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EROTETIC ARGUMENTS FROM INCONSISTENT PREMISES*

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Abstract

The aim of this paper is to generalize two basic concepts of Wiśniewski's theory of questions, namely question evocation and question generation, to the inconsistent case. For both concepts, I shall present three alternative definitions. Each of these is based on a prioritized adaptive logic. I shall show that, for the consistent case, each of the alternative definitions leads to the same results as the original ones, and that, for the inconsistent case, no other changes are introduced than those required for the sensible handling of inconsistencies. I shall also show that, in the generalized case, a new kind of question evocation can be defined (here called strong evocation). I shall demonstrate that if a question is strongly evoked by some inconsistent set of premises, then each of its direct answers provides guidance on how the inconsistencies should be resolved.

1. *Aim and Survey*

One of the most interesting problems in erotetic logic concerns the way in which questions arise from sets of statements. A central contribution in this respect is Wiśniewski's theory of questions (see [22], [23], and especially [24]). This theory provides a semantic explication of all the main concepts. It moreover has important applications, not only with respect to understanding erotetic arguments in natural language, but also with respect to the study of problem solving, discovery, and creativity.

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Wiśniewski's theory also exhibits a shortcoming. Some of the most important applications concern erotetic arguments that have inconsistent premises. For instance, it is typical of many creative processes that questions arise from inconsistencies. However, as Wiśniewski's theory is based on Classical Logic (henceforth CL), it can only handle cases in which one is dealing with consistent sets of premises.

The aim of this paper is to present a generalization of two central concepts of Wiśniewski's theory, namely question evocation and question generation, to the inconsistent case. An important requirement for this generalization will be that for the consistent case the alternative definitions lead to exactly the same results as the original ones, and that for the inconsistent case, no other changes are introduced than those required for the sensible handling of inconsistencies. Wiśniewski's definitions of question evocation and question generation are briefly discussed in Section 2. In Section 3, I examine the requirements for the generalization to the inconsistent case.

As we shall see, the generalization will be obtained by replacing CL by an appropriate prioritized adaptive logic. A logic is prioritized iff its consequence relation is defined with respect to n-tuples of sets of closed formulas, $\Sigma = \langle \Gamma_0, \ldots, \Gamma_n \rangle$, in which each Γ_i has a different preference ranking. As we shall see below, the logics required for the present application need to handle only couples of subsets $\langle \Gamma_0, \Gamma_1 \rangle$ in which Γ_0 and Γ_1 are subsets of $\mathcal{W}$, the set of closed formulas of the standard predicative language $\mathcal{L}$, and in which Γ_0 is preferred over Γ_1. I shall present three different adaptive logics that satisfy this requirement, and show for which situation each of them is best suited.

A brief introduction to adaptive logics is presented in Section 4, and an intuitive characterization of the three logics in Section 5. In Section 6, I present the first logic, and in Sections 7–9, I discuss the definitions of question evocation and question generation to which this logic leads. The alternative definitions (in terms of the second and third logic) are discussed in Section 10. Section 11 contains some conclusions and open problems.

The three logics presented here are special cases of those presented in [9]. In view of the specific purpose of this paper, I shall restrict the discussion to the semantics. The proof theories of the three systems (as well as the Soundness and Completeness proofs) may be obtained by straightforwardly transforming the results from [9].

2. *Wiśniewski's analysis*

In [24], Wiśniewski makes a distinction between two kinds of erotetic arguments. They differ from one another with respect to the semantical relation between the questions (which are the conclusions) and the premises. In arguments of the first kind, the question is *evoked* by the premises; in those of the second kind, it is *generated* by it.

A question Q is said to be evoked by a set of declarative sentences Γ iff two conditions are met. First, Q should be sound relative to Γ: if all members of Γ are true, then Q should have a true direct answer. Next, Q should be informative relative to Γ: none of the direct answers to Q should be derivable from Γ. If an evoked question is *risky* (in the sense that it is not truly answerable in every case), it is said to be generated by Γ.

As Wiśniewski shows, this general characterization can be applied to any logic of questions that satisfies some minimal conditions (see [24, pp. 226–230]). Its language $\mathcal{L}$ should consist of a declarative part (some standard formalized language) and an erotetic part (that allows for the formation of questions). The declarative part should be provided with a proper semantics that is rich enough to define some concept of truth (with regard to models of some kind, algebraic structures, games, ...). The only condition the erotetic part should meet is that to each question an at least two-element set of (declarative) sentences is assigned that form its direct answers.

Let QL be any logic that satisfies the above requirements, and let $\mathcal{L}$ be the language of QL in which $\mathcal{W}$ is the set of declarative wffs (henceforth, d-wffs) of $\mathcal{L}$. Using the concept of partitions from [20], the semantics of QL can be characterized in a general way. I shall use A, B, C, $A_1, \ldots$ as metalinguistic letters for d-wffs and Greek capital letters as metalinguistic letters for sets of d-wffs. The letters Q_1, Q_2, Q_3, ... will be used as metalinguistic variables for questions.

A partition of (the declarative part of) $\mathcal{L}$ is a couple $P = \langle T, F\rangle$ in which T and F are sets of d-wffs such that $T \cap F = \emptyset$ and $T \cup F = \mathcal{W}$. A QL-partition is a partition of $\mathcal{L}$ that is determined by the semantics of QL. A d-wff A is true in a QL-partition $P = \langle T, F\rangle$ iff $A \in T$; otherwise A is false. Both entailment and multiple-conclusion entailment (henceforth, mc-entailment)[1] are defined with respect to the QL-partitions in which all the premises are true. Thus, $\Gamma \models_{QL} A$ iff A is true in each QL-partition $P = \langle T, F\rangle$ such that $\Gamma \subseteq T$. And, where $\Gamma \mid\models \Delta$ stands for "Γ mc-entails Δ", $\Gamma \mid\models_{QL} \Delta$ iff at least one member of Δ is true in every QL-partition such that $\Gamma \subseteq T$.[2]

[1] The notion of multiple-conclusion entailment helps to define the concepts of question evocation and question generation in a way that is as general as possible (that is, for instance, not restricted to sets of premises that are ω-complete).

[2] In [24], Wiśniewski defines entailment and mc-entailment with respect to the *admissible* partitions of QL (a non-empty subclass of the QL-partitions). The admissible partitions may be, for instance, those that guarantee ω-completeness. However, as admissible partitions can be defined whenever partitions can, and as the discussion in the present paper does not require

The adequacy requirements for erotetic arguments are now easily defined. Let QL-soundness (respectively QL-informativeness) refer to soundness (respectively informativeness) according to QL, and let dQ stand for the set of direct answers to Q.

Definition 1: *A question Q is QL-sound relative to a set of d-wffs Γ iff $\Gamma \mid\models_{QL} dQ$.*

Definition 2: *A question Q is QL-informative relative to a set of d-wffs Γ iff for each $A \in dQ$, $\Gamma \mid\not\models_{QL} \{A\}$.*

Also the definitions of question evocation and question generation are straightforward. Let $\Gamma \models_{E_{QL}} Q$ stand for "Q is evoked by Γ according to QL" and $\Gamma \models_{G_{QL}} Q$ for "Q is generated by Γ according to QL".

Definition 3: $\Gamma \models_{E_{QL}} Q$ *iff*
(i) $\Gamma \mid\models_{QL} dQ$, *and*
(ii) *for each $A \in dQ$, $\Gamma \mid\not\models_{QL} \{A\}$.*

Definition 4: $\Gamma \models_{G_{QL}} Q$ *iff*
(i) $\Gamma \mid\models_{QL} dQ$,
(ii) *for each $A \in dQ$, $\Gamma \mid\not\models_{QL} \{A\}$, and*
(iii) $\emptyset \mid\not\models_{QL} dQ$.

Although this is not required by the above definitions, Wiśniewski's entire theory is based on logics of questions that are extensions of CL. As a consequence, the theory is inadequate to understand how questions arise from inconsistent premises. It is easily observed why. Let CL^* be any logic of questions based on CL. Definitions 1–4 entail that all possible questions are CL^*-sound relative to an inconsistent set of d-wffs Γ, but that none of them is CL^*-informative relative to Γ, and hence, that, according to CL^*, no question is evoked or generated by an inconsistent set of d-wffs.

Wiśniewski himself draws attention to this problem ([24, pp. 215–216]). The situation he considers is the following. Suppose that some accepted set of premises $\{A_1, \ldots, A_n\}$ entails B, and that there are also good reasons (for instance, empirical ones) to accept $\neg B$. As $\Gamma = \{A_1, \ldots, A_n, (A_1 \wedge \ldots \wedge A_n) \supset B, \neg B\}$ is inconsistent, no question is evoked by Γ on the above

that a set of admissible partitions is isolated, I keep the definitions as simple as possible. For a discussion of the importance of admissible partitions, see [24, pp. 104–105]—the discussion there proceeds in terms of "normal interpretations".

analysis. Still, when confronted with a set of premises like this, inquirers may infer questions from it, for instance, “Is $\neg A_1$ the case or ... or $\neg A_n$?”.

The way out proposed by Wiśniewski is the following. In situations like this, he claims, inquirers do not use the sentences $A_1, \ldots, A_n$ in their erotetic inferences, but ‘keep them in suspense’. The set of premises that *is* used for the erotetic inferences, namely $\Gamma' = \{(A_1 \wedge \ldots \wedge A_n) \supset B, \neg B\}$, is consistent. Moreover, the question “Is $\neg A_1$ the case or ... or $\neg A_n$?” is evoked by Γ' (according to any logic of questions based on CL). In view of this, Wiśniewski suggests that the definitions of evocation should be changed in such a way that the question “Is $\neg A_1$ the case or ... or $\neg A_n$?” is evoked by the inconsistent set Γ. Although the details are not spelled out, the underlying idea seems to be that a question Q is evoked by an inconsistent set of premises Γ iff (i) it is evoked by a consistent $\Gamma' \subset \Gamma$, and (ii) every member of $\Gamma - \Gamma'$ is ‘kept in suspense’.

At first sight, this proposal seems highly attractive. However, as it stands, there are several problems with it. The first is that it presupposes that one is able to distinguish beforehand (that is, before the inconsistencies are resolved) between premises that should be accepted unequivocally (and that together form a consistent subset), and those that should be kept in suspense. In interesting cases, this is usually far from evident. For instance, when dealing with an inconsistent scientific theory, it often takes years before one is able to make this distinction. Meanwhile, however, questions are derived from the inconsistent theory.[3]

The second problem is related to this. Wiśniewski’s analysis seems to suggest that the decision which premises should be kept in suspense and which not is merely taken on external grounds. (There are no logical reasons why $A_1, \ldots, A_n$ should be kept in suspense rather than $(A_1 \wedge \ldots \wedge A_n) \supset B$ or $\neg B$.) This, however, may lead to unwanted results. Consider, for instance, the set Γ that consists of

(1) $(\forall x)(Px \supset Qx)$
(2) Pa
(3) $Pb \vee Rb$
(4) $\neg Pa$
(5) $\neg Pb$
(6) $(\forall x)(Qx \supset (Rx \vee Sx))$

and suppose that (1)–(3) are (on the basis of non-logical grounds) clearly preferred over (4)–(6). In view of this, it seems reasonable to keep (4) in suspense in favour of (2). But what about (5) and (6)? As these are logically independent from the inconsistency between (2) and (4), it seems unjustified not to use them in one’s erotetic inferences, even if they do not have the same

[3] Interesting illustrations of this phenomenon can be found in [21], [11], and [15].

preference ranking as (1)–(3). Evidently, the choices made in this respect determine which questions arise from an inconsistent set of premises. If (4)–(6) are kept in suspense, then, on Wiśniewski's proposal, the question "Is Pb the case or Rb?" would be both sound and informative relative to Γ, and hence, would be evoked by it. The question "Is Ra the case or Sa?" would under the same conditions not be sound relative to Γ, and hence, would not be evoked by it. If, however, (5) and (6) are used in the erotetic inferences, then the former question would not be evoked, but the latter would.

A final problem is related to the fact that Wiśniewski's proposal is extremely sensitive to the formulation of the premises. Consider again (1)–(6), and suppose that it can be decided, on the basis of external grounds, that only (4) should be kept in suspense. Suppose now that (4)–(6) are replaced by

(7) $(\neg Pa \wedge \neg Pb) \wedge (\forall x)(Qx \supset (Rx \vee Sx))$

In that case, Wiśniewski's proposal leaves no other alternative than to keep (7) in suspense. But then, there is also a change in the set of evoked questions. For instance, the question that has Ra and Sa as its direct answers is no longer evoked. If the different premises derive from different sources, this sensitivity may be seen as desirable. However, where this is not the case, it may lead to arbitrary results.

3. *Generalizing to the Inconsistent Case*

When dealing with an inconsistent set of premises, the requirements for erotetic arguments are not clear. What does it mean that a question is sound relative to a set of inconsistent premises? Can we require that the question must have a true direct answer if all the premises are true? And, what does it mean that it is informative? Can we demand that it should be possible for each direct answer to be false, even if all the premises are true?

To some readers these questions may seem rhetorical. How is it possible that *all* members of an inconsistent set of premises are true? Several observations are important here. First, whether a statement A is true depends not only on the state of the world, but also on the language $\mathcal{L}$ that is used to express A, and on the correspondence relation $\mathcal{R}$ that links $\mathcal{L}$ to the world. Secondly, no matter how the world looks like, it is possible to design a language $\mathcal{L}$ and a correspondence relation $\mathcal{R}$ such that the true description of the world is inconsistent. Thirdly, inconsistent theories may be empirically successful—they may, for instance, provide explanations for a wide range of phenomena, and moreover lead to important predictions. Fourthly, finding a consistent alternative for an inconsistent scientific theory (that is at least as successful) may take years, even decades. If the inconsistent theory is sufficiently successful, it will in the meantime be used for explaining, predicting, and problem solving. Finally, by using a logic that does not validate *Ex Falso Quodlibet*, reasoning from an inconsistent theory can be just as rational as that from a consistent one.[4]

In view of all this, it should not surprise us that in some cases inquirers make inferences from an inconsistent set of premises, *without* being interested (at least for the time being) in modifying that set. In cases like this, it makes perfectly good sense to say that all members of the inconsistent set are accepted as true.

There is, however, also another type of situation. When dealing with an inconsistent set of premises, one may want to replace this set by a consistent alternative. This may be because one is convinced that, given the language $\mathcal{L}$ and the correspondence relation $\mathcal{R}$, some premises must be false. It may also be because one wants to change the underlying language and correspondence relation in such way that a consistent description becomes possible. Also in this second type of situation, inferences are made from an inconsistent set (at least as long as it is unclear which premises should be rejected or modified in order to restore consistency).[5] The difference is, however, that not all members of the inconsistent set are accepted as true, and that the subset of premises that *are* accepted as true (if any) is intended to be consistent.

In order to answer the questions from the beginning of this section, it is important that we make a distinction between these two types of situation. In the first type, all premises are accepted as true. Hence, soundness and informativeness can be defined in the same way as in the consistent case (provided, of course, that one chooses a logic of questions that can sensibly handle inconsistencies). So, the only change needed is that in the Definitions 1–4, the underlying logic does not validate Ex Falso Quodlibet.[6] The second type of situation is somewhat more complex. It not only requires that one chooses an appropriate logic that can handle inconsistencies, but also that one reconsiders the notions of "soundness" and "informativeness". In the rest of this paper, I shall only deal with situations of this second type.

[4] Some readers may begin to wonder why scientists bother about consistency. One possible explanation is that consistent theories, everything else being equal, are more economical than inconsistent theories—see also [7].

[5] Several case studies from the history of the sciences show that even in cases where scientists aim at finding a consistent alternative, consistency is not restored by simple excision. To the contrary, as long as there are no good reasons to resolve the inconsistencies in a particular way, scientists continue to make inferences from the inconsistent set—see, for instance, [15] and [21].

[6] Preferably, the logic should guarantee that, for the consistent case, one obtains the same results as Wiśniewski. As is shown in [12], this may be realized by replacing *CL* by an appropriate inconsistency-adaptive logic.

If it is unclear which members of an inconsistent set should be accepted as true and which not, it seems attractive to interpret all members of that set as *possibly true*. This reinterpretation has the advantage that the inconsistent set becomes consistent (and is thus safeguarded from triviality). It moreover expresses that the truth of the premises is no longer taken at face value.[7]

Though very natural, the reinterpretation faces two problems. The first is that it does not take into account external preferences. Even if it is unclear how the inconsistencies should be resolved, one may have external grounds to accept some premises as true. The second problem is that one may obtain an interpretation that is much too poor—even if external preferences would be taken into account. Consider, for instance, $\Gamma = \{p, \neg p, q, q \supset (r \vee s)\}$. Even if no member of Γ has a higher external preference than any of the others, it seems intuitively unjustified to accept both q and $q \supset (r \vee s)$ as possibly true only. As both formulas are logically independent from the inconsistency between p and $\neg p$, they can be retained in any consistent alternative to Γ, and hence, there seems to be no reason to question them.

So, what we seem to need is an interpretation of the premises that is "as rich as possible". If one has external reasons to accept some sentence as true, then this sentence should come out true. Moreover, if it can be decided, on the basis of a logical analysis, that some sentence is likely to be retained in any consistent alternative, then this sentence too should come out true.

Evidently, the logical grounds for deciding that some sentence is likely to be retained when the inconsistencies are resolved are dependent on the application context. Suppose, for instance, that $\Gamma = \{p \wedge q, \neg p \wedge (q \supset (r \vee s)), s \vee t\}$, and that each member of Γ stands for the information that originates from a certain source. In some cases, the fact that the first two sources contradict each other may be seen as a reason to question the credibility of those sources, and hence, to suspend judgment on every item of information that originates from them. In other cases, one will question only those items that are explicitly contradicted. Here, unlike in the former kind of cases, one will accept q and $q \supset (r \vee s)$ as true.[8]

[7] This is the so-called discussive approach to inconsistent theories that was first proposed by Stanisław Jaśkowski—see [14]. The idea behind discussive logics is quite simple: A follows 'discussively' from a (possibly inconsistent) set of premises Γ iff $\Diamond A$ follows from $\{\Diamond A \mid A \in \Gamma\}$ by some appropriate modal logic (for instance, $S5$). It is easily observed that discussive logics are paraconsistent ($\Diamond A$, $\Diamond\neg A \nvDash B$), and that they do not allow for the derivation of contradictions. A disadvantage is, however, that they invalidate all genuine multiple-premise rules, and hence, that they are extremely poor. In [17], an adaptive version is presented for Jaśkowski's discussive logic $D2$. This logic, called $D2^r$, preserves all qualities of discussive logics, but moreover validates all multiple-premise rules for sentences that behave consistently. The logics presented here are prioritized versions of $D2^r$.

[8] Also in cases where the origin of the information is unimportant, one usually wants to retain as many items of information as possible.

So, when trying to find a consistent alternative for an inconsistent set of premises, one tends to accept all sentences as true that, relative to external preferences and the application context, are likely to be retained *after* the inconsistencies are resolved. This observation is important to answer the questions from the beginning of this section. When judgment is suspended on the truth or falsity of some of the premises, it makes no sense to define soundness and informativeness with respect to *all* premises. It does make sense, however, to define these notions with respect to the sentences that are likely to be retained after the inconsistencies are resolved. Let us use the term "the consistent core of Γ" to refer to the consequences of Γ that are likely to be retained in any consistent alternative.[9] The requirements of question evocation can be defined in a very natural way with respect to this consistent core. Thus, a question Q is sound relative to a (possibly inconsistent) set of premises Γ iff Q must have a true direct answer if all the members of the consistent core of Γ are true. Q is informative relative to Γ iff each of its direct answers can be false if all members of the consistent core of Γ are true.

Example 1. Consider $\Gamma = \{p \wedge q,\ \neg p,\ q \supset (p \vee r)\}$, and suppose that the consistent core of Γ consists of the CL-consequences of $\{q,\ q \supset (p \vee r)\}$. In that case, the generalized definitions warrant that the question Q that has p and r as its direct answers is sound and informative relative to Γ. This seems intuitively justified. If one has (internal or external) reasons to believe that $p \vee r$ will be retained after the inconsistencies are resolved, one has reasons to believe that Q is truly answerable. Moreover, as neither p nor r belongs to the consistent core of Γ, one has reasons to believe that each of the direct answers to Q may be false.

Given these generalized definitions of soundness and informativeness, the situation is very similar to the consistent case. For instance, like in the consistent case, questions that have only contradictions as direct answers should neither be evoked nor generated—see also Section 7.[10] There are, however, two important differences.

The first difference concerns the conditions under which questions arise. In the consistent case, questions are evoked because the available information exhibits certain 'gaps'. If a question Q is evoked by a set of premises

[9] In view of the above considerations, it is evident that the consistent core of an inconsistent set of premises may vary from one application context to another.

[10] Note that this is different in situations in which all members of an inconsistent set are accepted as true. There, it makes perfectly good sense to infer questions that have only contradictions as direct answers.

Γ, then Γ is incomplete, and each direct answer to Q provides information that cannot be obtained from Γ itself. This property is retained in the inconsistent case, but some of the evoked questions satisfy an additional requirement, namely that each of their direct answers provides information on how the inconsistencies should be resolved. This is illustrated by the following example.

Example 2. Consider $\Gamma = \{p \supset r,\ \neg q \supset \neg r,\ p,\ \neg q\}$, and suppose that the consistent core of Γ consists of CL-tautologies only. In that case, the question that has $p \wedge q$, $p \wedge \neg q$, $\neg p \wedge q$, and $\neg p \wedge \neg q$ as its direct answers is sound and informative relative to the consistent core of Γ. However, it also has the property that each of its direct answers, when added to the consistent core of Γ, limits the choices that have to be made in order to restore consistency. As long as no logically contingent sentence is added to the consistent core of Γ, it can only be inferred that at least one of the members of Γ has to be rejected. If $p \wedge q$ (respectively $p \wedge \neg q$) is added to it, it can be inferred that $\neg q$ (respectively $p \supset r$ or $\neg q \supset \neg r$) has to be rejected. (The situation is analogous for the two other direct answers.)

By lack of a better term, I shall say that a question that satisfies the additional requirement (namely that each of the direct answers provides guidance on how the inconsistencies should be resolved) is *eliminative* relative to Γ. If a question Q is sound, informative and eliminative with respect to Γ, I shall say that it is *strongly evoked* by Γ. In Section 8, I shall define both notions in a more precise way.

The second difference with the consistent case concerns the requirement that an answered question should no longer arise. In the consistent case, this is realized in a very simple way: whenever a question is answered, its answer is added to the set of premises. The only complication in the generalized case is that answers should be added to the consistent core of the premises. Compare also with the discussion of Example 2: if the true answer would not be added to the consistent core, then the same question would continue to arise.

Let me end this section with a short summary. When generalizing the idea of erotetic arguments to the inconsistent case, a distinction has to be made between two types of situations: those in which the inconsistencies are (for the time being) accepted as true, and those in which this is not the case. In this paper, I only deal with the second type of situation. Important for this type is that the requirements for erotetic arguments have to be defined, not with respect to the inconsistent set Γ, but with respect to the *consistent core* of Γ. This consistent core is defined by a combination of external and internal criteria, and is dependent on the application context.

One of the main properties of the logics discussed below is that they enable one to *localize* the consistent core of an inconsistent set Γ. In Section 5, we shall see how this is realized. But first, I need to explain the basic concepts of adaptive logics.

4. *Adaptive Logics*

The first adaptive logic was designed by Diderik Batens around 1980 (see [1]), and was an inconsistency-adaptive logic. As their name indicates, inconsistency-adaptive logics localize the specific inconsistencies that follow from a theory, and 'adapt' themselves to these. If some consequences of a theory behave inconsistently, applications of the rules of inference to these consequences are restricted. However, for consequences of the theory that behave consistently, all rules of *CL* can be applied in their full strength.[11] Later, the idea of an adaptive logic was generalized to other types of logical abnormalities (such as negation-incompleteness)—see [5] for an overview. An important recent development within the adaptive logic programme is the generalization to ampliative forms of reasoning. All three logics presented in this paper are ampliative adaptive logics.

All adaptive logics are based on the idea that a specified set of presuppositions is followed 'as much as possible', that is, *unless and until* they are explicitly violated. If a presupposition is violated, the rules of inference are restricted in order to avoid triviality. However, where this is not the case, the rules can be applied in their full strength. Adaptive logics are thus especially suited for the formal study of reasoning processes that are non-monotonic and/or dynamic.[12] It is indeed typical of such processes that inferences are made on the condition that some presupposition is satisfied. If this condition is no longer fulfilled, there may be a restriction of the rules of inference, and hence, a revision of previously derived conclusions. For instance, when dealing with an inconsistent set of premises Γ, inferences may be drawn on the presupposition that a sentence A behaves consistently. If this presupposition

[11] What this comes to is that inconsistency-adaptive logics do not invalidate a set of rules of inference, but invalidate specific applications of such rules. Inconsistency-adaptive logics thus differ in important respects from monotonic paraconsistent logics. Most of these are obtained by dropping the consistency requirement of *CL*, and by restricting the rules of inference accordingly.

[12] A reasoning pattern is called dynamic if the mere analysis of the premises may lead to the withdrawal of previously drawn conclusions. Note that a dynamic reasoning process is not necessarily non-monotonic. In [6], for instance, Batens shows that the pure logic of relevant implication can be characterized by a dynamic proof theory.

is violated—that is, when it is discovered that A behaves inconsistently—conclusions that were previously drawn may be rejected.

Adaptive logics differ from each other with respect to the kinds of presuppositions that can *safely* be violated (that is, without arriving at triviality). Thus, an inconsistency-adaptive logic can handle theories that are inconsistent, but not necessarily theories that are negation-incomplete.[13] Moreover, adaptive logics differ from each other with respect to the interpretation of the phrase "as much as possible". Thus, some inconsistency-adaptive logics lead to a richer consequence set than others.

All currently available adaptive logics are defined in terms of three elements: an upper limit logic, a lower limit logic, and an 'adaptive strategy'. The upper limit logic is an extension of the lower limit logic. The former thus introduces a set of presuppositions on top of those of the latter. These additional presuppositions are the ones that are defeasible: they are followed 'as much as possible', but are abandoned when necessary to avoid triviality. The third element, the adaptive strategy, determines the interpretation of the phrase "as much as possible".

When a set of premises violates one of the presuppositions of the upper limit logic, it will be said to behave abnormally with respect to the upper limit logic. It is important to note that "abnormality" does not refer to the purported standard of reasoning, say CL. It refers to properties of the application context—to presuppositions that are considered desirable, but that may be overruled.

Adaptive logics can be divided into two categories: corrective and ampliative. In a corrective adaptive logic, the standard of reasoning is determined by the *upper limit logic*; specific *deviations* from this standard are *minimized*. All currently studied corrective adaptive logics have CL as their upper limit logic, and hence, adapt themselves to specific violations of CL-presuppositions. Examples in this category are the inconsistency-adaptive logics from [2] and [16]. In an ampliative adaptive logic, the standard of reasoning is determined by the *lower limit logic*; specific *extensions* of this standard (that are considered desirable within the application context at issue) are *maximized*.[14] Examples of ampliative adaptive logics are the logic of compatibility (see [8]), logics of diagnosis (see [9]), and logics of abduction (see [18] and [19]).

[13] Some adaptive logics adapt themselves to violations of several kinds of presuppositions. In [3], an example is discussed of an adaptive logic that can handle violations of every presupposition of CL.

[14] Formally, this is realized by choosing some upper limit logic that incorporates the desired presuppositions, and by minimizing the deviations from these presuppositions. So, from a formal point of view, corrective and ampliative logics are very similar.

The semantics of an adaptive logic is obtained in the following way. Let AL be an adaptive logic, and let LL and UL be its lower limit logic and its upper limit logic. The AL-models of a set of premises Γ are obtained by selecting a subset of the LL-models of Γ. The selection is determined by the adaptive strategy. The Minimal Abnormality Strategy, for instance, selects those LL-models of Γ that are minimally abnormal (in a set-theoretical sense) with respect to the upper limit logic. If some theory Γ behaves normally with respect to the upper limit logic, the AL-models of Γ coincide with the UL-models of Γ.

It is important to note that, although adaptive models are always defined with respect to a set of premises Γ, a logic of questions based on an adaptive logic would satisfy the requirements discussed in Section 2. The only change needed is that, in an adaptive logic, entailment and mc-entailment are defined with respect to the models of the lower limit logic that validate the premises, and that are moreover selected by the adaptive strategy.

5. *The General Idea*

As mentioned in Section 3, the adaptive logics presented here enable one to localize the consistent core of a possibly inconsistent set of premises. This is realized in two steps. First, their consequence relation is defined, not with respect to a set of premises Γ, but with respect to a *couple* of sets of premises $\Sigma = \{\Gamma_0, \Gamma_1\}$ such that Γ_0 contains all members of Γ that should be accepted as true on the basis of external grounds, and Γ_1 contains all members of Γ for which such grounds are missing. Next, the logics are designed in such a way that their interpretation of Σ is 'as rich as possible'. What this comes to is that they add to Γ_0 as many CL-consequences of Γ_1 as possible. The logics thus guarantee that all sentences come out true that are accepted as true on the basis of external reasons or that should be accepted as true on the basis of logical considerations, and hence, that all sentences come out true that belong to the consistent core of Γ.

The three logics differ from each other regarding the interpretation of the phrase "as rich as possible". As a consequence, the logics define (in general) a different consequence set for a given Σ. Importantly, however, all three systems define the same consequence set as CL, whenever $\Gamma_0 \cup \Gamma_1$ is consistent. The logics thus warrant that, for the consistent case, the generalized definitions of question evocation and question generation lead to the same results as Wiśniewski's definitions.

Where $\Sigma = \{\Gamma_0, \Gamma_1\}$ is a couple of sets of d-wffs, all three logics interpret Σ in terms of *S5*.[15] An *S5*-model $\mathcal{M}$ will be regarded as an *S5*-model of $\Sigma = \{\Gamma_0, \Gamma_1\}$ iff it verifies A for every $A \in \Gamma_0$, and verifies $\Diamond A$ for every $A \in \Gamma_1$. This is in line with the idea that the members of Γ_0 are accepted as true, and those of Γ_1 as possibly true.

As mentioned at the beginning of this section, the logics add to Γ_0 as many *CL*-consequences of members of Γ_1 as possible. This is realized by presupposing that A is derivable from $\Diamond A$ unless and until proven otherwise. Where this presupposition is violated, the logics behave like *S5*. So, their lower limit logic is *S5*. Their upper limit logic is the trivial system *Triv*—compare p. 65 of [13]. It is obtained by adding to *S5* the axiom "$\Diamond A \supset A$". The upper limit logic thus presupposes the normal situation—the one in which A is derivable from $\Diamond A$.

To semantically characterize the logics, we need a criterion for selecting a subset of the *S5*-models of Σ. In order to do so, we first need to specify which formulas are abnormal with respect to the upper limit logic. Next, we have to define the 'abnormal part' of a model: the set of (open and closed) abnormalities that are verified by the model. Finally, we have to choose an adaptive strategy to interpret the ambiguous phrase "as normally as possible".

In all three logics, a formula behaves abnormally iff it violates the presupposition that A is derivable from $\Diamond A$. Hence, as a first approximation, we may say that some formula A behaves abnormally iff $\Diamond A \wedge \neg A$ is *S5*-derivable from the premises. There are, however, three small complications.

The first is that, whenever $\Sigma = \{\Gamma_0, \Gamma_1\}$ is abnormal (in the sense that its *Triv*-consequence set is trivial), it is possible that no single abnormality is *S5*-derivable from Σ, but that some disjunction of abnormalities is. Suppose, for instance, that $\Sigma = \langle \emptyset, \{p \vee q,\ \neg p,\ \neg q\}\rangle$. In that case, no formula of the form $\Diamond A \wedge \neg A$ is *S5*-derivable from Σ, but several disjunctions of such formulas are—for instance, $(\Diamond\neg p \wedge p) \vee (\Diamond\neg q \wedge q)$. In view of this, a formula A will be said to behave abnormally iff $\Diamond A \wedge \neg A$ is a disjunct of a 'minimal' disjunction of abnormalities that is *S5*-derivable from Σ. A disjunction of abnormalities that is *S5*-derivable from Σ will be called a "*Dab*-consequence"; a *Dab*-consequence will be called "minimal" iff no result of dropping some disjunct from it is an *S5*-consequence of Σ.

The second complication concerns the generalization to the predicative case. Consider, for instance, $\Sigma = \langle\{(\forall x)Px\}, \{(\exists x)\neg Px\}\rangle$. From this, no quantifier-free *Dab*-consequence is *S5*-derivable, but $(\exists x)(\Diamond\neg Px \wedge Px)$

[15] As I mentioned in the introduction, the logics presented here are special cases of the ones presented in [9]. The latter interpret n-tuples of sets of premises $\Sigma = \{\Gamma_0, \ldots, \Gamma_n\}$ in terms of the modal logic T. When only *couples* of sets of premises are considered, as is the case here, the interpretation in terms of *S5* leads to the same results as that in terms of T.

is. This suggests that, where $\exists A$ abbreviates A preceded by a sequence of existential quantifiers (in some preferred order) over the variables that occur free in A, abnormalities are of the form $\exists(\Diamond A \wedge \neg A)$.

The last complication is that, in all three systems, abnormalities have to be restricted to *atoms* (primitive formulas and their negations). This is related to the fact that their lower limit logic spreads abnormalities. If, for instance, $\Diamond p \wedge \neg p$ is true in an $S5$-model $\mathcal{M}$, then so is either $\Diamond(p \wedge q) \wedge \neg(p \wedge q)$ or $\Diamond(p \wedge \neg q) \wedge \neg(p \wedge \neg q)$. This problem is well known from other adaptive logics (see, for example, [4] and [17]), and will be explained in some more detail below.

In view of all this, the abnormal part of an $S5$-model $\mathcal{M}$ is easily defined. It is the set of atoms A, such that $\exists(\Diamond A \wedge \neg A)$ is verified by $\mathcal{M}$. The formulas that behave abnormally and the abnormal part of an $S5$-model are the same for all three logics. The only difference between them concerns the choice of the adaptive strategy.

As explained above, the idea behind all three logics is to validate as many applications of $\Diamond A \ / \ A$ as possible. It immediately follows from this that an atom A is derivable from Σ, whenever $\Diamond A$ is $S5$-derivable from it, and A behaves normally. It also follows that an atom A is *not* derivable from Σ, whenever Σ has $S5$-models and $\Diamond A \wedge \neg A$ is true in all of them. In neither of these cases, the interpretation of the phrase "as much as possible" causes any difficulty. What is less clear, however, is how the phrase should be interpreted in cases where one is dealing with disjunctions of abnormalities that cannot be reduced to single abnormalities. It is with respect to cases like this that the three logics lead to different results. This will be illustrated by means of the following example.

Example 3. Suppose that $\Sigma = \langle\{\neg p \vee \neg q,\ \neg p \vee \neg r\}, \{p,\ q,\ r\}\rangle$. From this, two minimal disjunctions of abnormalities are $S5$-derivable, namely $(\Diamond p \wedge \neg p) \vee (\Diamond q \wedge \neg q)$ and $(\Diamond p \wedge \neg p) \vee (\Diamond r \wedge \neg r)$. What this comes to is that the abnormal behaviour of p is connected to that of q and also to that of r: it can be inferred that at least one of p and q (respectively p and r) behaves abnormally, but it cannot be inferred which ones. As we shall see, each of the three logics is based on a different strategy to handle cases like this.

The first logic, DR, follows the most cautious strategy. Whenever a formula behaves abnormally, it is considered as *unreliable*, and the selected models are those in which only unreliable formulas behave abnormally. So, in example 3, no DR-model verifies other abnormalities than $\Diamond p \wedge \neg p$, $\Diamond q \wedge \neg q$, and $\Diamond r \wedge \neg r$. However, some DR-models verify both $\Diamond p \wedge \neg p$ and $\Diamond q \wedge \neg q$, others verify both $\Diamond p \wedge \neg p$ and $\Diamond r \wedge \neg r$. This is the so-called Reliability Strategy.

The second logic, DM, is based on the Minimal Abnormality Strategy. A model $\mathcal{M}$ is selected according to this strategy iff there is no model $\mathcal{M}'$ such that the abnormal part of $\mathcal{M}'$ is a proper subset of the abnormal part of $\mathcal{M}$. It is easily observed that, whenever the abnormal behaviour of one formula is connected to that of another, DM leads to a richer consequence set than DR. In the above example, for instance, there are only two kinds of DM-models of Σ: those that verify one abnormality, namely $\Diamond p \wedge \neg p$, and those that verify two abnormalities, namely $\Diamond q \wedge \neg q$ and $\Diamond r \wedge \neg r$. As a consequence, all DM-models verify $p \vee q$ as well as $p \vee r$. What this comes to is that DM *minimizes* the number of abnormalities. As $(\Diamond p \wedge \neg p) \vee (\Diamond q \wedge \neg q)$ is $S5$-derivable from Σ, at least one of p and q must behave abnormally. However, unlike DR, DM excludes the situation in which *both* p and q behave abnormally. For similar reasons, DM also excludes the situation in which *both* p and r behave abnormally.

Also the third logic, DC, minimizes the number of abnormalities. However, where DM does so in a *set-theoretical* sense, DC selects the models that are minimally abnormal in a *numerical* sense. Thus, the DC-models of Σ are its $S5$-models that verify the *smallest number* of abnormalities. So, in the above example, the DC-models of Σ are those $S5$-models that verify $\Diamond p \wedge \neg p$ as the only abnormality. As a consequence, all DC-models verify both q and r.

I end this section with two remarks. The first is related to the ampliative character of the three logics. Each of the logics is based on the idea that the premises of an inconsistent set should be accepted as possibly true only, unless there are external reasons to accept them as true. This functions as the standard of correct reasoning for the application contexts at issue. At the same time, however, the logics enable one to *extend* this standard as much as possible: whenever it is justifiable (within the given context) to accept a sentence as true, the logics warrant that it is interpreted as such.

The second remark concerns a comparison with Wiśniewki's proposal to handle the inconsistent case. Central to Wiśniewksi's proposal is that some premises of the inconsistent set are 'kept in suspense'. In the present approach, this is formalized by reinterpreting these premises as *possibly true*. However, unlike in Wiśniewski's proposal, sentences kept in suspense are used in one's inferences, unless and until they behave abnormally. As we shall see below, this guarantees that, even in cases where *all* premises are kept in suspense, one is able to make erotetic inferences. Another important difference is that the approach presented here does not proceed in terms of a selection of the premises. We shall see that, because of this, erotetic arguments are less sensitive to the formulation of the premises than in Wiśniewski's proposal.

6. *The Logic DR*

As explained in the previous section, all three logics add to the CL-consequences of Γ_0 as many CL-consequences of members of Γ_1 as possible. In DR, this is realized by making a minimal number of assumptions about the application context—it is not assumed that the inconsistencies can, in some way or the other, be minimized. The idea is that each $\Sigma = \langle \Gamma_0, \Gamma_1 \rangle$ defines a set of unreliable formulas, and that no (CL-consequence of an) unreliable formula is added to Γ_0. If no formula is unreliable with respect to Σ, then all CL-consequences of Γ_1 are added to Γ_0.

I mentioned above that DR has $S5$ as its lower limit logic, and $Triv$ as its upper limit logic. As I shall rely on rather unusual semantics for $S5$ that characterizes the $S5$-models in terms of CL-models, I briefly describe this semantics.[16]

Let $\mathcal{L}$ be the standard language of CL, and $\mathcal{F}$, $\mathcal{F}^p$ and $\mathcal{W}$ the sets of formulas, primitive formulas and wffs (closed formulas) of $\mathcal{L}$. Let $M = \langle D, v \rangle$ be a standard CL-model, with D the domain and v the assignment function. In order to simplify the clauses for the quantifiers, the latter are handled in terms of the pseudo-language $\mathcal{L}^+$. This is obtained by extending the set of constants $\mathcal{C}$ of $\mathcal{L}$ with a (non-denumerable) set of pseudo-constants $\mathcal{O}$, and by requiring that any member of D is named by at least one member of $\mathcal{C} \cup \mathcal{O}$:

$$v : \mathcal{C} \cup \mathcal{O} \longrightarrow D \text{ (where } D = \{v(\alpha) \mid \alpha \in \mathcal{C} \cup \mathcal{O}\})$$

The standard modal language $\mathcal{L}^M$ is extended to $\mathcal{L}^{M+}$ by introducing the set of pseudo-constants $\mathcal{O}$ next to the set of constants $\mathcal{C}$. $\mathcal{W}^M$ refers to the set of wffs of $\mathcal{L}^M$.

A $S5$-model is a couple $\mathcal{M} = \langle \Sigma_\Delta, M_0 \rangle$, where Δ is a set of wffs of $\mathcal{L}$, Σ_Δ is the set of CL-models of Δ, and $M_0 \in \Sigma_\Delta$. The valuation function determined by a $S5$-model $\mathcal{M}$ is defined by the following clauses:

C1 where $A \in \mathcal{F}^p$, $v_\mathcal{M}(A, M_i) = v_{M_i}(A)$
C2 $v_\mathcal{M}(\neg A, M_i) = 1$ iff $v_\mathcal{M}(A, M_i) = 0$
C3 $v_\mathcal{M}(A \wedge B, M_i) = 1$ iff $v_\mathcal{M}(A, M_i) = v_\mathcal{M}(B, M_i) = 1$
C4 $v_\mathcal{M}((\forall \alpha)A(\alpha), M_i) = 1$ iff $v_\mathcal{M}(A(\beta), M_i) = 1$ for all $\beta \in \mathcal{C} \cup \mathcal{O}$
C5 $v_\mathcal{M}(\Box A, M_i) = 1$ iff $v_\mathcal{M}(A, M_j) = 1$ for all $M_j \in \Sigma_\Delta$.

The other logical constants are defined as usual. A model $\mathcal{M}$ verifies $A \in \mathcal{W}^M$ iff $v_\mathcal{M}(A, M_0) = 1$. A is valid iff it is verified by all models. A model $\mathcal{M}$ is an $S5$-model of $\Sigma = \langle \Gamma_0, \Gamma_1 \rangle$ iff, for all $A \in \mathcal{L}$, $\mathcal{M}$ verifies A if $A \in \Gamma_0$, and $\mathcal{M}$ verifies $\Diamond A$ if $A \in \Gamma_1$. I shall write $\Sigma \models_{S5} A$ to denote that all $S5$-models of Σ verify A.

[16] This $S5$-semantics was first presented in [8].

The above $S5$-semantics is easily adjusted to characterize the semantics of the upper limit logic. The $Triv$-models are the $S5$-models $\mathcal{M} = \langle \Sigma_\Delta, M_0 \rangle$, such that, for some maximal consistent subset $\Theta \subset \mathcal{W}$, $\Sigma_\Delta = \Sigma_\Theta$.

Let us now turn to the semantics of DR. The DR-models of $\Sigma = \langle \Gamma_0, \Gamma_1 \rangle$ are its $S5$-models in which only unreliable formulas behave abnormally. If $\Gamma_0 \cup \Gamma_1$ is consistent, then the DR-models of Σ are its $Triv$-models.

Let $\mathcal{F}^a$ be the set of atoms (primitive open and closed formulas and their negations) and let $Dab(\Delta)$ refer to the disjunction $\bigvee\{\exists(\Diamond A \wedge \neg A) \mid A \in \Delta\}$ *provided that* $\Delta \subset \mathcal{F}^a$. I shall say that $Dab(\Delta)$ is a Dab-consequence of Σ iff all $S5$-models of Σ verify $Dab(\Delta)$. A Dab-consequence $Dab(\Delta)$ of Σ will be called *minimal* iff there is no $\Delta' \subset \Delta$ such that $Dab(\Delta')$ is a Dab-consequence of Σ.

I first define the abnormal part of a model:

Definition 5: $Ab(\mathcal{M}) =_{df} \{A \in \mathcal{F}^a \mid \mathcal{M}$ *verifies* $\exists(\Diamond A \wedge \neg A)\}$.

The set of formulas that are unreliable with respect to Σ is defined by:

Definition 6: $U(\Sigma) = \bigcup\{\Delta \mid Dab(\Delta)$ *is a minimal Dab-consequence of* $\Sigma\}$.

DR-entailment is defined in terms of the reliable models of Σ:

Definition 7: *A $S5$-model $\mathcal{M}$ of Σ is reliable iff* $Ab(\mathcal{M}) \subseteq U(\Sigma)$.

Definition 8: *Where $A \in \mathcal{W}$, $\Sigma \models_{DR} A$ iff all reliable $S5$-models of Σ verify A.*

Note that Definition 8 restricts DR-entailment to non-modal formulas. In view of the specific application discussed in this paper, these are the only consequences that we are interested in. In view of the next section, I also define the notion of mc-entailment for DR:

Definition 9: *Where $\Delta \subset \mathcal{W}$, $\Sigma \mid\models_{DR} \Delta$ iff, for every reliable $S5$-model $\mathcal{M}$ of Σ, there is some $A \in \Delta$ such that $\mathcal{M}$ verifies A.*

In order to see what the DR-semantics comes to, consider the following simple example.

Example 4. Suppose that $\Sigma = \langle \{\neg r \vee t\},\ \{p \wedge (\neg q \vee r),\ \neg p \wedge q,\ p \vee s\}\rangle$. In that case, all $S5$-models verify $\neg r \vee t$ as well as $\Diamond(p \wedge (\neg q \vee r))$, $\Diamond(\neg p \wedge q)$ and $\Diamond(p \vee s)$. But then, as $U(\Sigma) = \{p, \neg p\}$, all DR-models of Σ verify q, r and t. Note, however, that some DR-models verify s and others verify

$\neg s$. Hence, $p \vee s$ is not a DR-consequence of Σ. All this is in line with the idea that the CL-consequences of members of Γ_1 should be added to the CL-consequences of Γ_0, except for those that follow by CL from an unreliable formula.

Note that if the abnormalities were not restricted to atoms, selecting the required models would be impossible. This is illustrated by the following example.

Example 5. Consider $\Sigma = \langle \{\neg p\},\ \{p, q\}\rangle$. As q behaves normally, it should be verified by all DR-models of Σ. However, if abnormalities were not restricted to atoms, one of the minimal disjunctions of abnormalities would be $\bigvee\{\Diamond(p \vee \neg q) \wedge \neg(p \vee \neg q),\ \Diamond q \wedge \neg q\}$. Hence, q would be unreliable, and according to the Reliability Strategy, models that verify $\Diamond q \wedge \neg q$ would not be eliminated. As a consequence, q would not be a DR-consequence of Σ.

As I mentioned above, DR is a special case of one of the logics presented in [9] (there called T^{sr}). The proofs of the following two theorems are obtained by straightforwardly transforming the corresponding proofs for T^{sr} from [9]. A wff A is said to be contradictory iff there is no $S5$-model $\mathcal{M}$ that verifies A.

Theorem 1: *If Γ_0 is consistent, and no $A \in \Gamma_1$ is contradictory, then $\Sigma = \langle \Gamma_0, \Gamma_1\rangle$ has DR-models (*Reassurance).

Theorem 2: *If Γ is consistent, then, for every Γ_0, Γ_1 such that $\Gamma_0 \cup \Gamma_1 = \Gamma$, $\langle \Gamma_0, \Gamma_1\rangle \models_{DR} A$, iff $\Gamma \models_{CL} A$.*

The proofs of the following theorems are obvious in view of the DR-semantics, and are left to the reader:

Theorem 3: *If Γ_0 is inconsistent or some $A \in \Gamma_1$ is contradictory, then $\Sigma = \langle \Gamma_0, \Gamma_1\rangle$ does not have DR-models.*

Theorem 4: *If $\Sigma = \langle \Gamma_0, \Gamma_1\rangle$ has DR-models, then, for every $A \in \mathcal{W}$, $\Sigma \not\models_{DR} A$ or $\Sigma \not\models_{DR} \neg A$.*

Theorem 5: *$A \models_{CL} B$ iff $\langle A, \emptyset\rangle \models_{DR} B$ iff $\langle \emptyset, A\rangle \models_{DR} B$.*

I shall say that a sentence $A \in \mathcal{W}$ belongs to the consistent core of Σ (relative to DR) iff $\Sigma \models_{DR} A$. Note that the consistent core of a (possibly inconsistent) sets of premises is thus defined in the most cautious way:

whenever a sentence follows (by CL) from an unreliable sentence, it is not included in the consistent core.

7. *Evocation*

In this section, I present the generalized definition of question evocation, and show that all the basic properties of Wiśniewski's definition are retained. The definition proceeds in terms of the logic of questions DR^* that has DR as its declarative part. The language of DR^* is $\mathcal{L}^*$, and is obtained by enriching $\mathcal{L}$ with questions. Like Wiśniewski, I do not prejudge on the way in which questions are constructed in $\mathcal{L}^*$, but only assume that some minimal conditions are satisfied. I assume (i) that to each question of $\mathcal{L}^*$ an at least two-element set of sentences of $\mathcal{L}$ is assigned that form its direct answers, and (ii) that each finite, at least two-element set of sentences is the set of direct answers to some question of $\mathcal{L}^*$. Here is the definition of question evocation on the basis of DR^*:

Definition 10: $\Sigma = \langle \Gamma_0, \Gamma_1 \rangle \models_{E_{DR^*}} Q$ *iff*
(i) $\Sigma \mid\models_{DR^*} dQ$, *and*
(ii) *for each* $A \in dQ$, $\Sigma \mid\not\models_{DR^*} \{A\}$.

In view of the definition of DR^*, Definition 10 warrants that a question Q is evoked by an inconsistent set of premises $\Gamma = \Gamma_0 \cup \Gamma_1$ iff it is sound and informative relative to the consistent core of Γ. In the following theorem (and henceforth), it is assumed that the declarative part of CL^* is CL, and that its erotetic part is the same as that of DR^*. The proof is obvious in view of Theorem 2.

Theorem 6: *If* $\Gamma = \Gamma_0 \cup \Gamma_1$ *is consistent,* $\langle \Gamma_0, \Gamma_1 \rangle \models_{E_{DR^*}} Q$ *iff* $\Gamma_0 \cup \Gamma_1 \models_{E_{CL^*}} Q$.

Theorem 6 warrants that, for the consistent case, Definition 10 leads to exactly the same results as Wiśniewski's definition of evocation.

Let us now turn to the basic properties of evocation examined by Wiśniewski. Like Wiśniewski, I shall make a distinction between three types of properties.

7.1. *The evoking sets*

The first type of properties is related to the kinds of sets that evoke questions. An important result in this respect is that the notion of incompleteness can be defined in terms of evocation—a set of premises Γ is said to be incomplete iff for some sentence A, neither A nor $\neg A$ is entailed by Γ (see [24, p. 130]). This result is retained in the generalized case. The only difference is that the evoking set may be prioritized.

Theorem 7: $\Sigma = \langle \Gamma_0, \Gamma_1 \rangle$ *is incomplete iff, for some question* Q, $\Sigma \models_{E_{DR^*}} Q$.

Proof. Suppose first that Σ is incomplete. In that case, there is some sentence A, such that $\Sigma \not\models_{DR^*} A$ and $\Sigma \not\models_{DR^*} \neg A$. Hence, where Q is the question that has A and $\neg A$ as its direct answers, $\Sigma \models_{E_{DR^*}} Q$.

Suppose next that, for some question Q, $\Sigma \models_{E_{DR^*}} Q$. It follows that, for some $A \in dQ$, $\Sigma \not\models_{DR^*} \neg A$ (by (i) of Definition 10), and $\Sigma \not\models_{DR^*} A$ (by (ii) of Definition 10). ■

Note that, in the inconsistent case, the question whether some set of premises Γ is complete is dependent on the preference ranking of the members of Γ. Suppose, for instance, that Δ is a maximal consistent subset of $\mathcal{W}$, and that for some $A \in \mathcal{W}$, $\Delta \cup \{A\}$ is inconsistent. In that case, $\langle \Delta, \{A\} \rangle$ is complete, but $\langle \emptyset, \Delta \cup \{A\} \rangle$ is not.

On Wiśniewski account each evoking set is consistent. The reason is that, according to CL, any inconsistent set is complete. Evidently, this property no longer obtains in the generalized case. Note, however, that it still holds true that $\langle \Gamma_0, \Gamma_1 \rangle$ is complete whenever Γ_0 is inconsistent or some $A \in \Gamma_0 \cup \Gamma_1$ is contradictory (see Theorem 3). Hence, we have:

Theorem 8: *If* $\Sigma = \langle \Gamma_0, \Gamma_1 \rangle \models_{E_{DR^*}} Q$, *then* Γ_0 *is consistent and no* $A \in \Gamma_1$ *is contradictory.*

7.2. *The evoked questions*

Also the properties concerning the kinds of questions that can be evoked are retained in the generalized case. Here, I shall only discuss the evocation of so-called self-rhetorical questions.[17] Intuitively, the term "self-rhetorical" is used to refer to any question that is rhetorical for 'logical reasons'. In the consistent case, examples of rhetorical questions are questions that have only contradictory direct answers, and questions that have tautologies as direct answers.

I shall proceed in two steps. First, I shall show that in the generalized case the same questions are considered as self-rhetorical as in the consistent case.

[17] Other important properties in this category concern the evocation of so-called normal questions and proper questions (see [24, pp. 131–132]). In view of the results presented here, it can easily be shown that also these properties are retained in the generalized case.

Next, I shall show that, like in the consistent case, no self-rhetorical question is evoked.

In order to define the notion of a self-rhetorical question, I first need to define the notion of a presupposition of a question. Usually, a sentence A is said to be a presupposition of a question Q iff, for every $B \in dQ$, $B \models A$. When generalizing to DR^*, we have to take into account that entailment is defined with respect to couples of sets of premises. However, in view of Theorem 5, $\langle B, \emptyset\rangle \models_{DR^*} A$ iff $\langle \emptyset, B\rangle \models_{DR^*} A$. Hence, where $Q^p_{DR^*}$ refers to the set of presuppositions of Q according to DR^*, it is safe to stipulate:

Definition 11: *For every $A \in \mathcal{W}$, $A \in Q^p_{DR^*}$ iff, for every $B \in dQ$, $\langle B, \emptyset\rangle \models_{DR^*} A$.*

Theorem 5 and Definition 11 yield:

Theorem 9: *For every question Q, and every $A \in \mathcal{W}$, $A \in Q^p_{DR^*}$ iff $A \in Q^p_{CL^*}$.*

Definition 11 also yields:

Theorem 10: *For every question Q, $Q^p_{DR^*}$ is inconsistent iff some $A \in dQ$ is contradictory.*

Hence, in view of Theorem 10, 2 and 3 we have:

Theorem 11: *For every question Q, $\langle Q^p_{DR^*}, \emptyset\rangle \models_{DR^*} A$ iff $\langle \emptyset, Q^p_{DR^*}\rangle \models_{DR^*} A$ iff $Q^p_{CL^*} \models_{CL^*} A$.*

I can now define when a question is self-rhetorical according to DR^*. In the standard account, a question Q is said to be self-rhetorical iff, for some $A \in dQ$, A is entailed by Q^p. In view of Theorem 11, A is entailed by $\langle Q^p_{DR^*}, \emptyset\rangle$ iff it is entailed by $\langle \emptyset, Q^p_{DR^*}\rangle$. Hence, I stipulate:

Definition 12: *A question Q is self-rhetorical according to DR^* iff, for some $A \in dQ$, $\langle Q^p_{DR^*}, \emptyset\rangle \models_{DR^*} A$.*

In view of Theorem 11 and Definition 12, we have:

Theorem 12: *A question Q is self-rhetorical according to DR^* iff it is self-rhetorical according to CL^*.*

Note that Theorem 12 is not evident. If, for instance, DR^* would allow for the derivation of contradictions, then Theorem 12 would not hold true.[18] I now show that, according to DR^*, no evoked question is self-rhetorical.

Lemma 1: *If* $\Sigma \models_{E_{DR^*}} Q$*, then, for every* $A \in Q^p_{DR^*}$, $\Sigma \models_{DR^*} A$.

Proof. Suppose that for some $A \in Q^p_{DR^*}$, $\Sigma \not\models_{DR^*} A$. In view of Definition 11, it follows that $\Sigma \mid\not\models_{DR^*} dQ$. But then, $\Sigma \not\models_{E_{DR^*}} Q$. ∎

Lemma 2: *If* $\langle Q^p_{DR^*}, \emptyset\rangle \models_{DR^*} A$ *then* $A \in Q^p_{DR^*}$.

Proof. Obvious in view of Definition 11. ∎

Theorem 13: *If* $\Sigma \models_{E_{DR^*}} Q$*, then* Q *is not self-rhetorical.*

Proof. Suppose that $\Sigma \models_{E_{DR^*}} Q$, and that Q is self-rhetorical. By Definition 12, it follows that for some $A \in dQ$, $\langle Q^p_{DR^*}, \emptyset\rangle \models_{DR^*} A$, and hence, by the supposition and Lemma 1 and 2, that $\Sigma \models_{DR^*} A$. But then, $\Sigma \not\models_{E_{DR^*}} Q$. ∎

The following two corollaries immediately follow from Definition 12 and Theorem 13:

Corollary 1: *If* $\Sigma \models_{E_{DR^*}} Q$*, then* Q *has at least two direct answers that are not contradictory.*

Corollary 2: *If* $\Sigma \models_{E_{DR^*}} Q$*, then no direct answer to* Q *is a tautology.*

Let a question that has only contradictory direct answers be called a "completely contradictory question", and one that has only tautologies as direct answers a "completely tautological question". In view of Corollaries 1 and 2, we have that, like in the consistent case, the following properties hold:

Theorem 14: *If* $\Sigma \models_{E_{DR^*}} Q$*, then* Q *is not a completely contradictory question.*

Theorem 15: *If* $\Sigma \models_{E_{DR^*}} Q$*, then* Q *is not a completely tautological question.*

[18] In some cases, it may be justified to change the meaning of self-rhetorical questions. If, for instance, all members of an inconsistent set are accepted as true, then a question that has only contradictions as direct answers should no longer be considered as self-rhetorical.

7.3. *Relations between evoking sets and evoked questions*

The third type of properties examined by Wiśniewski concern relations between evoking sets and the effects of these on the relations between evoked questions. Here, the generalization to the inconsistent case yields some interesting new results.

As is clear from Definition 3, a question that is evoked by a set Γ, is not evoked by every subset, respectively superset, of Γ. In the consistent case, and with *CL* as the underlying logic, the conditions under which Q is evoked by a subset or a superset of Γ are quite trivial. If Q is evoked by Γ, then Q is evoked by a *subset* of Γ iff Q is sound with respect to this subset; it is evoked by a *superset* of Γ iff it is informative with respect to this superset (see [24, p. 132]). One of the consequences is that, in the consistent case, only informativeness is a non-monotonic notion. A question may be informative relative to Γ, whereas it is not informative relative to some $\Gamma' \supset \Gamma$. However, a question that is sound relative to Γ is sound relative to any superset of Γ. When generalizing to the inconsistent case, things become a bit more complicated. We shall see, however, that all complications are intuitively justified.

In the inconsistent case, we are dealing with couples of sets of premises $\Sigma = \langle \Gamma_0, \Gamma_1 \rangle$. Let us first consider the cases where Γ_1 is kept constant. Here, the situation is the same as for the consistent case. Deleting members of Γ_0 may have the effect that some questions are no longer sound with respect to Σ. However, it cannot have the effect that some questions are no longer informative with respect to Σ. Adding members to Γ_0 may have the effect that some questions are no longer informative with respect to Σ, but not that some questions are no longer sound with respect to Σ. Hence, we have:

Theorem 16: *If* $\Sigma = \langle \Gamma_0, \Gamma_1 \rangle \models_{E_{DR^*}} Q$, $\Gamma'_0 \subset \Gamma_0$, *and* $\Gamma'_1 = \Gamma_1$, *then* $\Sigma' = \langle \Gamma'_0, \Gamma'_1 \rangle \models_{E_{DR^*}} Q$ *provided that* $\Sigma' \mid\models_{DR^*} dQ$.

Theorem 17: *If* $\Sigma = \langle \Gamma_0, \Gamma_1 \rangle \models_{E_{DR^*}} Q$, $\Gamma'_0 \supset \Gamma_0$, *and* $\Gamma'_1 = \Gamma_1$, *then* $\Sigma' = \langle \Gamma'_0, \Gamma'_1 \rangle \models_{E_{DR^*}} Q$ *provided that for each* $A \in dQ$, $\Sigma' \mid\not\models_{DR^*} \{A\}$.

Let us now turn to the cases where Γ_1 is not kept constant. Evidently, the properties expressed by Theorem 16 and 17 also obtain, *mutatis mutandi*, for these cases. But, there is something more. A question that is informative with respect to $\langle \Gamma_0, \Gamma_1 \rangle$ may no longer be informative with respect to $\langle \Gamma_0, \Gamma'_1 \rangle$, where $\Gamma'_1 \subset \Gamma_1$. It is not difficult to see why: taking subsets of Γ_1 may have the effect that $U(\Sigma') \subset U(\Sigma)$. When this happens, the number of statements that should be accepted as true increases, and hence, some questions may no longer be informative relative to Σ'. Consider, for instance,

$\Sigma = \langle \emptyset, \{p, \neg p\}\rangle$, and $\Sigma' = \langle \emptyset, \{\neg p\}\rangle$. The question that has p and $\neg p$ as its direct answers is informative with respect to Σ, but not with respect to Σ'. For similar reasons, a question that is sound with respect to $\langle \Gamma_0, \Gamma_1\rangle$ may fail to be sound with respect to $\langle \Gamma_0, \Gamma'_1\rangle$, where $\Gamma'_1 \supset \Gamma_1$. This typically happens when $U(\Sigma') \supset U(\Sigma)$. For instance, the question that has q and r as its direct answers is sound with respect to $\langle\{p\}, \{\neg p \wedge (q \vee r)\}\rangle$, but not with respect to $\langle\{p\}, \{\neg p \wedge (q \vee r)\} \cup \{\neg q,\ \neg r\}\rangle$—$q \vee r$ is DR^*-derivable from the former, but not from the latter. In view of all this, we have:

Theorem 18: *If* $\Sigma = \langle \Gamma_0, \Gamma_1\rangle \models_{E_{DR^*}} Q$, $\Gamma'_0 = \Gamma_0$, *and* $\Gamma'_1 \subset \Gamma_1$, *then* $\Sigma' = \langle \Gamma'_0, \Gamma'_1\rangle \models_{E_{DR^*}} Q$ *provided (i)* $\Sigma' \mid\models_{DR^*} dQ$, *and, (ii)* $U(\Sigma') = U(\Sigma)$.

Theorem 19: *If* $\Sigma = \langle \Gamma_0, \Gamma_1\rangle \models_{E_{DR^*}} Q$, $\Gamma_0 = \Gamma'_0$, *and* $\Gamma'_1 \supset \Gamma_1$, *then* $\Sigma' = \langle \Gamma'_0, \Gamma'_1\rangle \models_{E_{DR^*}} Q$ *provided (i) that for each* $A \in dQ$, $\Sigma' \mid\not\models_{DR^*} \{A\}$, *and, (ii)* $U(\Sigma') = U(\Sigma)$.

What Theorem 19 comes to is that, in the inconsistent case, not only informativeness is a non-monotonic notion, but also soundness. This, however, is in line with what one might expect. Adding new inconsistencies may result in a decrease of the number of statements accepted as true. Hence, for some questions it may no longer be justified to believe that they must have a true direct answer.

8. *Strong Evocation*

I mentioned in Section 3 that, in the generalized case, some evoked questions are *eliminative* relative to their evoking set: each of their direct answers provides guidance on how the inconsistencies should be resolved. Questions that satisfy this additional requirement are said to be *strongly evoked*. Let $\Sigma \models_{SE_{DR^*}} Q$ denote that Q is strongly evoked by Σ according to DR^*. Here is the definition:

Definition 13: $\Sigma \models_{SE_{DR^*}} Q$ *iff*
(i) $\Sigma \mid\models_{DR^*} dQ$,
(ii) *for each* $A \in dQ$, $\Sigma \mid\not\models_{DR^*} \{A\}$,
(iii) $dQ \subseteq U(\Sigma)$, *and*
(iv) *for each* $A \in dQ$, $\Sigma \mid\not\models_{DR^*} \{\neg A\}$.

It is easily observed that strong evocation is a special case of evocation:

Theorem 20: *If* $\Sigma \models_{SE_{DR^*}} Q$, *then* $\Sigma \models_{E_{DR^*}} Q$.

I leave it to the reader to check that the equivalents of Theorems 8, 13, 14, and 15 hold true for strong evocation.

I explained in Section 7 that completeness can be defined in terms of evocation. An important result is that consistency can be defined in terms of strong evocation. I shall say that a set of premises is contradictory iff some member of Γ is contradictory.

The proofs of the following two lemmas are obvious:

Lemma 3: *If Γ is inconsistent, but non-contradictory, then every DR^*-model of $\langle \emptyset, \Gamma \rangle$ verifies at least one formula of the form $\exists(\Diamond A \wedge \Diamond\neg A)$, such that $A \in \mathcal{F}^p$.*

Lemma 4: *A S5-model $\mathcal{M}$ verifies $\exists(\Diamond A \wedge \Diamond\neg A)$ iff it verifies $\exists(\Diamond A \wedge \neg A) \vee \exists(\Diamond\neg A \wedge A)$.*

Theorem 21: *Γ is inconsistent, but non-contradictory iff, for some question Q, $\Sigma = \langle \emptyset, \Gamma \rangle \models_{SE_{DR^*}} Q$.*

Proof. For the left-right direction, suppose that the antecedent holds true. By Theorem 1, it follows that $\Sigma = \langle \emptyset, \Gamma \rangle$ has DR^*-models. But then, by Lemma 3 and 4 and the definition of $U(\Sigma)$, there is some $A \in \mathcal{F}^a$, such that $A, \neg A \in U(\Sigma)$, and $\Sigma \not\models_{DR^*} \exists A$ and $\Sigma \not\models_{DR^*} \exists\neg A$. Hence, the question that has $\exists A$ and $\exists\neg A$ as its direct answers fulfills the conditions (i)–(iv) of Definition 13.

For the right-left direction, it suffices to observe that, if the antecedent holds true, Σ has DR^*-models, and $U(\Sigma) \neq \emptyset$. Hence, Σ is non-contradictory, but inconsistent. ■

I now show that, if some question Q is strongly evoked by Σ, then each direct answer to Q can be used to transform Σ in a Σ' that is less abnormal.

Theorem 22: *If $\Sigma = \langle \Gamma_0, \Gamma_1 \rangle \models_{SE_{DR^*}} Q$, then for every $A \in dQ$, (i) $\Sigma' = \langle \Gamma_0 \cup \{A\}, \Gamma_1 \rangle$ has DR^*-models, and (ii) there is a minimal Dab-consequence $Dab(\Delta)$ of Σ, such that, for some $\Delta' \subset \Delta$, $Dab(\Delta')$ is a minimal Dab-consequence of Σ'.*

Proof. Suppose that the antecedent holds true, and choose some arbitrary $A \in dQ$. By (iv) of Definition 13, it follows that $\Sigma' = \langle \Gamma_0 \cup \{A\}, \Gamma_1 \rangle$ has DR^*-models.

As $dQ \subseteq U(\Sigma)$, there is some $\Theta \subseteq \mathcal{F}^a$, such that $Dab(\{A\} \cup \Theta)$ is a minimal Dab-consequence of Σ. Moreover, as A is verified by some DR^*-models of Σ, Θ is non-empty. But then, as the DR^*-models of Σ' are the

DR^*-models of Σ that verify A, and hence falsify $\Diamond A \wedge \neg A$, $Dab(\Theta)$ is a minimal Dab-consequence of Σ'. ■

9. *Generation*

As explained above, question generation is a special kind of question evocation. A question is generated by a set of sentences iff it is evoked by that set, *and* is risky. In view of the previous sections, the generalized definition of question generation is straightforward:

Definition 14: $\Sigma = \langle \Gamma_0, \Gamma_1 \rangle \models_{G_{DR^*}} Q$ *iff*
(i) $\Sigma \,|\!\models_{DR^*} dQ$,
(ii) *for each* $A \in dQ$, $\Sigma \,|\!\not\models_{DR^*} \{A\}$, *and*
(iii) $\langle \emptyset, \emptyset \rangle \,|\!\not\models_{DR^*} dQ$.

In [24, pp. 155–160], Wiśniewski examines a series of properties of question generation. Each of these is retained in the generalized case or is subject to changes analogous to those discussed in Section 7. I only list those properties that highlight the main differences with question evocation.

Like in the consistent case, it is easy to prove that the empty set does not generate questions:

Theorem 23: *If* $\Sigma = \langle \Gamma_0, \Gamma_1 \rangle \models_{G_{DR^*}} Q$, *then* $\Gamma_0 \cup \Gamma_1 \neq \emptyset$.

In the consistent case, it can moreover be proved that sets of tautologies do not generate questions. In the generalized case, something stronger holds. Sets that entail only tautologies do not generate questions:

Theorem 24: *If* $\Sigma \models_{G_{DR^*}} Q$, *then for some* $A \in \mathcal{W}$, $\Sigma \models_{DR^*} A$, *and* $\langle \emptyset, \emptyset \rangle \not\models_{DR^*} A$.

Proof. Obvious in view of Definition 14 and the fact that, if the consequent would not hold true, then, for every $A \in \mathcal{W}$, $\Sigma \models_{DR^*} A$ iff $\langle \emptyset, \emptyset \rangle \models_{DR^*} A$. ■

As an illustration of Theorem 24, consider $\Sigma = \langle \emptyset, \{p, \neg p\} \rangle$. Even though the second element of Σ contains sentences that are not tautologies, no such sentence is entailed by Σ. Hence, no question is generated by Σ.

Theorem 25: $\Sigma = \langle \Gamma_0, \Gamma_1 \rangle$ *generates at least one question iff* Σ *is incomplete, and, for some* $A \in \mathcal{W}$, $\Sigma \models_{DR^*} A$, *and* $\langle \emptyset, \emptyset \rangle \not\models_{DR^*} A$.

Proof. The left-right direction is obvious in view of Theorem 7 (and the fact that each generated question is an evoked question) and Theorem 24. For the other direction, suppose that the antecedent holds true. In that case, there is some $A \in \mathcal{W}$, such that A is entailed by Σ and A is not a tautology, and there is some $B \in \mathcal{W}$, such that neither B nor $\neg B$ is entailed by Σ. But then, the question that has $A \wedge B$ and $A \wedge \neg B$ as its direct answers fulfills the conditions (i)–(iii) of Definition 14, and hence is generated by Σ. ■

Like in the consistent case, the generalized definition warrants that the non-contradictory direct answers to a generated question do not contradict each other:

Theorem 26: *If* $\Sigma = \langle \Gamma_0, \Gamma_1 \rangle \models_{G_{DR^*}} Q$*, then there are no non-contradictory* $A, B \in dQ$ *such that, for every* $S5$*-model* $\mathcal{M}$*,* $\mathcal{M}$ *verifies* A *iff* $\mathcal{M}$ *falsifies* B.

Proof. It suffices to observe that if the consequent would not hold true, then $\langle \emptyset, \emptyset \rangle \models_{DR^*} dQ$. ■

10. *The Logics DM and DC*

In this section, I briefly present the logics DM and DC, and discuss the alternative definitions of question evocation and question generation to which these logics lead.

As explained in Section 5, DM is obtained by selecting those $S5$-models of Σ that are minimally abnormal in a set-theoretical sense. We thus have:

Definition 15: *A* $S5$*-model* $\mathcal{M}$ *of* Σ *is minimally abnormal iff there is no* $\mathcal{M}'$ *such that* $Ab(\mathcal{M}') \subset Ab(\mathcal{M})$.

As for DR, entailment and mc-entailment are restricted to non-modal formulas:

Definition 16: *Where* $A \in \mathcal{W}$*,* $\Sigma \models_{DM} A$ *iff all minimally abnormal* $S5$*-models of* Σ *verify A.*

Definition 17: *Where* $\Delta \subset \mathcal{W}$*,* $\Sigma \mid\models_{DM} \Delta$ *iff, for every minimally abnormal* $S5$*-model* $\mathcal{M}$ *of* Σ*, there is some* $A \in \Delta$ *such that* $\mathcal{M}$ *verifies* A.

In order to define the logic DC, we need some additional definitions. First, let $f(A)$ be the result of reletttering the free variables in A in such a way that they occur in some standard order (for instance, the first occurring free variable is always x, the second always y, …). Next, let $A \prec B$ denote that $\exists B$ follows by (non-zero applications of) existential generalization from $\exists A$, and $g(\Delta)$ that $\{f(A) \mid A \in \Delta$; for no $B \in \Delta, f(B) \prec f(A)\}$. Finally, let Φ_Σ be the set of all $g(\Delta)$ such that Δ contains one factor of each minimal Dab-consequence of Σ.

The logic DC is obtained by selecting those members of Φ_Σ that are *numerically* smallest. Let $\Phi^{\#}_{\Sigma}$ be the set of members of Φ_Σ that do not have a larger cardinality than any other member of Φ_Σ.

Definition 18: *A S5-model $\mathcal{M}$ of Σ is a DC-model of Σ iff $Ab(\mathcal{M}) \in \Phi^{\#}_{\Sigma}$.*

Definition 19: *Where $A \in \mathcal{W}$, $\Sigma \models_{DC} A$ iff all DC-models of Σ verify A.*

Definition 20: *Where $\Delta \subset \mathcal{W}$, $\Sigma \mid\models_{DC} \Delta$ iff, for every DC-model $\mathcal{M}$ of Σ, there is some $A \in \Delta$ such that $\mathcal{M}$ verifies A.*

The alternative definitions for question evocation are obtained from Definition 10 by replacing DR by DM, respectively DC. Those for strong evocation and generation are obtained in the same way from Definitions 13 and 14. I leave it to the reader to check that all theorems proved in sections 7–9 are also provable for the alternative definitions.

The alternative definitions on the basis of DM are adequate for situations in which one has reasons to minimize *set-theoretically* the number of things that went wrong. In situations like this, DM leads to results that are intuitively more justified than those obtained by DR. The following example illustrates this.

Example 6. Consider $\Sigma = \langle\{\neg p \vee \neg q,\ p \supset r,\ q \supset r,\ p \supset (s \vee t),\ q \supset (s \vee t)\},\ \{p, q\}\rangle$. The only minimal Dab-consequence that follows from Σ is $(\Diamond p \wedge \neg p) \vee (\Diamond q \wedge \neg q)$. If one has reasons to believe that p and q cannot both be mistaken, then the question "Is p or q the case?" should be evoked by Σ. Under the same conditions, the question "Is r the case or $\neg r$?" should *not* be evoked by Σ—if either p or q is true, then so is r—but the question "Is s or t the case?" should. All three results are obtained by DM, but none of them is obtained by DR.

In some situations, one has reasons to minimize the number of abnormalities in a *numerical* sense. This may be, for instance, because one wants to restore consistency by a minimal number of changes. In cases like this, the logic DC leads to better definitions of question evocation and question generation than the logics DR and DM. Also this is best illustrated by means of an example.

Example 7. Suppose that $\Sigma = \langle\{\neg p \vee \neg q, \neg p \vee \neg r, q \supset (s \vee t)\}, \{p, q, r\}\rangle$. The minimal Dab-consequences that are $S5$-derivable from Σ are $(\Diamond p \wedge \neg p) \vee (\Diamond q \wedge \neg q)$ and $(\Diamond p \wedge \neg p) \vee (\Diamond r \wedge \neg r)$. What this comes to is that p or q is mistaken, and that p or r is mistaken. If one has reasons to minimize the *number* of things that are mistaken, then one should conclude that p is mistaken, and that neither q nor r is. But then, neither "Is q or $\neg q$ the case?" nor "Is r or $\neg r$ the case?" should be evoked by Σ. Both questions are evoked by DR and DM, but not by DC. Under the same conditions, the question "Is s or t the case?" should be evoked by Σ. Also this result can only be obtained by DC.

11. *Conclusions and Open Problems*

In this paper, I presented an analysis of how questions arise from inconsistent premises. I started from the theory of Wiśniewski, and showed in what way it can be generalized to the inconsistent case. For the consistent case, the generalized theory leads to exactly the same results as Wiśniewski's.

I distinguished between two different kinds of situations. When dealing with an inconsistent set of premises, one may accept all members of this set as true (at least for the time being), or one may suspend judgment on the truth of at least some of the premises. In this paper, I only examined the second type of situation in detail.

The generalization presented here is based on a specific type of prioritized adaptive logics. One of the main properties of these logics is that they localize the consistent core of a (possibly inconsistent) set of sentences. They do so in a way that takes into account not only logical considerations but also external preferences.

The generalization revealed some interesting new properties of question evocation. It became clear, for instance, that in the inconsistent case, a distinction has to be made between two kinds of evocation. A question may be evoked because the information exhibits certain 'gaps', or it may be evoked because of the inconsistencies. The second type of evocation I called strong evocation. I showed that if a question is strongly evoked from an inconsistent set of premises, then each of its direct answers is helpful in resolving the inconsistencies.

There are several open problems that deserve further study. A first important open problem concerns the comparison of the two types of situation. In this paper, I mentioned already several differences between them—for instance, with respect to the question whether completely contradictory questions should be allowed for. It would be interesting to study this in some more detail. A second series of open problems concerns a further analysis of the notion of strong evocation. According to the definition presented here, the direct answers to strongly evoked questions are always atoms. It would be interesting to generalize this definition, and to examine some further properties of this concept. A final series of problems is related to Wiśniewski's notion of question generation. In this paper, I briefly showed how this concept can be generalized to the inconsistent case, but did not distinguish between two types of question generation. In line with the results on question evocation, this distinction definitely seems worthwhile to explore.[19]

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