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ORDINARY MODALITIES

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Abstract
Using terms like *necessary*, *possible* and alike in the contexts outside the logical professional jargon we practically never mean logical modalities. It is shown that the option of exploiting modalities as defined in other systems than S5 is not acceptable; instead, the approach offered by transparent intensional logic (TIL) is chosen, in particular the fact that for TIL the modal variability is independent of the temporal modality, which makes it possible to define some further logical modalities and to derive some 'empirical modalities'. Two such modalities are defined: the 'ordinary necessity', based on the assumption that natural laws are on the one hand empirical, on the other hand eternal, and 'ordinary possibility', derived by means of De Morgan. The intuitive character of these explications is demonstrated.

1. *Logical modalities*

Since 1918, when C.I. Lewis 'invented' logical systems that should handle modalities, a host of such systems have been formulated. All these systems use one and the same symbol for necessity ($\Box$) and all of them use the same symbol for possibility ($\Diamond$), although the concepts connected with these symbols are distinct in distinct axiomatizations due to distinct formal properties of the 'accessibility' relation as defined by Kripke.

We set aside the possible criticism of the mentioned approach to modalities, see, however, [Tichý 1988]. What is relevant for our problem is only the following fact:

All the concepts 'of necessity' and 'of possibility' modelled in particular systems S1 through S5 (including the other, 'intermediary' systems) share the following property: A sentence of the form $\Box A$ or $\Diamond A$ is interpreted as a non-empirical, analytic sentence (true or false) or else its non-analyticity is given by dependence on possible worlds.

Well, one could object that there are interpretations of $\Box$ (or of $\Diamond$) where the dependence concerns time (see, e.g., [Chellas 1999, p. 69]), but in this case 'worlds' is simply interpreted as 'time points'. We will see in 3. that then an essential point is ignored.

Much more relevant to the problem of modality vs. temporality is Montague's IL, where the 'extension of an expression' is dependent on both worlds and times. One of the reasons why the present analysis is based on TIL (see below) rather than on Montague consists in the fact that the explicit intensionalisation used in TIL, i.e., using variables for worlds and for times, makes it possible to analyse cases where modality and temporality do not go 'side by side', so that, e.g., some expressions are modally *de dicto* being at the same time temporally *de re* etc. (See, e.g., the excellent analysis in Tichý 1986.)

To explicate the notion of *ordinary modality* we accept the approach suggested by the general approach to logic as articulated in *transparent intensional logic* ('TIL', see, e.g., Tichý's monograph 1988). We cannot reproduce here the apparatus and all the principles of TIL, so we will (in a rather simplifying way) formulate only some points indispensable for our purposes.

2. *Constructions of propositions*

TIL is a type-theoretical system based on four atomic types (truth-values, type o, individuals, type ι, time points/real numbers, type τ, possible worlds, type ω), where the composed types are sets of partial functions over this base. *Constructions* are abstract procedures denoted in the manner inspired by (typed) λ-calculus. *Intensions* are functions associating possible worlds with chronologies of a given type, so their type schema is

$\omega \to (\tau \to \alpha)$ where α is an arbitrary type. Their constructions usually have the structure

$\lambda w \lambda t A$, where w is a variable ranging over ω and t is a variable ranging over τ (A is a construction that constructs — possibly dependently on valuation — an object of type α). Thus *propositions*, which are objects of type $\omega \to (\tau \to o)$ (written in TIL as $((o\tau)\omega)$, abbreviated as $o_{\tau\omega}$), are constructed by constructions of the form $\lambda w \lambda t A$ where A is a construction which contains w and t and constructs — dependently on w and t — a truth-value. (A can fail to construct a truth-value because of the partiality of some function.)

Necessity is modelled in TIL as a class containing just one proposition, viz. the logically true proposition (which associates every possible world with such chronology of truth-values which associates every time point with T). Possibility is a class of those propositions which are true in at least one possible world at some time point. So the type of both modalities is

$((\omega \to (\tau \to o)) \to o)$. (In the notational jargon of TIL this is written $(oo_{\tau\omega})$.)

So both modalities are *extensions*: they are *classes* rather than *properties* of propositions. In contrast with modalities in other systems than S5, they cause that any construction of the form $\Box A$ or $\Diamond A$ constructs always T or F, never a proposition. The concepts 'of necessity' or 'of possibility' in those 'non-S5' systems are simply other concepts than those of *logical* modalities.

It could seem as if just those latter concepts could correspond to 'ordinary modalities' that we are prepared to explicate. In 3. we will try to show that this is probably not the case.

One important feature of TIL is that variables of possible worlds and time points are explicitly used. It can be shown (here and, e.g. in [Materna 2003]) that the expressive power of the systems using these variables exceeds the expressive power of the other systems. See also [Muskens 1989, p. 10, 15] where it is demonstrated that a Montague's system without such variables does not have the Rosser-Church property.

In particular, the history of Quine's challenge to modal logicians (see, e.g., the excellent overview in [Lindström 2000]) convincingly shows that the solution to such problems like whether

$$\forall x \forall y (x = y \supset \Box(x = y))$$

is valid would be much easier achieved if variables of possible worlds (or, if you like, 'indices') were used; moreover, the 'cooperation' of such variables with λ-abstraction immediately results in such analyses of empirical sentences which make it clear that the latter denote propositions rather than truth-values. And the sentence

The number of (major) planets equals 9 and not necessarily so,

one of the series of the famous number-of-planets examples, can be analysed as follows (with some notational simplifications):

$$\lambda w \lambda t [\mathrm{N}_{wt} = 9 \wedge \neg \forall w \forall t (\mathrm{N}_{wt} = 9)],$$

so that what is constructed as the denotation of the sentence is a proposition, i.e., a *function*, which in our case returns T(ruth) in those worlds (and time points) where the function N (type $\omega \rightarrow (\tau \rightarrow \tau)$) returns the value 9. (The second member of the conjunction is true because N, corresponding to the expression *the number of (major) planets* takes values dependently on worlds and times: it is no mathematical function.)

3. *The way we normally use* possible *and* necessary

When the terms *possible*, *necessary*, and all the derived terms are used outside logical contexts, then we can see that nearly never they express logical modalities. To illustrate this claim we adduce two 'paradigmatic' examples. Consider the sentences

a) *It is possible that it will rain in the afternoon.*

b) *If a pebble is thrown into water it necessarily sinks.*

If *possible* and *necessarily* expressed *logical* modalities, then a) as well as b) were analytic sentences bearing no empirical information:

Logical possibility means that the proposition that is logically possible is true in at least one possible world-time, in other words, it is sufficient not to be a contradiction. So a) would be analytically true, since it is not contradictory.

Logical necessity means that the proposition that is logically necessary is true in all possible worlds-times, so it is the logically true proposition and b) would be analytically false, since there is no *logical* necessity in the physical behaviour of material bodies.

Yet both a) and b) are informative, empirical claims: nobody will say "What an uninteresting tautology!" when hearing a) or "Surely not!" hearing b).

But then, since a) and b) are empirical, non-analytic claims, we could perhaps interpret the respective modal expressions in terms of some non-S5 system, could we? Now which of such systems should be taken into account?

I fear that there is no definite criterion of deciding due to the way the modal systems have been defined. Therefore, I will try another way of interpreting this kind of modalities (say, "ordinary modalities"), the way which is offered by TIL (see 2.).

4. *Modal and temporal variability*

We have seen that TIL distinguishes between *propositions*, which are taken to be just functions (mappings) in the sense of possible-world semantics, and *constructions of propositions*, which are abstract procedures that construct propositions. (Therefore, TIL does not need to *replace* the set-theoretical notion of propositions by the notion of *structured propositions* as it is done, e.g., in [King 1997].) Further, propositions associate possible worlds with *chronologies* of truth-values, so that possible worlds are conceived of as *possible histories*. (Thus all interesting intensions are such functions from possible worlds to chronologies.) The intuition underlying this conception can be illustrated as follows:

Let us consider any empirical sentence, say, *Warsaw is the capital of Poland*. Whereas the proposition denoted by this sentence is fix, independent of the given state of the world, its truth-values can be said to vary. We can distinguish between two factors of this variability.

First: Given a definite time point where the proposition (and so the sentence) is, say, true, we can even then say that *it could be otherwise*: the contingency of the empirical sentence cannot be ignored by claiming that the truth-value of the proposition at the moment t is — at the same moment — the same in all the possible worlds. This fact, viz. that the truth-value of the proposition at the moment t, is not necessary, i.e., that there are possible worlds where the truth-value of the proposition at the same moment t is another one, defines what we call *modal variability*.

Second: Given a definite possible world the truth-value of an empirical proposition (and so of the sentence) can change. (In our case, even in the actual world the sentence was false or even lacking any truth-value.) This fact defines what we call *temporal variability*.

The brute fact that the modal variability differs from temporal variability implies that *possible worlds should not be interpreted as symbols admitting once modal, once temporal interpretation*.

5. *Type-theoretical characteristics of logical and ordinary modalities*

Thus modal and temporal variability are mutually independent. An interesting consequence thereof is that we can construct more *logical* modalities than we are used to. All these modalities share one type: $((\omega \to (\tau \to o)) \to o)$ (in TIL $(oo_{\tau\omega})$). They are *classes of propositions*. The most used modalities (classical necessity and possibility in S5) are constructed as follows (p ranges over propositions, p_{wt} abbreviates the application $[[pw]t]$; brackets mean that a given function is applied to arguments. This is the way of notation used by TIL; one of the distinctions between the latter and the presently used notation is that TIL uses *prefix* notation, which is given by the fact that TIL is based on functional approach):

$\lambda p \forall w \forall t p_{wt}(\Box)$, $\lambda p \exists w \exists t p_{wt}(\Diamond)$.

Now we can construct the other *logical modalities*:

i) $\lambda p \forall w \exists t p_{wt}$, ii) $\lambda p \exists w \forall t p_{wt}$, iii) $\lambda p \forall t \exists w p_{wt}$, iv) $\lambda p \exists t \forall w p_{wt}$.

(Clearly, the class ii) is a subclass of iii), and the class iv) is a subclass of i).)

None of the new modalities can be used to interpret modalities connected with the modal terms in the sentences a) and b) above. Not only that the former are rather strange: first of all, they are *classes of propositions*, i.e., *logical modalities*. Thus what we need are 'empirical modalities'. Their type would be either $(\omega \to ((\omega \to (\tau \to o)) \to o))$ (in TIL $((oo_{\tau\omega})\omega)$) or perhaps $(\tau \rightarrow ((\omega \rightarrow (\tau \rightarrow o)) \rightarrow o))$ (in TIL $(oo_{\tau\omega})\tau$). We can construct four such modalities:

i′) $\lambda w \lambda p \forall t p_{wt}$, ii′) $\lambda w \lambda p \exists t p_{wt}$, iii′) $\lambda t \lambda p \forall w p_{wt}$, iv′) $\lambda t \lambda p \exists w p_{wt}$.

(i′ constructs the following function F: F associates any possible world (λw) with a function from propositions (λp) to truth-values $(\forall t p_{wt})$, in other words, F associates possible worlds with the classes of propositions, so the type of F is $(oo_{\tau\omega})_\omega$. In a given possible world w a proposition belongs to this type and satisfies F iff it is always true in w.

The function F′ constructed by ii′ is of the same type; in a given possible world w a proposition satisfies F′ iff it is at least sometimes true in w.

The function F″ constructed by iii′ is of the type $(oo_{\tau\omega})_\tau$; at a given time moment t a proposition satisfies F″ iff it is true (at t) in all possible worlds.

The function F‴ constructed by iv is of the same type; at a given time moment t a proposition satisfies F‴ iff it is true (at t) in at least one possible world.)

It does not seem that iii′) or iv′) could be used as interpretations we need to analyse such sentences like a) or b). Let us however consider i′)

First of all let us examine in which way the logical distinction between mathematical truths and natural laws can be characterised. Mathematical truths are sentences whose truth-value is independent of possible worlds. The constructions underlying mathematical sentences construct simply truth-values. On the other hand any scientist (physicist, biologist etc.) has to examine reality. Thus his/her claims cannot be independent of the given state of the world, i.e., of possible worlds. Yet the propositions denoted by law-like sentences of empirical sciences are (maybe implicitly) supposed to differ not only from truth-values (being functions which are defined on possible worlds) but also from such propositions which are denoted by other than law-like sentences. This latter distinction can be formulated as follows:

Let us suppose that a law-like sentence A is an ideal approximation of what we call *natural law*. Then the proposition denoted by A is (or should be) an *eternal* proposition: in all possible worlds where the respective natural law holds it does so at every time point. Thus it differs from 'mathematical propositions' in that the class of possible worlds where the law holds is a *proper subclass* of the class of possible worlds, and it differs from other empirical propositions in that the latter are not eternal. (See, however, 7.) Thus every 'law-like proposition' defines a class of possible worlds, i.e., an object of the type $\omega \rightarrow o$.[1] In a sense we can say that propositions stating some natural law lack the temporal variability. This property is inherited by

[1] Fred I. Dretske in his [1977] offered a really ingenious analysis of natural laws. Among other important claims he says (p. 266):

Statements of law... have a far wider scope than any true generalization about the actual world. Their scope extends to those possible worlds in which the extensions of our terms differ but the connections between properties remain invariant.

any proposition that states an event which is implied by the natural law in question.

Now we can return to the modality i′). Let the world dependent pseudonecessity defined by i′) be denoted by N. The type of N is $\omega \rightarrow ((\omega \rightarrow (\tau \rightarrow o)) \rightarrow o)$, in TIL $((oo_{\tau\omega})\omega)$; it could be called an *atemporal property of propositions*.

Further, let a simplified scheme of the construction underlying the sentence

b′) *If a pebble is thrown into water it sinks.*

be

b″) $\lambda w \lambda t \forall x[[[\mathrm{P}_{wt}x] \wedge [\mathrm{TW}_{wt}x]] \supset [\mathrm{S}_{wt}x]]$.

Such a construction constructs the proposition which is true for all individuals in those pairs $< \mathrm{W}, \mathrm{T} >$ where the individual which is a pebble and is thrown into water sinks. Now observe that whereas temporal variability is present in the case of empirical sentences which are not law-like no temporal variability is supposed in the case of the sentence b′). Once you throw a pebble into water and 'our' natural laws hold it will *always* sink. On the other hand, in a well imaginable world where 'our' natural laws do not hold the pebble could swim or even jump out of water etc. Thus any natural law defines a class of worlds, viz. such where the law holds.

So the sentence b) gets the following analysis (given b″) as the scheme of the analysis of b′):

c) $\lambda w[\mathsf{N}_w[\lambda w \lambda t \forall x[[[\mathrm{P}_{wt}x] \wedge [\mathrm{TW}_{wt}x]] \supset [\mathrm{S}_{wt}x]]]]$

We can clearly see the distinction from the case of logical modalities. The latter when applied to a proposition return a truth-value. The N in c) has to

Dretske rightly sees the necessity of intensional approach to analysing natural laws. From the viewpoint of intensional semantics there are two problems with his proposal.

First, Dretske believes that (mere) universal generalizations do not say anything — unlike statements of law — about those possible worlds where the extensions of the respective predicates differ from the actual extensions. This is not true. An empirical universal generalisation also denotes a proposition which is true in some possible worlds, but if it is not a statement of law then the distinction consists in the fact that it is true in some worlds *at some (not all) time points*. The statement *All swans are white* denotes a proposition which is true in some worlds at some time points; if it is true in some worlds eternally, then it is so in virtue of a natural law which holds in such worlds (not in the actual one).

Second, Dretske's scheme of a natural law, viz.

F-ness $\rightarrow$ G-ness (p. 263)

is interpreted as claiming "an *extensional* relation between *properties*...". As I understand extensional relations this would mean that the relation between F and G would hold *a priori*, independently of possible worlds. But then there would obtain no difference between natural laws and mathematical truths. The present paper makes it clear that what distinguishes natural laws and mathematical truths is that laws are dependent on worlds. What distinguishes natural laws and other empirical propositions (including generalisations) is that the former are eternal in such a group of possible worlds a member of which is the actual world.

be applied first to a possible world, and by λ-abstraction we get a *class of possible worlds*, viz. of those ones where the proposition denoted by b$'$) is eternal (holds at all time points). (Contemporary possible-world semanticists mostly identify propositions just with sets of possible worlds; for us the construction c) constructs a 'semiproposition', since 'normal propositions' embody modal *and* temporal variability as for their truth-values.)

6. *De Morgan*

The case of our sentence b) seems to be explained *via* defining the 'ordinary necessity' as the modality N. The modality ii$'$) is obviously a dual counterpart of N. It could be called 'ordinary possibility' and we will denote it by P. Let C construct a proposition. The construction whose form is

d) $\lambda w[\mathsf{P}_w\mathrm{C}]$

constructs the class of those worlds where the proposition constructed by C holds at some time points. We can see that the propositions which in a world W do not fulfil this condition are those ones which are incompatible with the natural laws holding in W (we could perhaps speak about 'semicontradictions'), further, the 'contingently not realised propositions' (e.g., the proposition that a person XY comes at 12 o'clock on Monday September 1$^{\text{st}}$ 2001) and, of course, contradictions (as a subclass of the first group).

Now we can explain the semantics of the sentence a). Its underlying construction has just the form d), where C constructs the proposition denoted by the sentence

e) *It will rain in the afternoon.*

In the light of what we said above this sentence says, properly speaking, that an afternoon rain is nothing what would be incompatible with the natural laws of the given world *and* that it will be realised. (For the sake of simplicity we suppose here that the 'afternoon' has been defined in some 'absolute' way, i.e., by referring to some temporal interval defined in terms of a calendar. Actually, "afternoon" is a function of the type $\tau \rightarrow (\tau \rightarrow o)$.)

Well, such an information is rather meagre but it is no tautology; we can imagine a world such that among the natural laws holding in it we find a following principle: *It never rains in the afternoon.* In such a world the sentence e) would be eternally false. Further, if it does not rain in the afternoon, the proposition is simply false.

Comparing N and P we can expect that modified De Morgan's laws will hold for them. Indeed: preserving the simplified symbolism like in d) we can easily verify that the following equivalences hold:

$[\mathsf{N}_w\mathrm{C}] = \neg[\mathsf{P}_w\lambda w\lambda t\neg\mathrm{C}_{wt}]$.

(Observe: $\lambda p\forall t... = \lambda p\neg[\exists t\neg...)$

$[\mathsf{P}_w\mathrm{C}] = \neg[\mathsf{N}_w\lambda w\lambda t\neg\mathrm{C}_{wt}]$.

Indeed, saying that the pebble necessarily sinks we say that it's *not* sinking is impossible (in the 'ordinary sense'), and saying that raining in the afternoon is possible we say that not raining in the afternoon is not necessary (in the 'ordinary sense').

7. *Revision*

Unfortunately, attractive as this explication may seem, it is inadequate. To see it, consider the following *fact*:

To be eternally true is a necessary but not the sufficient condition for a sentence to be law-like (or *necessary* in the "ordinary sense").

Indeed, let I be a definite interval of time points (let $< t >$ be also such an interval). Let a sentence express a construction of the schematic form (I use a notation not used by TIL; this simplification cannot influence our analysis):

$$\lambda w \lambda t[[\mathsf{I}t] \supset Q_{wt}],$$

where Q is a construction of a proposition q. In any possible world where this construction constructs a true proposition the latter is *eternally true*. Yet the respective sentence need not be law-like. Take a typical example:

A. *At 12 o'clock* (or, say, *from 12.00 to 13.00*) *on September 15, 2000 it rains* (or: *it will rain*, no analysis of tenses is necessary here).

In the case that this sentence is true it will be always true, but we would not say that such a sentence is law-like. What is important here is that we could say *It is possible that ...* but, of course, hardly *It is necessary that ...*, not even in the intuitive sense to be explicated here.

On the other hand, some sentences of the above form could be taken to be necessary (even if not *law-like*). Our example is:

B. *If a pebble is thrown to water between 12.00 and 13.00 ..., it sinks.*

Thus our definition of 'ordinary necessity' has to be modified in such a way that sentences (propositions) like A. were ruled out while sentences like B. remained to be 'necessary'. The 'ordinary possibility' could be defined using De Morgan.

In the following text we will abbreviate $\lambda w \lambda t$ A by λA. Further: we use the standard symbolism known from predicate logic (which can be easily 'translated' to the symbolism used in TIL); in particular, infix notation (rather than the prefix one) is used. Any construction C (including variables) provided with the index $_{wt}$ is to be understood as application of the respective intension to a possible world and (of the result) to a time point, so (as in TIL) C_{wt} abbreviates $[[\mathrm{C}w]t]$. (In particular C_w abbreviates $[\mathrm{C}w]$.) The variables p, q range over propositions.

Our new definitions will be (N, P are again of the same type $\omega \rightarrow ((\omega \rightarrow (\tau \rightarrow o)) \rightarrow o)$, in TIL $((oo_{\tau\omega})\omega)$):

$$^{*}\mathsf{N}_w p = \forall t p_{wt} \wedge \forall q[[p = \lambda[[\mathsf{I}t] \supset q_{wt}]] \supset \forall t q_{wt}]$$

We can see that the propositions denoted by the sentences of the kind A. above will be ruled out. Let us show it using a simplified construction for the sentence

At 12 o'clock on September 15, 2000 it will rain.

Let the time point so defined be t. R let be the proposition that it rains. Then the construction will look somehow like

$$\lambda[[\{\mathsf{t}\}t] \supset \mathrm{R}_{wt}]$$

Now if the respective proposition is true in a world W, then it is eternally true in W. Yet even then the condition given by the second member of the conjunction defining N above is not fulfilled (at least if W is not such a strange world where it rains always due to some natural law), so our sentence/ proposition cannot be said to be 'necessary' in W.

On the other hand, the sentences of the kind B., like

If a pebble is thrown to water at 12.00 ..., it sinks.

are *not* ruled out. The respective construction is (t is again the so defined time point)

$$\lambda[[\{\mathsf{t}\}t] \supset [\forall x[[[\mathrm{P}_{wt}x] \wedge [\mathrm{TW}_{wt}x]] \supset [\mathrm{S}_{wt}x]]]]$$

and this time both members of the defining conjunction hold (in such worlds where pebbles thrown to water sink due to some natural law) so that the sentence/ proposition is necessary in such worlds.

Unfortunately, *counterexamples* of the following kind can be found:

Charles is not a teacher.

Suppose that Charles (an individual) is never a teacher. The only relevant part of the *definition is the first member of the conjunction. Then Charles' not being a teacher would be nomically necessary. Absurd.

Solution:

Ascribing a property P to an individual would be nomically necessary only if it were a consequence of ascribing P to all individuals that share a property Q whose definition does not mention a particular individual.

A generalization leads to the following definitional scheme:

Let $\lambda w \lambda t \Phi^n$ be a scheme of propositional constructions that contain n free individual variables and no particular individual. Let $\lambda w \lambda t \Phi^n_{[xj \rightarrow aj]}$ be the result of replacing the variables by individuals. Now the following scheme captures the essence of nomic necessity:

$[\mathsf{N}_w \lambda w \lambda t \phi^n_{[xj \to aj]}] =$
$[\forall t \forall x_1 ... x_n \Phi^n \wedge \forall q[[\lambda w \lambda t \forall x_1 ... x_n \Phi^n = \lambda w \lambda t \forall x_1 ... x_n [[\mathsf{I} t] \supset q_{wt}]] \supset \forall t \forall x_1 ... x_n q_{wt}]]$
(I.e., a proposition P constructed by a construction containing n individuals is nomically necessary in a world w iff a) the generalization of P in w is true eternally ($\forall t$) and b) in the case that P is given by the construction that could guarantee the eternality due to a temporal fixation ($[\mathsf{I} t] \supset q_{wt}$) the consequent q is in w also eternally true.)
Example:

The velocity of Charles’ and Peter’s fall in the vacuum is the same.

The analysis shows that this sentence is nomically necessary since

$$[[\mathrm{V}_{wt} \text{ Charles}] = [\mathrm{V}_{wt} \text{ Peter}]]$$

logically follows from

$$\forall t \forall xy[[\mathrm{V}_{wt} x] = [\mathrm{V}_{wt} y]],$$

so that our definition applies.

Nomic possibility

Obviously, the way from nomic necessity to nomic possibility is mediated by De Morgan.

$$[\mathsf{P}_w p] = \neg[\mathsf{N}_w \lambda w \lambda t \neg p_{wt}]$$

Thus we have

$[\mathsf{P}_w \lambda w \lambda t \Phi^n_{[xj \to aj]}] = \neg[\mathsf{N}_w \lambda w \lambda t \neg \Phi^n_{[xj \to aj]}] =$
$[\forall t \forall x_1 ... x_n \neg \Phi^n \supset \exists q[[\lambda w \lambda t \forall x_1 ... x_n \neg \Phi^n = \lambda w \lambda t \forall x_1 ... x_n [[\mathsf{I} t] \supset q_{wt}]] \wedge \exists t \exists x_1 ... x_n \neg q_{wt}]]$

We now show by our afternoon-rain example that this definition is in harmony with our intuition and that *nomic possibility is really a dual counterpart of nomic necessity*. We will once again ignore the complications connected with tenses (we could speak about a ‘tenseless determination of time’) and suppose that the area where it could rain was specified. Further, let *the afternoon* denote a definite afternoon (say, on September 15, 2000).

Let I be a definite interval denoted by *the afternoon* (see above). Then our sentence can be analysed as

$$\lambda w \lambda t[[\mathrm{I}t] \wedge \mathrm{R}_{wt}].$$

The respective negation ($\lambda w \lambda t \neg p_{wt}$) is then analysed as

$$\lambda w \lambda t[[\mathrm{I}t] \supset \neg \mathrm{R}_{wt}].$$

According to our definition, to say that this is *nomically possible* means to say that although it holds that for every time moment in the afternoon it does not rain ($\forall t \neg p_{wt}$), there is a proposition q (here: $\lambda w \lambda t \neg \mathrm{R}_{wt}$) such that *sometimes* it holds that $\neg q_{wt}$, i.e., that it rains (in the given area). In other words,
if it does not rain during the given interval then it is only contingently so, not because of some law-like necessity.
It is easily provable that our 'ordinary modalities' obey the 'classical laws of modalities', in particular it holds in all worlds that

$$\mathsf{N}_w p \text{ implies } p_{wt},$$

and, of course,

$$p_{wt} \text{ implies } \mathsf{P}_w p.$$

Remark:
Sharing nomic necessity as a Kripkean accessibility relation

Since nomic necessity has been defined as a (semi-)property of propositions, i.e.,

$$\mathsf{N}_w = \lambda p[\forall t \forall x_1 ... x_n p_{wt} \wedge \forall q[[p = \lambda w \lambda t[[\mathrm{I}t] \supset q_{wt}]] \supset \forall t \forall x_1 ... x_n q_{wt}]],$$

we can easily define an equivalence relation (S5!) between worlds:

$$[\mathrm{R}ww'] = [\mathsf{N}_w = \mathsf{N}_{w'}].$$

This relation links those worlds that share nomically necessary propositions. From our definitions it follows that such worlds share also propositions that are nomically possible. This is a confirmation of the (logically trivial) fact that if the accessibility relation is an equivalence relation it does not mean that it connects every world with every world. Thus S5 can work not only for logical necessity but also for an empirical necessity.[2]

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[2] My anonymous referee mentioned a surely important problem that the present article has not taken into account so that the proposed solution — as it is articulated here — could not be defended (unless some not entirely *ad hoc* changes would be made). The point is that "the value of the Cosmological Constant may have changed over time" (as some scientists think). Then the eternal character of nomically necessary propositions — which is the core of our argumentation — would be cast doubt on. I am aware of the fact that the problem of *defining* natural laws is a non-trivial and obviously difficult problem. I only hope that solving the conceptual character of natural laws from the viewpoint of modal behaviour need not take into account all components of a possible definition and that the eternal character of natural laws is a consequence of any such definition. I am just now not able to propose the way how to harmonise this conviction about eternal character of the nomically necessary propositions with the possibility that even Cosmological Constant could have changed. Does the possibility of such a change mean that we could not use '$\forall t$' when speaking about nomic necessity? My guess is that we cannot expect such radical consequences, but my guess is no argument, of course.

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