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DEEP MANY-VALUEDNESS

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1. *Origin and motivation of the idea*

The fundamental topic of this article is the notion of deep many-valuedness. This concept was made explicit for the first time by Professor Hubert Marraud in his manuscript (1997) for the introduction course to many-valued logic held at the Summer School at the University of the Basque Country; the idea of deep many-valued logic appears as a complement to the notion of superficial many-valued logic. In investigations about logical many-valuedness the latter idea has always prevailed, being understood as a kind of logic with an arbitrary number (bigger than two) of truth-values but incorporating a *binary* consequence relation. It would seem very natural to spread the perspective to logics with a given number of alethic values equal to the cardinal of the characteristic function that assigns the values to the consequence operation. Therefore, deep many-valued logic that logic whose consequence relation is $n > 2$-valued. This concept opens a new route which certainly has precedents (one aim of this essay to study them) but I think that, by its radicalism, has no comparison. Besides, it has the virtue of generating a theoretical approach to many-valued logics out of those traditional views that see them as interesting but simple combinatorial games[1].

Let $\mathcal{P}$ be a prolanguage[2], that is, an artificial language composed of a table of signs and a set of formation rules, the combination of which creates a recursive definition of formula. Let $\langle \mathcal{S}, I \rangle$ be an admissible interpretation of $\mathcal{P}$ so that $\mathcal{S}$ is a model-theoretic structure and I is a function of interpretation of $\mathcal{S}$. If V is the set of values assigned, under the admissible interpretations, to the formulas of the set F generated by $\mathcal{P}$, then a bivalent logic is one whose set V has two values versus a many-valued logic in which V has a number $n \geq 3$ of truth values. This point of view banishes many-valuedness to the notion of truth being a generalized orientation between logicians. Thus, the

[1] A very significant author for his defence of the proper philosophical and mathematical meaning of many-valued logics is Lorenzo Peña (see (1992)).

[2] For the introduction to the concept, cf. Marraud (1998bis, p. 11).

recent but classical handbook of Bolc and Borowik (1992, p. 23) affirms: "(...) The idea of accepting sentences which (in a given instance) fail to be either absolutely true or absolutely false aroused contention between the Epicureans, on the one side, and the Stoics (including Chrysippus) on the other. The latter represented the standpoint of extreme determinism, with its orthodox bivalence in logic. The former, rejecting absolute determinism, admitted the possibility that neither of two statements, one of which negated the other, must necessarily be true (...)".

Marraud wonders why many-valuedness has been excluded from the consequence operation, that is, if we agree that the nuclear idea of the science of logic is based on the correction of the arguments and henceforth a logical system must be understood as a structure $\langle F, C\rangle$, where C is a consequence operation, it is difficult to understanding the reason for considering the metalanguage of a logic as bivalent although its object language may be many-valued. In fact, many authors hold the existence of logics with a number of more than two truth-values but in which the sentences of the kind "S_{n+1} follows from S_1, ..., S_n" only can take the assignation of two truth-values. In this way, Marraud distinguishes between many-valued logics considered in a traditional sense, e.g., as logics with $n > 2$ truth-values and a *binary* consequence operation (they are called superficial many-valued logics) and logics integrated by $n > 2$ truth-values and an $n > 2$-valued consequence relation, named deep many-valued logics (Marraud, 1998, p. 59). We firmly believe that the frequent accusation made against many-valued logics of lack of philosophical foundation is motivated by the nonexistent development of deep many-valued logics. Thus, a very salient logician such as Urquhart (1986, p. 114) said:

> "Of course, much ingenious and attractive work has been done on many-valued logic considered as a purely mathematical structure. In this light, many-valued logic is simply the study of the functions definable on a finite set. Obviously, such research has considerable importance as pure combinatorial mathematics which in no way depends for its value on dubious philosophical motivation".

In the more restricted realm of partial logic, Fenstad (1997) in his excellent compendium on Partiality shows his uncertainty (he has no doubts about the philosophical or epistemological value) regarding the necessity of using partial logical systems perfectly translatable to classical logic. Fenstad appeals, in a certain sense, to the comparison with the heuristic significance which several decades ago had constructive logic generating new subjects and applications in the computational field. I sincerely believe that the scepticism of Erik Fenstad and other great logicians[3] about the genuine nature of partial logic and many-valued logic in general, would decrease if many-valued logics were also thought of as deeply many-valued logics.

Out of this context but in the superficially many-valued line, Michael Dummett (1959) has considered that the subdivisions in values propitiated by the many-valued machinery are of purely instrumental interest. According to Professor Dummett, when the dichotomy true/false is used for uttering assertions, it does not seem that a particular sentence must be classified in a different category. But let us allow our author to state his case:

> "We need to distinguish those states of affairs such that if the speaker envisaged them as possibilities he would be held to be either misusing the statement or misleading his hearers, and those of which this is not the case: and one way of using the words 'true' and 'false' would be to call states of affairs of the former kind those in which the statement was false, and the others those in which the statement was true".

From this declaration it may be inferred that sentences initially not described as true or false will find (I suppose that it will be a matter of time or of adequate precisation, although nothing of this is elucidated by Dummett) a place in the compartment of the true or false. This perspective hides a persistency in seeing many-valued logics in a superficial way behind the division between designated (faithful image of the true) and antidesignated (a transcript of the false) elements of a matrix. Lloyd Humberstone (1998), explaining the arguments of Dummett, differentiates (see (1998), p. 86) between many-valued logics in a narrow sense (logics conceived as sets of theses) and many-valued logics in a broad sense (logics conceived as consequence relations). This categorization agrees with Marraud's proposal but Professor Humberstone commits with the reduction of many-valuedness to bivalence appealing to the difference between the concepts of evaluation and valuation. Given a matrix M, an M-evaluation is an assignment of elements of M to formulas of the language while the idea of valuation is reserved for the bivalent case. If D is the set of designated values of the matrix and h is an M-evaluation, h verifies the sequent $X \vdash Y$ when if $h(X) \subseteq D(M)$ then $h(Y) \cap D(M) \neq \emptyset$ (cf. op. cit., p. 86). But if, for the bivalent valuation v_h and for every formula A, $v_h(A) = 1$ iff $h(A) \in D(M)$, then, in opinion of Humberstone, a sequent is valid in a many-valued sense if it is valid in the bivalent sense on v_h (art. cit., p. 87). As this influential theorist recognizes:

[3] See Busch (1996, p. 68) for the case of three-valued logics:
"The mathematical nature of the mapping of three-valued into two-valued logic could hardly be simpler. I think it warrants the philosophical conclusion also, that three-valued logic is no real alternative to two-valued logic. In fact, rightly understood, it is just ordinary logic, viewed a bit differently. This is hardly a new opinion".

> "Our introduction (...) of the valuation v_h induced by a matrix evaluation h, serves as a reminder of the twofold division which is always present in the background of the many-valued setting, in the form of the ubiquitous designated/undesignated distinction" (Humberstone (1998, p. 88)).

But dispensing with other judgements about the superficial treatment of many-valued logics if we look for early concrete intuitions founding the idea of many-valued logics with an operation of consequence n-valued for $n > 2$, Marraud unerringly affirms[4] that in Logic of Paradox LP of Graham Priest there is a certain sign of a deep many-valued logic, a three-valued logic with the strong connectives of Kleene but with the intermediate value ("truefalse") interpreted genuinely as a value of the same status as the values true or false. In his article "Logic of Paradox" (1979) the genial philosopher interprets the formulas of a paraconsistent logic using non-empty sets of classical sentences. Logical values of the set $\{1, 0, 1/2\}$ are represented in three-valued matrices as a non-empty set of classical values $\{1\}, \{0\}$ and $\{1, 0\}$ being $\{1, \{1, 0\}\}$ the set of designated values. This does not at all mean that Professor Priest directly formulated a paraconsistent consequence relation for his logic but perhaps he had to do it because, although he tolerated the simultaneous truth and falsity of a sentence, he followed supporting a classical or bivalent consequence relation: $\Gamma \vdash A$ or not $(\Gamma \vdash A)$, but why not bother? Actually, the path conducting the superficially paraconsistent logic of Priest towards deep paraconsistence could be so synthesized via argument and codification of Gödel sentence (cf. Marraud (1998, p. 60)):

Gödel's sentence is truefalse
There is not distinction between object language and metalanguage
———————————————————————
Gödel's sentence is and is not arithmetically provable

This example of Marraud induces us to think that the transition towards a three-valued notion of consequence seems more natural from a partial approach (not computability of some arguments) than from a paraconsistent perspective (some arguments are simultaneously provable and refutable?). In any case, from here to a definition of deep many-valued logic there is a long course and we have to arrive at the article of G. Malinowski, "Q-Consequence Operation" (1990) to find a very close reference.

[4] See Marraud (1998, p. 60).

2. *Malinowski's inferential many-valuedness*

Malinowski anticipates the concept of a metalogical relation of inference being three-valued in its spirit. The genealogy of the idea of quasi-consequence in Professor Malinowski is influenced by the semantic theory of Roman Suszko. Suszko avoids the Fregean approach which placed the reference of the sentences on its truth-value. For the Polish logician the reference of a sentence is what is said about a given situation. Suszko introduces situations into its semantics influenced by Wittgenstein's notion of "Sachlage". In Suszko's situational semantics[5] is not necessary, as in Frege's semantics, for two true sentences to have the same reference: they can have an identical truth-value but different reference. It is an aspect in which the concept of reference of Suszko is more close to the Fregean idea of "Sinn". Basic postulates of Suszko's semantics are: (i) every sentence has its reference (he does not agree with Wittgenstein, as the Viennese philosopher thought that not all sentences have a reference interpreted as sense —that is, "sinnlose" sentences as tautologies or inconsistent logical sentences—); (ii) true sentences are referred to positive facts (they denote existent situations) while false sentences are related to negative facts; (iii) acceptance of the principle of compositionality.

It is obvious that the fulfilment of these conditions requires the existence of at least two situations. This circumstance is reflected by an axiom accepted by Frege and named (AF) by Roman Suszko: $(A \equiv B) \vee (A \equiv\sim B)$. This axiom could be read as saying that in the world there would be a positive fact identifiable with the Truth and a negative fact comparable to the False. Suszko rejects the axiom for situations although it does not mean its negation. But the rejection of (AF) is a key point of the sentential calculus with identity of the Polish logician and by this reason, his logic is called non-Fregean logic (NFL). The system of Suszko has a partial flavour because the values in NFL are related with situations but not with the world considered as a whole. Malinowski opposes the concept of inferential two-valuedness to the referential many-valuedness one showing the way for obtaining many-valuedness also in the inferential side (see Malinowski (1994, p. 75)).

Let $\mathcal{M} = (U, D)$ be a matrix for a sentential logic, D being a non-empty subset of the universe U. D is the set of the designated elements of the matrix. To every matrix $\mathcal{M}$ is associated a set of formulas taking only designated values: $E(\mathcal{M}) = \{A \in \text{For} : hA \in D \text{ for every } h \in \text{Hom}(\mathcal{L}, U)\}$ (Malinowski, of course, speaks about a sentential language $\mathcal{L}$ and he does not use the notion of prolanguage). Then, the consequence relation $C_{\mathcal{M}}$ for

[5] For more complete information, see Wójcicki (1984).

a set X of formulas and a formula A is defined as $X \vdash_{\mathcal{M}} A$ iff for every homomorphism, $hA \in D$ whenever $hX \subseteq D$. According to Suszko (1977), Malinowski (1994, pp. 78 and 79) opposes logical valuations, that is, functions defined on For, to homomorphisms associated with sentences and having a referential character. Given the set of valuations $TV_{\mathcal{M}} = \{t_h : h \in \text{Hom}\}, t_h(A) = 1$ if $h(A) \in D$ and $t_h = 0$ if $h(A) \notin D$. The author of the University of Łodz exposes[6] that logical bivalence is connected to the division of the universe of interpretation in the subsets of distinguished and non-distinguished elements and that this distinction, under the assumption of the property of structurality, determines that the Tarskian notion of consequence can be judged as an inferential bivalent operation. But nothing is opposed to the possibility of getting logics based on three-element algebras (op. cit. in the footnote, p. 83) satisfying a generalized version of the axiom of Frege (AF) in which there is an identification between the logical three values and the corresponding three semantic correlates or situations. This property is faithfully reflected in the construction of q-matrices whose universe is divided in three subsets. The triple $\mathcal{M}_q = (u, \overline{D}, D)$ is a q-matrix (art. cit. p. 81) in which $\overline{D}$ (the set of rejected values) and D (the set of distinguished values) are subsets of U such that $\overline{D} \cap D = \emptyset$. And a q-consequence relation is so defined:
$X \vdash_{\mathcal{M}_q} A$ iff for every $h \in \text{Hom}(\mathcal{L}, \mathcal{M})$, if $hX \cap \overline{D} = \emptyset$ then $hA \in D$. For a three-valued function k_h : For$^{\rightarrow}\{0, 1, 1/2\}$ such that $k_h(A) = 0$ if $h(A) \in \overline{D}, k_h(A) = 1/2$ if $h(A) \in \mathcal{M} - (\overline{D} \cup D)$; $k_h(A) = 1$ if $h(A) \in D$. Malinowski provides a three-valued description of a q-consequence relation such that it can be described the quasi-inference of a formula from a set of formulas receiving indeterminate assignments or not belonging to $\overline{D} \cup D$. And so, $X \vdash_{\mathcal{M}_q} A$ iff for every three-valued valuation k_h, if $k_h(X) \cap \{0\} = \emptyset$ then $k_h(A) = 1$ (op. cit., p. 82).

Malinowski is aware of the relation between a three-valued characterization of the consequence operation and the subdivision of the universe of the matrix into three subsets, breaking the superficial dichotomy which translates the classical case to the other ones. But the most fascinating aspect of the task of Malinowski is when he suggests spreading the three-valued case to the n-valued one affirming

> "(...) which obviously tempts us to introduce the notions of logical n-valuedness (for each natural $n > 3$). On the whole the solution being intuitive and to some extent natural looks quite promising. Why not to discern e.g. different degrees of rejection and further why not to define 'matrix' inference relation and operation in a very

[6] See "Inferential Many-Valuedness", p. 80.

much similar manner? A little more that I can say about the last step is left till another occasion (...)" (Malinowski (1994, p. 83).

The closeness of the valuable undertaking of Malinowski and the planning of Marraud becomes evident. But while the former follows Suszko's thesis for proposing the idea of many-valued consequence, Marraud simply emphasizes the theoretical disarrangement of managing a number n of truth-values in a logic and working its metatheory with only two categories. Definitively, Marraud's approach is more radical than Malinowski's proposal. Because, in a first moment, the brilliant Polish logician obtains an instrumental adaptation for allowing inferential rules which regulate the transition from non-rejected hypotheses to accepted conclusions (see Malinowski (1997, p. 66)). In fact, Malinowski conjectures that perhaps a simple weakening of the concept of rule of inference is enough for his project of many-valuedness inference (see 1998, p. 213). Marraud, on the contrary, propounds graded systems of sequents[7] which mixing different consequence relations (in a similar way to the labelled deductive systems of Dov Gabbay (1996) although Gabbay only uses labels for formulas) to express the idea of deep many-valuedness. For a given ordered set $\langle I, \leq \rangle$, a graded system of sequents would have rules of the form:

$$\frac{X_1 \vdash_i Y_1, ..., X_n \vdash_j Y_n}{X \vdash_k Y}$$

with $i, ..., j, k \in I$. Intuitively, the subindices joined to the symbol of assertion can be interpreted as meaning accuracy of the inferences. In any case, such a system does not define a unique consequence relation but a family of them, although its definitions can be dependent between them.

3. *Sentential partial logic of Abdallah*

Areski Nait Abdallah (1995) enunciates a sentential logic which is the basis of his ionic partial logic. On pages 22 and 23 of his book, Professor Abdallah distinguishes two varieties of truth: truth in a classical sense and potential truth[8]. These two interpretations are translatable by means of two operators (following the above considerations we will use subindices annexed): $\models_1$ for truth and $\models_2$ for potential truth. The connection between these operators is

[7] See Marraud (1998, p. 64).

[8] Abdallah says that "(...) Essentially, something is potentially true if it is not outright false, i.e. if it either true (...), or undefined (...)" (*The Logic of Partial Information*, p. 23).

established by means of the following couple of equivalences: (a) $i \models_1 \neg A$ iff not $i \models_2 A$, (b) $i \models_2 \neg A$ iff not $i \models_1 A$.

If the connective $\neg$ corresponds to negation of Kleene, $i \models_1 A$ expresses that the formula A is true under the interpretation i; not $i \models_2 A$ affirms that A is definitely false under i while not $i \models_1 A$ tells that A is false or undefined under i. This behaviour of the turnstiles induces to Marraud to affirm[9] that Abdallah's system can be seen, in a certain sense, as a deep many-valued logic of three values: $X \vdash_1 A$ (A is derived from the set X of formulas), $X \vdash_2 A$ (A is potentially derived from X) and not $X \vdash_2 A$ (A is refuted by the formulas of the set X). Actually there are rules[10] in the sentential partial logic of Abdallah combining the symbols $\vdash_1$ and $\vdash_2$ which prelude the deductive tool of Marraud based on graded systems of sequents. So, the rule of left semi-modus ponens : $\vdash_1 (A \rightarrow B); \vdash_2 A / \vdash_1 B$ and the rule of right semi-modus ponens : $\vdash_2 (A \rightarrow B); \vdash_1 A / \vdash_2 B$.

The partial logic of Abdallah is a splendid referent for all those researchers interested in default reasoning and it is worthy of detailed study. Meanwhile we hope that this brief note gives an idea of the deeply many-valued nature of this logic.

4. *An example of deep many-valuedness: definition of deep partial logic*

Let us see how the idea of Marraud can be understood if we confine ourselves to the realm of partial logic.

A total function is a relation f such that $\forall x \in \text{Do } f, \exists^1 y$ such that $\langle x, y \rangle \in f$. This unique element y is the xth value of f. A partial function f' is a function such that, for every n-termed sequence of terms $x_1, ..., x_n$ as arguments, takes at most one element $f'(x_1, ..., x_n)$ of its range. If for $x_1, ..., x_n$, f' takes a value, $f'(x_1, ..., x_n)$ is defined; if for $x_1, ..., x_n$, f' does not take a value of its range, it is said that $f'(x_1, ..., x_n)$ is not defined. If $f'(x_1, ..., x_n)$ is a partial function whose range is the set $\{0, 1\}$ then it can be substituted by a total function $f(x_1, ..., x_n)$ with range $\{0, 1, 1/2\}$ if for any x_i of the n-termed sequence, $f(x_i) = 1/2$ iff $f(x_i)$ is not defined.

A logic is superficially partial iff it is partial, that is, it verifies the condition (a) of partiality (see Alonso (1992)) and it holds the clause (b) of superficial many-valuedness:
(a) $\forall i \in I$ and for any formula A, $\{A : (A, 1) \in i\} \cup \{A : (A, 0) \in i\} \neq S$,

[9] Cf. Marraud (1998, p. 69).

[10] The referee makes me any comments on the early influence in this sense of Guccione and Tortora (1981). The authors use sundry inference-rules which are sorted out in different ways.

S being the set of well-formed formulas of the prolanguage $\mathcal{P}$.
(b) A consequence relation is *binary* iff it is a total function $C : \mathrm{Sub}(F) \times F \to \{0,1\}$ between a set X of formulas belonging to F and a formula A such that there are two possibilities:
$C(\langle X, A\rangle) = 1$, being derivable the argument,
$C(\langle X, A\rangle) = 0$ and the argument is refutable.

A consequence relation is partial iff it is a partial function C': $\mathrm{Sub}(F) \times F \to \{0, 1, 1/2\}$ such that, for a set X of formulas and a formula A, there are three possibilities (cf. Marraud (1998, p. 59)):
$C'(\langle X, A\rangle) = 1$ and the argument is derivable,
$C'(\langle X, A\rangle) = 0$ and the argument is refutable, or
$C'(\langle X, A\rangle) = 1/2$ and the argument is neither derivable neither refutable.

A logic is deeply partial iff it is a partial logic with a partial consequence relation. A notion of partial consequence respecting the strong connectives of Kleene is so defined (see Marraud (1998, p. 59, note 4)):
$C'(\langle X, A\rangle) = 1$ iff for any interpretation i, if $i(B) = 1$ for every formula $B \in X$, then $i(A) = 1$;
$C'(\langle X, A\rangle) = 0$ iff for some interpretation i, $i(B) = 1$, for every formula $B \in X$, and $i(A) = 0$;
$C'(\langle X, A\rangle) = 1/2$, otherwise.

5. *Beyond this article*

This article is merely of an introductory nature, and an inmense panorama may be glimpsed under the title of deep many-valuedness. I suggest the following tasks among many others:
a) To establish a general mathematical theory fibring at least three consequence operators.
b) To construct graded systems of sequents expressing deductive accuracy.
c) To create metalogical systems reflecting metaproperties for deep many-valued logics, for instance, theorems of relative consistency with respect to several consequence operators.
d) To elaborate deeply many-valued algorithms, that is, algorithms with three or more outputs for different classes of inputs.

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REFERENCES

ABDALLAH, A.N. (1995): *The Logic of Partial Information*, Berlin, Springer-Verlag.

ALONSO, E. (1992): *Contenido, Verdad y Consecuencia Lógica*, Doctoral Dissertation (Universidad Autónoma de Madrid).

BOLC, L. and P. BOROWIK (1992): *Many-Valued Logics 1 (Theoretical Foundations)*, Berlin, Springer-Verlag.

BUSCH, D. (1996): "Sequent Formalizations of Three Valued Logic", in Doherty, P. (1996, ed.: 45–75).

CRAIG (1998): *Encyclopaedia of Philosophy*, London, Routledge.

DOHERTY, P. (1996): *Partiality, Modality and Nonmonotonicity*, CSLI, Stanford.

DUMMETT, M. (1959): "Truth", *Proceedings of the Aristotelian Society* 59: 141–162.

FENSTAD, J.E. (1997): "Partiality", in van Benthem, J. and A. ter Meulen (1997, ed.: 651–682).

GABBAY, D. and F. GUENTHNER (1986): *Handbook of Philosophical Logic* (vol. 3), Dordrecht, D. Reidel.

GABBAY, D. (1996): *Labelled Deductive Systems* I, Oxford, Clarendon Press.

GUCCIONE, S. and R. TORTORA (1981): "Deducibility in Many-Valued Logics", *Proceedings of the 12th ISMVL*, Paris.

HUMBERSTONE, Ll. (1998): "Many-Valued Logics, Philosophical Issues in", in Craig (1998, ed.: 84–91).

MALINOWSKI, G. (1990): "Q-Consequence Operation", *Reports on Mathematical Logic* 24: 49–59.

MALINOWSKI, G. (1994): "Inferential Many-Valuedness", in Wolenski, J. (1994, ed.: 75–84).

MALINOWSKI, G. (1997): "On Many-Valuedness, Sentential Identity, Inference and Łukasiewicz Modalities", *Logica Trianguli* 1: 59–71.

MALINOWSKI, G. (1998): "Modes of Many-Valuedness", in Martínez, C., Rivas, U. and L. Villegas-Forero (1998, eds.: 199–213).

MARRAUD, H. (1997): "Lógicas polivalentes" (manuscript).

MARRAUD, H. (1998bis): *Introducción a la teoría de los sistemas deductivos*, Madrid, ediciones de la Universidad Autónoma (28).

MARRAUD, H. (1998): "Razonamiento aproximado y grados de consecuencia", *Éndoxa* 10: 55–70.

MARTÍNEZ, C, RIVAS, U. and L. VILLEGAS-FORERO (1998): *Truth in Perspective: recent issues in logic*, Ashgate.

PEÑA, L. (1992): "Algunas aplicaciones filosóficas de lógicas multivalentes", *Theoria* 16-17-18: 141–160.

PRIEST, G. (1979): "The Logic of Paradox", *Journal of Philosophical Logic* 8: 219–241.

URQUHART, A. (1986): "Many-Valued Logic", in Gabbay, D. and Guenthner, F. (1986, eds.: 71–116).

van BENTHEM, J. and ter MEULEN, A. (1997): *Handbook of Logic and Language*, Amsterdam, Elsevier.

WÓJCICKI, R. (1984): "R. Suszko's Situational Semantics", *Studia Logica* 43: 323–340.

WOLENSKI, J. (1994): *Philosophical Logic in Poland*, Amsterdam, Kluwer.