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ADAPTIVE LOGIC AND COVERING LAW EXPLANATIONS*

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Abstract

In his theory of explanation Hempel introduced two basic types of covering law explanations for particular events: deductive-nomological and inductive-statistical. In this article we argue that there is more than one reason why adaptive logics provide the right tools for analyzing the argument patterns involved in these covering law explanations. To this end we claim that in the case of inconsistent knowledge systems, neither classical logic, nor a paraconsistent logic suffice to capture the right class of permissible arguments that can make up a deductive-nomological explanation, whereas an adaptive logic gives just the right results. The arguments behind inductive-statistical explanations face the well-known problem of inductive ambiguities, which Hempel tried to solve with his *requirement of maximal specificity*. We show how this requirement can be nicely incorporated in a logic for these arguments, again using an adaptive logic (which we describe in some detail).

1. *Introduction*

The distinction between two basic types of so-called covering law explanations for particular events, *deductive-nomological* (D-N) and *inductive-statistical* (I-S), goes back to the godfather of philosophy of explanation, Carl Gustav Hempel (for an overview of the modern history of theories of explanation, see Salmon 1989 — for Hempel's own ideas, see Hempel 1965). A D-N explanation can be characterized as a valid *deductive* argument having as conclusion the statement that the event did occur. An I-S explanation,

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on the other hand, consists in an *inductive* argument with the same conclusion. Of course, the premises will have to be subject to further restrictions, which we will analyze in some detail. The most well known restriction is the demand that the premises contain a sentence stating a law, whence the general term of *covering law* explanations.

At first glance it might seem that the logical analysis of D-N explanations is straightforward, but in section 2 we will recapitulate an earlier argument from Weber & De Clercq (2002), showing that there are common cases that cause trouble. As we will explain in section 3, this introduces a first reason to approach the logic of explanation from the perspective of adaptive logics (as was already claimed in Weber & De Clercq (2002)). The general ideas behind these logics will also be presented in that section.

That I-S explanations cannot be handled with tools from classical logic is of course no surprise — this is a counterpart of Hume's problem with inductive predictions. But as we will show in section 5, adaptive logics give us just the right tools to analyze the logic of I-S explanations. Section 4 will contain the needed preliminaries in which we discuss the general structure of these explanations.

The general lesson will be that classical logic is not well-suited to handle standard problems in philosophy of science. Adaptive logics, on the other hand, turn out to be more fruitful in this respect.

2. *D-N Explanations*

2.1. *The general structure*

Let us begin with a clear informal characterization, as given by Hempel:

> ... a D-N Explanation answers the question '*Why* did the explanandum-phenomenon occur?' by showing that the phenomenon resulted from certain particular circumstances, specified in $C_1, C_2, \ldots,$ C_k, in accordance with the laws $L_1, L_2, \ldots, L_r$. By pointing this out, the argument shows that, given the particular circumstances and the laws in question, the occurrence of the phenomenon *was to be expected*; and it is in this sense that the explanation enables us to *understand why* the phenomenon occurred. (1965, p. 337; italics in original)

In this approach, explanation is to be identified with *expectability*. Since the conclusion of a deductive inference follows with certainty from the premises, this clearly gives us the right sort of argument.

In the traditional D-N model some further conditions besides deductive validity are added to the argument. Most importantly, the premises that make up the *explanans* must contain at least one sentence stating a general law: it is in virtue of this law that the other particular facts cited in the premises can be considered explanatory relevant to the *explanandum*. In what follows we will ignore the problems that surround the characterization of so-called lawlike sentences, and concentrate ourselves on the general structure of D-N explanations, assuming that all such sentences can be represented by generalized sentences containing only universal quantifiers. It is clear from the preceding that all D-N explanations fall under something like the following schema (L stands for the lawlike sentence, C_i for sentences describing particular facts that together with L make up the explanans, and E for the explanandum-sentence):

L: $(\forall x)(\mathrm{P}_1x \& \mathrm{P}_2 \& \ldots \& \mathrm{P}_\mathrm{n}x \rightarrow \mathrm{Q}x)$
C_1: P_1b
C_2: P_2b
...
C_n: $\mathrm{P}_\mathrm{n}b$

E: $\mathrm{Q}b$

A very simple example is the following explanation of the fact that Mary has blood group A:

L: All humans who belong to category $\mathrm{I^AI^A} \times \mathrm{I^AI^O}$ have blood group A.
C_1: Mary is a human
C_2: Mary belongs to category $\mathrm{I^AI^A} \times \mathrm{I^AI^O}$

E: Mary has blood group A.

So far, so good. Now, one minor complication has to be handled. The following would not be considered a satisfactory explanation by anyone:

L: All humans who belong to category $\mathrm{I^AI^A} \times \mathrm{I^AI^O}$ have blood group A.
C_1: Mary is a human
C_2: Mary has blood group A.

E: Mary has blood group A.

An obvious way to repair this defect is to demand that the law sentence is *really* needed in the derivation. Hempel takes care of this by adding the requirement that the deletion of L makes the argument invalid. So, if want to summarize the idea behind D-N explanations in a more formal way, we get something like the following set of conditions (CL standing for 'Classical Logic' — and a knowledge system being the set of all statements accepted at a given time):

> Knowledge system K provides a *deductive-nomological explanation* for E if and only if there are sentences $C_1, \ldots, C_n$ describing particular facts, and a lawlike sentence L such that
> (i) $C_1, C_2, \ldots, C_n, L$ is consistent,
> (ii) $C_1 \& C_2 \& \ldots \& C_n$ and L are CL-derivable from K,
> (iii) E is CL-derivable from $C_1 \& C_2 \& \ldots \& C_n \& L$, and
> (iv) E is not CL-derivable from $C_1 \& C_2 \& \ldots \& C_n$.

The first condition is needed to exclude trivial explanations: CL validates the rule *ex contradictione sequiter quodlibet*, so if we have inconsistent premises, they could explain everything (something that may please mystical minds, but doesn't help much if we want to understand more down-to-earth instances of explanation). Besides this technical problem, it is also questionable if one could ever consider a set of contradictory sentences as a good explanation for some event. However, this condition (i) is not enough to avoid all problems that can arise from inconsistencies.

2.2. *A problem*[1]

Suppose that our knowledge system contains the following sentences:

(K_1) All Quakers are pacifists.
All republicans are non-pacifists.
Nixon is a Quaker and a republican.

Applying the preceding definition leads to the conclusion that K_1 provides a D-N explanation for both "Nixon is a pacifist" *and* "Nixon is not a pacifist":

L_1: All Quakers are pacifists
C_1: Nixon is a Quaker
———————————————
E_1: Nixon is a pacifist

L_2: All republicans are non-pacifists
C_2: Nixon is a republican
———————————————
E_2: Nixon is not a pacifist

[1] The following examples are taken from Weber & De Clercq (2002).

This kind of situations is common enough: most real-life knowledge systems do contain rules of this kind that can lead to inconsistencies. However, the possibility of explaining both a sentence and its negation at the same time should somehow be excluded (what do we learn by knowing that E was expectable, if it turns out that $\sim E$ was expectable as well?).

One way to secure this would be the requirement that the knowledge system be closed under logical implication *and* consistent.[2] However, we believe this too stringent a demand. There are both empirical and logical reasons for this suspicion. The empirical ones are the apparent unavoidability — due to the pragmatic limitations of human reasoning — of inconsistencies in everyday knowledge (as already illustrated by our simple example K_1), as well as in scientific theories (consult e.g. Meheus 1993, Norton 1993, Smith 1988). The logical sources of suspicion are well known, and among the most surprising results of twentieth century philosophical investigations, i.e. Gödel's theorems.

In view of this, we believe that the D-N account of explanation needs another way to circumvent the problem — if it is to be interesting as an analysis of real-life explanations. A solution would be to expand the set of conditions on admissible D-N explanations, so as to prevent the possibility of giving at the same time a D-N argument for E *and* for $\sim E$. If we add a fifth condition to the set of criteria in 2.1.,

(v) $\sim E$ is not CL-derivable from K,

then K_1 does not provide an explanation for "Nixon is a pacifist", nor for its negation. Indeed, whenever both E and $\sim E$ are derivable, neither can be given a D-N explanation, as a consequence of this extra condition.

However, it turns out that, as it stands, this solution coincides with the first one. Indeed, consider what happens if we have a contradiction in a knowledge system. Since in CL the rule *ex contradictione quodlibet* holds, this means that we can deduce any sentence, so for *any* E, we also have that $\sim E$ is CL-derivable — and as a result condition (v) implies that *no D-N explanations are possible* from an inconsistent knowledge system.

That this is counterintuitive can best be illustrated with a further example. Consider the following knowledge system:

(K_2) All birds fly.
Penguins don't fly.
Tweety is a bird and a penguin.
Billy is a bird but not a penguin.

[2] No doubt, this would have been Hempel's own answer to the problem (e.g. Hempel 1965, p. 396).

We think that almost everybody would agree that on this basis one *can* explain why Billy flies, even if we have contradictory information about Tweety. So adding condition (v) seems by no means the most promising way to safeguard our theory of explanation of becoming trivial. As it stands, it clearly forbids too much. This example illustrates the general problem with our expanded definition: though the information we have about some facts (e.g. that Billy flies) is consistent, the inconsistency of the knowledge system as a whole implies that *nothing* can be explained. It seems that it is not possible to analyze D-N explanations in a satisfying way, using CL as the underlying logic. In what follows we will claim that this problem can be solved by replacing CL by an adaptive logic in the conditions (i)–(v) (notice that condition (v) will be retained, since we still want to exclude the possibility of explaining E if $\sim E$ was expectable as well, *but* that by using an adaptive logic this no longer coincides with the first solution). As we will show this entails that K_2 allows one to give a D-N explanation for Billy's flying, but not for Tweety's not flying.

3. *Adaptive logics*

3.1. *The general idea*

An adaptive logic is a logic that tries to take the best from two worlds. Put more formally: it uses the rules from one strong logic, but when these cause problems, it uses the rules from another, weaker logic. It 'oscillates' between an upper limit logic (ULL) and a lower limit logic (LLL). (Adaptive logics were first introduced and developed by Diderik Batens; for a survey see Batens 1998,2000,200+.)

When one is confronted with a knowledge system containing inconsistencies, the safest thing to do is to use a paraconsistent logic. Such a logic avoids triviality (*ex contradictione quodlibet* is not generally valid in it), by dropping some inference rules of CL (e.g. *modus tollens* and *disjunctive syllogism*). But the fact that some inference rules are dropped also means that in general a set of premises will have less conclusions. Consider the following simple example: if we use a paraconsistent logic that invalidates *modus tollens*, then '$\sim p$' will not follow from the set of premises $p \supset q, \sim q, r, \sim r$. So, avoiding the triviality that arises from the inconsistency comes at a price; and a price that is hard to pay, since p and q have nothing to do with the inconsistency!

Inconsistency-adaptive logics were developed to handle this kind of situations. The intuitive idea is that one can use all rules from the ULL (in this case CL), unless a specific application causes trouble. Whenever trouble arises, one can still use the rules from the LLL (in this case a paraconsistent logic). One easily sees how this can generalize to other sorts of situations, where “trouble” arises from other sources than inconsistencies (in these cases the LLL will often be CL, and the ULL will contain extra inference rules — e.g. to take inductive steps — which can cause their own kind of trouble). A concrete example of this will be given in section 5 where we will introduce an adaptive logic for I-S explanation.

The “unless”-clause is formalized by introducing *conditional rules*: something can be derived from a set of sentences *on a condition*. If at a later stage of the proof it turns out that the condition is violated, then the sentence that was derived at an earlier stage is *marked* as invalid: the line of the proof containing this sentence is out. “Derived” and “invalid” thus have to be interpreted as “at a stage”; this does not prevent us from also introducing the notion of *final derivability*: a sentence is finally derived at a line in a proof if and only if any extension of the proof in which the line is marked may be further extended in such a way that the line is non-marked. It is clear that all the rules from the LLL can be treated as *unconditional rules*. But of course, if the sentences to which we apply such unconditional rules were derived on a condition, then the sentence that we introduce at a new line in the proof will also contain a condition: the union of all the conditions of the earlier sentences of which it is a consequence.

Adaptive logics thus always have something like the following proof format. Every line of the proof consists of five elements:

(i) a line number,
(ii) the sentence derived,
(iii) the line numbers of the sentences from which (ii) is derived,
(iv) the rule of inference that justifies the derivation,
(v) the set of sentences on the normal behavior of which we rely in order for (ii) to be derivable by (iv) from the sentences of the lines enumerated in (iii).

By normal behavior we mean to indicate that these sentences cause no trouble — a notion that always will be specified according to the particular adaptive logic that is studied. Besides a *structural rule* by which you can introduce premises in a proof (always with an empty fifth element), there are two kinds of *inference rules*: the unconditional ones, and the conditional ones. For example, an application of the rule *unconditional modus ponens* (MP) looks like the following:

j	$p \supset q$	...	...	Δ
$j+1$	p	...	...	Θ
$j+2$	q	$j, j+1$	MP	$\Delta \cup \Theta$

And an application of the rule *conditional modus tollens* (MTC) looks like the following:

j	$p \supset q$	$\ldots$	$\ldots$	Δ
$j+1$	$\sim q$	$\ldots$	$\ldots$	Θ
$j+2$	$\sim p$	$j, j+1$	MTC	$\Delta \cup \Theta \cup \{q\}$

Besides these inference rules, there is one special type of rules: the *marking rules* that indicate when a line in the proof has to be marked. For example, when you discover that B is causing trouble (e.g. because it behaves inconsistently), than these rules tell you to mark all lines of the proof containing B in its fifth element.

One will recognize this general format in the adaptive logic that we will propose in section 5.

3.2. *How to solve the problem with D-N explanations*

In the concluding remarks of his (1965) Hempel states his main objective as being an *explication* of the concept "explanation", implying that:

> Like any other explication, the construal here put forward has to be justified by appropriate arguments. In our case, these have to show that the proposed construal does justice to such accounts as are generally agreed to be instances of scientific explanation, and that it affords a basis for a systematically fruitful logical and methodological analysis of the explanatory procedures used in empirical science. (1965, p. 489)

The examples in 2.2. were intended to throw doubt on the fruitfulness of Hempel's original explication: there is an important class of situations for which it fails. An improved construal should avoid the problem that arises with inconsistent knowledge systems, while at the same time doing justice 'to such accounts as are generally agreed to be instances of scientific explanation'.

It is clear from the foregoing that a paraconsistent logic would avoid the problem with the explanation of Billy's flying (we can no longer use *ex contradictione quodlibet* to derive that Billy does not fly). But at the same time we believe that this would not be the right way to explicate the concept of explanation, for we think it undeniable that scientists don't refrain from using a rule like *modus tollens* in constructing explanations, *wherever* it doesn't cause trouble. (A very nice example of an actual case in which this happened is found in Meheus 1993, in which Clausius's reasoning leading up to modern thermodynamics is analyzed — reasoning that had to deal with inconsistencies.) Consider for example what would happen if we would restrict permissible explanations to arguments based solely on a paraconsistent logic invalidating *modus tollens*, and if our knowledge system would be $K_3 : \{p \supset q, \sim q\}$. In this case it would be impossible to explain the fact that $\sim p$ — clearly contrary to what is 'generally agreed to be instances of scientific explanation'! Of course, someone could reply to this that one can work with two sets of conditions on D-N explanations, one for consistent knowledge systems, and another one for inconsistent knowledge systems. That this will not do, is evident when one considers that it is not always clear from the outset whether a system contains no contradictions (remember Frege's despair after receiving Russell's letter), but most importantly that this still seems to give the wrong result with knowledge systems like the following $K_4 : \{p \supset q, \sim q, r, \sim r\}$. When confronted with K_4, one feels that one can explain '$\sim p$' as much as when confronted with K_3!

These remarks point to an inconsistency-adaptive logic as the right tool for explicating D-N explanations. Indeed, using an inconsistency-adaptive logic it is always possible to derive $\sim p$ from K_3 and K_4, on the condition that q behaves consistently. So, all we have to do is replace CL by $\mathsf{I\text{-}AL}$ (standing for a suitable inconsistency-adaptive logic)[3] in the extended set of conditions (i)–(v) from section 2. (Notice that this replacement renders condition (v) unproblematic, since *ex contradictione quodlibet* is no longer valid.) With this revised definition in place, it follows that K_2 provides a D-N explanation for 'Billy flies' (this sentence is $\mathsf{I\text{-}AL}$-derivable, whereas its negation is not), and at the same time excludes an explanation for 'Tweety does not fly' (since 'Tweety flies' is $\mathsf{I\text{-}AL}$-derivable as well as 'Tweety does not fly'). For more on this, we refer the reader to Weber & De Clercq (2002), where she can find a detailed description of an inconsistency-adaptive logic for D-N explanations.

4. *I-S Explanations*

4.1. *The general structure*

Once again, let us begin with a characterization given by Hempel:

> Explanations of particular facts or events by means of statistical laws thus present themselves as arguments that are inductive or

[3] Depending on the paraconsistent logic that one uses as LLL, and on the specific marking rules, one can choose the adaptive logic one thinks best suited. In Weber & De Clercq (2002) $\mathsf{ACLuN_1}$ is used for this purpose.

> probabilistic in the sense that the explanans confers upon the explanandum a more or less high degree of inductive support or of logical (inductive) probability; they will therefore be called inductive-statistical explanations; or I-S explanations. (1965, pp. 385–386)

Explanation is still linked with expectability, but in this case expectability comes in degrees. The idea of lawlike sentences thus has to be extended to account for *statistical laws* that have the conditional form $Prob(\mathrm{G} \mid \mathrm{F}) = r$, where r denotes the probability that an object of the set F is also a member of the set G. The set F is called the *reference class* of this statistical law.

An I-S explanation will be an argument with the following structure, analogous to D-N explanations:

$$\begin{array}{ll} L: & Prob(\mathrm{G} \mid \mathrm{F}) = r \\ C_1: & \mathrm{F}b \\ \hline\hline E: & \mathrm{G}b \end{array} \quad [\mathrm{r}]$$

The notation is borrowed from Hempel: the double line indicates that the argument is inductive rather than deductive, and "[r]" represents the degree of inductive support that is conferred upon the conclusion by the premises. This argument explains the fact that object b has property G by showing that this could be expected with probability r, given the fact that the statistical law L holds, and that b has property F. (Of course this structure can be extended to a more general schema in which the reference class of the conditional probability is determined by a conjunction of properties $\mathrm{F}_1\&\mathrm{F}_2\&\ldots\&\mathrm{F}_n$, and in which b has the properties $\mathrm{F}_1, \ldots, \mathrm{F}_n$.)

Once again, there are further restrictions on this argument for it to count as an I-S explanation. As in the case of D-N explanations we have to rule out the possibility of circular explanations (this can be done in exactly the same way). As an extra condition we can also require $r > t$ with t a chosen limit on the degree of inductive support.

A very simple example of an I-S explanation would be the explanation of why John Jones (j) recovered (R) from a streptococcus infection (S), when our knowledge system also contains the information that John was administered penicillin (P), and the probability (r, which is close enough to 1) of recovery from an infection given that penicillin is administered:

$$\begin{array}{ll} L: & Prob(\mathrm{R} \mid \mathrm{S\&P}) = r \\ C_1: & \mathrm{S}j \\ C_2: & \mathrm{P}j \\ \hline\hline E: & \mathrm{R}j \end{array} \quad [\mathrm{r}]$$

Hempel, after introducing this example, immediately remarks that an important problem remains to be solved.

4.2. *A problem*

Not all streptococcal infections can be cured by administrating penicillin, and some streptococcus strains are even resistant to penicillin. The probability of recovery among the people who are treated with penicillin *and* are infected by a resistant strain is a number s very close to 0 (equivalently, we could say that the probability of not recovering among these people is a number $1 - s$ very close to one). If we would know that John Jones was infected by such a strain (Z), then we could give the following argument:

$$\begin{array}{ll} L\text{:} & Prob(\sim\text{R} \mid \text{S\&P\&Z}) = 1 - s \\ C_1\text{:} & \text{S}j \\ C_2\text{:} & \text{P}j \\ C_3: & \text{Z}j \\ \hline\hline E: & \sim\text{R}j \end{array} \quad [1 - \text{s}]$$

But now we are confronted with two strong inductive arguments, the premises of which could all be true at the same time, that give contradictory conclusions. This phenomenon is dubbed the *ambiguity of I-S explanations* by Hempel.

The problem clearly has to do with the choice of the right reference class. This problem was well-known from the attempts at constructing an inductive logic. But with I-S explanations another solution is needed, since *the requirement of total evidence* that was favored by inductive logicians is totally inappropriate. Hempel cites the following formulation by Carnap:

> in the application of inductive logic to a given knowledge situation, the total evidence available must be taken as basis for determining the degree of confirmation. (1965, p. 397)

Consider what happens if we transpose this requirement to the problem at hand: the *explanandum* must be incorporated among the premises of the argument (since it is already known when constructing an explanation and thus is part of the total evidence), but this means that all explanations are rendered trivial!

Hempel's solution is the so-called *requirement of maximal specifity (RMS)* ('b', 'F' and 'r' as introduced in 4.1.)[4]:

> ... if K is the set of all statements accepted at the given time, let k be a sentence that is logically equivalent to K ... Then, to be

[4] We adapted the notation in the quotation to ours — we also slightly altered Hempel's original condition, in which he allows for premises not contained in the knowledge system at the time of the explanation, since this doesn't make a difference to our discussion.

> rationally acceptable in the knowledge situation represented by K, the proposed explanation ... must meet the following condition (the requirement of maximal specificity): if ... k implies that b belongs to a class F_1, and that F_1 is a subclass of F, then ... k must also imply a statement specifying the statistical probability of G in F_1, say
>
> $$Prob(G \mid F_1) = r_1$$
>
> Here, r_1 must equal r unless the probability statement just cited is simply a theorem of mathematical probability theory. (1965, p. 400)

The unless-clause is needed to exclude the necessity of introducing $Prob(G \mid F\&G) = 1$ as the statistical law with the maximally specific reference class.

If we turn back to John Jones's recovery, it is clear that the first explanation does not satisfy RMS, whereas the second does: we have a correct I-S explanation for $\sim Rj$, but not for Rj. We always have to subsume an object under the class that gives the most specific information that is available in K; in this case this is 'S & P & Z'.

There are some further complications associated with Hempel's original formulation of RMS; we refer the reader to Salmon (1989) for this. In what follows, our intention is to show how to formalize the arguments that make up I-S explanations, incorporating RMS in the logic itself.

5. *An adaptive logic for I-S explanations*

5.1. $\mathsf{AL_{ISE}}$

Trouble is spelled 'ambiguity'; marking is governed by RMS. These are the basics for the adaptive mechanism we want to propose to formalize Hempel's I-S explanations.

Once the adaptive logic, which we will call $\mathsf{AL_{ISE}}$, is defined, the set of conditions for I-S explanations can be easily defined in an analogue way as for D-N explanations. All we have to do is replace CL by $\mathsf{AL_{ISE}}$ in the conditions (iii) and (iv) in the characterization of D-N explanations given in section 2 (for the moment disregarding problems that can arise with an inconsistent knowledge system).

The LLL of the adaptive logic is classical predicate logic CL (with an extended language). The ULL is an inductive logic that allows you to conclude 'Gb' when you already have '$Prob(G \mid F) = r$', 'Fb', and when $r > t$ (t being the chosen limit on the degree of inductive support).

The adaptive logic works as follows: one is allowed to give the arguments that make up I-S explanations *unless* one discovers that an inductive ambiguity can arise. If this is the case, one can still give plain deductive arguments. So, after one has analyzed all the information contained in a knowledge system, all explanations that give rise to inductive ambiguities are excluded, whereas all explanations using maximally specific reference classes (relative to the knowledge system) can be retained.

Let us make all this more precise.

THE LANGUAGE

The language consists of the language of CL, with the equality sign added, together with a two-place probability function $Prob(\cdot \mid \cdot)$[5], the arithmetic function symbols +, $\times$, >, and the real numbers in the interval $[0,1]$[6].

STRUCTURAL RULE

We introduce a *premise rule* that at once recalls the structural properties of lines in a proof (as defined in section 3).

PREM At any stage of a proof one may add a line consisting of (i) an appropriate line number, (ii) a premise, (iii) a dash, (iv) 'PREM', (v) 'ϕ'.

THE LOWER LIMIT LOGIC

The LLL, $\mathsf{CL}^\star$, consists of CL, with equality, to which are added the standard axioms of arithmetic and probability theory. This immediately gives us the following *unconditional rule*.

RU If $A_1, \ldots, A_n \vdash_{\mathsf{CL}^\star} B$, and $A_1, \ldots, A_n$ occur in the proof, then one may add B to the proof. The fifth element of the new line is the conjunction of the fifth elements of the lines in its third element.

THE UPPER LIMIT LOGIC

The ULL, as always, contains all the inference rules from the LLL plus some extra rules. The ULL that will make possible the inductive arguments of section 4 can take several forms, depending on the degree of inductive support one demands for the arguments to go through. Here we opt for the simplest form (albeit maybe not the most attractive one), with as extra rules (Γ stands

[5] A one-place probability function can always be defined as: $Prob(\cdot) = Prob(\cdot \mid \lambda)$, with λ a tautology.

[6] It is not desirable to name all these. We could restrict ourselves to rational numbers, arguing that these are all that show up in real-life cases; or we could assume, as we implicitly do, that the real numbers are defined by way of rational numbers using Dedekind cuts and the like.

for the conjunction of the statements making up K and the newly introduced premises, as yet not included in K):

RF^{+} If $Prob(\varphi \mid \theta) = r$ and θb occur in the proof, and if $\vdash_{\mathsf{CL}^*} r > 0.5$, then one may add φb to the proof.

RF^{-} If $Prob(\varphi \mid \theta) = r$ and θb occur in the proof, and if $\vdash_{\mathsf{CL}^*} r < 0.5$, then one may add $\sim\varphi b$ to the proof.

As was explained in section 3, these rules will appear as *conditional rules* in our adaptive logic $\mathsf{AL}_{\mathsf{ISE}}$. The condition is the couple of the statistical law being used in the inference, with the name of the object to which it is applied (the reason for this will become clear when we discuss the marking rules). As a result of this, the fifth element of the lines in an $\mathsf{AL}_{\mathsf{ISE}}$-proof where we write down the conclusions of these inductive inferences, will look like this: $< Prob(\varphi \mid \theta) = r, b >$. Lines in a proof have to be retracted when these conditions are troublesome, and since in $\mathsf{AL}_{\mathsf{ISE}}$ trouble spells ambiguity, the marking rules will have to reflect this.

MARKING RULE

A line in an $\mathsf{AL}_{\mathsf{ISE}}$-proof will be marked when a not maximally specific statistical law was used to derive the sentence on this line (possibly in an indirect way via earlier lines in the proof). This is the case when it is known that (i) the object to which the law was applied is an element of a subset of the reference class of that law, that (iia) there is no statistical law, with this subset as reference class, giving a conditional probability for the property mentioned in the conclusion, or that (iib) this subset is the reference class of a statistical law (which is not a theorem of probability theory) that gives another conditional probability for the property mentioned in the conclusion. This gives us the following definition, and as a consequence the marking rule MR:

A statistical law $Prob(\varphi \mid \theta) = r$ is not the maximally specific statistical law for object b, iff there is a γ such that:

(i) γb and $(\forall x)(\gamma x \rightarrow \theta x)$ occur in the proof,

one of the following conditions is satisfied:

(iia) $Prob(\varphi \mid \gamma) = r'$ does not occur in the proof,

(iib) $Prob(\varphi \mid \gamma) = r'$ does occur in the proof, is not a theorem of probability theory, and $r' \neq r$,

MR If $Prob(\varphi \mid \theta) = r$ is not the maximally specific statistical law for the object b, then mark all lines the fifth element of which contains $< Prob(\varphi \mid \theta) = r, b >$.

It is important to remark that the property of being a maximally specific statistical law is a relational property holding between a law and an object. So one law can at the same time be maximally specific with respect to one object, and not maximally specific with respect to another object. This is the reason why we introduced couples in the conditions of the conditional rules.

This relativity is a result of the epistemic character of this property: RMS is defined with respect to the knowledge system and not the world. Most importantly, this cannot cause inductive ambiguities to arise, since 'Fa' and '$\sim$Fb' are not contradictory sentences. So the following would be a correct $\mathsf{AL}_{\mathsf{ISE}}$-proof ($r$ is supposed to be a number close to 1, and s a number close to 0):

	1.	$Prob(\mathrm{F} \mid \mathrm{G}) = r$	-	PREM	φ
	2.	$Prob(\mathrm{F} \mid \mathrm{H}) = s$	-	PREM	φ
	3.	Ga	-	PREM	φ
	4.	Gb	-	PREM	φ
	5.	Fa	1,3	RF^{+}	$< Prob(\mathrm{F} \mid \mathrm{G}) = r, a >$
$\surd$	6.	Fb	1,4	RF^{+}	$< Prob(\mathrm{F} \mid \mathrm{G}) = r, b >$
	7.	Hb	-	PREM	φ
	8.	$(\forall x)(\mathrm{H}x \rightarrow \mathrm{G}x)$	-	PREM	φ
	9.	$\sim$Fb	2,7	RF^{-}	$< Prob(\mathrm{F} \mid \mathrm{H}) = r, b >$

Line 6 was marked when line 8 was written down. So before we had introduced this information, 'Fb' was a justified conclusion, and at this stage of the proof (or at this stage of our knowledge, if at the moment we cannot introduce any further relevant premises) 'Fa' still is. This allows us to give two IS-explanations, one for a having the property F, and one for b not having this property.

The fact that in one explanation we use a statistical law that is considered not maximally specific in the other explanation does not cause any trouble. An example could be that you explain John Jones's lasting sickness by citing his having the properties S, P, and Z together with the relevant law, whereas you explain the fact that his friend Mary did recover by citing her having the properties S and P together with the relevant law. It may well be that you find this an unsatisfactory situation, and that you wish to explain Mary's recovery by the properties S, P, and $\sim$Z and a statistical law. But to do this, you have to introduce '$\sim$Zm' and the law in the proof, so first you will have to ascertain that these facts hold in the world.[7] The important thing is that if you can ascertain this, you will mark the earlier conclusion and favor the more specific derivation of the conclusion, but if you cannot ascertain this, you can still explain her recovery (although you may find the explanation not as good as it could be): after all, if you only know that Mary has properties S

[7] Another option would be that you would introduce these as premises *because* you think it would be a better explanation. Here we will not enter upon a discussion of inferences to the best explanation.

and P, it could have been expected that she recovered (and you have no more specific information concerning this fact).

The foregoing remarks nicely illustrate the dynamic aspects of our adaptive logic, which it shares with all other adaptive logics. In view of newly acquired information earlier conclusions sometimes have to be withdrawn. This is the reason why adaptive logics are particularly well-suited to analyze I-S explanations. Indeed, classical logic has the property of *monotonicity*, which means that adding premises cannot invalidate earlier conclusions. The example of John Jones made clear that the logic behind I-S explanations cannot be monotonic. Also in Mary's case this shows up, although in a slightly different way: the conclusion ('Rm') remains valid in the light of new premises, but now we prefer another argument leading up to this conclusion. When the topic is explanation this can be highly relevant: we are not just interested in the conclusion (remember that this is already known), but in the *arguments* we can give for this conclusion.

5.2. *Some alternatives*

The logic we presented here naturally suggests some alternatives (whence the indefinite pronoun in the title of our article).

As already mentioned, the rules RF$^+$ and RF$^-$ could be modified to set other restrictions on the degree of inductive support.

We could also change the RMS, so as to have an alternative definition in which a line only gets marked when there is a more specific alternative to the law used, giving a different conditional probability. The following is a suggestion:

A statistical law $Prob(\varphi \mid \theta) = r$ is not the maximally specific statistical law for object b, iff

(i′) γb and $(\forall x)(\gamma x \rightarrow \theta x)$ occur in the proof,

(ii′) $Prob(\varphi \mid \gamma) = r'$ does occur in the proof, and is no theorem of probability theory, and $r' \neq r$.

With this alternative definition, it could be the case that we know that an object is a member of a subset of a reference class (condition (i′) holds), and still use the statistical law with this broader reference class, because there is no statistical information concerning this more specific set (there is no condition (iia) as in the earlier definition, but only (iib) — now renamed (ii′) — which doesn't hold).

The most important modification, from a logical point of view, would be the introduction of an inconsistency-adaptive logic as LLL. As our discussion of D-N explanations made clear, this is by no means an unnecessary modification. The only reason why we did not introduce this in $\mathsf{AL}_{\mathsf{ISE}}$ was that we did not want to make matters too complicated, and rather wanted to concentrate ourselves on the specific problems surrounding the inductive ambiguity.

6. *Conclusion*

We do not want to claim that Hempel's D-N and I-S explanations are the final words on explanation; far from it. But we do claim that if you want to get a better grip on the intuitions behind this proposals, adaptive logics are a much more natural tool than classical logic is. Among the advantages of adaptive logics over classical logic are the nice way in which inconsistent knowledge systems can be handled, and the natural non-monotonicity that arises. At the same time adaptive logics do have the advantages classical logic has; advantages that were well known by the logical positivists. Most importantly, there is a clear and unambiguous proof-theory, as well as a semantics (on which we were silent here, but see e.g. Batens 1995).

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