Logique & Analyse **165–166** (1999), 127–138

COMBINED DA COSTA LOGICS (WORLD ACCORDING TO N. C. A. DA COSTA)

VLADIMIR L. VASYUKOV

Abstract
It is known that N.A. Vasiliev distinguished two levels in logic assuming inconsistency on the ontological level, but denying it on the logical one. Getting started from these ideas V.A. Smirnov introduced several combined calculi of sentences and events (cf. [Smirnov 1988]) consisting of two parts: the abstract (external) logic depending on epistemological assumptions and the empirical (internal) logic depending on ontological ones. Early on the author proposed to approach an algebra of events as the discursive system of S. Jaśkowski by treating an algebra of events as a $S5$-modal algebra, introducing a Jaśkowski's type conditional and then θ-translating it into sentential calculus (cf. [Vasyukov 2001]). There is another opportunity to fulfill Vasiliev's program if we take a da Costa's algebra [Carnielli Alcantara 1984] to reflect most of the logical properties of the da Costa's systems C_n as the internal logic. The resulting system of combined logic would then be inconsistent (paraconsistent) on the ontological level but consistent on the logical one.

1. *Introduction*

It is known that N.A. Vasiliev distinguished two levels in logic assuming inconsistency on the ontological level, but denying it on the logical one. He wrote: "...in logic there are removable, hence, empirical elements. If we remove what is removable and empirical, there remains irremovable rational logic. This logic I shall call metalogic". A metalogic is precisely "a logic valid for every world no matter how peculiarly its objects have been constructed, for it contains only laws of pure thought, of judgement and inference in general, it reflects only the nature of cognizing subject". The other way round, our empirical (Aristotelian) logic "is not a formal logic, but a science where the formal, the metalogical of thought is mixed up with the content of thought; its law of contradiction, its negation are based on the fact of incompatibility, on something cognized, hence material". In fact, our logic "is a mixture, a hybrid form, something in between formal metalogic and material natural science. The laws of our logic are partially the laws of metalogic, partially natural laws; such is, for example, the law of contradiction" [Vasiliev 1993, p. 344–345]. Moreover, metalogic "may be used to construct the entire content of our empirical logic; by means of empirical logic it is possible to construct the world of imagination alien to it" [Vasiliev 1993, p. 348].

Getting started from these ideas V.A. Smirnov introduced several combined calculi of sentences and events (cf. [Smirnov 1988]) consisting of two parts: the abstract (external) logic and the empirical (internal) logic. The former depends on epistemological assumptions while the ontological one determines the latter. Smirnov points out that another source of his inspiration was Frege's differentiation of mental process (*Gedanke*) and assertion statement (*Urteil*) that resulted in the introduction of the special sign $\vdash$. By Smirnov's opinion it is precisely the conception of two levels that underlies our contradistinction of acts of assertion (the relation of mental content with the way things are) and acts of predication (the synthesis of a property with the object).

The language of Smirnov's combined calculi usually includes two sorts of variables: event variables (terms) and propositional ones. If a and b are terms then $a \cup b$, $a \cap b$, $\sim a$ will be also terms (complex events) while θa, θb are the formulas along with the formulas $\theta a \vee \theta b$, $\theta a \wedge \theta b$, $\neg \theta a$. Proceeding from some equivalencies like $\theta(a \cup b) \equiv \theta a \vee \theta b$, $\theta(a \cap b) \equiv \theta a \wedge \theta b$ etc., we arrive at different combinations of algebras of events and propositional calculi in the framework of one logic.

Let us remind ourselves that Vasiliev's conception of imaginary (non-aristotelian) logic was developed in order to cope with the problem of contradiction in logic. Pursuing Vasiliev's approach to this issue, it would be interesting to tackle the combined systems dealing with violations of the law of contradiction in a non-standard way. Hence, we need combined logics in which the algebra of events or an interrelation of events and sentences would be unusual (e.g. paraconsistent) from the point of view of the law of contradiction.

Early on the author proposed to approach an algebra of events as a discursive system a notion that goes back to S. Jaśkowski (cf. [Vasyukov 2001]). In this case, treating an algebra of events as a $S5$-modal algebra we are in a position to cope with the contradictory character of our ontological level by introducing a Jaśkowski's type conditional in our algebra of events and then θ-translating it into our sentential calculus. Moreover, if we would be bent on "pure" discursive event algebra as the algebraic counterpart of Jaśkowski's discursive logic, then it might be better to tackle the Jaśkowski's algebra, that is to be found in [Kotas 1975, p. 158]. If in the first case we accept paraconsistency by means of some properties of our θ-operator, then in the second case we, in effect, directly exploit an embedding of a paraconsistent event-ontology into a consistent logical system (as a model of our reasoning).

2. *A System of da Costa Combined Logics: syntax and axiomatics*

There is another opportunity to fulfill Vasiliev's program. Instead of Jaśkowski's algebra in a role of event-ontology, we can try to adopt more elaborated paraconsistent theories describing different cases and models of paraconsistency.

Following this course of consideration we propose to accept a da Costa algebra [Carnielli Alcantara 1984] that reflects the most of the logical properties of the da Costa systems C_n as internal logic in our assumed combined system. In this case the resulting system of combined logic also would be inconsistent (paraconsistent) on the ontological level but consistent on the logical one. Formally it goes as follows.

Firstly, since our theoretical constructions are essentially based on da Costa algebra, for the further proceedings we adduce the complete definitions.

Definition 1: [Carnielli Alcantara 1984, p. 81]. *By a* da Costa algebra *we mean a structure*

$$\mathrm{A} = \langle S, 0, 1, \leq, \cap, \cup, \rightarrow, \sim \rangle$$

such that for every a, b, c, x *in* S *the following conditions hold:*

(1) $\leq$ *is a quasi-order;*
(2) $a \cap b \leq a$, $a \cap b \leq b$;
(3) *if* $c \leq a$ *and* $c \leq b$ *then* $c \leq a \cap b$;
(4) $a \cap a = a$, $a \cup a = a$;
(5) $a \cap (b \cup c) = (a \cap b) \cup (a \cap c)$;
(6) $a \leq a \cup b$, $b \leq a \cup b$;
(7) *if* $a \leq c$ *and* $b \leq c$ *then* $a \cup b \leq c$;
(8) $a \cap (a \rightarrow b) \leq b$;
(9) *if* $a \cap c \leq b$ *then* $c \leq (a \rightarrow b)$;
(10) $0 \leq a$, $a \leq 1$;
(11) $x^0 \leq (\sim x)^0$, *where* $x^0 = \sim(x \cap \sim x)$;
(12) $x \cup \sim x \leftrightarrow 1$, *where* $a \leftrightarrow b$ *iff* $a \leq b$ *and* $b \leq a$;
(13) $\sim\sim x \leq x$, *where* $\sim\sim x$ *abbreviates* $\sim(\sim x)$;
(14) $a^0 \leq (b \rightarrow a) \rightarrow ((b \rightarrow \sim a) \rightarrow \sim b)$;
(15) $x^0 \cap \sim(x^0) \leftrightarrow 0$.

If there exists a $x \in S$ such that it is not true that $x \cap \sim x \longleftrightarrow\!\!\!\!\!/\;\; 0$ the algebra A is said to be a *proper da Costa algebra*.

In order to obtain a da Costa combined logic we enrich the axiom schemata of positive classic sentential logic and the rule *modus ponens* with the following schemes:

A1. $\theta a \wedge \theta b \equiv \theta(a \cap b)$;
A2. $\theta a \vee \theta b \equiv \theta(a \cup b)$;
A3. $\theta a \wedge \theta(a \to b) \supset \theta b$;
A4. $(\theta(a \cap c) \supset \theta b) \supset (\theta c \supset \theta(a \to b))$;
A5. $\theta(a^0) \supset \theta(\sim a)^0$, where $a^0 = \sim(a \cup \sim a)$;
A6. $\theta(\sim\sim a) \supset \theta a$;
A7. $\theta(a^0) \supset \theta((b \to a) \to ((b \to \sim a) \to \sim b))$;
A8. $\theta b \supset \theta(a \cup \sim a)$;
A9. $\theta(a^0 \cap \sim(a^0)) \supset \theta b$.

It is easy to check that all the axioms of da Costa algebra are fulfilled in such a formulation. As a consequence we arrive at the system where an algebra of events is a da Costa algebra.

Now the question immediately arises concerning the notion of event in such an algebra and the reading of the θ-operator. It seems that the wrong choice of such a reading would cause many unpredictable consequences.

Let us recall that by Smirnov's opinion precisely the act of assertion means the relation of mental content with the way things are. And we quickly discover that many theorists speak of events in terms of actuality or reality. For example, U. Meixner claims: "For an event to be actual or real is to *happen*. Not all events happen (but only events happen); hence some events are not actual, but merely possible" [Meixner 1994, p. 30]. He puts the following analytical postulate for *to happen* among the others: "*For all x: x happens iff x is an actual (real) event*".

Should we interpret the relation of mental content with the way things are as a kind of the confirmation of the event happened? If so then in this case we can speak of a "factuality" of some event a (and not the situation) purporting that this factuality is our (mental) acceptation of a, which would be claimed as "a is an actual (real)". An advantage of such a way consists in the respective reading of θa as "a is actual" according to Meixner's claim.

There is one more argument in favour of the reading of θa as "a is actual". It can be seen easily that in da Costa combined logic we have a derived rule

$$\frac{\theta(a \to b)}{\theta a \supset \theta b}$$

which implicitly means that one can conclude from the internal implication to the external one, i.e. we owe by our conclusion to the way things are connected. In other words, an actuality of the event-ontological implication involves the acceptation of a logical (epistemological) implication which would be read according to our proposal as "from the fact that it is the case that ($a \rightarrow b$ is actual) it follows that it is the case that (if a is actual then b is actual)".

Do we need to accept in a da Costa combined logic only the axiom schemata of positive classic sentential logic (and the rule *modus ponens*), or can we state that the axioms dealing with negation also are involved? In fact, we can do it, but in this case we ought always to distinguish two kinds of negation (internal and external) and their usage. The latter means that we must pay attention to such peculiarities, e.g., that external negation does not figure in our axiomatic formulations.

A more interesting question is whether a mental process leading to the formulation of an external formula would be considered as an event too. Following Frege's differentiation of mental process and assertion statement Smirnov in his paper from 1989 proposed to approach our thesis itself as events [Smirnov 1989]. Formally it leads to the enrichment of our language with the help of a [-]-operator which acts as follows: *if α is a formula then $[\alpha]$ is a term.*

Previously the author mentioned (cf. [Vasyukov 2001]) that the idea of the [-]-operator could be traced back to the J. Slupecki's idea from [Slupecki 1971]. Slupecki proposed, namely, to enrich the language of modal logic with the expressions $p * x$ which might be read as follows:

(1) saying that p, we state (the event) x;
(2) sentence p states the event x.

Adapting such a proposal for the case of a combined language we conclude that $[\alpha]$ would be read as "sentence α states the event $[\alpha]$".

A less complicated reading should be easily obtained if we would borrow the concept needed from the causal theory of events. For example, in U. Scheffler's paper on this topic we discover the following passage: "Events are usually given by description, including a descriptive sentence about what is the case and about the spatiotemporal region in which it is the case" [Scheffler 1994, p. 36]. He illustrates his assumptions with the following examples: "The event, that a tyrant was killed", "The event, that Good overcomes Evil" etc. Clearly, if we would accept the idea of a descriptive stating of events then this quickly prompts us to the plain reading of $[\alpha]$ as "The event, that α".

All these considerations force us to accept the following axiom schemata:

A10. $\alpha \supset \theta[\alpha]$
A11. $\theta[\alpha \vee \beta] \equiv \theta([\alpha] \cup [\beta])$
A12. $\theta[\alpha \wedge \beta] \equiv \theta([\alpha] \cap [\beta])$

If we would consider formula θa as, in a sense, a description of the event a then following N. C. A. da Costa [Da Costa 1975] we can define a theory T of event interpretation when the following conditions are satisfied:

(1) If a is an event, then $\theta a \in T$;
(2) T is closed under θ-detachment: if $\theta a \in T$ and $\theta(a \rightarrow b) \in T$, then $\theta(b) \in T$.

Let Γ be a set of formulas. (Γ) denotes the least theory, containing all elements of Γ. Then the following proposition will be true:

Proposition 1: *There exist internally-inconsistent theories of event interpretation which are not over-complete (i.e. if T is such a theory then it is not always the case that $T = F$ where F is the set of all formulas).*

Proof. If $\Gamma = \{\theta a, \theta(\sim a)\}$, then Γ is internally-inconsistent but not over-complete, since $\theta(a \cup \sim a \rightarrow b)$ is not a thesis of T, where b is any event distinct from a. ∎

3. *Semantics of Combined da Costa Logics*

Pursuing Smirnov's approach to combined logics the first version of a semantics of combined da Costa logic could be described as follows. Let W be a non-empty set of possible worlds. Events will be identified with subsets of W. In order to introduce respective operations on the subsets of W we need some additional notions because the usual set-theoretical apparatus is insufficient in view of its boolean (hence, classical against the paraconsistent) character. The remedy will be an exploitation of a notion of paraconsistent algebra of sets.

Definition 2: [Carnielli Alcantara 1984, p. 83]. *A paraconsistent algebra of sets is a structure*

$$\mathrm{A} = \langle S, \emptyset, I, \leq, \bigcap, \bigcup, \Rightarrow, ' \rangle$$

where

(1) $\bigcap$ *and* $\bigcup$ *are the set operations of intersection and union;*
(2) $\leq$ *is a preorder;*
(3) $S \subseteq \mathsf{P}(I)$*;*
(4) S *is closed with respect to the binary operations* $\bigcap$, $\bigcup$, *and the unary operation* $'$*;*
(5) $a \bigcap b \leq a$, $a \bigcap b \leq b$*;*
(6) *if* $c \leq a$ *and* $c \leq b$ *then* $c \leq a \bigcap b$*;*
(7) $a \leq a \bigcup b$, $b \leq a \bigcup b$*;*

(8) $a \bigcap (a \Rightarrow b) \leq b$;
(9) *if* $a \bigcap c \leq b$ *then* $c \leq (a \Rightarrow b)$;
(10) $\emptyset \leq a$, $a \leq I$;
(11) $x \bigcup \sim x \Leftrightarrow I$, *where* $a \Leftrightarrow b$ *iff* $a \leq b$ *and* $b \leq a$;
(12) $x'' \leq x$;
(13) $x^0 \leq (y \Rightarrow x) \Rightarrow ((y \Rightarrow x') \Rightarrow y')$, *where* $x^0 = (x \bigcap x')'$;
(14) $x^0 \bigcap (x^0)' \Leftrightarrow \emptyset$.
(15) $x^0 \leq (x')^0$.

Let φ be a function assigning to event variables subsets of W. The function φ will be extended for all terms in the usual way:

$\varphi(a \cap b) = \varphi(a) \bigcap \varphi(b)$
$\varphi(a \cup b) = \varphi(a) \bigcup \varphi(b)$
$\varphi(a \rightarrow b) = \varphi(a) \Rightarrow \varphi(b)$
$\varphi(\sim a) = \varphi(a)'$

Here $\bigcap$, $\bigcup$, $\Rightarrow$, $'$ are the operations of a paraconsistent algebra

$$\mathrm{W} = \langle S, \emptyset, W, \leq, \bigcap, \bigcup, \Rightarrow, ' \rangle.$$

The notion of truth can be described in a standard way:

$w \models_\varphi \theta a \Leftrightarrow w \in \varphi(a)$ (*that an event occurred, is true in a possible world* w *if and only if this world belongs to the event*);
$w \models_\varphi \alpha \vee \beta \Leftrightarrow w \models_\varphi \alpha$ or $w \models_\varphi \beta$;
$w \models_\varphi \alpha \wedge \beta \Leftrightarrow w \models_\varphi \alpha$ and $w \models_\varphi \beta$;
$w \models_\varphi \alpha \supset \beta \Leftrightarrow w \models_\varphi \alpha$ implies $w \models_\varphi \beta$;
$w \models_\varphi \neg\alpha \Leftrightarrow$ not $w \models_\varphi \alpha$.

Obviously, the function φ ought to be also extended for terms of [-]-type:
$\varphi([\alpha]) = \{\mathrm{w} : w \models_\varphi \alpha\}$

Theorem 1: *Axioms PC+(A1-A10) are valid in the semantics above.*

Proof. Straightforward. ■

Obviously, the provability of this theorem does not mean that combined da Costa logic is (internally) sound with respect to the semantics above. In order to show this let us recall that if K is a Kripke structure (in a sense, $\langle W, \models \rangle$ will be one of them) then K is a model for Γ iff for every $\gamma \in \Gamma$ there is a world w of K such that $w \models_\varphi \gamma$.

Proposition 2: *There are (internally) inconsistent sets of formulas, which have models.*

Proof. Apply the semantics above for combined da Costa logic (consider the case of $\Gamma = \{\theta a, \theta(\sim a)\}$). ■

In an algebraic way another semantics of da Costa combined logic might be obtained just by approaching propositions and events as two different kinds of entities. Then an algebraic da Costa bundle would be a triple $\langle \mathrm{A}, \mathrm{B}, f\rangle$ where $\mathrm{A} = \langle A, +, \circ, -\rangle$ is a Boolean algebra (A contains two elements at least), $\mathrm{B} = \langle B, 0, 1, \leq, \cap, \cup, \rightarrow, \sim\rangle$ is a da Costa algebra (B contains three elements at least), $f : \mathrm{B} \rightarrow \mathrm{A}$ is an embedding function. Let 0 and 1 be defined in both algebras as usual. For f the following conditions are fulfilled:

$f(a \cup b) = f(a) + f(b)$;
$f(a \cap b) = f(a) \circ f(b)$;
$f(a) \circ f(a \rightarrow b) \leq f(b)$;
$(f(a \cap c) \supset f(b)) \leq (f(c) \supset f(a \rightarrow b))$;
$f(a^0) \leq f(\sim a)^0$, where $a^0 = \sim(a \cup \sim a)$;
$f(\sim\sim a) \leq f(a)$;
$f(a^0) \leq f((b \rightarrow a) \rightarrow ((b \rightarrow \sim a) \rightarrow \sim b))$;
$f(b) \leq f(a \cup \sim a)$;
$f(a^0 \cap \sim(a^0)) \leq f(b)$.

where $x \supset y = -x + y$ and $a, b, c \in B$.

If F is a set of well-formed formulas and E is a set of events then a valuation v is defined by:
$v : F \bigcup E \rightarrow A \bigcup B$,
$v(\alpha \vee \beta) = v(\alpha) + v(\beta)$,
$v(\alpha \wedge \beta) = v(\alpha) \circ v(\beta)$,
$v(\neg\alpha) = -v(\alpha)$
(where α, β are wffs and $v(\alpha), v(\beta) \in A$),
$v(a \cup b) = v(a) \cup v(b)$,
$v(a \cap b) = v(a) \cap v(b)$,
$v(a \rightarrow b) = v(a) \rightarrow v(b)$,
$v(\sim a) = \sim v(a)$,
$v(\theta a) = f(v(a))$,
(where a, b are events and $v(a), v(b) \in B$).

Theorem 2: *Axioms PC+(A1-A9) are valid in any da Costa bundle* $\langle \mathrm{A}, \mathrm{B}, f\rangle$.

Proof. Straightforward. ■

In order to encompass axioms A10-A12 we need to modify our notion of da Costa bundle. Now an algebraic da Costa bundle would be a 4-tuple $\langle \mathrm{A}, \mathrm{B}, f, g\rangle$ where $\mathrm{A} = \langle A, +, \circ, -\rangle$ is a Boolean algebra (A contains two elements at least), $\mathrm{B} = \langle B, 0, 1, \leq, \cap, \cup, \rightarrow, \sim\rangle$ is a da Costa algebra (B contains three elements at least), $f : \mathrm{B} \rightarrow \mathrm{A}$, $g : \mathrm{A} \rightarrow \mathrm{B}$ are embedding functions. For g the following conditions must be fulfilled:

$g(x + y) = g(x) \cup g(y)$,
$g(x \circ y) = g(x) \cap g(y)$,
$x \leq f(g(x))$,
where $x, y \in A$.

A valuation v now is additionally defined by:
$v([\alpha]) = g(v(\alpha))$.

Theorem 3: *Axioms PC+(A1-A12) are valid in any da Costa bundle* $\langle \mathrm{A}, \mathrm{B}, f, g\rangle$.

Proof. Straightforward. ■

The more interesting version of da Costa bundle is obtained if we change the second component. In this case in a $\langle \mathrm{A}, \mathrm{B}, f, g\rangle$-bundle instead of B we use a paraconsistent algebra $\mathrm{C} = \langle S, \emptyset, I, \leq, \bigcap, \bigcup, \Rightarrow, \sim\rangle$ (which provides us with the set of events as fibres) and for f the following conditions are fulfilled:

$f(a \bigcup b) = f(a) + f(b)$;
$f(a \bigcap b) = f(a) \circ f(b)$;
$f(a) \circ f(a \Rightarrow b) \leq f(b)$;
$(f(a \bigcap c) \supset f(b)) \leq (f(c) \supset f(a \Rightarrow b))$;
$f(a^0) \leq f(\sim a)^0$, where $a^0 = \sim(a \bigcup \sim a)$;
$f(\sim\sim a) \leq f(a)$;
$f(a^0) \leq f((b \Rightarrow a) \Rightarrow ((b \Rightarrow \sim a) \Rightarrow \sim b))$;
$f(b) \leq f(a \bigcup \sim a)$;
$f(a^0 \bigcap \sim(a^0)) \leq f(b)$.
(where $x \supset y = -x + y$ and $a, b, c \in S$).

For g the following conditions should be fulfilled:
$g(x + y) = g(x) \bigcup g(y)$,
$g(x \circ y) = g(x) \bigcap g(y)$,
$x \leq f(g(x))$,
where $x, y \in A$.

A valuation v will be defined by:
$v: F \bigcup E \to A \bigcup S$,
$v(\alpha \vee \beta) = v(\alpha) + v(\beta)$,
$v(\neg\alpha) = -v(\alpha)$
(where α, β are wffs and $v(\alpha), v(\beta) \in A$),
$v(a \cup b) = v(a) \bigcup v(b)$,
$v(a \cap b) = v(a) \bigcap v(b)$,
$v(a \to b) = v(a) \Rightarrow v(b)$,
$v({\sim}a) = {\sim}v(a)$,
$v(\theta a) = f(v(a))$,
$v([\alpha]) = g(v(\alpha))$.
(where a, b are events and $v(a), v(b) \in S$).

Corollary 1: *Axioms PC+(A1-A12) are valid in any da Costa* $\langle \mathrm{A}, \mathrm{C}, f, g\rangle$*-bundle.*

Should our bundle semantics be exclusively algebraic? Detailed analysis shows that Smirnov's approach to a Kripke-type semantics of combined logics allows itself to be reformulated in terms of so-called Kripke bundles. We define a Kripke bundle as a 4-tuple $\langle \mathrm{W}, \mathrm{E}, f, g\rangle$ where $f : \mathrm{E} \to \mathrm{W}$ (fibration), $g : \mathrm{W} \to \mathrm{E}$ (indexing) are surjective mappings. Thus, sets of events are formed now by possible worlds while possible worlds are indexed by events.

Again, let φ be a function assigning to an event variable a a subset $\varphi(a) \subseteq \mathrm{E}$ (fiber) and we extend function φ in usual way:

$\varphi(a \cap b) = \varphi(a) \bigcap \varphi(b)$
$\varphi(a \cup b) = \varphi(a) \bigcap \varphi(b)$
$\varphi(a \to b) = \varphi(a) \Rightarrow \varphi(b)$
$\varphi({\sim}a) = {\sim}\varphi(a)'$
$\varphi([\alpha]) = \{a : a \in g(\alpha)\}$
where $\bigcap, \bigcup, \Rightarrow, '$ are the operations of a paraconsistent algebra $\mathrm{S} = \langle S, \emptyset, I, \leq, \bigcap, \bigcup, \Rightarrow, '\rangle$ and $S \subseteq P(\mathrm{E})$.

A relation $\models$ is said to be a valuation on a Kripke bundle $\langle \mathrm{W}, \mathrm{E}, f, g\rangle$ if it is a binary relation between each element $w \in \mathrm{W}$ and each atomic formula. We extend $\models$ inductively as follows:

$w \models_\varphi \theta a \Leftrightarrow \varphi(a) \cap f^{-1}(w) \neq \emptyset$ (*an event* a *belongs to a fiber* $f^{-1}(w)$)
$w \models_\varphi \alpha \vee \beta \Leftrightarrow w \models \alpha$ or $w \models \beta$
$w \models_\varphi \alpha \wedge \beta \Leftrightarrow w \models \alpha$ and $w \models \beta$
$w \models_\varphi \alpha \supset \beta \Leftrightarrow$ if $w \models \alpha$ then $w \models \beta$

$w \models_\varphi \neg\alpha \Leftrightarrow$ not $w \models \alpha$
Besides, the function φ ought to be also extended for terms of [-]-type:
$\varphi([\alpha]) = \{a : w \models_\varphi \alpha$ and $w \in g^{-1}(a)\}$ (*a set of indexes of possible worlds in which* α *is true*).

A formula α is said to be valid in a Kripke bundle $\langle \mathrm{W}, \mathrm{E}, f, g\rangle$ if for every valuation $\models$ on $\langle \mathrm{W}, \mathrm{E}, f, g\rangle$ and every $w \in \mathrm{W}$, one has $w \models \alpha$.

From these definitions the following results easily follow.

Theorem 4: *Axioms PC+(A1-A2) are valid in any Kripke bundle* $\langle \mathrm{W}, \mathrm{E}, f, g\rangle$.

Corollary 2: *Axioms PC+(A1-A2) are valid in any Kripke bundle* $\langle \mathrm{W}, \mathrm{S}, f, g\rangle$.

Department of Logic
Institute of Philosophy
Russian Academy of Sciences
Volkhonka 14
119992 Moscow
Russia
E-mail: vasyukov@logic.ru

REFERENCES

[Carnielli Alcantara 1984] W. A. Carnielli, L. P. Alcantara, "Paraconsistent algebras", *Studia Logica*, XLIII, No 1/2, 1984, pp. 79–87.
[Kotas 1975] J. Kotas. "Discussive Sentential Calculus of Jaśkowski", *Studia Logica*, XXXIV (1975), No 2, pp. 149–168.
[Meixner 1994] U. Meixner. "Events and Their Reality", *Logic and Logical Philosophy*, No 2, 1994, pp. 23–34.
[Scheffler 1994] U. Scheffler. "Events as Shadowy Entities", *Logic and Logical Philosophy*, No 2, 1994, pp. 36–53.
[Slupecki 1971] J. Slupecki. "A Generalization of Modal Logic", *Studia Logica*, v. 28, 1971, pp. 7–17.
[Smirnov 1988a] V. A. Smirnov. "Statement and Predication. A Combined Calculus of Sentences and Events" - [In:] *Intensional Logic, History of Philosophy and Methodology*, Budapest, 1988, pp. 101–108.
[Smirnov 1989] V. A. Smirnov. "Combined Calculi of Sentences and Events and von Wright's Logic of Truth" - [In:] *Investigations on Non-Classical Logics*, V. A. Smirnov (ed.), Moscow, 1989, pp. 16–29. (in Russian)

[Smirnov 1989a] V. A. Smirnov. "Assertion and Predication. Combined Calculi of Sentences and Events" - [In:] *Syntactical and Semantic Investigations of Non-Extensional Logics*, V. A. Smirnov (ed.), Moscow, 1989, pp. 27–35. (in Russian)

[Smirnov 1988] V. A. Smirnov. "Internal and External Logics", *Bull. Sect. Logic*, v. 17, No 3/4, 1988, pp. 170–181.

[Vasiliev 1993] N. A. Vasiliev. "Logic and Metalogic", *Axiomathes*, IV, No 3, 1993, pp. 329–351.

[Vasyukov 2001] V. L. Vasyukov. *Jaśkowski-Vasiliev Combined Discursive Logic*. - To appear.