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BEGGING THE QUESTION AS A FORMAL FALLACY*

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Abstract
The aim of this paper is to define the fallacy of begging the question in the formal language of the theory of consequence. Its main assumption claims this fallacy depends not only on the form of an argument but also on its context. On the ground of recent developments in informal logic the contextual theory of argumentation, suitable for formalisation, is propounded. According to it there are two relevant factors in argumentational contexts: beliefs of those who take part in argumentation and their inferential tools. Due to the variety of logical abilities of such participants two sets of definitions are submitted: simplified and extended. The 'wickedness' of this fallacy is explained by means of theorems involving definitions of probative efficiency of arguments.

In his recent paper 'Begging The Question As A Pragmatic Fallacy' Douglas Walton (Walton 1994) has outlined the history of the logical analysis of the fallacy of begging the question. The moral of it is that the context of an argument plays an essential role in evaluating its epistemological value. To grasp the contextual factors of argumentation various kinds of sophisticated formal systems were applied, Charles Hamblin's theory of formal dialogues (Hamblin 1971) is a *locus classicus*. In this article I intend to turn to a much simpler tool, the theory of consequence, to express David Sanford's account of this fallacy. Its first section briefly summarises the elements of this theory. The second section develops Sanford's epistemological theory of argumentation in order to render it suitable for being translated into the formal language. The translation is contained in the following two sections. The last part gives several more important theorems substantiating Walton's claim that question-begging arguments miss the aim of probative argumentation.

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I. *Formal background*[1]

I will briefly recollect some basic definitions and theorems from the theory of consequence. Let L denote a language that contains the language of the lower predicate calculus. Let *FIN* be the set of all finite subsets of L. A *consequence operation* in L is a function C: $\wp(L) \to \wp(L)$ that satisfies:

(D1) For all sets $X, Y \subseteq L$:
(i) $X \subseteq C(X)$,
(ii) $X \subseteq Y \to C(X) \subseteq C(Y)$,
(iii) $C(C(X)) \subseteq C(X)$,
(iv) $\forall \alpha \in L[\alpha \in C(X) \to \exists Y \subseteq X(Y \in \mathit{FIN} \wedge \alpha \in C(Y))]$.

Let C denote the set of all consequence operations in L. They may be ordered in C with respect to their strength:

(D2) $C_1 \leq C_2 \equiv \forall X \subseteq L(C_1(X) \subseteq C_2(X))$.
(T1) The pair $\langle \mathsf{C}, \leq \rangle$ is a complete lattice.

The strongest operation in C is the *inconsistent consequence INC*:

(D3) $C = \mathit{INC} \equiv \forall X \subseteq L\, C(X) = L$,

and the weakest operation is called the *idle consequence ID*:

(D4) $C = \mathit{ID} \equiv \forall X \subseteq L\, C(X) = X$.

A set of sentences X is C-*inconsistent* if $C(X) = L$; otherwise X is C-*consistent*.

An *inference* in L will be viewed as a pair $\langle X, \alpha \rangle$ such that $X \in \mathit{FIN}$ and $\alpha \in L$. Instead of "$\langle X, \alpha \rangle$", I will also write "$X \vdash \alpha$". An inference $\langle X, \alpha \rangle$ is an *inference of* C if and only if $\alpha \in C(X)$. A *rule of inference* r is a non-empty set of pairs $\langle X, \alpha \rangle$. In other words,

(D5) $\emptyset \neq r \subseteq \mathit{FIN} \times L$.

A consequence operation 'decides' which rules of inference are valid with respect to it:

(D6) $r \in \mathit{RULE}_C \equiv \forall X \in \mathit{FIN}\, \forall \alpha \in L(\langle X, \alpha \rangle \in r \to \alpha \in C(X))$.

[1] Definitions D1–D8 and theorems T1–T4 are taken with some slight modifications from (Wójcicki 1984, 39–57).

Each consequence operation is determined uniquely by its rules of inference. More rigorously,

(T2) $C_1 = C_2 \equiv \mathit{RULE}_{C_1} = \mathit{RULE}_{C_2}$.

Thus we may define the consequence operation C_R *determined by* a set of rules R.

(D7) $C = C_R \equiv \forall r \in R\, r \in \mathit{RULE}_C \wedge \forall C' \in \mathsf{C}(\forall r \in R\, r \in \mathit{RULE}_{C'} \rightarrow C \leq C')$.

It can be proved that:

(T3) C is a consequence operation iff there exists a set R of rules of inference such that $C = C_R$.

As a result, consequence operations may be identified with sets of rules that determine them. If $\alpha \notin C_R(X)$, then I will say that α is C_R-*independent* from X.

Due to D1(iv) it holds that

(T4) $\alpha \in C_R(X) \equiv \exists n \in \omega\, \exists \alpha_1, \alpha_2, \ldots, \alpha_n \in L[\alpha = \alpha_n \wedge \forall i \in \{1, 2, \ldots, n\}(\alpha_i \in X \vee \exists r \in R\, \exists Y \subseteq \{\alpha_1, \alpha_2, \ldots, \alpha_{i-1}\}\langle Y, \alpha_i\rangle \in r)]$.

A rule of inference is *derivable* from a set of rules if it does not enlarge the consequence operation determined by this set.

(D8) $r \in \mathit{DERV}_R \equiv C_{R \cup \{r\}} = C_R$.

It follows immediately from D8 and T4 that if $X \in \mathit{FIN}$, then

(T5) $\alpha \in C_R(X) \rightarrow \{\langle X, \alpha\rangle\} \in \mathit{DERV}_R$.

The consequence operation determined by a set of rules represents one's global inferential pattern: it settles what one may infer using these rules a limitless number of times. A *partial consequence operation* settles what one may infer if one uses them only a limited number of times.

(D9) (i) $C_R^0(X) := X$.
(ii) $C_R^{n+1}(X) := C_R^n(X) \cup \{\alpha \in L : \exists r \in R\, \exists Y \subseteq C_R^n(X)\, \langle Y, \alpha\rangle \in r\}$.
(iii) $C_R^\omega(X) := \bigcup_{n \in \omega} C_R^n(X)$.

In other words, partial consequence operations correspond to steps in deriving consequences of sentences by means of some fixed set of rules. Being partial they satisfy D1(i), (ii), (iv), but not D1(iii). T4 entails that

(T6) $C_R(X) = C_R^{\omega}(X)$.

Observe further that

(T7) $C_R^n(C_R^m) = C_R^{n+m}$.

Proof:
T7 follows from two lemmas:

(L1) $C_R^{n+1}(X) = C_R^n(C_R^1(X))$.
(L2) $C_R^{n+1}(X) = C_R^1(C_R^n(X))$. ■

Proof of L1:
If $n = 0$, then L1 is $C_R^1(X) = C_R^1(X)$. Assume that $C_R^{n+1}(X) = C_R^n(C_R^1 (X))$. I will show that $C_R^{n+2}(X) = C_R^{n+1}(C_R^1(X))$. If $\alpha \in C_R^{n+2}(X)$, then either $\alpha \in C_R^{n+1}(X) \subseteq C_R^{n+1}(C_R^1(X))$ or $\exists r \in R\, \exists Y \in C_R^n(C_R^1(X))\langle Y, \alpha\rangle \in r$. The latter implies that $\alpha \in C_R^{n+1}(C_R^1(X))$. On the other hand, if $\alpha \in C_R^{n+1}(C_R^1(X))$, then either $\alpha \in C_R^n(C_R^1(X)) = C_R^{n+1}(X) \subseteq C_R^{n+2} (X)$ or $\exists r \in R\, \exists Y \in C_R^{n+1}(X)\langle Y, \alpha\rangle \in r$. The latter also implies that $\alpha \in C_R^{n+2}(X)$. ■

Proof of L2:
The proof is similar. ■

By analogy with D9 I introduce *partially derivable rules of inference*. While I gradually become aware of consequences of my beliefs, I recognise that some new rules of inference are at my disposal (cf. D8):

(D10) (i) $\mathit{DERV}_R^0 := R$,
(ii) $r \in \mathit{DERV}_R^{n+1} \equiv C_{R\cup\{r\}}^{n+2} = C_R^{n+2}$,
(iii) $\mathit{DERV}_R^{\omega} := \bigcup_{n\in\omega} \mathit{DERV}_R^n$.

Consider the following example. Let r_1 be the rule of inference that consists of inferences of the form $\langle\{\alpha_1 \wedge \alpha_2\}, \alpha_1\rangle$. If $R := \{r_1\}$, D9 yields that

$C_R^0(\{((p \wedge q) \wedge r) \wedge s\}) = \{((p \wedge q) \wedge r) \wedge s\}$,
$C_R^1(\{((p \wedge q) \wedge r) \wedge s\}) = \{((p \wedge q) \wedge r) \wedge s, (p \wedge q) \wedge r\}$,

$C^2_R(\{((p \wedge q) \wedge r) \wedge s\}) = \{((p \wedge q) \wedge r) \wedge s, (p \wedge q) \wedge r, p \wedge q\}$,
$C^3_R(\{((p \wedge q) \wedge r) \wedge s\}) = \{((p \wedge q) \wedge r) \wedge s, (p \wedge q) \wedge r, p \wedge q, p\}$,
and if $n > 3$, $C^n_R(\{((p \wedge q) \wedge r) \wedge s\}) = C^3_R(\{((p \wedge q) \wedge r) \wedge s\})$.

Let $\beta_1 := (\alpha_1 \wedge \alpha_2)$ and for $n > 1$, $\beta_n := (\beta_{n-1} \wedge \alpha_{n+1})$. r_n will denote the rule that consists of all pairs of the form $\langle\{\beta_n\}, \alpha_1\}\rangle$. D10 yields that

$\mathit{DERV}^0_R = \{r_1\}$,
$\mathit{DERV}^1_R = \{r_1, r_2\}$,
$\mathit{DERV}^n_R = \{r_1, \ldots, r_{n+1}\}$.

T4 entails that

(T8) $\mathit{DERV}_R = \mathit{DERV}^\omega_R$.

and due to T7 we have that

(T9) $\mathit{DERV}^n_{\mathit{DERV}^m_R} = \mathit{DERV}^{n+m}_R$.

Partial consequence operations and partially derivable rules of inference are correlated in the following way:

(T10) $\forall n \geq 1 \, \forall \, C^n_R = C^1_{\mathit{DERV}^{n-1}_R}$.

Proof:
If $n = 1$, T10 is $C^1_R = C^1_R$. Assume that for all $k \leq n \; C^k_R = C^1_{\mathit{DERV}^{k-1}_R}$. If $\alpha \in C^{n+1}_R(X)$, then either $\alpha \in C^n_R(X)$ or $\exists r \in R \, \exists Y \subseteq C^n_R(X) \langle Y, \alpha\rangle \in r$. In the former case $\alpha \in C^1_{\mathit{DERV}^{n-1}_R}(X) \subseteq C^1_{\mathit{DERV}^n_R}(X)$. In the latter case $\exists r \in R \, \exists Y \subseteq C^1_{\mathit{DERV}^{n-1}_R}(X) \langle Y, \alpha\rangle \in r$. Since $R \subseteq \mathit{DERV}^{n-1}_R$, this implies $\alpha \in C^2_{\mathit{DERV}^{n-1}_R}(X)$. Since the assumption of our induction is general, we have that $C^2_{\mathit{DERV}^{n-1}_R} = C^1_{\mathit{DERV}^1_{\mathit{DERV}^{n-1}_R}}$, and consequently, $\alpha \in C^1_{\mathit{DERV}^1_{\mathit{DERV}^{n-1}_R}}(X)$. The required result, i.e. $\alpha \in C^1_{\mathit{DERV}^n_R}(X)$, is obtainable via T9. On the other hand, if $\alpha \in C^1_{\mathit{DERV}^n_R}(X)$, then $\alpha \in X \subseteq C^{n+1}_R(X)$ or $\exists r \in \mathit{DERV}^n_R \, \exists Y \subseteq X \langle Y, \alpha\rangle \in r$. The latter implies that $\alpha \in C^{n+1}_R(Y) \subseteq C^{n+1}_R(X)$.
To see this observe that

(L3) $r \in \mathit{DERV}^n_R \wedge \langle X, \alpha\rangle \in r \rightarrow \alpha \in C^{n+1}_R(X)$.

This concludes the proof that $C_R^{n+1} = C^1_{DERV_R^n}$. ■

Moreover, T4 and T10 imply that

(T11) $C_R = C^1_{DERV_R}$.

Finally, it is easy to verify that partial consequence operations and partially derivable rules of inference are well-ordered chains:

(T12) $n \leq m \rightarrow \forall X \subseteq L(C_R^n(X) \subseteq C_R^m(X)) \wedge DERV_R^n \subseteq DERV_R^m$.

II. *Non-formal assumptions of formalisation*

The aim of this section is to develop an epistemological conception of argumentation in which the definition of begging the question may be embedded. The ruling assumption of my attempt is the contention that epistemological values of argumentation depend not only on its formal features but also on the context in which it occurs.

Let Walton's concluding remarks provide a starting point on which to build the conceptual frame of my formalisation:

> [...] begging the question is a fallacy where 'fallacy' means an argument that fails to perform a useful function in contributing to a goal of dialogue. So conceived, begging the question is a pragmatic fallacy, a failure that needs to be evaluated in relation to how an argument has been used in a context of dialogue.
>
> In particular, one function of argument is the probative or doubt removing (or doubt reducing) function which presuppose the following framework of dialogue. One participant, the questioner, has doubts or question concerning a particular conclusion. The other participant, the arguer or proponent, has the job or role in the dialogue of proving this conclusion to the satisfaction of the questioner, according to the requirements of burden of proof appropriate for the type of dialogue and the particular case. (Walton 1994, 127)

Since, as Walton claims, persuasion dialogue is a generic type of dialogue, I want to focus my analysis on this type of discourse, in which:

> [...] one arguer has the goal or burden of proving his thesis from another arguer's premises. (ibid., 116)
>
> The argument should fulfil the probative function of proving to the respondent that the proposition queried is true by inferring it [ij] from premises that the respondent accepts as non-doubtful, or at least can be brought to accept as non-doubtful. [...] Now we can

> begin to see why begging the question is a fallacious move in this context of dialogue, a kind of illegitimate sophistical tactic. Giving a circular argument in answer to a request to prove a proposition (in the sense of fulfilling a probative function in a dialogue) is fallacious if the very same doubts attach to one of the premises that were already raised by the respondent in questioning the conclusion to be proved. (ibid., 119)

This broad pragmatic theory needs a trim to fit the theory of consequence. Contrary to Hamblin's account I will treat arguments as separate units.[2] Retractions, questions and resolutions will be excluded from contexts of argumentation and I will proceed as if these contexts contained only two kinds of relevant epistemic elements: beliefs[3] of the person to whom an argument is addressed and her inferential tools. Given my formal background the simplest way of expressing them is to identify the first with a set of sentences and the second with rules of inference. Beliefs and rules of inference determine rejected and neutral sentences. A participant in a dialogue *rejects* sentences that together with her beliefs form an inconsistent set. Towards every other sentence she adopts *neutral* attitude (she abstains).

Among my beliefs I distinguish between explicit and implicit beliefs. My *explicit beliefs* are propositions I am willing to accept without turning to inference. In other words, I am so familiar with my explicit beliefs that there is no point in presenting me arguments for them. On the other hand, I *implicitly accept a proposition* if I concede it only after being shown that it is obtainable from my explicit beliefs by means of my rules of inference. Within the set of rules of inference it is possible to draw a similar distinction. My *explicit rule of inference* comprises only those inferences that I accept without reflecting on my previous inferential activities (cf. D10). My *implicit rules* emerge as side effects of applying my explicit rules.

Given these distinctions it is evident that every person is committed to accept the conclusion of an argument such that

(i) she explicitly accepts all its premises,
(ii) the argument belongs to some of her explicit rules of inference.

Whether an argument whose premises belong to her (explicit or implicit) beliefs or that belongs to some of her (explicit or implicit) rules of inference is

[2] This means that a chain of arguments will be interpreted as one argument whose premises comprise premises of all arguments in the chain and the conclusion of which is the conclusion of the last argument in the chain. Given the terminology of (Woods and Walton 1977) I treat molecular arguments as atomic.

[3] In what follows I will treat "sentence", "belief", and "proposition" as equivalents.

a reasonable ground for accepting its conclusion depends, in my opinion, on her logical and metalogical abilities. A person is *potentially logically omniscient* if we are justified in presenting her arguments whose premises belong to her (explicit or implicit) beliefs. A *potentially metalogically omniscient* person is committed to every argument that belongs to some of her (explicit or implicit) rules of inference.[4] Therefore, one is permitted to refer to all beliefs and to the operation of consequence determined by rules of inference while advancing arguments towards questioners who are both logically and metalogically potentially omniscient. Such construed omniscience is postulated in the simplified forms of definitions below. All of them are relative to two epistemological determiners of a participant: explicit beliefs and rules of inference.

However, it seems reasonable to allow that some questioners are neither potentially logically nor metalogically omniscient. Arguments presented to them will have to be evaluated against the extended versions of the definitions. Now it is presupposed that only some of implicit beliefs and only some of implicit rules of inference are available for arguers. Given the set of a questioner's explicit rules of inference the definition of partial derivability determines the well-ordered chain of sets of implicit rules of inference (cf. D10 and T12). Given the set of her explicit beliefs and some element from this chain, partial consequence operations are steps in applying these rules to those beliefs. Again owing to T12 these operations are also ordered in a well-ordered chain. I assume that for every questioner there is some set in the former chain such that

(i) all its elements (i.e. all derivable rules of inference) may be referred to as her patterns of inference,
(ii) none its superset in the chain has this property.

I will call the number of this set (i.e. the superscript of $DERV^n_R$) the *metalogical depth* of a questioner. Similarly, I assume that for every questioner there is some set of implicit beliefs in the latter chain such that

(i) all its elements are admissible as premises of arguments advanced against her,
(ii) none its superset in the chain has this property.

The number of this set will be referred to as the *logical depth* of a questioner.

[4] A person is *actually logically* (*metalogically*) *omniscient* if and only if the set of her implicit beliefs (rules of inference) is empty. However, is anyone of us actually logically or metalogically omniscient?

Let us develop our example. Suppose that a person S explicitly accepts only that $((p \wedge q) \wedge r) \wedge s$ and explicitly allows only inferences of the form of r_1. If she is potentially logically and metalogically omniscient, you may resort to $\{((p \wedge q) \wedge r) \wedge s, (p \wedge q) \wedge r, p \wedge q, p\}$ for premises of arguments and you may reason in any of the ways defined by the rules from $\{r_n : n \in \omega\}$. However, if her logical depth and metalogical depth equal one, you may use $(p \wedge q) \wedge r$, but not $p \wedge q$, as a premise. Analogously, you may argue according to r_1 and r_2, but not, for example, according to r_3.

As a result the extended definitions will be relative to four epistemological determiners of participants: explicit beliefs, explicit rules of inference, metalogical depth, and logical depth.

In this framework Walton's pragmatic account could be confined to the epistemological description of argumentation given by David Sanford. In a series of papers (Sanford 1972, 1977, 1981) he exposes the position according to which the epistemological value of an argument depends not only on its form and on the truth-values of its premises but also on the beliefs of the person to whom it is addressed and on her inferential tools. In his view

> A primary purpose of inference is to increase the degree of reasonable confidence that one has in the truth of the conclusion. (Sanford 1981, 150)
>
> In dealing with inferences, our questions are not limited to the logical relations between the premisses and the conclusion. We may also ask about the epistemic relations between the inferer, the premisses, and the conclusion. [...] Despite a certain obscurity, I find W. E. Johnson's distinction between the constitutive and epistemic conditions of inference illuminating. I will provide my own formulation of his distinction rather than discuss his [...].
> (1) *Constitutive Conditions*: (i) p; (ii) p implies q.
> (2) *Epistemic Conditions*: (i) S believes that p; (ii) S believes that p implies q; (iii) S does not have either of these beliefs because he already believes that q. (ibid., 149)

I suggest we should slightly modify Sanford's account. Suppose that an arguer S_1 advances an argument "Since $\beta_1, \beta_2, \ldots, \beta_n$, therefore α" to increase another person's, a questioner S_2, degree of reasonable confidence in its conclusion α. I will say that an argument is *efficient for* S_2 if all its premises belong to her beliefs (that is Sanford's first epistemic condition), S_2 acknowledges its validity (his 2(ii)) and does not accept its conclusion with equal or more confidence than its premises. An argument satisfying the first condition will be called *grounded for* the questioner. An argument satisfying the second condition will be called *valid for* the questioner. An argument satisfying the last condition, which corresponds to 2(iii) from Sanford's account, will be called *non-superfluous for* the questioner.

I have substituted the last condition for 2(iii) for the following reasons:

(a) 2(iii) is the conjunction of
(iiia) S believes that p not because she believes that q, but for some other reasons,
(iiib) S believes that (p implies q) not because she believes that q, but for some other reasons.
The non-superfluity requirement is more general, i.e. if S accepts the conclusion of an argument with less reasonable confidence than its conclusions, then (iiia) and (iiib) hold, but not *vice versa*.

(b) If an argument fails to meet (iiib), it seems to commit the semantic fallacy of mixing levels of language rather than some epistemological fallacy. The usual nonformal account of validity says that it is impossible that the premises are true and the conclusion false (Haack 1977, 14). If asserting the conclusion of an argument were a ground for asserting the validity of this very argument, then the conclusion would conditionally assign the truth-value to itself.
Even if I am wrong, for the sake of homogeneity I prefer to separate beliefs form inferential tools in such a way that it is impossible on the ground of logic for the conclusion of an argument to influence the validity of this very argument. This, to be sure, does not exclude other (meta-) arguments whose conclusions refer to the validity of the initial argument.

(c) Non-superfluity seems to me more immediately linked with the goal of probative argumentation than 2(iii).

Sanford's degree of reasonable confidence (*DRC*) in a sentence may be expressed in my framework by its 'implicitness'. Explicit beliefs have the maximum 'positive' *DRC*. The more implicit a sentence is, the lesser is its *DRC*. Neutral sentences have the 'zero' *DRC* and the rejected are of some 'negative' *DRC*.

What about other Sanford's conditions? Well, if I read him correctly, 1(i) requires all premises of an argument be true. Since in most cases it is not a matter of logic to evaluate the truth-values of premises I ignore this condition.

As I want to allow a variety of modes of inference, 1(ii) will be met by imposing rather loose restrictions: D1, non-triviality, and consistency of consequence operations (see the first paragraph of the next section). However, some more constraints may be introduced if you wish to reduce arbitrariness of these modes. Eventually, you may end up with the classical consequence operation as the only plausible pattern of inference.

I hold that all efficient arguments provide reasonable grounds for changing a questioner's reasonable confidence in their conclusions.

Sanford has also presented us with one of the clearest account of context-dependent question begging.

> An argument formulated on Smith's benefit [ij] begs the question either if Smith believes one of the premises only because he already believes the conclusion or if Smith would believe one of the premises only because he already believed the conclusion. (Sanford 1972, 198)

More generally, an argument begs the question against S if and only if S can infer some of its premises only from its conclusion and S can infer a premise β of an argument only from its conclusion α iff

(QB1) an argument "Since α, therefore β" is valid for her,
(QB2) if S accepts β, then when she is asked to justify β she has to resort to α,
(QB3) if S does not accept β, than she would have to accept α in order to accept β.

The first condition guarantees that S can infer β from α. The second together with the third that she can infer β only from α. QB2 and QB3 express the idea that to beg the question is to reason in a circle: if you wish to accept the conclusion, you must first accept one of the premises, but if you wish to accept this premise, you must eventually accept the conclusion. Observe that our distinction: explicit beliefs – implicit beliefs makes it possible to interpret "accept" in QB either as "explicitly accept" or as "explicitly or implicitly accept". I find the former interpretation to be more in the spirit of the distinction than the former. After all, you may object to inflicting on you as "accepted" all consequences of your beliefs.

Notice further that both of these conditions express the indispensability of acceptance of α to acceptance of β. In QB2 no other explicit belief of S but α entails β, in QB3 no other sentence does. What is meant here by "no other sentence"? The phrase is clearly ambiguous in respect of the range (i.e. the set) of sentences that are checked for the possibility of inferring β from them. Roughly speaking, the more inclusive set is taken into account, the less arguments beg the question. One extreme is L. If "no other sentence" is interpreted literally, QB3 is hardly ever satisfied. For example, given that you accept the rule r_1 from our example you may choose any sentence $\gamma \in L$ such that $(\beta \wedge \gamma) \neq \alpha$ to avoid circularity in inferring β from α. Consequently, there seems to be numerous kinds of the fallacy in question.[5] However, my suggestion is that there are two marking points in this variety:

[5] One of the major objections to this conception points at its vagueness resulting from the lack of clarity in determining this set (see Wilson 1988).

all beliefs of S, and all sentences which are not rejected by S. Circularity in the former set determines the weak version of the fallacy and circularity in the latter the strong version.

In accordance with a well-established tradition (cf. Wilson 1988, 38–40) I supplement Sanford's conditions with the requirement that only valid arguments are eligible to commit the fallacy in question.

Summarising, an argument "Since $\beta_1, \beta_2, \ldots, \beta_n$, therefore α" *weakly begs the question against S* iff

(WQB0) the argument is valid for S,

and there is a premise β_i such that

(WQB1) an argument "Since α, therefore β_i" is valid for S,
(WQB2) if S explicitly accepts β_i, then no other explicit belief of S but α entails (for S) β_i.
(WQB3) if S does not explicitly accept β_i, then no other belief of S but α entails (for S) β_i.

An argument "Since $\beta_1, \beta_2, \ldots, \beta_n$, therefore α" *strongly begs the question against S* iff

(SQB0) the argument is valid for S,

and there is a premise β_i such that

(SQB1) an argument "Since α, therefore β_i" is valid for S,
(SQB2) if S explicitly accepts β_i, then no other explicit belief of S but α entails (for S) β_i.
(SQB3) if S does not explicitly accept β_i, then no other non-rejected sentence of S but α entails (for S) β_i.

The (succedents of) conditions WQB2–3 and SQB2–3 contain the relational predicate "no sentence from ... but a ... entails a ...(for ...)". To be more perspicuous I substitute "β is circular for S in relation to α in a set of sentences Z" for "No sentence from a set Z but α entails β (for S)". The next section provides with the analysis of this relation in terms of the theory of consequence.

III. *Simplified definitions of begging the question*

Suppose that an arguer S_1 submits a questioner S_2 an argument "Since $\beta_1, \beta_2, \ldots, \beta_n$, therefore α" in order to increase S_2's reasonable confidence in α. Let L be the language in which they both formulate their beliefs. Arguments of this form will be interpreted in the theory of consequence as inferences $X \vdash \alpha, X = \{\beta_1, \beta_2, \ldots, \beta_n\}$. S_2 is referred to by means of the pair $\langle E, R\rangle$:

(i) E is the set of her explicit beliefs,
(ii) R is the set of her explicit rules of inference.

C_R is then her consequence operation. The second of Sanford's constitutive conditions requires at least that C_R should be neither inconsistent nor idle to be someone's inferential basis. Moreover, I require that E should be C_R-consistent and nonempty to be someone's set of beliefs.

All that can be said, i.e. the language L, is divided for $\langle E, R\rangle$ into three exclusive subsets:

(D11) $\mathit{BELIEF}^{\langle E,R\rangle} := C_R(E)$.
(D12) $\mathit{REJECT}^{\langle E,R\rangle} := \{\gamma \in L : C_R(\mathit{BELIEF}^{\langle E,R\rangle} \cup \{\gamma\}) = L\}$.
(D13) $\mathit{NEUTRAL}^{\langle E,R\rangle} := L \backslash (\mathit{BELIEF}^{\langle E,R\rangle} \cup \mathit{REJECT}^{\langle E,R\rangle})$.

Since explicit beliefs occupy a special position among $\langle E, R\rangle$'s beliefs I suggest introducing strong-groundedness as a subtype of groundedness:

(D14) $X \vdash \alpha \in \mathit{GROUND}^{\langle E,R\rangle} \equiv X \subseteq \mathit{BELIEF}^{\langle E,R\rangle}$.
(D15) $X \vdash \alpha \in \mathit{STRONG\text{-}GROUND}^{\langle E,R\rangle} \equiv X \subseteq E$.

In order to take into account the epistemological difference between enthymemes and non-enthymemes some of the definitions below contain two versions, which, for the sake of brevity, are expressed in one formula. In the case of enthymemes all explicit beliefs of $\langle E, R\rangle$ are presumed. In the case of non-enthymemes validity of arguments is evaluated separately form these beliefs. An auxiliary set Y will be used: the definitions of enthymemes have $Y = E$, and the others have $Y = \emptyset$. An argument is valid for $\langle E, R\rangle$ if and only if its conclusion belongs to the set of C_R-consequences of its premises (in the case of enthymemes: of its premises and her explicit beliefs):

(D16) $X \vdash \alpha \in \mathit{VALID}^{\langle E,R\rangle} \equiv \alpha \in C_R(X \cup Y)$.

The degree of $\langle E, R\rangle$'s reasonable confidence will be measured by the following function:

$$\text{(D17) } \mathit{DRC}^{\langle E,R\rangle}(\gamma) := \begin{cases} 1 & \text{if } \gamma \in E, \\ \frac{1}{2} & \text{if } \gamma \in \mathit{BELIEF}^{\langle E,R\rangle} \backslash E, \\ 0 & \text{if } \gamma \in \mathit{NEUTRAL}^{\langle E,R\rangle}, \\ -1 & \text{if } \gamma \in \mathit{REJECT}^{\langle E,R\rangle}, \end{cases}$$

An argument is superfluous for $\langle E, R\rangle$ if and only if it cannot increase her *DRC* in its conclusion just because this *DRC* is greater than or equal to the

DRC of the premise in which she has the least confidence.

(D18) $X \vdash \alpha \in$ *NON-SUPERFLUOUS*$^{\langle E,R\rangle} \equiv$ *DRC*$^{\langle E,R\rangle}(\alpha)$
$< \min \{x : x =$ *DRC*$^{\langle E,R\rangle}(\beta) \wedge \beta \in X\}$.

Arguments' efficiency for $\langle E, R\rangle$ could be now defined as follows:

(D19) *EFFICIENT*$^{\langle E,R\rangle}$:= *VALID*$^{\langle E,R\rangle} \cap$ *GROUND*$^{\langle E,R\rangle}$
$\cap$*NON-SUPERFLUOUS*$^{\langle E,R\rangle}$.
(D20) *STRONG-EFFICIENT*$^{\langle E,R\rangle}$:= *VALID*$^{\langle E,R\rangle}$
$\cap$*STRONG-GROUND*$^{\langle E,R\rangle} \cap$ *NON-SUPERFLUOUS*$^{\langle E,R\rangle}$.

Now I turn to the definition of our fallacy. Let "*CIRC*$^R(\alpha, \beta, Z)$" mean "β is circular for R in relation to α in a set of sentences Z". Since I have separated beliefs from rules of inference, explicit beliefs of S_2 are irrelevant for this kind of circularity. How to express *CIRC*^{R} in the language of the theory of consequence?

I begin with an apparently obvious stipulation:

(1) *CIRC*$^R(\alpha, \beta, Z) \equiv \forall Z' \subseteq Z(\beta \in C_R(Z') \rightarrow \alpha \in Z')$.

β is circular for R in relation to α in a set of sentences Z iff whenever R obtains β using sentences from Z she always resorts to α.

Observe however that D1(i) entails $\beta \in C_R(\{\beta\})$. Therefore, given $\beta \in Z$ it is not the case that *CIRC*$^R(\alpha, \beta, Z)$ unless $\alpha = \beta$. Consequently, the definitions WQB and SQB imply that only those grounded arguments one premise of which is identical to their conclusions beg the question. In other words, strict orthographic identity (between a premise and the conclusion) is necessary for begging the question.

But *1* may be improved:

(2) *CIRC*$^R(\alpha, \beta, Z) \equiv \forall Z' \subseteq Z(\beta \in C_R(Z') \wedge \beta \notin Z' \rightarrow \alpha \in Z')$.

Unfortunately, it is now the case that:

(2.1) *CIRC*$^R(\alpha, \alpha, E\backslash\{\alpha\}) \wedge \alpha \in C_R(E) \rightarrow \alpha \in E$.

Subsequently, if one premise of an argument is identical to its conclusion and belongs to implicit beliefs of $\langle E, R\rangle$ (i.e. $\alpha \notin E$ and $\alpha \in C_R(E)$), then *2.1* renders the argument non-circular. In other words, strict ortographical identity is not sufficient for begging the question.

I find both of these consequences utterly counterintuitive.[6] For that reason I propose weakening *2* by means of the disjunct $\alpha = \beta$ (the identity sign denotes strict orthographic identity):

(3) $\mathit{CIRC}^R(\alpha,\beta,Z) \equiv \forall Z' \subseteq Z(\beta \in C_R(Z') \wedge \beta \notin Z' \rightarrow \alpha \in Z')$
$\vee \alpha = \beta$.

This definition guarantees that strict identity is sufficient but not necessary for circularity. Moreover, it can be proved that the right-hand side of *3* is equivalent to:

(4) $\forall Z' \subseteq Z(\beta \in C_R(Z') \wedge \beta \notin Z' \rightarrow \beta \notin C_R(Z' \backslash \{\alpha\})) \vee \alpha = \beta$.

and to:

(5) $\forall Z' \subseteq Z\ \forall r \in \mathit{DERV}_R(\langle Z',\beta\rangle \in r \wedge \beta \notin Z' \rightarrow \alpha \in Z' \vee \langle Z',\alpha\rangle \in r)$.

Proof:
Assume that

(*) $\forall Z' \subseteq Z(\beta \in C_R(Z') \wedge \beta \notin Z' \rightarrow \alpha \in Z')$,
(**) $\exists Z' \subseteq Z(\beta \in C_R(Z') \wedge \beta \notin Z' \wedge \beta \in C_R(Z' \backslash \{\alpha\}))$.

Since if $\beta \notin Z'$, then $\beta \notin Z' \backslash \{\alpha\}$, (**) entails that $\beta \in C_R(Z' \backslash \{\alpha\}) \wedge \beta \notin Z' \backslash \{\alpha\}$. Thus, (*) give us that $\alpha \in Z' \backslash \{\alpha\}$. This contradiction proves that the right-hand side of *3* implies *4*.

Suppose now that

(*) $\forall Z' \subseteq Z(\beta \in C_R(Z') \wedge \beta \notin Z' \rightarrow \beta \notin C_R(Z' \backslash \{\alpha\})) \vee \alpha = \beta$.
(**) $\exists Z' \subseteq Z\ \exists r \in \mathit{DERV}_R(\langle Z',\beta\rangle \in r \wedge \beta \notin Z' \wedge \alpha \notin Z' \wedge \langle Z',\alpha\rangle \notin r)$.

If $\alpha = \beta$, the contradiction is straightforward. Since $\alpha \notin Z'$, $Z' \backslash \{\alpha\} = Z'$. (**) implies that $\beta \in C_R(Z')$ and $\beta \notin Z'$. From this conjunction and the left disjunct of (*) it follows that $\beta \notin C_R(Z' \backslash \{\alpha\}) = C_R(Z')$. This concludes the proof that *4* implies *5*.

Finally, in order to prove that the right-hand side of *3* follows from *5* let

(*) $\forall Z' \subseteq Z\ \forall r \in \mathit{DERV}_R(\langle Z',\beta\rangle \in r \wedge \beta \notin Z' \rightarrow \alpha \in Z' \vee \langle Z',\alpha\rangle \in r)$,
(**) $\exists Z' \subseteq Z(\beta \in C_R(Z') \wedge \beta \notin Z' \wedge \alpha \notin Z') \wedge \alpha \neq \beta$.

[6] For examples supporting this intuition see (Barker 1978, 491–492) and (Sanford 1972, 197–198).

Owing to D1(iv), if $\beta \in C_R(Z')$, then there is some finite set Y such that $Y \subseteq Z'$ and $\beta \in C_R(Y)$. T5 and (**) imply then that one of the R-derivable rules of inference has $\langle Y, \beta\rangle$ as its only element, consequently the disjunction $\alpha \in Y \vee \langle Y, \alpha\rangle \in r$ follows from (*). The right disjunct is false because $\alpha \neq \beta$ and the rule contains only one element, and the left disjunct contradicts $\alpha \notin Z' \supseteq Y$ from (**). ■

I hope these equivalences add some more plausibility to my claim that *3* is the adequate approximation of circularity. This is not, however, the whole story. *3* does not exclude the possibility that β is circular in respect of α in Z only because the antecedent of the right-hand side of *3* is never fulfilled, i.e. only because β is C_R-independent from every subset of Z which does not include β. In this case you would not say that one has to resort to $\alpha \in Z$ in order to infer β, after all, there is no sentence in Z (apart perhaps from β itself) from which you may obtain β. *3* needs, then, the conjunct:

(6) $\exists Z' \subseteq Z(\beta \in C_R(Z') \wedge \beta \notin Z')$.

Finally, I submit the following definition of the probative circularity:

(D21) $\mathit{CIRC}^R(\alpha, \beta, Z) \equiv [(\forall Z' \subseteq Z(\beta \in C_R(Z') \wedge \beta \notin Z' \rightarrow \alpha \in Z') \vee \alpha = \beta) \wedge \exists Z' \subseteq Z(\beta \in C_R(Z') \wedge \beta \notin Z')]$.

Subsequently (cf. WQB and SQB),

(D22) $X \vdash \alpha \in \mathit{WEAK\text{-}QB}^{\langle E,R\rangle} \equiv X \vdash \alpha \in \mathit{VALID}^{\langle E,R\rangle} \wedge \exists \beta \in X [\beta \in C_R(Y \cup \{\alpha\}) \wedge (\beta \in E \rightarrow \mathit{CIRC}^R(\alpha, \beta, E)) \wedge (\beta \notin E \rightarrow \mathit{CIRC}^R(\alpha, \beta, \mathit{BELIEF}^{\langle E,R\rangle}))]$.

(D23) $X \vdash \alpha \in \mathit{STRONG\text{-}QB}^{\langle E,R\rangle} \equiv X \vdash \alpha \in \mathit{VALID}^{\langle E,R\rangle} \wedge \exists \beta \in X [\beta \in C_R(Y \cup \{\alpha\}) \wedge (\beta \in E \rightarrow \mathit{CIRC}^R(\alpha, \beta, E)) \wedge (\beta \notin E \rightarrow \mathit{CIRC}^R(\alpha, \beta, \mathit{BELIEF}^{\langle E,R\rangle} \cup \mathit{NEUTRAL}^{\langle E,R\rangle}))]$.

IV. *Extended definitions of begging the question*

The definitions D11–D23 will be now extended by making them relative to the logical depth of a questioner, referred to by the variable n, and to her metalogical depth, referred to by m:

(ExD11) $\mathit{BELIEF}^{\langle E,R\rangle}_{n,m} := C^n_{\mathit{DERV}^m_R}(E)$.

$\langle E, R\rangle$ may derive sentences from $\mathit{BELIEF}^{\langle E,R\rangle}_{n,m}$ in the n[th] step of inference if he uses rules from DERV^{m}_{R}.

(ExD12) $\mathit{REJECT}^{\langle E,R\rangle}_{n,m} := \{\gamma \in L : C^{n}_{\mathit{DERV}^{m}_{R}}(\mathit{BELIEF}^{\langle E,R\rangle}_{n,m} \cup \{\gamma\}) = L\}$.

(ExD13) $\mathit{NEUTRAL}^{\langle E,R\rangle}_{n,m} := L \backslash (\mathit{BELIEF}^{\langle E,R\rangle}_{n,m} \cup \mathit{REJECT}^{\langle E,R\rangle}_{n,m})$.

Observe that due to T12 it holds that

(T13) (i) $n_1 \leq n_2 \rightarrow \mathit{BELIEF}^{\langle E,R\rangle}_{n_1,m} \subseteq \mathit{BELIEF}^{\langle E,R\rangle}_{n_2,m}$.
(ii) $m_1 \leq m_2 \rightarrow \mathit{BELIEF}^{\langle E,R\rangle}_{n,m_1} \subseteq \mathit{BELIEF}^{\langle E,R\rangle}_{n,m_2}$.
(T14) (i) $n_1 \leq n_2 \rightarrow \mathit{REJECT}^{\langle E,R\rangle}_{n_1,m} \subseteq \mathit{REJECT}^{\langle E,R\rangle}_{n_2,m}$,
(ii) $m_1 \leq m_2 \rightarrow \mathit{REJECT}^{\langle E,R\rangle}_{n,m_1} \subseteq \mathit{REJECT}^{\langle E,R\rangle}_{n,m_2}$.

The distinction between strongly grounded and grounded arguments now disappears while $\mathit{BELIEF}^{\langle E,R\rangle}_{0,m} = E$.

(ExD14) $X \vdash \alpha \in \mathit{GROUND}^{\langle E,R\rangle}_{n,m} \equiv X \subseteq \mathit{BELIEF}^{\langle E,R\rangle}_{n,m}$.
(ExD16) $X \vdash \alpha \in \mathit{VALID}^{\langle E,R\rangle}_{n,m} \equiv \alpha \in C^{n}_{\mathit{DERV}^{m}_{R}}(Y \cup X)$.

The extension of the definition of *DRC* (D17) is not so trivial. Let me remind that our guiding intuition identifies the *DRC* of a sentence with the level of its implicitness. As the latter is relative to the metalogical depth of a questioner, so should be the former. Subsequently, since $\{\mathit{BELIEF}^{\langle E,R\rangle}_{n,m} : n \in \omega\}$ and $\{\mathit{REJECT}^{\langle E,R\rangle}_{n,m} : n \in \omega\}$ are well-ordered, we may express our intuition by the following definition:

(ExD17)

$$\mathit{DRC}^{\langle E,R\rangle}_{m}(\gamma) := \begin{cases} \frac{1}{\min\,\{n:\gamma\in \mathit{BELIEF}^{\langle E,R\rangle}_{n,m}\}+1} & \mathit{if}\ \gamma \in \mathit{BELIEF}^{\langle E,R\rangle}_{\omega,m}, \\ 0 & \mathit{if}\ \gamma \in \mathit{NEUTRAL}^{\langle E,R\rangle}_{\omega,m}, \\ -\frac{1}{\min\,\{n:\gamma\in \mathit{REJECT}^{\langle E,R\rangle}_{n,m}\}} & \mathit{if}\ \gamma \in \mathit{REJECT}^{\langle E,R\rangle}_{\omega,m}. \end{cases}$$

Observe that the assumption on consistency of $\langle E, R\rangle$'s beliefs implies that if $C_R(E) \not\subseteq E$, then $\mathit{REJECT}^{\langle E,R\rangle}_{0,m} = \emptyset$.
Therefore, the function $-\frac{1}{\min\,\{n:\gamma\in \mathit{REJECT}^{\langle E,R\rangle}_{n,m}\}}$ is well defined if you admit that there are no actually logically omniscient questioners.

T8 and T11 entail that

(T15) (i) $\mathit{BELIEF}^{\langle E,R\rangle}_{1,\omega} = \mathit{BELIEF}^{\langle E,R\rangle}$,

(ii) $\mathit{BELIEF}^{\langle E,R\rangle}_{\omega,\omega} = \mathit{BELIEF}^{\langle E,R\rangle}$.

(T16) (i) $\mathit{REJECT}^{\langle E,R\rangle}_{1,\omega} = \mathit{REJECT}^{\langle E,R\rangle}$,

(ii) $\mathit{REJECT}^{\langle E,R\rangle}_{\omega,\omega} = \mathit{REJECT}^{\langle E,R\rangle}$.

Thus, my contention that ExD17 is a reasonable extension of D17 is supported by the theorem

(T17) $\mathit{DRC}^{\langle E,R\rangle}_{\omega} = \mathit{DRC}^{\langle E,R\rangle}$,

that follows from T12, T15, and T16.

Subsequently,

(ExD18) $X \vdash \alpha \in \mathit{NON\text{-}SUPERFLUOUS}^{\langle E,R\rangle}_{m} \equiv \mathit{DRC}^{\langle E,R\rangle}_{m}(\alpha) < \min \{x : x = \mathit{DRC}^{\langle E,R\rangle}_{m}(\beta) \wedge \beta \in X\}$.

(ExD19) $\mathit{EFFICIENT}^{\langle E,R\rangle}_{n,m} := \mathit{VALID}^{\langle E,R\rangle}_{n,m} \cap \mathit{GROUND}^{\langle E,R\rangle}_{n,m} \cap \mathit{NON\text{-}SUPERFLUOUS}^{\langle E,R\rangle}_{m}$.

(ExD21) $\mathit{CIRC}^{R}_{n,m}(\alpha, \beta, Z) \equiv [(\forall Z' \subseteq Z(\beta \in C^{n}_{\mathit{DERV}^{m}_{R}}(Z') \wedge \beta \notin Z' \rightarrow \alpha \in Z') \vee \alpha = \beta) \wedge \exists Z' \subseteq Z(\beta \in C^{n}_{\mathit{DERV}^{m}_{R}}(Z') \wedge \beta \notin Z')]$.

(ExD22) $X \vdash \alpha \in \mathit{WEAK\text{-}QB}^{\langle E,R\rangle}_{n,m} \equiv X \vdash \alpha \in \mathit{VALID}^{\langle E,R\rangle}_{n,m} \wedge \exists \beta \in X [\beta \in C^{n}_{\mathit{DERV}^{m}_{R}}(Y \cup \{\alpha\}) \wedge (\beta \in E \rightarrow \mathit{CIRC}^{R}_{n,m}(\alpha, \beta, E)) \wedge (\beta \notin E \rightarrow \mathit{CIRC}^{R}_{n,m}(\alpha, \beta, \mathit{BELIEF}^{\langle E,R\rangle}_{n,m}))]$.

(ExD23) $X \vdash \alpha \in \mathit{STRONG\text{-}QB}^{\langle E,R\rangle}_{n,m} \equiv X \vdash \alpha \in \mathit{VALID}^{\langle E,R\rangle}_{n,m} \wedge \exists \beta \in X [\beta \in C^{n}_{\mathit{DERV}^{m}_{R}}(Y \cup \{\alpha\}) \wedge (\beta \in E \rightarrow \mathit{CIRC}^{R}_{n,m}(\alpha, \beta, E)) \wedge (\beta \notin E \rightarrow \mathit{CIRC}^{R}_{n,m}(\alpha, \beta, \mathit{BELIEF}^{\langle E,R\rangle}_{n,m} \cup \mathit{NEUTRAL}^{\langle E,R\rangle}_{n,m}))]$.

V. *Some theorems*

The following four theorems 'explain' fallaciousness of question begging:

(T18) $\mathit{WEAK\text{-}QB}^{\langle E,R\rangle}_{n,m} \cap \mathit{EFFICIENT}^{\langle E,R\rangle}_{n,m} = \emptyset$.

Proof:

Assume otherwise. Then there is an inference $X \vdash \alpha$ such that it belongs both to $\textit{WEAK-QB}^{\langle E,R\rangle}_{n,m}$ and to $\textit{EFFICIENT}^{\langle E,R\rangle}_{n,m}$. I will show that the inference is superfluous, what contradicts its efficiency.

Let β be the 'circular' premise. If $\beta \in E$ and $\alpha = \beta$, then $\alpha \in E$, and the argument is superfluous because $\textit{DRC}^{\langle E,R\rangle}_{n,m}(\alpha) = \textit{DRC}^{\langle E,R\rangle}_{n,m}(\beta) = 1$. If $\beta \in E$ and $\alpha \neq \beta$, then $\forall Z' \subseteq E(\beta \in C^n_{\textit{DERV}^m_R}(Z') \wedge \beta \notin Z' \rightarrow \alpha \in Z')$ and for some set $Z' \subseteq E$, $(\beta \in C^n_{\textit{DERV}^m_R}(Z') \wedge \beta \notin Z')$. Therefore, $\alpha \in Z' \subseteq E$, and the argument is superfluous. If $\beta \notin E$, then $\forall Z' \subseteq \textit{BELIEF}^{\langle E,R\rangle}_{n,m}(\beta \in C^n_{\textit{DERV}^m_R}(Z') \wedge \beta \notin Z' \rightarrow \alpha \in Z')$ or $\alpha = \beta$. For $\alpha = \beta$, the argument is superfluous. Suppose then $\alpha \neq \beta$. Since $E \subseteq \textit{BELIEF}^{\langle E,R\rangle}_{n,m}$ and $\beta \in C^n_{\textit{DERV}^m_R}(E)$ (the argument is grounded), therefore $\alpha \in E$. Consequently, the argument is superfluous, for now $\textit{DRC}^{\langle E,R\rangle}_{n,m}(\beta) < \textit{DRC}^{\langle E,R\rangle}_{n,m}(\alpha) = 1$. ■

(T19) $\textit{STRONG-QB}^{\langle E,R\rangle}_{n,m} \cap \textit{EFFICIENT}^{\langle E,R\rangle}_{n,m} = \emptyset$.

The proof of T19 is similar. It is evident that T18 and T19 hold for potentially omniscient questioners as well.

(T20) $S \subseteq \textit{BELIEF}^{\langle E,R\rangle} \rightarrow \textit{WEAK-QB}^{\langle E,R\rangle} \cap \textit{EFFICIENT}^{\langle E\cup S,R\rangle} = \emptyset$.

Proof:

Observe first that if $S \subseteq \textit{BELIEF}^{\langle E,R\rangle}$, then
$\textit{GROUND}^{\langle E\cup S,R\rangle} = \textit{GROUND}^{\langle E,R\rangle}$.

Suppose that for some set of sentences $S \subseteq \textit{BELIEF}^{\langle E,R\rangle}$ there is an inference $X \vdash \alpha$ included both in $\textit{WEAK-QB}^{\langle E,R\rangle}$ and in $\textit{EFFICIENT}^{\langle E\cup S,R\rangle}$. Let β be the 'circular' premise. If $\beta \notin E$, then $\forall Z' \subseteq \textit{BELIEF}^{\langle E,R\rangle}(\beta \in C_R(Z') \wedge \beta \notin Z' \rightarrow \alpha \in Z')$ or $\alpha = \beta$. Assume the left disjunct. Since $E \subseteq \textit{BELIEF}^{\langle E,R\rangle}$ and $\beta \in C_R(E)$ (the argument is grounded for $E \cup S$, and thus for E), therefore $\alpha \in E$, and the argument is superfluous for $\langle E \cup S, R\rangle$. The remaining cases may be established along the lines of the previous proof. Consequently, the inference is not efficient for $\langle E \cup S, R\rangle$. ■

(T21) $S \subseteq \textit{BELIEF}^{\langle E,R\rangle} \cup \textit{NEUTRAL}^{\langle E,R\rangle} \rightarrow \textit{STRONG-QB}^{\langle E,R\rangle}$
$\cap \textit{EFFICIENT}^{\langle E\cup S,R\rangle} \cap \{X \vdash \alpha : S \cap X = \emptyset\} = \emptyset$.

Proof:

Assume that there is a set of sentences $S \subseteq \textit{BELIEF}^{\langle E,R\rangle} \cup \textit{NEUTRAL}$ ${}^{\langle E,R\rangle}$ and an inference $X \vdash \alpha$ such that it belongs both to $\textit{STRONG-QB}^{\langle E,R\rangle}$

and to *EFFICIENT*$^{\langle E\cup S,R\rangle}$, and $S \cap X = \emptyset$. Let β be the 'circular' premise. If $\beta \notin E, \forall Z' \subseteq$ *BELIEF*$^{\langle E,R\rangle} \cup$ *NEUTRAL*$^{\langle E,R\rangle}(\beta \in C_R(Z') \wedge \beta \notin Z' \rightarrow \alpha \in Z')$ or $\alpha = \beta$. Since $E \cup S \subseteq$ *BELIEF*$^{\langle E,R\rangle} \cup$ *NEUTRAL*$^{\langle E,R\rangle}$ and $\beta \in C_R(E \cup S)$ (the argument is grounded for $E \cup S$), and $\beta \notin E \cup S$, therefore $\alpha \in E \cup S$, and as a result the argument is superfluous for $\langle E \cup S, R\rangle$. The remaining cases may be established along the lines of the proof of T18. ■

T18 and T19 confirm Walton's claim that to beg the question is to miss the probative goal of argumentation. T20 points out that in the case of weakly question begging arguments advanced against a potentially omniscient questioner even some enlargement of the set of her explicit beliefs may not result in reversing their epistemological status, i.e. they remain non-efficient afterwards. T21 determines the range of enlargements preserving non-efficiency of strongly question begging arguments.

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