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IMAGINARY (NON-ARISTOTELIAN) LOGIC*

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[53] The aim of this paper** is to show the possibility of a logic and of logical operations different from those we use and to show how our Aristotelian
[54] logic is only one of the many possible logical systems. This new logic will not be a novel account of the old one. It differs from it not as an account, but in the very train of its logical operations; this is a "new logic" and not a new treatise on logic.

Different treatises on logic differ in their contents, but all have the same subject matter: our logical world, our logical operations. Imaginary (non-Aristotelian) logic is different from our logic — which I call 'Aristotelian' after its first systematist — in its very subject matter.

The subject matter of imaginary (non-Aristotelian) logic is a logical world and logical operations different from ours. Formulae of both logics are mutually contradictory: the truth of the formulae of imaginary logic excludes the truth of formulae of our Aristotelian logic, and vice versa. Consequently, they both cannot be true in one and the same world; if Aristotelian logic is true in our world, then non-Aristotelian logic can be true only in a different world. This contradictory, mutually exclusive relation between the two logics, their difference not only in content but also in subject matter, is a reason to call this "new logic" non-Aristotelian logic. In calling it 'imaginary', we would (also) want to point to another peculiarity. Our logic is the logic of reality, in the sense that it is a tool for knowledge of this reality, and thus is closely connected with it. The new logic does not have such a connection with our reality; it is a purely ideal construction. Only in a world different from ours, in an imaginary world (the basic properties of which we can, nevertheless, exactly define) imaginary logic could be a tool for knowledge.

* Numbers between square brackets indicate the page numbers in V.A. Smirnov's edition of Vasil'ev's work: *N.A. Vasil'ev. Imaginary Logic. Selected Works (V.A. Smirnov, ed.)* Moscow: Nauka, 1989. Vasil'ev's style is somewhat peculiar at times. In our translation we have tried to stay as close to the original as possible. Italics in the text are Vasil'ev's (unless otherwise indicated). (*R.V. – E.Z.*).

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It is not difficult to see that these designations are analogous to those of the "new geometry" created by Lobachevski. He called it imaginary geometry; later on, the name 'non-Euclidean geometry' has been adopted. To the analogy of names, there corresponds an inner analogy between non-Aristotelian logic and non-Euclidean geometry, which consists in a logical identity of their methods.

Non-Euclidean geometry is a geometry without the 5th postulate, [that is] without the so-called axiom of parallels. Non-Aristotelian logic is a logic without the law of contradiction. It is worth mentioning here that it was precisely non-Euclidean geometry that has served us as a model for the construction of non-Aristotelian logic.

[55] The very idea of a logic different from ours, [the idea] of [the existence of] several [different] logics, should appear to be an absurd one, since we are so accustomed to the idea of one logic common to all, that we cannot imagine the opposite. However, this is only a psychological explanation of our belief in the uniqueness of (our) logic — since no one has [ever] proved it. Before Lobachevski, the idea of a geometry different from ours seemed equally absurd, but today non-Euclidean geometry is generally recognized. We are simply accustomed to believe in the uniqueness of logic. We believe in a single logic exactly in the same way as a people in the primitive stage of their cultural development believe that their language is the only one possible. When such a people is confronted with a neighboring one, the latter give them the impression of being a people without a language, a people of mutes. It would be an offense to our intellectual maturity if we too, encountering logical operations different from ours, would arbitrarily deny them the right to be called "logical."

However unusual the idea of a different logic may be, there is nothing implausible in it. That which is obvious for us, [i.e.] in our world with our structure of mind and our faculty of perception, may be not only not evident, but also completely wrong in a different world, for beings with a different kind of mental structure.

Is it really true that God should necessarily think according to Aristotelian logic, following the canons of the syllogism and Mill's rules of induction? From very early onwards religious thought has conceived of the idea of a God whose mind was infinitely superior to the human mind. Consequently, there is nothing implausible or absurd in the idea that divine logic would be different from human logic.

Therefore, it is quite conceivable, that there may exist systems of logical thinking and logical operations altogether different from ours. The same conclusion can be drawn from the [following] analysis of different points of view on the nature of logic and logical laws.

In contemporary logic, there are three major points of view on the basic logical laws (the law of identity, [the law] of contradiction, [the law] of

the excluded middle, and [the law] of sufficient reason). Some logicians consider them as psychological, natural laws of thought. According to Heymans*, for example, logical laws are natural laws of thought, just as the law of inertia and [the law] of the parallelogram of forces are natural laws of motion. It is clear that from such a point of view it is impossible to defend
[56] the uniqueness and invariance of the laws of logic. We have to conceive of different laws of logic as soon as we imagine a world with different natural laws of thought, if we imagine beings with a different intellectual structure.

According to other logicians, as, for example, *Göring***, the laws of logic constitute norms of correct thought. Such a point of view assumes that the laws of logic are analogous to moral or legal laws. Moral and legal laws are not natural causal laws underlying our acts, but only norms by conforming to which our acts become moral or lawful.

Accordingly, logical laws are neither causal nor psychical laws of thought, but norms by conforming to which thought becomes correct. It is clear that within the framework of such a concept of logic and logical laws it is impossible to defend their uniqueness and absolute universality, since we can easily imagine different norms of correct thinking under different conditions. Indeed, we see that precisely moral and legal laws, whose normative character is undeniable, display a wide range of diversity and variability. At different epochs and in different countries humanity has held different legal and moral norms. Why not to imagine, then, that different rational beings may have different norms for correct thinking?

Finally, according to the third major point of view (for example, Husserl's) logical laws are ideal truths, which are right regardless of their psychological realization in our mind. In such a conception, the laws of logic come close to the axioms of mathematics. But then it is completely impossible to defend the unicity of logic. Precisely mathematics provides us with rigorous scientific instances of 'imaginary disciplines', as, for example, non-Euclidean geometry. At each step (in its development) mathematics involves a generalization of its operations and the extension of the field of its objects. In this way it moves, for example, from the real numbers to the imaginary numbers.
[57] It is impossible to extract the square root of a negative number, since every squared real number is positive; but mathematicians, by introducing imaginary numbers, can extract the square root of a negative number and [thus] give a more general characterization of the operation of 'taking the root of'.

* Heymans, Gerard (1857–1930). Dutch philosopher and psychologist. Proponent of 'Psychic Monism' and of Psychologism in logic (cfr. Eisler, E. Philosophenlexikon p. 266, Berlin 1912, 1977^2) (*R.V. – E.Z.*).

** Göring, Carl (1841–1872). German Critical Empirist and Positivist (cfr. Eisler, E. Philosophenlexikon p. 210, Berlin 1912, 1977^2) (*R.V. – E.Z.*).

Exactly in the same way as mathematical operations can be generalized; logical operations can be generalized too, and in both cases this generalization may lead to the creation of imaginary objects.

Thus, whichever of the three major points of view on logical laws we accept, from each one of them there follows (in its own way) the possibility of the existence of a logic different from ours. This possibility also follows from the fourth, old, and now nearly abandoned point of view of Mill, according to which the laws of logic are generalizations from experience. If this is true, then, even more, we can imagine a world in which the generalizations taken from experience, and consequently logic, will be different from ours.

Therefore, the controversy about the nature of the laws of thought does not have much importance for our aim; whatever solution of this controversy is accepted, we have to conclude to the possibility of a logic different from ours. So, we will not deal now with this very complex and difficult matter.

The possibility of the existence of a logic different from ours can be also demonstrated in a different way.

Had all of the content of logic been analytically contained in some unique characterization or definition of logic, then, of course, a different logic would have been impossible. All the content of logic would have arisen from this single definition and nothing could have been changed in the content of logic without a violation of this basic logical principle, and consequently of logic in general.

But let us assume that not all of the content of logic arises from one logical definition and description of [the notion of] the logical, [but] that logic is reducible to several propositions which are themselves not further reducible. Let us assume that logic does not arise from the simple analysis of a definition of logic, but from a synthesis of several independent axioms. Then we can consider the following case. Suppose we retain some of the axioms of logic and reject some [other] axioms, which are synthetic additions to the former ones. By virtue of the admitted independence of the axioms, the corollaries of the retained axioms will remain valid here too, and [they] will give us the possibility to construct a logic without the rejected axioms.

[58] This is exactly what happens in logic when it is considered as founded upon the synthesis of several independent axioms[1]. This also corresponds to the traditional idea of four independent laws of thought ([the law] of identity, [the law] of contradiction, [the law] of the excluded middle, and [the law] of

[1] Mathematical logic, which is based on several axioms and postulates, can be considered as an elegant demonstration of this [thesis].

sufficient reason). Therefore, since logic is a synthesis of several independent axioms, we are to conclude that the rejection of some axioms and the construction of a logic without them is quite conceivable.

Exactly in the same way, the construction of non-Euclidean geometry was possible only because the 5th postulate of Euclid is an axiom and not a theorem, the demonstration of which had not yet been found, as it was believed prior to Lobachevski. Non-Euclidean geometry is possible because the rejected axiom of parallels is not reducible to the other Euclidean axioms which are preserved in non-Euclidean geometry. In its turn, the construction of a system of imaginary geometry by Lobachevski served as a demonstration of the independence of the 5th postulate and of the impossibility of its reduction to other axioms. It may be said, then, that the axiomatic character of the 5th postulate is the *ratio essendi* of non-Euclidean geometry, and non-Euclidean geometry is the *ratio cognoscendi* of the axiomatic character of the 5th postulate.

These preliminary remarks are intended to show that a logic different from ours is conceivable, [and] to demolish the prejudice in favor of the unicity of our logic[2]. But our aim is more ambitious. It consists in showing the 'knowability' of this "alternative logic." We can admit that God's logic, for example, is completely different from ours, but at the same time we can be certain that this divine logic is completely beyond the grasp of the human mind. Our aim is to show that by means of reason it is possible, too, to construct a logic different from ours and to construct a system — or at least the foundation of a system — of imaginary logic.

Let us, now, turn to this task.

II

[59] Imaginary logic is a logic without the law of contradiction. Therefore, before proceeding to the construction of an imaginary logic we have to formulate exactly the rejected axiom in order to avoid any misunderstandings. At the

[2] Benno Erdmann* defends the possibility of a different logic and of different logical laws**. Although my views differ from his in many ways, they differ even more from those of Husserl, who has criticized Erdmann's ideas in detail in his "Logical investigations." See Husserl E., Logical Investigations, St. Petersburg, 1909, Vol. 1 (in Russian translation).

* Erdmann, Benno (1851–1921). German philosopher and logician esp. known for his work on Kant (cfr. Ziegenfuss, W. Philosophen-Lexikon, p. 292ff. Berlin 1949). (*R.V. – E.Z.*).

** See, Erdmann B. Logik. B., 1907. Bd. 1. S. 527sq.

same time, from an analysis of the law of contradiction the possibility of its rejection and of carrying out logical reasoning without it will transpire.

The law of contradiction expresses the incompatibility between an assertion and [its] negation. A cannot be non-A. No object contains a contradiction, [i.e.] allows us to at once make an affirmative and a negative proposition* (about it).

But if we ask ourselves what in fact negation is, we can define it only in one way: *negation is that which is incompatible with affirmation*. We call 'red' the negation of 'blue' and say that a red object is not a blue one, because red is incompatible with blue. Where there is no incompatibility, we are not allowed to speak about negation.

Thus, in the case of a simple difference we cannot speak about negation. When we think about something that is not blue, we think about something red, white, orange, etc., which is all that which is incompatible with blue, but we do not think about something dry, and dry can in no way be called the negation of blue.

Exactly in the same way, we cannot speak about negation in the case of the simple absence of a predicate. What does it mean that a given object A does not have the predicate B? I cannot convince myself of this in a direct way, since we have no sense of absence, i.e. [we have] no means to convince ourselves directly, via perception, of the absence of a predicate. I can only be convinced in a mediate way, by comparing my perception, or conception, of the object A with the predicate B. *But the simple absence of the predicate B in my perception or conception of the object A cannot serve as a logical ground for a negative proposition.* Suppose I have never noticed in a person any sign of moral nobleness. This would (by itself) not constitute a logical reason to call him ignoble. I can call him/her ignoble with sufficient reason only then if I know that some of his/her acts are incompatible with moral nobleness. Suppose, furthermore, that I don't see a certain object in a room, then this does not mean yet that it is not there. I can only say with sufficient
[60] reason that an object is not in the room if in every place in the room I have found objects that were different from the sought one. Here, the presence in the room of these other objects excludes the possibility of the presence in it of the object sought, since two things cannot occupy the same place. Moreover, the simple absence of a visual perception of the object sought is not yet a guarantee that it is not [there]. It can be there, but at the same time be absent from my visual perception or impression.

* Vasil'ev uses the word 'suzhdenie', which can be translated as 'judgment'. We have, however, chosen to translate it as 'proposition', which is the more modern terminology. We are aware of the more or less subtle differences in meaning that may exist here (cfr. Kline 1965, p. 318, who also uses 'proposition') (*R.V. – E.Z.*).

About absence [in general] the following should be noticed. Expressions like — "a (certain) property is absent," "I don't see the property," "I don't hear a (certain) word," are extremely vague. It is impossible "not to see," "not to hear." The mind has no negative functions. "Not to see something" — means to see something different or, in other words, it means to hear, to think, to perceive something definite. The expressions "I don't see, I don't perceive a certain predicate," or "a certain predicate is absent" all mean that I observe something different and compare it with the given predicate. Having determined the difference between what I have seen and the given predicate, I then subsequently can say: "I don't see, I don't observe, the given predicate." But the simple difference between the real and the expected images of the object, as every simple difference, cannot constitute a reason for negation. Only if in the real image of the object there are properties which exclude the expected image, I can say that the expected image is, in fact, absent. If, e.g., I have no gustatory sensation of a glass with a colorless liquid, if I have no taste of 'water', I cannot affirm, that what is in the glass is not water; only when I taste it and get the gustatory sensation of vodka, which excludes the possibility of there being water in the glass, I can say: [that, which is] in the glass is not water. Thus, absence can serve as a reason for a negative proposition only when it can be reduced to incompatibility. *In general, it can, then, be said that the only logical basis for negation is incompatibility.*

All negative propositions about objects and perceptions of our world are obtained as inferences derived from propositions about the incompatibility of two properties. I cannot see in a direct way that a given object is not white. We have no negative perceptions, as e.g. the perception of [being]
[61] "not white." I can have only definite positive perceptions of e.g. red, blue, black, etc. . . When I assert that a certain object is not white, I have undoubtedly made an inference. I *saw* that a certain object was red, and I *have inferred* — knowing that red cannot be white — that the object was not white. Here we are dealing with an inference, namely with a syllogism of the first figure: what is red cannot be white (major premiss). This object is red (minor premiss). Therefore, this object is not white (conclusion). In general, in a similar way all negative propositions about objects, perceptions, or facts are obtained. *We can negate the property P in the object only when we affirm [the presence] in it [of] a property N, which excludes P.* Here again, we have a syllogism:

N excludes P, [that is, N] is incompatible with P (incompatibility clause)
S is N (minor premiss)

S is not P (inferred negative proposition).

As a rule, we do not notice that in negative propositions we are dealing with inferences. This is because such an inference is something we are very much used to perform and thus it is performed so quickly and automatically, that it is not even a conscious operation. The major premisses of the negation, e.g. propositions about incompatibility like 'red cannot be white', being absolutely obvious, drop out as unnecessary links in our psychological train of thoughts, so that we at once go from the perception of the red color of an object to the proposition: "it is not white." But with respect to logic, such an abridgement of the psychological process cannot serve as an argument against its syllogistic nature. Links that are psychologically unnecessary, may be necessary logically, e.g. for the soundness of thought, [(and) for] its conclusive character.

Thus, in our logic negative propositions are of two kinds: firstly, major premisses of negation (red is not white, the property N excludes the property P); *secondly, inferred negative propositions*, which are obtained from the former ones by means of a syllogism.

Let us, now, briefly review what has been said before[3]. The law of contradiction states the incompatibility between an affirmation and a negation, and negation is that which is incompatible with affirmation. *From that, it is clear that the law of contradiction is already implied in the definition of negation.* It is, then, not difficult to see, why the law of contradiction is never violated in our logic.

[62] Should the proposition A and its negation B ever coincide, we would by no means say that the law of contradiction is violated, but we would conclude that we incorrectly called B the negation of A, since negation is by definition that which cannot coincide with affirmation. The law of contradiction is as stable as the truth that the Earth rotates around its axis in twenty-four hours. Should its rotation be faster or slower, it would still be accomplished in twenty-four hours, since by 'twenty-four hours' we just mean the time of the rotation of the Earth around its axis.

But both the proposition about the rotation of the Earth around its axis in twenty-four hours and the law of contradiction are not simple tautologies. The former presupposes the rotation of the Earth around its axis, and the latter presupposes the existence of incompatible predicates. Without the existence of incompatible predicates, there would be no negation in our sense of the word and consequently no law of contradiction, which is only a corollary of the definition of negation, and of its main property which is that it is reducible to incompatibility.

[3] Unfortunately, the lack of space prevents us from giving a detailed account of the logic of negative propositions.

Since the law of contradiction is a consequence of the definition of negation, constructing a logic without the law of contradiction amounts to constructing a logic without our negation which is reducible to incompatibility.

That is where imaginary logic originates. Its method consists in constructing a negation different from ours and in the generalization of the concept of a negative proposition. A negative proposition such as "S is not P" has two aspects. The first is a formal one: a negative proposition states the falsehood of the affirmative one, of "S is P." The second is a material one: a negative proposition is based upon the incompatibility of predicates; it is either a proposition of incompatibility or a consequence of such a proposition. One should accurately distinguish between these two aspects. The formal aspect manifests [the fact] that the truth of a negative proposition implies the recognition of the falsehood of the affirmative one, but it leaves open the question on what grounds we can ascertain the truth of negative propositions. The material aspect gives an answer to this question. Therefore, the formal aspect manifests the properties of negation; the material aspect manifests the grounds for negation. While preserving the formal aspect, we can change the material one and then obtain a different kind of negation.

[63] Only our affirmative propositions about objects and facts are immediate, that is, based on perception and sensation; the negative ones are always inferred.

That is how matters stand in our world and in our logic. But let us imagine a different world where negative propositions would be just as immediate as affirmative propositions [are] for us, where experience itself would convince us without any inference that "S is not P". Such propositions would remain negative, since they would retain the formal property of our negative propositions, which consists in stating the falsehood of the affirmative ones. But this negation would be different from ours, since it would be based on immediate perception, and not on propositions about incompatibility or inferences from them. It would have a different material aspect. In other words, in our world immediate perception provides us with only one kind of propositions — the affirmative ones — but we can image a logical world and a logic in which immediate perception generates two kinds of propositions: affirmative ones and negative ones.

But then it is possible that in some object the grounds for both affirmative and negative propositions coincide. This is impossible in our Aristotelian logic, because of the link between negation and incompatibility, a link which is severed in imaginary logic. Therefore, imaginary logic is a logic with a different kind of negation and without the law of contradiction. Later on, we will give a hypothetical interpretation of this different negation and show what properties the world has to possess in order for such a "different negation" to be possible in it. At that moment, the very concept of this different negation will become clear. But let us for a moment stick to the purely logical domain without resorting to any additional hypotheses. In order to remain as general as possible, we will not further specify negation in imaginary logic and we will say that a negative proposition is a proposition which declares the affirmative one to be false but not, however, on the grounds of incompatibility. The introduction of such a negation is, in fact, equivalent to the rejection of the law of contradiction.

In order to avoid any misunderstanding, it is necessary to distinguish now between the rejected law of contradiction and another one which is (sometimes) confused with it and which cannot be rejected. We would like to call
[64] this law *the law of absolute difference between truth and falsehood*, which can be formulated as follows: "*One and the same proposition cannot be true and false simultaneously.*" It is impossible to reject this law, since anyone who would reject it, and therefore confuse truth and falsehood, would stop to reason logically at all. Therefore, this law remains valid in imaginary logic as well.

Usually, one does not distinguish between the law of contradiction and the law of absolute difference between truth and falsehood since one believes that they are just different formulations of the same law of contradiction. In fact, this is at the origin of a well-known controversy about the formulation of the law of contradiction.

There exist two major formulations of this law. One is that of Kant: "Nothing can have a predicate which contradicts it."[*] It is this formulation of the law that we have in mind.

The other formulation is stated by Sigwart[*][4] in the following way: "The propositions, "A is B," and "A is not B," cannot both be true at the same time." Since for Sigwart a negative proposition "A is not B" is an expression of the falsehood of the affirmative [proposition] "A is B", it is clear that in his formulation of the law of contradiction he does not allow the simultaneous acceptance of the proposition "A is B" in the affirmative statement "A is B," and the denial of the proposition "A is B" in the negative statement "A is not B,": in other words is, he is actually speaking about the law of absolute difference between truth and falsehood itself.

It is not difficult to see that the law of contradiction and the law of absolute difference between truth and falsehood are not two formulations of one and

[*] I. Kant, "Critique of Pure Reason," Petrograd, 1915, p. 124.

[*] Sigwart, Christoph von (1830–1904). German logician and a representative of psychologism (cfr. Eisler E., Philosophenlexikon p. 677ff., Berlin 1912, 1977[2]) (*R.V. – E.Z.*).

[4] The former formulation is usually called anti-Leibnizean, the latter Aristotelian.

the same law at all, but two completely different laws[5]. The law of absolute difference between truth and falsehood applies to the cognizing subject and forbids him/her to contradict him/herself; [it] indicates that a true proposition is always true, and a false one always false, and that therefore he/she cannot declare one and the same proposition now true, now false. This law
[65] forbids self-contradiction; [it] imposes "self-consistency", the coherence of propositions of the cognizing subject. Therefore, it could be called the law of "non-self-contradiction".

Contrariwise, the law of contradiction applies to the world of objects, and implies that contradictions cannot be realized in them, i.e. that in no object can contradictory predicates be realized, [that] there cannot exist at the same time grounds for [both] affirmative and negative propositions [about them]. The [latter] law banishes contradictions from the world, as the former one banishes them from the subject. The law of contradiction has an objective value, whereas *the law of absolute difference between truth and falsehood [has] a subjective one*[6]. Therefore, it is clear, that one can —

[5] In Sigwart, there are some vague hints to this extent. "One requires no lengthy argument [to understand] that Kant speaks about something completely different from what was meant by the original law of contradiction". (*Sigwart Ch.*, Logic, SPb, 1908, V. 1, p. 163, in Russian translation). This difference is even more evident in Spir[**].

[**] Spir, Afrikan (1837–1890). Ukrainian neo-Kantian philosopher (cfr. Eisler, E. Philosophenlexikon p. 704, Berlin 1912, 1977[2]) (*R.V. – E.Z.*).

[6] Indicating the ambiguity of the traditional law of contradiction to the extent that there are in fact not one but two laws will help us to shed light upon a problem which in Russia has recently attracted much attention. It is the problem of whether logical laws, and in particular the law of contradiction, preserve their validity in the intelligible world for example in the world of the Things-in-Themselves (Dinge-an-sich). It is known that Prof. Vvedenski and Lapshin (*on Vvedenski and Lapshin, see 'The Worlds of Logic and the Logic of Worlds' (*Logique et Analyse, *this issue) R.V. – E.Z.*). answer this problem negatively; in their view, the application of the law of contradiction to the intelligible world becomes highly problematic.
However, all depends on which of the two laws we mean. If we mean the law of contradiction, then, of course, this law loses its validity in the intelligible world since it derives its force from experience, from the experience of the ascertained existence of incompatible predicates and it is, in fact, unquestionable only in the world of experience. But the Thing-in-Itself or God can be considered as the union of contradictory predicates, as a *coincidentia oppositorum*, a view which has been held more than once in the history of philosophy.
But the law of absolute difference between truth and falsehood, the law of non-self-contradiction preserves its validity also in the case when we argue about the intelligible world. Within God, contradictory predicates may be realized, but there can be no self-contradictions in our affirmations about God. As soon as I say: "It is certain (right) that the law of contradiction is not applicable to God," I cannot say afterwards "It is necessary that the law of contradiction is not applicable to God." *(Indeed, since the modalized proposition starting with "It is necessary that", if true, states that it cannot be not the case that the law of contradiction is not applicable to God and the proposition starting with "it is certain (right) that"*

without violating the law of absolute difference between truth and falsehood or the law of non-self-contradiction — violate or reject the law of contradiction. If I affirm, that this NN is at the same time both a human being and not-a-human-being, I do, of course, violate the law of contradiction, but if
[66] I always affirm it and firmly hold it, without contradicting myself, I am not at all violating the law of absolute difference between truth and falsehood at all. Therefore, imaginary logic, which builds upon the negation of the law of contradiction, violates nowhere the law of absolute difference between truth and falsehood. It nowhere contradicts itself and is (thus) a system which is devoid of self-contradictions.

The law of absolute difference between truth and falsehood is not only not rejected in imaginary logic but, to the contrary, it is at its basis. We have seen that in imaginary logic negation is characterized by being justified factually or immediately, just like an affirmative proposition. Assume that fact a is the basis for the affirmative proposition "S is A," and fact b the basis for the negative proposition "S is not A." The relation between the facts a and b is not, as normally, the relation of incompatibility. Consequently, it is quite possible that the facts a and b can co-exist simultaneously. What will happen in this case? By virtue of the fact a, the affirmative proposition "S is A" is true; by virtue of the fact b, it is false. On the other hand, by virtue of the fact a, the negative proposition "S is not A" is false, while by virtue of the fact b, it is true. Thus, in this case, both propositions — the affirmative and negative one — turn out to be at the same time true and false. But that is not allowed by the law of absolute difference between truth and falsehood. Therefore, there should exist in the case of the simultaneous existence of the facts a and b a third (kind of) proposition, which will be true here. *We will call this third kind of proposition — which reveals the presence of a contradiction in the object* S*, [that is,] a coincidence within it [i.e. within this object] of the grounds for both an affirmative and a negative proposition — a proposition of contradiction, or, better, an indifferent proposition, and we will denote [it] as follows: "*S *is* A *and is not* A *[simultaneously]"*. In the case of imaginary logic, in which a negation is introduced [which is]

states that the law of contradiction does not apply to God, there is an inconsistency. If the latter proposition is true then it is also true that God falls outside of the scope of the law of contradiction. Consequently one cannot affirm anything with necessity about God, since if the law of contradiction does not hold in the case of God there is nothing impossible in this case. Put simply: if the law of contradiction does not hold in the case of God, nothing is logically impossible with God and therefore nothing can be affirmed with (logical) necessity about God. R.V. – E.Z.)

A bad philosopher would be he who would justify the self-contradictions in his/her metaphysics by referring to the fact that it deals with the intelligible world. A person should be consistent in his/her affirmations both about the empirical world and about the intelligible world.

different from ours, all propositions are divided according to quality not into two forms as usually, [that is,] affirmative and negative ones, but into three: affirmative "S is A," negative "S is not A" and the indifferent proposition "S is A and is not A [simultaneously]." Just as either the affirmative or the negative proposition is true, so in imaginary logic, in each given case one of the three forms is true: either the affirmative one or the negative one, or the indifferent one. *This is the most important peculiarity of imaginary logic.*
[67] *It introduces a threefold division of propositions according to their quality and a new kind of proposition, viz. an indifferent one, alongside with the affirmative and the negative ones.*

From this also other peculiarities follow. Normally, an affirmative proposition is false in the case of the negative proposition being true. However, in imaginary logic, an affirmative proposition is false in the case in which the (corresponding) negative and indifferent propositions are true. *Negative propositions in imaginary logic turn out to be only of one of the two forms declaring affirmative propositions false.* In general, each of these forms — affirmative, negative, or indifferent — is false, when one of the two remaining is true.

Let us dwell at some length on this major point of imaginary logic, [that is, on] the possibility of indifferent propositions, i.e. of propositions [which are] not submitted to the law of contradiction. We have up to now given a purely logical demonstration of such a possibility by generalizing the notion of a negative proposition and by introducing a negation different from ours. But the same point can be demonstrated epistemologically, by elucidating the nature of the law of contradiction and its epistemological value.

The law of contradiction is a law which is empirical and real. [It is] empirical, because it addresses the fact of the existence in our world of incompatible predicates, something which can be ascertained only through experience. If there had been no incompatible predicates, then "our" negation would not exist, and thus, the law of contradiction would not be true. "A is not non-A" is true only because in our world there exist predicates that are incompatible with A, and we call these predicates non-A. The law of contradiction is an abbreviated formula which in itself comprises infinitely many facts, such as e.g., that red is incompatible with blue, white, black, etc., silence with noise, rest with motion, etc. But all these particular incompatibilities can be asserted only from experience.

The law of contradiction is a real law, because it applies not to thoughts, but to reality. It applies not to propositions, but to objects. It states that in reality, no contradiction can exist, that contradictory properties cannot be present within objects. A red object cannot be blue, a circle cannot be square; such propositions, which follow from the law of contradiction, are propositions about red objects, circles... All of them are propositions characterizing these objects.

[68] *Contrariwise, the formal laws of thought apply to thought only, and not to reality; they apply to propositions, and not to objects.* They are laws about propositions and about propositions only. In contrast to the empirical changeability of things, the law of identity, e.g., establishes the logical constancy of concepts, i.e. of parts of propositions. It is a law about propositions and [it] tells absolutely nothing about objects. Exactly in the same way the law of absolute difference between truth and falsehood and the law of sufficient reason tell us about propositions only, and not about objects. Consequently, one should strictly distinguish between them and those real laws (about objects) they can easily be confused with. A formal law of propositions — e.g. the law of sufficient reason, "every proposition has to be justified" — should be distinguished from the "real" law of causality which says that "every phenomenon must have a cause." A formal law such as "propositions should not contradict each other" should be distinguished from a real law such as "there is no contradiction within objects." *Thus, the real law of contradiction stands to the formal law of absolute difference between truth and falsehood just as the real law of causality stands to the formal law of sufficient reason.* We have seen that one can deny the law of contradiction without denying the law of absolute difference between truth and falsehood. Exactly in the same way, the denial of the law of causality does not imply the denial of the law of sufficient reason. Indeterminism is the negation of the universal validity of the law of causality, but it too should be justified and demonstrated according to the law of sufficient reason.

Since the law of contradiction is an empirical and real law, we can reason without it as well, and then we will get an imaginary logic. *In fact, on empirical grounds I can arbitrarily build whatever imaginary objects and imaginary disciplines.* I can create centaurs, sirens, griffins and imaginary zoology. I can create utopias, an imaginary sociology, or an imaginary history — Uchronie —, as did Renouvier[*][7]. Empirical and real laws are about reality, but their opposite is always conceivable.

[69] In corroboration of our analysis of the law of contradiction, we can refer to Prof. Vvedenski. This author teaches that the law of contradiction is a natural law of our representation, since a contradiction cannot be conceived of. Thinking itself is not subject to this law, however, because we are able to think a contradiction. Thus we are able to think of, though not able to imagine, a round square or God's Trinity. Thinking is subject to this law as a

[*] Renouvier, Charles (1818–1903). French (neo)-Kantian philosopher (cfr. Eisler, E. Philosophenlexikon p. 591ff., Berlin 1912, 1977^2) (*R.V. – E.Z.*).

[7] On the other hand, our imagination has created these imaginary objects precisely by violating the law of contradiction, and by the unison of incompatible predicates within one object, for example, "human being" and "horse".

norm when we aim at adjusting our thoughts to our representation for which the law of contradiction is a natural law.

From Prof. Vvedenski's theory, which seems to us to be correct, the idea of an imaginary logic must follow as an unavoidable corollary. It also follows from this that if we do not adjust our thoughts to our representations, but instead think of an imaginary world, a world of different representations, we can think without the law of contradiction, and think a contradiction. Every real thought is always manifested in a proposition. Therefore, to think a contradiction actually means to form a special kind of proposition of contradiction, viz. an indifferent one, alongside with the affirmative and negative ones.

III

Now we only have to show that with this new form of propositions — the indifferent ones — we can operate logically, that they can enter as a constituent part in logical inferences; we have to show that in the case of the rejection of the law of contradiction, there remain logical rules and logical laws; then, our task — to show the possibility of an imaginary logic — will be completed. As soon as there are propositions and inferences that are subject to strict rules, and where there is a difference between truth and falsehood we have to speak about a logic, even if these rules are not similar to ours.

Thus, let us show that in the case in which there is no law of contradiction, there remains the possibility of logical inference and [that] logical rules can be constructed.

We will assume that all remains just as it was in our world and in our logic: the knowing subject and a known reality (remain). An internal and an external world (also remain). Sensory experience which gives rise to facts, and knowledge of facts which is obtained through concepts and rules. Finally, we have to think of a language or some kind of symbolization procedure for the logical operations. Further, we have to consider a knowing subject endowed
[70] with the same discursive structure of mind as ours. We accept all major logical categories: facts, propositions and inferences. Furthermore, we have to think of the logical laws of identity, the absolute difference between truth and falsehood, and of sufficient reason as preserved. Only one thing has changed: negative propositions will not be based upon incompatibility.

On the Proposition

"There can be no doubt that all our understanding begins with experience, when it processes the raw material of the sense-impressions,"* these words, that open "The Critique of Pure Reason"** can serve as the opening words for our imaginary logic. In it too, understanding builds propositions out of sense impressions; at the outset, [these are] propositions about an individual S. The predicates of these propositions are [constituted by] those sense impressions, that are transmitted to the mind by a given S. *They constitute the singular propositions, and they are the starting point for imaginary logic, just as for our logic. But in imaginary logic, these propositions — as all propositions in general — can be of three kinds. They can be affirmative, negative, or indifferent* (if in a given individual S the grounds for both an affirmative and a negative proposition coincide, i.e. if S at the same time is and is not P).

Consequently reason constructs the concept of S, the class S, and passes on to propositions about the concept [S], [and] about the class S. The process of cognition is then complete, and the knowing [subject] knows that either 1) all individuals S possess the predicate P, and he/she constructs [thus] a universal affirmative proposition, or 2) he/she knows that all S are not P, and constructs a universal negative proposition, or, finally, 3) he/she knows that all individuals S at the same time are and are not P, and constructs a universal indifferent proposition. These are the three kinds of so-called universal propositions and if a proposition in imaginary logic is universal, it will be of one of these three kinds.

It may happen, however, that not all S possess some given predicate; here, we are dealing with *accidental propositions* which can be of four kinds. Accidental propositions of the first kind occur when some S are P, and the others are not P. Accidental propositions of the second kind occur when
[71] some S are P, and the others are and are not P at the same time. Accidental

* In his citation, Vasil'ev fused into one, in an essentially abridged form (using a wrong translation?), the first two phrases from Kant's Critique of Pure Reason, which in German read as follows:
"Daß alle unsere *Erkenntnis* mit der Erfahrung anfange, daran ist gar kein Zweifel; denn wodurch sollte das Erkenntnisvermögen sonst zur Ausübung erweckt werden, geschähe es nicht durch Gegenstände, die unsere Sinne rühren und teils von selbst Vorstellungen bewirken, teils unsere *Verstandestätigkeit* in Bewegung bringen, diese zu vergleichen, sie zu verknüpfen oder zu trennen, und so den rohen Stoff sinnlicher Eindrücke zu einer Erkenntnis der Gegenstände zu verarbeiten, die Erfahrung heißt?" Thus, Kant uses in the first proposition Erkenntnis ("knowledge") and in the second one — Verstandestätigkeit ("understanding") (*R.V. – E.Z.*)

** Kant, I., Critique of Pure Reason, Spb 1902 (in Russian), p. 26.

propositions of the third kind occur when some S are not P, and the others are and are not P at the same time. Finally, there is a last case of accidental propositions of the fourth kind [which occurs] when some S are P, some are not P, and all the rest [of S], are P and are not P at the same time. *When we consider a class S in relation to a predicate P, all possible cases are, in fact, limited to these 7 ones: the 3 general and the 4 accidental ones.*

In addition to these forms of propositions, certain 'preliminary' or exclusive forms should be noticed as well.

In our (Aristotelian) logic, stating the falsehood of an affirmative proposition is equivalent to a negative proposition, and stating the falsehood of a negative proposition is equivalent to an affirmative one. The situation is different in imaginary logic. There, stating the falsehood of an affirmative proposition excludes an affirmative proposition, but leaves open the question as to which of the two (other) propositions — the negative or the indifferent (one) — is true. Just in the same way, stating the falsehood of a negative proposition excludes a negative [proposition], but leaves a vacillation between the affirmative and the indifferent propositions. Quite the same happens in the case of the falsehood of an indifferent proposition. All of this can be summed up in the following table.

[Mutually] exclusive forms:

1) The form of exclusion of an affirmative proposition = the falsehood of the affirmative proposition = vacillation between a negative and an indifferent proposition
2) The form of exclusion of a negative proposition = the falsehood of the negative proposition = vacillation between an affirmative and an indifferent proposition
3) The form of exclusion of an indifferent proposition = the falsehood of the indifferent proposition = vacillation between an affirmative and a negative proposition

We will call these [mutually] exclusive forms preliminary [ones], since they represent the beginning of the process of cognition, viz. a vacillation between two possibilities, one of which should be chosen in order to bring that process to an end. Therefore, these forms represent some analogy to the indefinite propositions of our logic which are of the form "Some, maybe all, S are P." This form also excludes, i.e. declares as false, the universal negative proposition "All S are not P," but leaves a vacillation between
[72] the universal proposition "All [S] are P" and the particular [proposition] —

"Only some S are P."[8] These [mutually] exclusive forms play an important role in imaginary logic, in particular in the doctrine of *conversion* of propositions; but I cannot dwell on them because of lack of time.

Thus, in imaginary logic, the doctrine of the proposition can — in general — be represented in the following way:

According to quality, propositions are divided into three kinds: affirmative, negative and indifferent ones.

According to quantity, they are divided into singular [propositions] and propositions about classes or concepts, which can be either universal or accidental, [the latter] corresponding to our particular [propositions].

In addition to that, there are [mutually] exclusive forms, analogous to our indefinite propositions.

Thus, in the division of propositions according to quality, a new class [consisting] of indifferent propositions is added, but in the division of propositions according to quantity, the same classes as in Aristotelian logic remain.

On the Syllogism

Let us pass to the doctrine of the syllogism in our logic and, for the sake of brevity, let us primarily deal with the first figure. *The principle of the syllogism of the first figure does not depend upon the law of contradiction.* Indeed, it should be clear by now that the law of contradiction expresses a relation between affirmative and negative propositions, whereas in the first figure only purely affirmative moods are possible. Such is, for example, the case for the main mood of the first figure — Barbara.

Negation does not enter in Barbara and consequently the law expressing the relation between affirmation and negation has no influence here. However, this is only a preliminary remark; when in a logic without the law of contradiction the possibility of inference according to the first figure is shown, its very independence from the law of contradiction will also become clear.

[73] Indeed, we have seen, that in a logic without the law of contradiction there can be universal propositions, and, incidentally, also universal indifferent [propositions]. *If, in the first figure, we take a universal indifferent proposition as the major premiss and an affirmative proposition as the minor premiss, we have an inference according to the first figure.* The following syllogism, then, results:

[8] For more details on this, see my paper "On particular Propositions, the Triangle of Oppositions, and the Law of Excluded Fourth" (Uchionye zapiski Kazanskogo universiteta (Scientific Notes from the University of Kazan), 1910, October).

All M are and are not P simultaneously.
S is M.
Therefore, S is and is not P simultaneously.

This indifferent conclusion is just as inescapable as are the conclusions — i.e. affirmative and negative ones — according to the first figure. What is the sense of the minor premiss "S is M?" Its sense consists in that all propositions, that are true with respect to M, are true with respect to S. When we consider the proposition "Caesar is a human being," we think that all that is true with respect to human beings is also true with respect to Caesar. All human beings are mortal, (therefore) Caesar is also mortal. In our (Aristotelian) logic only affirmative and negative propositions can be true with respect to M, and therefore conclusions with respect to S can be only affirmative or negative. In a logic without the law of contradiction, the indifferent proposition "M is and is not P simultaneously" can be true with respect to M, and since S is M, we are compelled to infer [that] "S is and is not P simultaneously." If there is a contradiction in the concept M, and S is subsumed under this concept, then S is to possess this contradiction as well. If a contradiction is intrinsic to the whole class M then it goes without saying that it will be also intrinsic to any S which belongs to this class.

The logical lawfulness of the indifferent mode of the first figure and of an indifferent conclusion can be demonstrated by a reductio ad absurdum. Let us consider the following syllogism:

All M are and are not P simultaneously.
All S are M.
Conclusion. All S are and are not P simultaneously.

It can be shown that, if we refuse to draw this conclusion, we are forced to contradict ourselves, that is, to violate the law of absolute difference between truth and falsehood. And that is why we are forced to draw this conclusion.

Let us try to reject the conclusion "All S are and are not P simultaneously," [that is,] let us consider it false. Then we will have to deduce two
[74] further propositions: 1) some, but maybe all, S are P and 2) some, but maybe all, S are not P. Indeed, in any given case, either the affirmative, the negative, or the indifferent proposition is true, and since we have denied the indifferent proposition, we will have to vacillate between the affirmative and negative ones. By taking these sentences as the major premisses and the minor premiss of the given syllogism as the minor [premiss], we obtain two syllogisms of the third figure with middle term S:

Disamis (for the affirmative case)
Some, but maybe all, S are P.
All S are M.
Conclusion: Some M are P.

Bocardo (for the negative case)
Some, but maybe all, S are not P.
All S are not M.
Conclusion: Some M are not P.

But these conclusions ascribe respectively an affirmative and a negative predicate to some M. Then, the major premiss of the given syllogism is false, whereas it [the major premiss] ascribes an indifferent predicate to all M. Therefore, we are to draw an indifferent conclusion in this syllogism, since otherwise we are contradicting ourselves.

In the case of an indifferent minor premiss, we don't obtain any inference at all, in the same way as we don't obtain one in our (Aristotelian) logic if the minor premiss is negative in the first figure. Quite in the same way, no inference can be drawn if we take a singular, accidental, or exclusive form as a major premiss; the major premiss must always be general. *Thus, in imaginary logic the general formal rule of the first figure is preserved: the major premiss should be general, and the minor [premiss] affirmative. But the number of moods of the figure has changed: to the four moods of our logic (two affirmative and two negative ones) two more, indifferent [moods] are added*, viz. Mindalin — the universal indifferent [mood] — and Kindirinp — the particular indifferent [mood].

Universal indifferent [mood]

All M are and are not P simultaneously.
All S are M.
All S are and are not P.

Particular indifferent [mood]

All M are and are not P simultaneously.
Some S are M.
Some S are and are not P.

[75] Thus, in imaginary logic the first figure has six moods. Other moods are impossible. Sorting out the impossible moods in imaginary logic is done by means of methods which are analogous to ours (i.e. methods in Aristotelian logic R.V. – E.Z.), but only more complicated.

Attention should be drawn to a symmetry that exists in the first figure: By replacing the affirmative quality of the proposition by the negative [quality] in the major premiss of the affirmative moods Barbara and Darii, we get the negative moods Celarent and Ferio; by replacing it by the indifferent [quality], we get the indifferent moods Mindalin and Kindirinp.

The 2nd figure [of the syllogism]. Something curious turns up when we consider the second figure. In imaginary logic, no new moods are added there, while the old ones — Cesare, Camestres, Festino, Baroco — do not result in an unambiguous conclusion. *The only thing that can be inferred in imaginary logic in the 2nd figure is that the conclusion cannot be affirmative.* Since besides affirmative [propositions] negative and indifferent ones are possible in imaginary logic, [the question of] which conclusion — a negative or an indifferent one — can be drawn in the 2nd figure, remains open. The formal rule governing the 2nd figure of imaginary logic runs as follows: 1) the major and the minor premisses should be of a different quality (affirmative and negative, affirmative and indifferent, or negative and indifferent) and 2) the major premiss should be universal.

The conclusion will always be [represented by] an exclusive form, namely a form which excludes affirmative propositions.

The proofs of all these properties of the 2nd figure are very complicated and require an exposition of many particular theorems of imaginary logic. Therefore, I will omit them here.

The 3rd figure. In the third figure there remain 6 moods of our (Aristotelian) logic and 3 new indifferent ones are added. Here too, the same symmetry shows up that we noticed in the 1st figure. If, in the affirmative moods of our 3rd figure the quality of the major premiss changes from affirmative to negative, then we accordingly get from the affirmative moods — Darapti, Disamis, Datisi — the corresponding negative moods Felapton, Bocardo, Ferison. If the quality of the major premiss is changed to indifferent, we accordingly get indifferent moods:

[76] 1) (from Darapti)

All M are P and are not P simultaneously.

All M are S.

Conclusion. Some S are and are not P simultaneously.

2) (from Disamis)

Some M are and are not P simultaneously.
All M are S.
Conclusion. Some S are and are not P simultaneously.

3) (from Datisi)

All M are and are not P simultaneously.
Some M are S.
Conclusion. Some S are and are not P simultaneously.

Thus, in the 3rd figure of imaginary logic, there will be 6 of our (Aristotelian) moods and 3 new indifferent [ones], all together 9 [moods]. But here too, the formal rule governing the 3rd figure remains valid: the minor premiss should be affirmative.

This difference in fortunes between the first and the third figures on the one hand, and the second [figure] on the other hand seem to us to be an additional argument for the correctness of prof. Karinski's theory*, who sharply distinguishes inferences according to the first and the third figures from inferences according to the second [figure].

A generalization of the idea of imaginary logic. The idea of imaginary logic can be extended even more. Our imaginary logic has affirmative, negative, and indifferent propositions. However, the question may emerge whether a logic with a greater number of qualitative differences between the propositions than these three is conceivable? That is indeed quite conceivable. As Spinoza conceived of God with an infinite number of attributes, of which only two — thinking and extension — are accessible to us, so we can imagine logical systems with an arbitrary number of qualitative differences between the propositions, of which only two — the affirmative and the negative ones — are accessible to us.

We can imagine a logical system with n kinds of qualitative differences between the propositions, and we shall call such a system a logical system of order n, or an n-dimensional [logical system]. Accordingly, our "terrestrial" logic will be a system of the 2nd order, or a two-dimensional [system], while imaginary logic without the law of contradiction will be a system of the 3rd order, or a three-dimensional [system].

* Karinski, Michael (1840–1917). Russian logician and epistemologist (cfr. Lossky, N.O., History of Russian Philosophy, London 1952, pp. 145–149 and Zenkovsky, V.V., A History of Russian Philosophy, London 1953, vol. 2, pp. 584–585) (*R.V. – E.Z.*).

Analogously, plane geometry is a two-dimensional geometry, and the geometry of space is a three-dimensional geometry. But we can imagine,
[77] though we cannot represent, a space of four or more, for example n, dimensions, and a geometry of $4 \ldots n$ dimensions. Thus, we can imagine a logic of n dimensions, or of the order n, though we are unable to clearly represent it. But we can not only imagine such a logic; we are even able to generalize some formulae for a logic of n dimensions. Since we imagine our space as three-dimensional, we must firstly imagine these 3 dimensions and secondly that there is no fourth dimension. Conversely, a being [living] on a plane would imagine two dimensions and he would think that there is no third one. In general, in order to imagine a space of n dimensions, we have to imagine n dimensions, and, secondly, that there is no (n+1)-th dimension.

In the same way, in a logical system of the order n, or an n-dimensional [system], we must first imagine all these n forms of qualitative differences between the propositions and, secondly, that there exists no (n+1)-th form. Without this latter condition, we could be certain that there exist n forms of propositions, but we could never be certain that there exist only n forms of them. In order to be certain of this we will posit *the special law of the excluded (n+1)-th form: a logical system with n qualitatively different kinds of propositions presupposes that the (n+1)-th form is impossible.* This is a pure law of thought, applicable to all logical systems.

Therefore, the laws of excluded third, fourth, etc. that exist in different logical systems will be particular cases of the law of the excluded (n+1)-th form. Thus, in our imaginary logic without the law of contradiction, there must (also) exist the special law of excluded fourth, which states that there is no 4^{th} kind of propositions besides the affirmative, the negative, and the indifferent ones. We have earlier made use of this law of the excluded 4^{th}, for example, in our proof by contradiction of the general indifferent mood in the 1^{st} figure.

Let us, now, dwell at some length on the relation between the law of excluded middle and the law of contradiction. This relation can be established by means of the method of imaginary logic, and this will be an example of its usefulness in solving problems in our Aristotelian logic. The law of
[78] contradiction precludes us from constructing a third form of propositions, in fact, a specific third form of propositions containing the combination of an affirmation and a negation (i.e. contradictions). The law of excluded middle has a broader scope, it forbids any third form of propositions, independently of its origin, and consequently also of contradictions. *Thus, the law of contradiction is a particular instance of, and a corollary of, the law of excluded middle*, and not the other way around, as many have thought.

Here we must interrupt our exposition of imaginary logic. Of course, imaginary logic is not limited to what has been presented. Its subject matter includes the subject matter of our (Aristotelian) logic, and chapters and parts of our logic will, therefore, be found there too. It is possible to present imaginary logic in the form of a system of logical rules, which is just as complete and closed as our [system is]. But we shall not bother the reader with that any longer. Our present aim does not consist at all in constructing a system of imaginary logic — that is quite a different task — but in showing the very principles, upon which it is built. For this purpose the little of the subject matter of imaginary logic we have presented is probably enough. Even that little shows that imaginary logic preserves the necessary character of inferences and the rigour of the logical rules.

IV

Let us, now, go into some analogies between non-Aristotelian logic and non-Euclidean geometry. I will not touch here upon the history of imaginary or non-Euclidean geometry. The only thing I want to say here is that the famous mathematician Gauss had already fostered the idea of a non-Euclidean geometry. It was, however, proposed independently by Lobachevski in Russia and Bolyai in Hungary. Lobachevski rejected the 5th postulate of Euclid about the necessary intersection of two straight lines. When intersected by a third [straight line] the sum of the interior angles is less than two right [angles]. The vicissitudes of this thesis are highly instructive. From Euclid to Lobachevski numerous sharp-minded mathematicians have tried in vain to prove it. Very different proofs were proposed, but each was based on a new thesis, which in turn had no proof. This means that they were not proof of the postulate, but rather substitutions with new equivalent postulates. In the
[79] first half of the 19th century, Lobachevski and Bolyai independently decided to examine what would happen if one rejects this postulate, if one infers geometrical propositions after substituting a different postulate for the 5th one[9]. It was discovered that a rigorous demonstrative geometry, which nowhere gets into any self-contradiction, is possible without the 5th postulate too; but this geometry is partly similar to ours, partly dissimilar. In our geometry, through a point outside a straight [line], one can draw only one straight line, which will not intersect with the first one; in Lobachevski's geometry infinitely many such lines can be drawn. In our geometry, the sum of the angles of a triangle is equal to two right [angles]; in Lobachevski's geometry it is always less than two right [angles], and the smaller the triangle the closer [the sum of the angles] is to two right [angles]. In our geometry there are similar triangles, there are none in Lobachevski's geometry. But both geometries

[9] Later on, other imaginary systems of geometry have been constructed. So, for example, Riemann's system of geometry.

have some propositions in common. They are precisely those propositions of our geometry that can be proven without the use of the 5th postulate.

It is not difficult to see that the relation of non-Aristotelian logic to imaginary logic is completely identical to the relation of non-Euclidean geometry to our geometry. Both non-Euclidean geometry and non-Aristotelian logic arise as a consequence of the rejection of an axiom; both are closed and consistent systems, though they constantly contradict our common sense and our immediate intuition. Therefore, as the former is called geometry the latter, by the same token, may be called logic. Non-Euclidean geometry and non-Aristotelian logic are in part similar to, and in part different from, our geometry and our logic. We can determine the part that is common to both Euclidean and Lobachevski's geometry; it comprises just those propositions of our geometry that do not depend upon the 5th postulate. In the same way, one can determine all propositions that are common to both logics, real and imaginary; this set consists of all those propositions of our 'real' (Aristotelian) logic that do not depend on the law of contradiction. The propositions that are common to both logics must be distinguished from the propositions of our real logic that make it different from imaginary [logic].
[80] Moreover, the greater complexity of non-Euclidean geometry as compared to Euclidean geometry and non-Aristotelian logic as compared to our Aristotelian logic has not escaped, I believe, the attentive reader. Therefore, we can, in general, say that the systems of geometrical and logical propositions that we use are the simplest ones possible. This could turn out to be very important for epistemology. There are here a number of interesting problems that we are unable to go into now.

Further, also the content of non-Aristotelian logic bears a certain analogy to the content of non-Euclidean geometry. Our geometry distinguishes two kinds of relations between two straight lines on a plane: either two [straight] lines intersect, or they do not intersect, that is, they are parallel. In Lobachevski's geometry, there are three kinds of such relations. We shall retain the terms used by Lobachevski in his "New principles of geometry."[10] Either two [straight] lines on a plane are convergent, if they intersect, or they are divergent, if they do not intersect, or they are, finally, parallel, and thus constitute a boundary which separates convergent [straight] lines from divergent [ones].[*] There are two [straight] lines that are parallel to any given

[10] Lobachevski N.I., Complete Works on Geometry, Kazan, 1883, Vol. 1, p. 301.

[*] Lobachevski had to construct his own technical terms: "svodnye" and "razvodnye," which can be translated into English only approximately. Formally, both are passive terms (but not such 'strong' passives as "svedionnye" and "razvedionnye"); however, their meaning is active. In modern expositions of Lobachevski's geometry the terms "skhodiaschtchiesia" and "raskhodiaschtchiesia" are used with an active meaning, which can be translated in English as "convergent" and "divergent". (*R.V. – E.Z.*)

straight [line]. N.I. Lobachevski says: "Two lines parallel to a given [straight line] divide a plane into four parts: convergent lines are situated in the opposite [parts of the plane], the divergent [lines] in the two other [parts of it]."[11] Divergent and parallel lines constitute the class of non-convergent, or non-meeting, lines, because they do not intersect with a given straight [line]. In other words, Lobachevski's geometry discriminates between divergent and parallel lines within the class of non-convergent lines; that is impossible in Euclidean geometry, in which, instead, [only] one class of parallel lines exists. If we now turn to logic, we can see that in Aristotelian logic two kinds of subject-predicate relations are possible, affirmative and negative. In imaginary logic, that relation is threefold: affirmative, negative, and indifferent. In imaginary logic the class of negative propositions, that is, the class of the propositions that assert the falsehood of affirmative propositions, fall apart into negative and indifferent ones.

[81] The formal analogy between the content of non-Euclidean geometry and non-Aristotelian logic can be summarized as follows: *the dichotomy characteristic to both our logic and our geometry turns into the trichotomy of the imaginary disciplines [of logic and geometry].* But the analogy between non-Euclidean geometry and non-Aristotelian logic is not limited to that.

We can give a "real" interpretation of non-Euclidean geometry; [we] can find certain structures in Euclidean space whose geometry is non-Euclidean. Thus, the geometry of the sphere, [that is,] on a surface of constant curvature — [which is called] spherical geometry — represents the "real" interpretation of Riemann's geometry. The "real" interpretation of Lobachevski's geometry can be [represented by] a geometry on a surface of constant negative curvature (a so-called pseudo-sphere) as was shown by the Italian mathematician Beltrami[12]. In the same way, in our world structures can be found whose logic is analogous to imaginary [logic]. More specifically, we here mean concepts. When we begin to construct a logic of concepts, we will see that it is different from the logic of things. *The law of excluded middle is applicable only to sense impressions and perceptions, to things and facts only.* A lamp is on or is not on; there is no third possibility. If we consider a thing, a fact, a sense impression or a perception, etc., as the subject of a proposition, any predicate will give rise either to a negative or an affirmative proposition about this subject. *Contrariwise, the law of the excluded middle is not applicable to concepts; however, the law of excluded fourth is applicable there.* Indeed, if we consider a concept as the subject of a proposition, any predicate is related to it either as: 1) a certain predicate is necessary

[11] Ibid.

[12] This is a special saddle surface, models of which can be manufactured and representations of which usually feature in books on non-Euclidean geometry.

w.r.t. the concept (e.g., for a triangle — to be closed), and we represent this by means of an affirmative proposition about the concept (a universal affirmative proposition of classical logic), or as: 2) a certain predicate is impossible w.r.t. this concept (e.g., for a triangle — to be virtuous), and we represent this by means of a negative proposition about the concept (a universal negative proposition of classical logic), or as: 3) a certain predicate is compatible with a given concept (e.g., for a triangle — to be equilateral). We
[82] represent this third case by means of a special accidental proposition about concepts. 'A concept S is compatible with a predicate P', or, 'S may be P'. 'Triangles may be equilateral'. This proposition has its own copula distinct from that of an affirmative and a negative one. Besides these three propositions, no fourth is, indeed, possible. And in each specific case one of them will be true. This is the law of excluded fourth. Thus, in relation to things and perceptions, one of two propositions is true: either the affirmative or the negative one. In relation to concepts, one of three propositions is true: the affirmative, the negative, or the accidental one. The law of excluded fourth, which is a law of imaginary logic, is at the same time also a law of our terrestrial logic of concepts. To an indifferent proposition in imaginary logic, there corresponds an accidental proposition about a concept: "S may be P." The latter proposition can be considered as a specific synthesis of an affirmation and a negation. Indeed, an accidental proposition such as "S may be P" or "triangles may be equilateral", is equivalent to the form: "Some S are P, [and] some S are not P." "Some triangles are equilateral, [and] some are not."

Therefore, it can be said that the logic of concepts bears a resemblance to imaginary logic[13].

[13] This was developed at some length in my paper "On Particular Propositions, the Triangle of the Oppositions, and the Law of Excluded Fourth". For the sake of clarity, I will give [here] its main conclusions. I. Propositions can be subdivided into propositions about concepts (rules) and propositions about facts. Each of these [two] kinds of propositions has its own formal logic. Thus, the triangle of oppositions and the law of excluded fourth are valid for propositions about concepts. The square of oppositions and the law of excluded middle [are valid] for propositions about facts. II. So-called universal, particular, and singular propositions, are all ambiguous forms, depending upon whether they are propositions about concepts or about facts. Singular propositions may be 1) Propositions about facts: if so, their subject matter is perceptions and their copula implies a certain temporal situatedness. 2) Propositions about rules: then their subject matter is individual concepts. Particular propositions can be: 1) Propositions about facts — indefinite numerical propositions, such as 'several particular S are P', 2) Propositions about rules such as 'some (but not all) S are P', which is a universal proposition and can be represented either in a disjunctive form (each S is P or is not P), or an accidental one (S may be P), 3) Indefinite propositions such as 'some, but maybe all, S are P', which represent a psychological, rather than a logical, form of transition from a proposition about facts to a proposition about a rule. Universal propositions
[83] too, can be propositions about facts or propositions about rules. The proposition "All of my friends abandoned me in my hour of need" does not represent any rule; [it] is a proposition

about the group of my friends and it is, then, a proposition about a fact. The proposition "All friends abandon one in one's hour of need" is a proposition about the class of friends and, as such, expresses a rule. III. As a matter of fact, there are no particular propositions. Indefinite numerical propositions are propositions about a group, about the whole of a given group. Accidental propositions are propositions about concepts, about the whole of a given class; they are incontestably universal propositions. Indefinite propositions are not propositions at all, but only the expression of a vacillation between two hypotheses. All three species of the so-called "particular proposition" turn out not to be particular propositions at all. Propositions about concepts and facts, propositions about individual objects, or about a group or a class are propositions about the whole of the concept and the whole of the fact; about the whole of the object, about the whole of the group, or about the whole of the class. There is no category of particular propositions. The subject is always distributed.

In the Journal of the Ministry of Education, 1911, March, C.A. Smirnov has published a review of my paper (apparently also published as a separate pamphlet, under the title, "N.A. Vasil'ev and His Law of Excluded Fourth"), which I would like to briefly comment upon (See Smirnov C.A., Review of the Paper by N.A. Vasil'ev: "On Particular Propositions, the Triangle of Oppositions, and the Law of Excluded Fourth": Journal of the Ministry of Education (New Series, March, Part 32. p. 144–154)). In my paper, I have strictly distinguished between propositions about facts and propositions about concepts or rules. Propositions about facts always, *implicitly* or *explicitly*, imply a certain moment in time and a certain location in space, such as e.g. the proposition "I broke the lamp." Rules or propositions about concepts imply neither a certain moment in time, nor a certain location in space; they remain valid anytime and anywhere. The proposition "All human beings are mortal" is equivalent to the proposition "All human beings are, always and everywhere, mortal." Propositions about facts express something existent, a fact, something '*was ist*'. Propositions about concepts express something significant, a rule, a law, something 'was gilt'. Conversely, propositions about facts cannot express laws or anything significant (i.e., in the sense of 'general' as with laws, *R.V. – E.Z.*). They are not valid outside the reality they designate. In turn, propositions about concepts cannot express existence, since they express laws, i.e. a connection between two existing things, and not existence itself. Every existing thing is always in a definite time and place, whereas propositions about concepts [are] timeless and spaceless and thus their objects can be timeless and spaceless laws or rules only. Consequently, propositions about facts should be sharply distinguished from propositions about concepts. As to accidental propositions such as "S may be P" (e.g. "The lamp may be broken"), it is clear that these propositions should be qualified as rules, since they are not bound up with a definite moment in time and space: they speak about any lamp, about the class of lamps or about the concept of a lamp.

[84] This is so evidently true that even C.A. Smirnov agrees. In fact, he writes on page 152: "Indeed, where many rules have the form of analytical propositions and are, therefore, propositions about concepts as such, as, for example, any geometrical or, in general, mathematical theorems are, other [rules] have the form of disjunctive (or accidental) propositions, and do not stand out for such a definiteness and logical perfection at all." Also, on p. 153: "many rules still remain, however, on the level of disjunctive or accidental propositions." This means that for C.A. Smirnov too, accidental propositions express rules. How could this be consistent with his statement on p. 150 (Cfr. also p. 151): "First of all and principally, it is impossible to agree with the author's relegating accidental propositions to propositions about concepts, and not to propositions about facts." I would be interested to know from C.A. Smirnov what an accidental proposition, then, is: is it a proposition about facts or is it a rule? If an accidental proposition is a proposition about facts, then it implies a certain moment in time; if it is a rule, then it does not imply such a moment. By criticizing me, C.A. Smirnov contradicts himself; thus, in one passage [he] relegates accidental propositions to propositions about facts,

Lobachevski's geometry finds its real interpretation in the geometry of the pseudo-sphere, imaginary logic in the logic of the concept. But geometry is not limited to that. It tries do determine the conditions under which (any) imaginary geometry could become a real geometry of space. It can
[85] be shown that such a condition is provided simply by the physical structure of our universe. The famous French physicist, mathematician, and philosopher Poincaré has excellently demonstrated the dependence of our geometry on the physical organization of our world (La science et l'hypothèse, p. 84–87)*. First, he proves that if there would be space (in the world) and a

whereas in another [he relegates them] to rules; but is clear that they cannot be both at the same time.

In general, the conjunction of contradictory propositions constitutes, as it seems, a favorite logical method of C.A. Smirnov. Thus, on p. 149 he accuses me of identifying propositions about concepts with rules, whereas on p. 154 he accuses me of vagueness in the distinction between propositions about concepts and propositions about rules. Had that been the case, I would have been guilty of a contradiction; but as I am not, and C.A. Smirnov does not accuse me of contradiction, then, apparently, he thinks in earnest that it is possible to both identify and vaguely discern at the same time.

Let me now go into another aspect of C.A. Smirnov's critical activities that has surprised me in a very unpleasant way. C.A. Smirnov reproaches me that I allegedly turn language into a criterion for logic. On p. 151 he writes: "The author thinks, absolutely without reason, that "logic should always beware of being less rigorous than language.", p. 22."

The quotation from my paper is incorrect, since the word "always" was inserted, which is absent in my paper. In the case in question, this mistake is all the more regrettable, because it completely distorts the meaning of the quotation. Without the arbitrarily inserted word "always", the whole phrase which the passage in question is taken from, reads as follows: "To the rigour of distinctions in language there corresponds a real distinction in thinking, and logic should beware of being less rigorous than language." What I really mean is the following: only those distinctions in language are important for logic, to which there correspond real distinctions in thinking; but since there exists such a relation between language and logic, it provides an additional argument for recognizing such a distinction in logic. This relation resembles the relation between philosophy and common sense; not all the truths of common sense remain valid in philosophy; but if a thesis is endorsed by both philosophy and by common sense, we can refer to that agreement as an *additional* argument in its favor and state that philosophy should beware of being less rigorous than common sense! But by inserting the word "always," C.A. Smirnov has imparted to my thought a sense which is contrary to that which it had. Therefore, all of C.A. Smirnov's subsequent arguments are a fight not with me, but with his own constructions. C.A. Smirnov writes: "At first sight the new logical theory of N.A. Vasil'ev seems to be very neat but, unfortunately, it suffers from basic shortcomings and collapses completely once these are removed" (p. 150). Was C.A. Smirnov not in too much a hurry to bury my theory? At least, the reviews of my paper in Rech, October 11, 1910 and in Logos, n2, express quite a different view. As to the review by C.A. Smirnov, after all that has been said above I leave it to the reader to decide whether his [own] arguments do not suffer from "basic shortcomings" and whether, instead of my theory, his [own] criticism does not "collapse."

* Cfr. Poincaré A., Science and Hypothesis. Moscow 1904.

geometer, gifted with a mind such as Euclid's, but no solid bodies, there would be no geometry. Subsequently, Poincaré conceives of the world as a sphere with a temperature which uniformly decreases from the center to the surface, and (assuming, also, the law of the expansion of solids under the influence of heat) he shows that in such a world geometry would necessarily be non-Euclidean.

In the same way *it can be shown that, assuming a certain structure of our world or our faculty of sense-perception, logic should necessarily be non-Aristotelian.*

Our world and our faculty of sense-perception are uniformly positive. Also sense-perceptions provoked by negative causes are positive: silence, darkness, rest — no less than sound, motion, light. Silence, rest, darkness, are, in themselves, just like the sense-perceptions of sound, motion, and light; they have their own meaning and nothing in them implies their negative nature. Darkness is the specific perception of darkness, and not the perception of the absence of light. Darkness becomes the negation of light only secondarily, through their incompatibility. In our world, two positive perceptions become each other's negations when there is a relation of incompatibility between them. Therefore, negativity is something external to perceptions.
[86] It is something which is added to them if one considers them in relation to other perceptions, but which is not inherent to these perceptions considered separately. Red in itself is a certain perception, and it has nothing negative; but if we oppose it to white, we can consider it as non-white. To sum up, negation, or the negative character, is not a property of any perception, but it is a relation that reveals itself in the process of comparative thinking. In themselves, all perceptions are positive.

But we could imagine a world with negative perceptions, with pure 'non-A's' that would have as their sole meanings to be the negations of A's. In this case, negation would be part of the nature of the perception of 'non-A'; it would not be a relation, which is added by comparative thinking. We could, then, think of pure perceptions of "non-white," just "non-white" and nothing more, which we do not have at present, since "non-white" means for us red, blue, etc. Such a negation would be an absolute negation, in contrast to our negation, which is a relation and which therefore we could call relative [negation]. This difference between absolute and relative negation is strictly logical, and it can be found in the work of several logicians, for example in Bolzano[14]. If one assumes [the existence] of negative perceptions in a world, the purely logical concept of an absolute negation would become true as a reality in that world.

[14] *Bolzano B.B.* Wissenschaftslehre. Sulzbach, 1837. Bd. 1. §89.

Thus, we could assume there to be in a world [the existence of] two kinds of perceptions, perceptions of type A, and perceptions of type non-A. When we have a perception A of a subject S, we can construct the proposition "S is A." When we have a perception of non-A, we can construct the proposition "S is not A". Here, as in the case of imaginary logic in general, negative propositions are directly based on perceptions. Now, let us think of an object S simultaneously causing in us positive perceptions A and negative perceptions non-A. That is quite conceivable, since in a world with nega-
[87] tive perceptions negation originates independently from the incompatibility with positive perception, and can, consequently be in conjunction with it. Since, some object S simultaneously causes the perceptions A and non-A, we would be able to construct the indifferent proposition "S is A and not A simultaneously." Thus, in the case of the existence of negative perceptions in a world, the logic of that world would be non-Aristotelian.

But other interpretations of imaginary logic could be given as well. Consider the following. Suppose an affirmative proposition designates the similarity of two phenomena. The affirmative propositions of our logic can be interpreted in that sense, too. The proposition "the rose is red," signifying the inclusion of the rose in the class of red objects, also expresses the rose's similarity to red objects. Let us assume, then, that negative proposition do not express an incompatibility, as with us, but a difference, an absolute difference or an absolute dissimilarity. Then, clearly, propositions are possible that simultaneously express similarity and dissimilarity between two phenomena, i.e., indifferent propositions. In our world too, two similar objects are at the same time also different.

However, (in our world) we have elaborated a logic of the presence and absence of predicates, i.e. a two dimensional logic. The logic described above, however, is a logic of similarity, difference and the conjunction of similarity and difference, i.e. a three-dimensional logic or an imaginary logic.

Furthermore, still another interpretation can be given of imaginary logic.

In our logic, by affirming that S is P, we affirm the whole of the meaning (intension) of the concept P, and also each of the properties of P separately. The syllogism Barbara (a property of a property of a thing is a property of the thing itself) is based upon this. But, by denying that S is P, by declaring this proposition to be false, we deny the totality of the properties of P, taken collectively, whereas we deny only some of the individual properties of P. Thus, by denying that Columbus was the first European who reached the shores of America, we are not denying that he was a European and that he reached the shores of America. In the proposition "Dogs are not human beings," not the whole of the meaning of the concept "dog"* is negated; for example, it is not negated that dogs are mammals[15].

But one could conceive of the concept of absolute falsehood and the concept of an absolute negation. One could imagine a 'non-A' that would have none of the properties of A. If the concept A has the properties p, q, r, s, $\dots$,
[88] then the concept 'non-A' would have the properties non-p, non-q, non-s, etc. Let us, furthermore, assume that our (regular) negation, which negates not all the properties of A but only some of them, is also preserved.

Then again, in each specific case there would be three possibilities: 1) the proposition 'S is A' would be true or false and, if false, it could be either 2) absolutely false, so that S would have none of the properties of A, or 3) simply false, so that only some of the properties of A would be negated.

In general, we can either 1) affirm all the properties of A, or 2) negate all the properties of A, or 3) affirm some of the properties and negate some others [of the properties of A]. The first case results in an affirmative proposition, the second case is the absolute negation and the third case results in a proposition with our [regular] negation. In all, there are, then, three subdivisions of the propositions according to their quality.

The class of negative propositions, viz. propositions declaring affirmative propositions false, is subdivided into 2 subclasses: propositions of absolute falsehood and propositions of simple falsehood.

It should be noted that in both of the aforementioned interpretations we get an imaginary logic, which is however somewhat different from the one the main features of which we have sketched in this paper. Thus it is, for example, not difficult to see, that within the latter interpretation the 1st figure of the syllogism with an absolute negation as a minor premiss[16] is possible (valid). I have given these interpretations in order to show how the meanings of the known logical operations could be modified in different ways and

* In the original text, the predicate "human being" is mistakenly used here. (*R.V. – E.Z.*)

[15] In order to negate a predicate of a subject, it suffices to negate at least one of the properties of the predicate of the subject.

[16] If the concept A has the properties p, q, r, s, $\dots$, then, as we have seen, its absolute negation non-A should have the properties non-p, non-q, non-r. From this, it is clear, that we can construct the following syllogism:

A is P	major premiss
S is non-A	minor premiss
S is non-P	conclusion.

Here, the formula "S is non-A" should be read as "A is absolutely negated with respect to S."

how in this way very substantial modifications of logical operations could be obtained.

In conclusion, I will try to reply in advance to some misunderstandings and objections that arise naturally.

[89] One might object here in the following way: "You refer to the possibility of imaginary geometry. But imaginary geometry is created by logic. What is your imaginary logic created by?" My reply to that is: *by logic*. In logic, there are principles and operations that are common to all logics in general, (principles and operations) transcending particular logics and transcending both imaginary and 'real' logic. It is material and not formal laws of thought that differentiate the one 'logic' into particular logics subordinated to it, and lead to differences between these logics. It is by dealing with a different material, with a different subject matter of knowledge — e.g. incompatibility in one case and absolute negation in other cases — that creative thought constructs different logics. This is essentially the same as what a zoologist or a botanist does when, by dealing with different empirical material, he constructs different scientific disciplines by means of the same logical apparatus. *Logic in the form in which we are used to apply it, is full of empirical elements; it is logic under the conditions of experience; it is adapted to the empirical.* Basic properties of our world affect our logic.

But one should not think that everything is empirical in our logic, that everything (in it) belongs to the world and to experience and that nothing (in it) belongs to reason and thought. First of all, the very form[s] of the propositions and the inferential process are functions of our reason, since in nature there are objects and perceptions, as you like, but there are no propositions and inferences. Therefore, the forms themselves of the propositions and inferences, which constitute the very essence of the logical, are beyond experience and are not derived from experience and reality. Furthermore, in nature there is neither truth nor falsehood. Only human beings distinguish between truth and falsehood. Therefore, there have to be (other) propositions that arise from the form of the propositions and inferences themselves and that do not have any empirical basis. *I would propose to call these extra-empirical logical propositions, and I would call a logic without any empirical elements metalogic.* The name "metalogic" is better suited for this discipline, as it indicates a formal analogy to metaphysics. Metaphysics is the knowledge of being regardless of the conditions of experience. Metalogic is the knowledge of thought regardless of the conditions of experience. Metaphysics is the science of pure being. It constitutes an abstraction from the world of phenomena, and it is the knowledge of that which is common to all empirical things. Metalogic is a discipline of pure thought. It is an abstraction from everything in thought that is empirical. There may be many
[90] worlds, but the essence of being is one. Such is the basic premiss of metaphysics. There may be Many logics, but they all have something in common

which is only One, viz. metalogic. Metalogic, then, is the discipline of the formal aspect of thought regardless of its content.

Therefore, the only formal logic is metalogic. Our so-called formal logic is, in fact, not formal, since it does not completely abstract from the content of thought. For example, the law of contradiction is a material principle. Therefore, we should carefully distinguish between metalogic and empirical logic.

One could also define these two disciplines as follows: Metalogic is the science of propositions and inferences in general. Empirical logic is the science of those kinds of propositions and inferences that correspond to our world. It is constructed in accordance with the main properties of our world. Therefore, in metalogic we get to know nothing besides thought. In empirical logic we get to know the main properties of our world as well. Empirical logic is a combination of the metalogical and the experiential, of the rational and the empirical. Therefore, empirical logic lies at the foundation of any 'terrestrial' reasoning, since scientists or representatives of any specific science will make mistakes if they are reasoning contrary to the main properties of our world. So, scientists have to reason and, in fact, do reason according to both the principles of metalogic and empirical logic. But philosophers can, and in certain cases have to, reason according to the principles of metalogic only. Metalogic in itself constitutes a certain logical minimum; it is that, which is contained in all possible logics, real and imaginary ones. It is that, which makes logic be logic. Empirical logic (but also any of the imaginary logics) is richer in content; it is more concrete, more definite; it contains all of the metalogical and something specific to empirical logic only. Therefore, a person who is reasoning in accordance with empirical logic (or any imaginary logic), is also reasoning in accordance with metalogic, but not at all vice versa.

We have a means to separate the propositions of metalogic from the specific propositions of empirical logic. This means is the method of the construction of imaginary logic. All those propositions that can be rejected and be replaced by other propositions while at the same time the whole remains 'logic' (be it an imaginary one) belong to empirical logic only. All those propositions that cannot be rejected without destroying the very possibility of logical reasoning belong to metalogic. Thus, for example, the law of con-
[91] tradiction is a law of empirical logic, the law of absolute difference between truth and falsehood is a law of metalogic. The same metalogic underlies both imaginary and empirical logic, and this explains the possibility of the construction of an imaginary logic. Metalogic is analogous to those most general propositions of geometry that are common to all geometries: Euclidean and imaginary ones; it corresponds to what Bolyai called absolute geometry. Lack of space does not allow us to draw at any length on the methods and content of metalogic and its difference from our empirical (terrestrial) logic.

It is worth noting here that, consequently, the word "logic" may have four meanings and can designate four different disciplines:
1) It can designate our 'terrestrial' logic, 'Aristotelian logic' as we call it in contrast to imaginary (non-Aristotelian) logic, or 'empirical logic', as we call it in contrast to metalogic. This is, of course, the original meaning of the word.
2) It can designate different systems of imaginary logic. And these all belong to the same kind of logic as empirical logic.
3) It can designate metalogic as that which is common to all logics and constitutes the formal preconditions of any logic.
4) Finally, it can designate logic as a concept, as a class or a genus, in relation to which the "logics" in the former three senses are species and individuals. It can simply designate "logical" anywhere, be it in metalogic, empirical logic, or imaginary logic.

One should not think that imaginary logic is a simple curiosity, a mental game, which has neither theoretical nor practical value. To the contrary, imaginary logic, it seems to me, should be considered of utmost importance. Here I will be brief and mention the most essential only. No one who is studying logic is ignorant about the chaotic condition in which the doctrine of the laws, or principles, of thought nowadays is. There is nothing firmly established in this domain. Schopenhauer, for example, reduces all logical laws to the following two: the law of excluded middle and the law of sufficient reason. Hartmann, on the contrary, retains the law of contradiction only, inferring all other logical laws from it.

[92] By contrast, other thinkers consider the law of contradiction as a corollary to the law of identity; the idea of the derivative character of the law of excluded middle is widespread as, for example, in Sigwart. The opinion that all four laws are independent is the most widespread one.

Moreover, it is not proven that there could not be further logical principles — that for that matter have yet to be explicitly formulated — that would belong to the foundations of our logic. The case of geometry may convince us of this possibility. The complete and systematical formulation of the axioms of geometry has been done only recently by Pasch, Hilbert and others; up to then, many unquestionable axioms had escaped the attention of scholars.

Finally, the formulations of the logical laws should be considered as being especially questionable and unfixed.

Therefore, it turns out that in the doctrine of the logical axioms (which as axioms should apparently be completely obvious and reliable) there is nothing obvious nor reliable.

The method of the construction of imaginary logic could serve to solve problems related to the laws of thought, since it allows for the analysis of the complex and intertwined fabric of "the logical," in which all threads are multiply interlaced, combined, crossed, and enmeshed in each other. This method offers the possibility of separating different strata of "the logical," of tracing back the most important threads — the canvas of the fabric — and their mutual interrelationships.

In other words, by means of this method one could, as it seems, first of all more reliably determine the axioms and postulates that constitute the foundation of logic; secondly, one could give them precise formulations, since by enumerating all the axioms we could avoid the constant recurring conflation of different axioms; thirdly, one could demonstrate that all the axioms discovered are independent and are not derived from each other, since independence is a basic property of the concept of an axiom or a basic principle; fourthly, one could determine which logical operations and propositions depend upon which axiom (for example, when, upon removal of the axiom, these operations themselves have to be abolished); finally, one could formulate a complete classification or system of axioms and postulates for logic.

In short, for logic the same kind of investigation should be carried out that has already been carried out for geometry, viz. an axiomatic one[17].

[93] Thanks to this axiomatic investigation, it will in the future be possible to determine all of the axioms and postulates of logic and all of its presuppositions. The method of imaginary logic is exactly the method that should be used in an axiomatic investigation. By removing the law of contradiction, we also remove from logic all that depends upon it and pick out all that which does not depend upon it.

The method of imaginary logic could also be used in solving other logical problems. We have already seen that it could be used in the solution of a problem concerning the 2nd figure of the syllogism and concerning the relation between the law of contradiction and the law of excluded middle. By means of this method also the meaning of the general law of *conversio* could be determined and many questions that arise in the study of *conversio* could be resolved. For the moment we can confine ourselves to these examples. The main advantage of imaginary logic is that it allows us to generalize the laws and formulae of our logic and to construct logical laws in their most general form.

Last but not least, there is also an epistemological, and thus more general, philosophical significance in imaginary logic and in a differentiation

[17] The term "axiomatic investigation" was apparently introduced by Hilbert. He writes: "By the axiomatic investigation of mathematical truth I understand an investigation which does not aim to discover new and more general propositions related to that truth but, rather, to reveal the place of that truth and its logical connections within the system of the already known truths in such a way, that it would be known for certain which propositions are necessary and sufficient for its justification." (Grundlagen der Geometrie. 3-te Auflage 1909, S. 129). On the axiomatic method in philosophy, cf. also: Ueber das sogenannte Erkenntnisproblem. 1908. p. 779.

between the logical and the empirical in logic. But this is a problem that should not be touched upon just in passing.

I understand very well that the ideas in support of a different logic put forward here contradict millennial convictions of mankind and could provoke numerous misunderstandings and objections. It is even more clear to me that this paper will not be able to do away with them, but will very likely generate additional ones. In publishing it, I have taken as a lead the words of Kant: "I think that with such a vague kind of knowledge as metaphysical [knowledge], it is more suitable to first formulate the idea for a general discussion
[94] (examination) in the form of vague attempts — and not at once in its full brightness, masterly solidness, and full persuasiveness — since in that case any improvement is usually rejected at once, and all that is bad in it becomes irremediable."

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