is a *constraint*.
$\vDash$ is extended to information channels and constraints as follows:
Where $c \in C$, $c \vDash \phi \to \psi$ iff for every s_1, $s_2 \in S$
such that $s_1 \overset{c}{\mapsto} s_2$, if $s_1 \vDash \phi$ then $s_2 \vDash \psi$.

(3.1) states that an information channel supports a constraint iff for each pair of situations s_1 and s_2 that are related by c, if the antecedent of the constraint is supported by s_1 then the consequent of the constraint is supported by s_2. (3.1) is central to the task of understanding relevance frames as modeling information flow. If we take c to be something distinct in kind from s_1 and s_2, to be something other than a situation, then it will make no sense to talk

[9] The authors' respective research projects into situation semantics have since branched. One branch has examined information in terms of situations and *infons* (see Israel and Perry (1990, 1991) and Devlin (1991)). The other branch has examined information flow in terms of *constraints* and information channels (see Barwise (1993) and Barwise and Seligman (1997)). The present essay is concerned only with the latter.

of $\rightarrow$ iteration. We *want* talk of $\rightarrow$ iteration to make sense. Another reason to secure an understanding of information channels as situations concerns the fact that (3.2) stipulates the condition for a channel's supporting a constraint. Constraints are conditional types, we note this explicitly via (3.2):

(3.2) $c \vDash \phi \rightarrow \psi$ iff for all s_1 and s_2 such that $s_1 \overset{c}{\longmapsto} s_2$, if $s_1 \vDash \phi$ then $s_2 \vDash \psi$

(3.2) is near identical to the evaluation conditions for $\rightarrow$ given by the fourth condition for an evaluation on a relevance frame, (2.5) in §2 above. By recognising the information channel c in the ternary relation $\mapsto$ to be a situation, the identity can be made complete.[10]

In this case the ternary relation $\mapsto$ of information structures is identical to the ternary relation R of relevance semantics. The ternary relation $Rxyz = y \overset{x}{\longmapsto} z$ means that the conditional information given by x when applied to y results in nothing more than z. The information channel theoretic analogue to (2.1) in §2 above is given by (3.3):

(3.3) If $y \overset{x}{\longmapsto} z$ and if $x' \sqsubseteq x$, $y' \sqsubseteq y$ and $z \sqsubseteq z'$, then $y' \overset{x'}{\longmapsto} z'$

As with (2.1), x and y are downward closed whilst z is upward closed. The proof of an interpretation of frame semantics in terms of information flow is given by Restall in his (*ibid.*). It proceeds via the notion of a *bare frame* $\langle 0, S, \mapsto, \sqsubseteq \rangle$ which is the information theoretic analogue to a *relevance frame* explicated in §2 above. The demonstrated tractability of the formal task of explicating frame semantics in terms of information flow does not in itself expose the extent to which such an achievement is philosophically illuminating. We originally turned to the theory of information flow

[10] Information channels are systematic regularities in the world, so taking them to be situations would appear legitimate *prima facie*. Barwise leaves it open as to whether or not information channels are situations (see his *basic notion 2.1* in his *op. cit.*, 13). He does note however, that his theory of information flow "suggests a way to think about the three-place accessibility relation semantics for [the] relevance logic of Routley and Meyer". Barwise also notes Restall's (*ibid.*) work on the project of interpreting frame semantics in terms of information flow. With respect to the particular task of securing an understanding of information channels as situations, Barwise (in Austinian (1961) terms) understands the conditional statement *If* S_1 *then* S_2 to take the constraint $\phi \rightarrow \psi$ as its *descriptive content*. It takes as its *demonstrative content* the information channel c. Barwise also takes the demonstrative content of other statements to be situations. We legitimately ask, along with Restall (*ibid.*, 471) why it is that conditional statements should be treated differently. If a conditional statement takes a situation s as its demonstrative content, then we take s itself to be an information channel linking the situations which constitute the demonstrative content of the conditional statement's antecedent and consequent respectively.

in order to assist us in answering the following questions: What are *situations?* What are *impossible situations?* What could 0, the *logic situation* conceivably be? What concepts underpin the ternary relation R? How are we supposed to understand the partial order relation $\sqsubseteq$? We have the necessary background on the theory of information flow to make *sense* of the issues. We also have, via Restall's formal work, the *license* required.[11]

4. *Impossible Situations and Psychological Information*

We have understood $\sqsubseteq$ as *informational development*, or *informational inclusion*. Any explication of the logic situation 0, *qua logic* situation, supervenes on an explication of *situation simpliciter*, and similarly with the notion of an *impossible situation*. We are left with the concepts underpinning the ternary relation R. With a little work, we can reduce this to the problem of explicating situations. The ternary relation R was interpreted as the ternary relation $\mapsto$. The concepts underpinning $\mapsto$ are that of an *information channel*, which we understand to be a situation, and that of *information flow itself*. The explication of *information flow* reduces to the explication of *situations* as follows: *Information flow* is a metaphor that Barwise cashed out in terms of *information structures*.[12] Leaving aside the familiar logical operations, information structures contain *types*, *information channels*, and *situations*. But information channels are situations, so this leaves types and situations. Types are simply abstractions over situations (tokens). Tokens ground their types. An explication of a type turns on an explication of its tokens. We are left then with the task of explicating *situations* and *impossible situations*. Any answer to these question will dictate the terms in which we explicate 0.

Starting with *situations*, note that the concern here is an instance of the concern regarding the ontological status of possible worlds. *Non-actual* situations figure in information structures and the resulting interpretation of frame semantics. Consider (4.1):

(4.1) If there is peace over all the Earth, then pigs fly.

A counterexample for (4.1) at a situation x requires the following: Some situations y and z such that $y \overset{x}{\mapsto} z$, and where $y \vDash$ *there is peace over all the earth* and $z \nvDash$ *pigs fly*. A counterexample for (4.1) requires the existence of

[11] See Mares (1997a) for a similar proof via the infonic thread (see fn. 9).

[12] Priest (2001, 198) comments that this metaphor is “hardly a transparent one”. This *prima facie* metaphorical opacity is conceded by Barwise (1993, 7) and Barwise and Seligman (1997, 4), however their work is obviously dedicated to clarifying the notion.

some situation y that supports the antecedent. (4.1) has a false antecedent, so the supporting situation y must be non-actual. Our dilemma is the admission of non-actual situations on the one hand, or the truth of (4.1) on the other. We are opting for the former horn, hence something must be said about non-actual situations. Note that strictly speaking this is an issue for truth theory rather than model theory, since the latter can survive on mathematical abstractions alone. The first thing we recognise is that *possible (non-actual) situations* are not part of the actual world in the same manner as are *actual situations* (such as the situation involving you reading this issue of *Logique et Analyse* say).

For the purpose of a purely formal transparency we may accept Barwise's position that the situations in question be taken "as in probability theory, as mathematical objects used for modeling possibilities, not as real-but-not-actual situations" (1993, 14, fn. 7). As noted, this is acceptable for model theory. If the interpretation of relevance frames in terms of information flow is to be philosophically transparent however, this cannot be left as it stands. The theory of information flow was appealed to in order to counter the charge that the ternary relation R (among the other related notions) was, although formally impressive, philosophically opaque. Countering this charge by replacing one set of mathematical abstractions with another is no counter at all. The function of an explanation is to provide *an explanation*. We note with Restall (*op. cit.*, 476) that for a model to have explanatory (as opposed to merely formal) force, the things being modeled must correspond to something real.

We take it as an assumption that avoiding modal realism (possibilism) is a goal. The proposal might be that we take the notion of possibility as a primitive. We would say that x models a possible situation if x *would* model an actual situation, were the world to be different from how it actually is. For a development of this proposal see Menzel (1990). It arguably captures what we do in fact mean when we speak about x modeling possible-but-not-actual situations. To this extent it is persuasive in a manner that modal realism is not. Despite this, it is circular in that it involves (in an uneliminated sense) the notion of *possibility*. Restall notes that an account such as this will fail to provide a reductive account of conditionals (*op. cit.*). In philosophy and elsewhere we are forced to admit new primitives in order to account for particular phenomena. The account of modeling sketched here is circular in a fashion, yet it is not viciously so. Taking possibility as a primitive in this sense allows the interpretation of frame semantics as modelling information flow to give us what Restall calls "a helpful regimentation of our intuitions about conditionals, and a new way to analyse their semantic content" (*ibid.*).

This proposal cannot do the work we require of it. We need an account not only of possible situations, but of *impossible situations*. There is no way that the world *could* differ such that an *impossible*-but-not-actual situation *would* be an actual situation. The notion of a possibly-actual-impossible-but-not-actual situation does not make sense. Recall that we are taking 'impossible situation' as shorthand for 'logically impossible situation' (see fn. 8). This is simply a result of the meaning of '*im*possible situation'. It is the reductive explication of this notion that will deliver to us a novel, non *ad-hoc* basis onto which we build a theory of *PI*.

Priest (*op. cit.*, 171) states that since worlds where the laws of physic fail are routine, worlds where the laws of logic fail "must be". This is moving much too quickly. Although Priest is using worlds and we are using situations (hence the level of abstraction differs) and his motivations for introducing impossibilities differ from ours, we can still (with a little license) use his observation for fruitful discussion. Priest makes the distinction between situations where the laws of logic differ, and those where the logically impossible occurs. We are considering situations where the logically impossible *does* occur, hence we get situations where the laws of logic differ for free. There is a conceptual gap between situations where the laws of *physics* fail (nomologically impossible situations) and those where the laws of *logic* fail (logically impossible situations). The nature of this gap is that which was expounded in the paragraph above; the notion of a possibly-actual-impossible-but-not-actual situation does not make *sense*. There is no way that an impossible situation could be actual, which is just to say that it is not possible that it be actual, by definition. By contrast, a situation where physically impossible things happen is entirely compatible with our claim that x models a possible situation if x *would* model an actual situation, were the world to be different from how it actually is. Here there is no conceptual barrier to the world differing in the required manner.

The recognition of this conceptual barrier is what motivates Gregory (2001, 4) to give the following explication of impossible situations: Impossible situations "correspond to confused epistemic states. In such a model each agent is allowed to be confused in their own way, without all such confused states being identified [with each other]". As it stands this is much too weak due to the naturalistic constraints on cognitive capacities.[13] Let us grant that such states may be combined up to arbitrary complexities. Let us also grant that epistemic states, whatever they are, must presumably be veridical. Since logical impossibilities are arguably never true (unless one holds to a dialethic paraconsistentism), *doxastic states* may be a more accurate way of putting things. Running with this explication gives us the following account of the

[13] There is also an important sense in which it is also too *permissive*, insofar as the number of impossibilities is concerned. We certainly don't want to allow there to be an impossible situation for every way in which an agent may be confused, as the resulting logic underlying the theory of *PI* would then have very few principles at all. This topic is dealt with in §5 below.

modeling of impossibilities: x models an impossible situation if x *would* model an actual situation, were the world to be as it is represented to be in the mind of an agent in a confused doxastic state. The confused doxastic state would be specified in terms of the impossible situation requiring modeling (or strictly speaking, *support*). It is this ineliminable presence of confused doxastic states that will deliver to us the novel, non *ad-hoc* basis onto which to build a theory of *PI*.

Gregory cites two alternative interpretations to the identification of impossible situations with (in some sense) irrationality: Barwise (1997) and Restall (2000a). The former is a misidentification on Gregory's part. Barwise's position is in fact near identical to Gregory's own. Barwise identifies impossible situations with "ways things cannot be, given the available information" (*op. cit.*, 495). Barwise includes logically impossible situations within epistemically possible situations, given the available information (*ibid.*, 498). 'Given the available information' in the case of epistemic possibility and logically impossible situations *means* limited rationality, an expressly psychological notion.

Restall's alternative explication differs. Echoing Belnap (1977a, 1977b), Dunn (1971), and Meyer and Martin (1986), Restall expounds the view that impossible situations are important for semantics because some important theories are inconsistent (he gives the examples of naïve set theory and *our beliefs*). Our semantics might need to make room for distinct ways in which things cannot be. These inconsistent situations will be descriptions of the different ways things cannot happen. In his (1997) Restall explicates impossible situations as inconsistent sets of possible situations. This is stable insofar as the individuation of impossible situations is concerned. Once we use one of them on the left hand side of the binary relation $\models$ however, we are back where we began with respect to philosophical transparency (or lack of it). A description of a particular way that things *cannot* be *cannot* play the *supports role*, as it is a description of a particular way that things cannot be that sits on the right hand side of the binary relation $\models$.

Any attempt to go primitive on impossibility will fail not on account of explanatory vacuity, but on account of *incomprehensibility*. Reiterating, the notion of a possibly-actual-impossible-but-not-actual situation does not make *sense*, at least not unless we alter the meaning of 'impossibility'. (Recall again that we are taking 'impossible' to mean 'logically impossible' *simpliciter*.) Any attempt to construct impossible situations out of property abstractions in a manner that makes them "real" or "existing" (in whatever sense people mean by this when they talk of the ontology of possible worlds) is ruined by the same observation. There are no such concreta. They are impossible.

With respect to playing the supports role, we must interpret impossible situations as (possibly existing) confused doxastic states. In this case the notion of impossibility will be in a sense *parasitic* on the notion of possibility. The suggestion is this: we understand *all* of the situations in frame semantics as corresponding to possible doxastic states. The explanation of modeling possible and impossible situations is adjusted to give us (4.2):

(4.2) x models a possible or impossible situation s if x *would* model an actual situation, were the world to be as it is represented to be in the mind of an agent in a non-confused or confused doxastic state respectively.[14]

Given (4.2), we understand the logic situation 0 as a logically omniscient *state*. Since situations are being understood as, both non-confused and confused, (possibly existing) doxastic states, the interpretation of frame semantics in terms of information flow is now recast in terms of the information flow between different possible doxastic states of an agent.[15]

As it stands, (4.2) specifies a sufficient but non-necessary condition on situation modelling. This is because situations may be adequately modeled by mathematical abstractions as per Barwise's suggestion just so long as we don't understand the notion of modelling to entail an *explanation*. If however, we *do* take the notion of modelling to entail a philosophically respectable explanation, and denote this with 'models*', then we can adjust

[14] It is important that this is read in line with a non-dialethic paraconsistentism whereby the *truth* of contradictions is denied. The admittance of inconsistent situations is *not* the same thing as understanding them to be possible, in the sense of possibly actual. This would be to change the meaning of *im*possible. There is no way that the world could be such that is as represented in the mind of an agent in a confused doxastic state, where this representational state is inconsistent. This is precisely why they're *impossible* situations (barring a dialethic paraconsistentism of course).

[15] The idea of plugging the basis of a semantic theory directly into doxastic notions is not new. Gärdenfors' (1988) involves *belief sets*, where such sets are sets of sentences. Importantly, Gärdenfors specifies a consistency constraint ((2.1) p. 22) and a deductive closure constraint ((2.2), p. 22) that idealize his agent. It is the mark of a theory of *PI* that it *not* be idealized (as idealized *PI just is CSI*).

(4.2) to give us the necessary and sufficient (4.3):

(4.3) x models* a possible or impossible situation s iff x *would* model an actual situation, were the world to be as it is represented to be in the mind of an agent in a non-confused or confused doxastic state respectively

Via an independent chain of reasoning, we have an interpretation of frame semantics in psychological terms. The independence of this interpretation gives us a novel, non *ad-hoc* basis via which to build a theory of *PI* on the back of frame semantics. The following section is a preamble for the logic we should accept as the basis of a theory of *PI*.

5. *Towards a Theory of Psychological Information*

Substructural logics come in varying strengths. Given our interest in building a theory of *PI* on the back of frame semantics, how strong should we want the substructural logic to be? The answer is 'not very'. The resulting logic will be so weak that it will collapse into a type of *linear logic*, DMALL (*Multiplicative Additive Linear Logic, with Distribution*, which is equivalent to the relevant logic R, without Contraction, see below). However, the constraints imposed upon us by the logic of information flow may force us to go weaker still. The substructural logic interpreted by Barwise's channel theory, called P by Restall (1994), is weaker than DMALL. The sense in which this is the case is brought out below.

Questions concerning the logical structure of impossible situations become relevant here. Whether or not we should understand impossible situations as having their internal structure constrained by some particular logic or other is a popular point of contention (see Mares (1997b), Mortensen (1997), Restall (1997) and Barwise (1997) for pro, and Zalta (1997), Vander Laan (1997) and Nolan (1997) for con). There is no sensible answer to this question outside of a specified application. Given an application we can ask what restrictions we might want on the impossibilities in question. This is a specific point from a more general logical pluralist perspective. Logical pluralism rejects the debate for *the correct logic simpliciter* as misguided, arguing instead that the application in question decides the logic for the job (Beall and Restall, 2006). Readers with a computer science background will see this as a trivial point, but philosophy is still concerned about the nature of the one true logic, with other logics being manifestly false for various reasons. Embracing logical pluralism, and noting that the information types built on the back of the various semantics will also differ, we have *informational pluralism* (Allo, 2005). Nolan's (*ibid.*, 547) comprehension principle, that there is an impossible situation for every way that we say things cannot be, is too permissive for the purposes of *PI*. He argues that on the basis of this principle, modifying our account of logical consequence to accommodate every impossible situation would be mistaken, because *then "there will be impossible situations where even the principles of subclassical logics fail"* (*ibid.*). The implicit logical monism at work is what is causing the trouble here. Twice. By freeing ourselves to use different logics for different applications, we allow that there be applications where principles of a logic we use for application a (*CSI* say) fail for application b (*PI*). Our account of logical consequence should be contingent upon the phenomena under analysis. It is only the adherence to a logical monism that causes the tension. It is argued below that the account of logical consequence that best fits our phenomena is that specified by a type of linear logic.

Generally, linear logics are relevance logics that lack the *structural rules* of *contraction* and *distribution*. Information, be it classical or psychological, is a semantic notion. Despite this, we begin with syntactic issues to gain a perspective. In order that the arguments for DMALL forming the basis of a theory of *PI* be transparent, we first expound the notion of a *structural rule*.[16]

Adopting a sequent style natural deduction proof theory, we steal the following definition from Restall (2000a, 24).[17] Where X, Y, $Z\ldots$ are structured bodies of premises in a language and A, B, $C\ldots$ are formulae in that same language, and '$X \vdash A$' (called a *consecution*) is read as 'A is a consequence of X', a structural rule is a rule such as (5.1):

$$(5.1) \quad \frac{X \vdash A}{X' \vdash A}$$

that has its formulae closed under substitution. This is just to say the following; pick any formula, B say, which occurs in either X or X' or both. Where an arbitrary structure Y uniformly replaces every occurrence of B, the result remains an instance of the rule. Similarly for uniform replacements of the formula A by any arbitrary formula C. A *structure* is made up of a *language*

[16] There exists a clear connection between structural rules and the ternary relation R that will be brought out below.

[17] There exists a large array of competing symbolisms, labels, and systemizations of substructural logics. No attempt is made to navigate between them here. We simply follow Restall's notation throughout. For a short proto-construction in the same notation, see Slaney (1990).

(composed of atomic formula and connectives) and *punctuation marks* of varying arities that stand in relation to structures in an identical manner to that in which connectives stand to formulas (see (Restall, *ibid.*, §2.2). Letting $X' \Leftarrow X$ stand for the structural rule of the form expressed by (5.2):

$$(5.2) \quad \frac{Y(X) \vdash A}{Y(X') \vdash A}$$

In (5.2) Y is "an arbitrary context in which structures may appear" (*ibid.*). The more structural rules a logic contains, the stronger that logic is.

The structural rules relevant to our discussion of a theory of *PI* are the following:

B:	Associativity	$X;(Y;Z) \Leftarrow (X;Y);Z$
C:	Strong Commutativity	$(X;Y);Z \Leftarrow (X;Z);Y$
W:	Strong Contraction	$(X;Y);Y \Leftarrow X;Y$
K:	Weakening	$X \Leftarrow X;Y$

In the present context, we understand the structured bodies of premises denoted by X, $Y\ldots$ as *structured bodies of psychological information for an agent*. Given this, we may think of $X;Y$ as the result of the application of one body of information (X) applied to another (Y). How we understand the behaviour of the punctuation mark ';' is decided by the structural rules we wish to accept given the application to *PI*. Accordingly, if we take the application of the result of applying X to Y, to Z ($(X;Y);Z$) to be the same body of information as the application of the result of applying X to Z, to Y ($(X;Z);Y$) then we will want to allow C. This is perhaps easier to appreciate in the case of *Weak Commutativity*, CI (which follows from C): $X;Y \Leftarrow Y;X$. Here the motivation is that the application of X to Y ($X;Y$) be understood as the same body of information as the application of Y to X ($Y;X$). Similar reasoning applies to B where information application is understood as associative. B and C simply involve the recombination of preexisting structured bodies of information. As such, the inclusion of B and C in the logic forming the basis of a theory of *PI* would appear justified. The inclusion of K and W is not.

Relevance and linear (but not intuitionistic) logics are distinguished by their rejection of K.[18] Rejecting K for a theory of *PI* is a given. K allows us to weaken inferences by adding in irrelevant premises. If K is allowed,

[18] As well as, strictly speaking, Commuted Weakening: $X \Leftarrow Y;X$.

then we can get $A; B \vdash A$ from $A \vdash A$, and hence $A \vdash B \rightarrow A$.[19] This *irrelevance result* causes the fact that A is what was actually used in the deduction of A to be lost. Relevance and linear logicians jettison K precisely because they want $A \rightarrow B$ to be an *encoding* of the fact the A was *used* in the deduction of B. The relevance logic R is comprised of the structural rules B, C, and W, plus distribution (D).[20] R is clearly much weaker than classical logic. What it lacks in strength it gains in subtlety, preserving deductive details otherwise lost. Given our interest in formulating a theory of *PI*, there may still be details lost that we would like to preserve.

W stands apart from B and C. B and C involve the *recombination* of premise structures, and involve neither their *introduction* (as is the case with K) nor, crucially, their *duplication* (as is the case with W). W allows the arbitrary duplication of pre-existing premise structures. This will not do if the notion of the information flow between doxastic states of an agent in a putative theory of *PI* is being taken seriously, and it is. We don't want to obscure the number of times a body of information is used. Neither do we want to take it for granted that a body of information may be re-used arbitrarily, where such re-use is guaranteed to be insignificant. This is not to say that we want to arbitrarily rule out the repetition of a body of information, only that we do not wish to loose track of it. In order to actually keep track of it, we require *exponentials* (see fn. 21 below). By rejecting W along with K we get a restricted class of relevance logics, linear logics. The fact that linear logics have traditionally been studied in computer science to track computational resource use is a strong indication that they will correctly lend themselves to tracking the *psychological* resource use at the heart of a theory of *PI*.[21]

[19] The rule for $\rightarrow$ introduction ($\rightarrow I$) is as follows:

$$\frac{X; A \vdash B}{X \vdash A \rightarrow B}$$

[20] Distribution: $A \wedge (B \vee C) \vdash (A \wedge B) \vee (A \wedge C)$ is kept separate from the structural rules as it is explicitly concerned with logical connectives.

[21] The manner by which linear logics keep track of information repetition turns on two things. Firstly, it turns on the formula-analogue of $X; Y$ above. The *fusion*, $A \circ B$ (or *intensional conjunction* often written as $\otimes$) of two *formulas* A and B is the result of applying the information in A to the information in B. Secondly, it turns on the truth constant t. t tracks the behaviour of 0, which keeps track of *theoremhood*, or *provability* in the object language (in this case linear logic). The classical tactic of the theoremhood of A being recorded by $\vdash A$ is disallowed in this context (see Restall, *ibid.*, 30). $X \vdash A$ is sensible only where X is a structure, and 'nothing' is not a structure. 0 is the zero-pace punctuation mark introduced for theorem labeling: $0 \vdash A$ states that A is a theorem. 0 is a *left identity*, when it satisfies the structural rules *Left Push* and *Left Pop* (Dunn, 1993):

The final syntactic aspect requiring exposition lies outside the scope of structural rules. Linear logic has commonly been considered in the absence of *distribution*, despite the presence of extensional conjunction and disjunction, $\wedge$ and $\vee$. Distribution allows us the following:

(5.4) $A \wedge (B \vee C) \vdash (A \wedge B) \vee (A \wedge C)$

We should allow distribution in any logic underpinning a theory of *PI*. It generates no paradoxes or counterintuitive consequences in line with those commonly attributed to classical implication. Logics lacking distribution are the hallmark of quantum logics. However non-classical the behaviour of psychological information may be, it is presumably not quantum.

What of negation? DMALL contains *strict de Morgan negation*. Strict de Morgan negation is *strict* due to it incorporating the following axiom relating negation to intensional structure:

(5.5) $$\frac{X; A \vdash {\sim}B \qquad Y \vdash B}{X; Y \vdash {\sim}A}$$

Left Push: $X \Leftarrow 0; X$
Left Pop: $0; X \Leftarrow X$

Here the application of 0 delivers no more information than does X itself. The zero-place punctuation mark 0 is the syntactic counterpart to the semantic notion of the logic world 0 expounded in §2 above. In this syntactic context, 0 is the logical body of information that permits inferences of the form $A \rightarrow A$. The zero-place connective t is a *truth-constant* for a left identity 0, and hence tracks its behaviour *in* the language, iff it satisfies the following rules, (tI) and (tE):

(tI) $0 \vdash t$
(tE) $\frac{X \vdash t \qquad Y(0) \vdash A}{Y(X) \vdash A}$

Now that we have fusion and t we can explicate how it is that linear logics keep track of information repetition. This turns on the non-normal modality !, read 'of course' and called an *exponential*). We write $A \circ A$ as A^2, and $A \circ (A \circ A)$ as A^3 etc. Generalising, $A \circ A^n$ is written as A^{n+1}. $!A$ may be rendered transparent by the equivalence stated in (5.3):

(5.3) $!A \dashv\vdash t \wedge A^1 \wedge A^2 \wedge A^3 \wedge \ldots$

$!A$ allows us access to as many repetitions of A as we require. That is, $A \vdash A^n$ for any n (*ibid.*, 56).

It is a De Morgan negation in that it allows the following (where $\dashv\vdash$ is *iff*):

(5.6) $\sim(A \wedge B) \dashv\vdash \sim A \vee \sim B$
(5.7) $\sim(A \vee B) \dashv\vdash \sim A \wedge \sim B$

De Morgan negation is a *simple negation* in that (5.8) is preserved:

(5.8) $$\frac{A \vdash \sim B \qquad X \vdash B}{X \vdash \sim A}$$

De Morgan negation brings with it the usual double negation rules, *double negation introduction* (DNI) (5.9) and *double negation elimination* (DNE) (5.10):

(5.9) $$\frac{X \vdash A}{X \vdash \sim\sim A}$$

(5.10) $$\frac{X \vdash \sim\sim A}{X \vdash A}$$

It is with negation that we breach the limit of what may be sensibly discussed with reference to the purely syntactic aspects of DMALL insofar as its forming the basis of a theory of *PI* is concerned.

Recalling the semantics frames from §2 above, and noting that frames may be thought of as a set of situations with accessibility relations defined over them, we return to frame semantics in more detail. So far all we have is a *bare frame* $\langle 0, S, \mapsto, \sqsubseteq \rangle$ for information flow (recall that the ternary relation R of relevance frames has been interpreted as an information channel $\mapsto$). We begin with Dunn's (1994, 1996) binary *compatibility relation* C.[22] xCy asserts that the situation x is *compatible* with the situation y. xCy holds when nothing rejected by x is accepted by y. We require this compatibility relation on our frame as it may be used to give a semantic definition of negation:

(5.11) $x \vDash\sim A$ iff for every y such that $xCy, y \nvDash A$.

The properties assigned to the compatibility relation will affect the nature of the negation type. Centrally, we ask whether or not the compatibility relation should be taken to be *symmetric*. In the absence of a particular application, there is no constraint here either way. Given the application to impossible

[22] Originally, Dunn used $\perp$ to stand for *incompatibility* to achieve identical results.

situations understood as confused doxastic states however, symmetry is delivered for free. The compatibility relation is an *inter* as opposed to *intra* relation between situations. The situations themselves may be impossible, which is just to say that we don't want the compatibility relation C to be *reflexive*. For x to be compatible with y, whilst at y *failing* to be compatible with x, it would need to be the case that what is *meant* by the compatibility of situations x and y is that whilst there is nothing rejected by x which is accepted by y, there is nonetheless something rejected by y which is accepted by x. Although there are doxastic states (confused or otherwise) that stand in such an asymmetric relation, this is not the relation picked out by *compatibility* when we talk of two (or more) situations being compatible. Given symmetry on compatibility, we have (5.12):

(5.12) $x \vDash \sim A$ iff for every y such that $yCx, y \nvDash A$.[23]

Symmetry on compatibility guarantees that we are working with the *simple negation* specified by (5.8) with respect to *PI*. Moreover, symmetry on compatibility guarantees that DNI holds (see Restall, 2000b, for the proof), hence DNI holds for *PI*. Given symmetry on compatibility, the interaction between compatibility and informational inclusion is specified by (5.13) which is as we would expect, and fully coherent with *PI*:

(5.13) If $x' \sqsubseteq x$ then if xCy then $x'Cy$

The case for DNE is more complex. Symmetry on compatibility does not guarantee DNE. In order to state the frame condition on DNE, we require the definition of the *inverse relation* on frames. Where xIy is read x is an inverse to y, we have (where $\leftrightarrow$ is read 'if and only if' in the metalanguage and not understood as importing any implicational force) (5.14):

(5.14) $xIy =_{df} \forall z(xCz \leftrightarrow z \sqsubseteq y)$

We use (5.14) to get (where $\rightarrow$ is read 'if-then' in the metalanguage, and not understood as importing any implicational force) (5.15):

(5.15) $\forall x \exists y(xIy \wedge \forall z(yIz \rightarrow z \sqsubseteq x))$

[23] In the absence of symmetry on compatibility, (5.11) and (5.12) would define distinct negation types (usually distinguished via $\sim$ and $\neg$).

(5.15) is Lemma 2.5 of Restall’s (*ibid.*, 859) where he gives a proof of the correspondence between DNE and (5.15). The plausibility of DNE for a theory of *PI* accordingly turns on the plausibility of (5.15). In *PI* terms, (5.15) defines the agent’s doxastic state x to be an inverse of the agent’s doxastic state y to be such that for any of the agent’s doxastic states z, they are such that x is compatible with them if and only if the information they contain is included in the information in y. Intuitively, we can think of the inverse of an agent’s doxastic state as a collation of all the information compatible with it. In *PI* terms, (5.15) can now be read as asserting the following: Each of an agent’s doxastic states has an inverse, and any information contained in the inverse of an inverse of such a state will be included in the information specified by that state. This makes sense in terms of *PI*, and hence DNE does also.

De Morgan negation is, a noted above, a simple negation with DNE. With simple negation (and hence DNI) and DNE made sensible in *PI* terms, we have it that De Morgan negation is sensible in *PI* terms also. We have one final step, and that is to demonstrate that *strict* de Morgan negation is a sensible notion in terms of *PI*. Reminding ourselves of the ternary relation R given by (2.1) in §2 above:

(2.1) If $Rxyz$ and if $x' \sqsubseteq x$, $y' \sqsubseteq y$, and $z \sqsubseteq z'$, then $Rx'y'z'$.

The frame condition corresponding to (5.5) is given in (5.16) (where $\leftrightarrow$ is read as in (5.14)):

(5.16) $\exists x(Ryzx \wedge xCw) \leftrightarrow \exists v(Rywv \wedge zCv)$

(5.16) is quite complex and the result of attempting to render it transparent in *PI* terms may unfortunately deliver a rather opaque result. Recalling (2.5) from §2, we explicate $Ryzx$ as the following: should $A \rightarrow B$ be true in y and should A be true in z, B is true in x. Couching Restall’s (2000a, 241) point in terms of *PI*, (5.16) states that there exists an agent’s doxastic state x such that it is the result of applying the rules in the state y to the information in z and is compatible with the state w, if and only if there exists a state v such that it is the result of applying the rules in the state y to the information in the state w and is compatible with z (since we are taking compatibility as symmetric, we can read this final conjunct in either direction). Insofar as perceiving acceptability is concerned however, the cashed out version of (5.16) is so complex that it is difficult to do so. Symmetry on compatibility at least wards off any obvious objection.

Returning to the substructural rules with their conditions in frame semantics; in order to specify the frame conditions on B, C and W, R is used to define two new terms:

(5.17) $R(xy)zw = \exists u(Rxyu \wedge Ruzw)$
(5.18) $Rx(yz)w = \exists u(Ryzu \wedge Rxuw)$

This done, the frame conditions on B, C and W are stated as follows (where $\rightarrow$ is read as in (5.15)):

B: $R(xy)xw \rightarrow Rx(yz)w$
C: $R(xz)yw \rightarrow R(xy)zw$
W: $Rxyz \rightarrow R(xy)yz$

We strengthen the motivation for the rejection of W in the theory of *PI* via reference to its frame condition. That W is too strong for our purposes is clarified by making its existential claim explicit. Unpacking the right hand side via (5.17) gives us (5.19):

(5.19) $Rxyz \rightarrow \exists w(Rxyw \wedge Rwyz)$

All that the *left hand side* of (5.19) is claiming is that, since $Rxyz$, then if $x' \sqsubseteq x$, $y' \sqsubseteq y$, and $z \sqsubseteq z'$, then $Rx'y'z'$. Recalling (2.5) from §2, we explicate this claim as the following: should $A \rightarrow B$ be true in x and should A be true in y, B is true in z. In terms of *PI*, an agent's doxastic state z contains everything you get from applying the rules in the agent's doxastic state x to the information contained in the agent's doxastic state y. If the information included in x is a development of the information included in x' ($x' \sqsubseteq x$) and the information included in y is a development of the information included in y' ($y' \sqsubseteq y$) then applying the rules in the agent's doxastic state x' to the information in the agent's doxastic state y' will *not* generate any more information for the agent. Accordingly, the entire informational output will be contained in the agent's doxastic state x, as well as any doxastic state x' of the agent such that the psychological information composing x' is a development of the psychological information composing x. This is as desired. However, as couched in such terms (of psychological information) it remains entirely cogent for this condition on the left hand side of (5.19) to hold, whilst the right hand side does not.

In the context of psychological information, the existential claim on the right hand side of (5.19) is, although potentially satisfiable, not something that should be admitted as a necessary condition on the satisfaction of the left hand side. Unpacking the right hand side of (5.19) is cumbersome, but illustrative: It claims that there *exists* a doxastic state w such that $Rxyw$, hence if $x' \sqsubseteq x$, $y' \sqsubseteq y$ and $w \sqsubseteq w'$, then $Rx'y'w'$, *and that* $Rwyz$, hence if $w' \sqsubseteq w$, $y' \sqsubseteq y$ and $z \sqsubseteq z'$, then $Rw'y'z'$. That is, should $A \rightarrow B$

be true in x and should A be true in y, B is true in w, *and that* should $A \rightarrow B$ be true in w and should A be true in y, B is true in z. Unpacking this in more explicit *PI* terms, the right hand side of (5.19) is claiming that *there exists* a doxastic state w of the agent such that this state w contains everything you get from applying the rules in the agent's doxastic state x to the information in the agent's doxastic state y, *and that* the doxastic state z is such that it contains all of the information you get from applying the rules in the agent's doxastic state w to the agent's doxastic state y. All of this is *required* for the satisfaction of the left hand side of (5.19), but (5.19) is too strong a condition to impose on psychological information, with the left hand side being too weak to ensure that the right hand side holds. That it is too strong is a function of the limits on natural cognition. Suppose that w is massively (or at least considerably) complex; then despite our having psychologically parsed the information in w *as the output* of applying x to y, we cannot guarantee that we have psychologically parsed the information in z as the output of *applying* w *to* y, on the basis of $Rxyz$ alone.

By contrast, the existence of a state satisfying the right hand sides of the frame conditions on B and C on the basis of their left hand sides is uncontroversial, as the left hand side takes the existence of the state as given. This is made transparent by unpacking the frame conditions on B and C, generating (5.20) and (5.21) respectively:

(5.20) $\exists u(Rxyu \wedge Ruxw) \rightarrow \exists u(Ryzu \wedge Rxuw)$

(5.21) $\exists u(Rxyz \wedge Ruyw) \rightarrow \exists u(Rxyu \wedge Ruzw)$

In contrast to W, the satisfaction of B and C does not involve the introduction of a state. It merely involves the innocuous recombination of preexisting states. This is merely a deeper explication of the facts that have been read off of B and C directly above. The reader may check the innocuous nature of the recombination in *PI* terms for herself.

This done, we have established a strong *prima facie* case for the use of DMALL to construct a theory of *PI*. But the case *is* only *prima facie*, as there remains the catch pointed out at the beginning of this section. The substructural logic P interpreted by Barwise's channel theory does not contain C, as channel composition is not commutative, although it is associative, hence it contains B (see Restall, 1994). So there exists a tension between the frame conditions it would appear that we would like, given a putative theory of *PI*, and those that we may get, given the constraints imposed by a theory of information flow. This remains an open problem.

6. *Conclusion*

The direction for future research is clear. The frame semantics of a substructural logic will be used to give information measures in much the same way as are the classical semantics underlying *CSI*. Echoing fn. 15 in §4 above, we can fruitfully think of *CSI* as non-refined, or idealized *PI*. Given the added complexity involved in obtaining information measures (of any sort) as opposed to information *individuations*, it is sensible to begin with the goal of deploying a substructural logic towards the *individuation* of *PI*. Yet the clarity of this direction for future research does not imply its triviality. The complexity of the work involved in building a theory of *PI* via frame semantics guarantees that there will almost surely be issues and obstacles to which we currently remain blind. An issue that is already apparent has been mentioned above, namely that the logic underlying channel theory, P, lacks the substructural rule C. A sensible approach would be to begin the construction of a theory if *PI* on the back of P + strict de Morgan negation. Only when this is done will we be in a position to properly evaluate C with respect to *psychological* information flow, and hence the formal tractability of *PI* on top of DMALL. It is the purpose of the future article that will form the sequel to the present one to identify and surmount such obstacles. It has been the purpose of the present article to propose a basis for the theory of *PI* and to identify and surmount the obstacles involved in this proposal. The proposal is in general, intuitive and tempting. The information assignments by *CSI* to logically true sentences in particular and logically equivalent sentences in general is a function of the classical modal space out of which *CSI* is built. Altering the information assignments by altering the underlying modal space is a natural and immediate response. This response itself has utilized the logical apparatus of frame semantics, which has not been philosophically uncontroversial in itself (the ternary relation R and *impossible situations* being just two examples). By appealing to interpretations in terms of information flow and (ir)rationality we have novel resolutions of such controversies that illustrate the non *ad-hoc* basis of the construction of a theory of *PI* in terms of frame semantics.

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