

SUBSTITUTIVITY*

Dedicated to the memory of Yehoraz Kasher

Alex BLUM

An earmark of twentieth century analytic philosophy is an all pervasive concern with issues of substitutivity. Whether the subject is modality, belief, explanation, intentionality, or the slingshot, to name but a few of the perennial concerns, a common assumption regarding permissible substitution is never far from the surface. I question this common assumption.

Quine gives this contention its classic formulation. He writes:

(Q) "When a singular term is used in a sentence purely to *specify* its object, and the sentence is true of the object, then certainly the sentence will stay true when any other singular term is substituted that *designates* the same object. Here we have a criterion for what may be called *purely referential position*: the position must be subject to the *substitutivity of identity*".¹

But how is one to determine whether a singular term is used in a sentence "purely to specify its object"?

The answer seems fairly clear. Expressions which say nothing about the objects they denote, which is generally the case with proper names and pronouns, purely specify their objects; while other expressions, like descriptions, do not.²

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¹Quine: 142. The double underlining is my own.

²I follow Kripke in taking the referent of a definite description to be its "semantic referent". (Kripke: 25) Regimentation via variables may test for anaphora and for gender implication, and be treated accordingly. I am however reminded of Wittgenstein's poignant double edged remark:

Hence Quine's attempt to show that substitutivity of singular terms used purely to specify its object (henceforth *purely referential singular terms*) fail in modal contexts is incorrect.

For Quine argues that since the premises of:

- (A) 1. $\Box 9 > 4$
 2. $9 = \text{the number of the planets}$
 \therefore 3. $\Box \text{the number of the planets} > 4$

are true and its conclusion false, the position of '9' in 1 is not purely referential.³ But surely 'the number of the planets' unlike, say, 'ix', or '9' does more than purely specify 9. It identifies it with the number of the planets. Hence while (A) is invalid:

- (B) 1. $\Box 9 > 4$
 2. $9 = ix$
 \therefore 3. $\Box ix > 4$

is sound. For again unlike the singular terms in (B) which are purely referential 'the number of the planets' in (A) is not.⁴

If referential position is what is needed for permissible substitution, then (Q) ought to be replaced by:

(Q') When a singular term is used in a sentence purely to refer to its object, and the sentence is true of the object, then certainly the sentence will stay true when any other singular term is substituted that purely refers to the same object. Here we have a criterion for what may be called *purely referential position*: the position must be subject to the *substitutivity of identity of purely referential singular terms*.

We replaced in (Q) "*specify*" by 'refers to' and "*designates*" by 'purely refers to'.

"A proposition like "this chair is brown" seems to say something enormously complicated, for if we wanted to express this proposition in such a way that nobody could raise objections to it on grounds of ambiguity, it would have to be infinitely long." Wittgenstein: 5e.

³Quine: 197.

⁴Hence testing for purely referential position in modal context should be carried out with singular terms which do no more than refer to the object in question. Quine writes: "... if in trying to settle whether a position is referential... we can always fall back on... substitutivity of identity for constant terms". (Quine: 168).

For, after all, what rationale other than Leibniz' principle of the indiscernibility of identity could possibly serve as the justification of (Q)? But Leibniz' principle is a *de re* principle. It's a principle about objects and not expressions. Extrapolation to linguistic contexts thus requires in the first instance the concept of a purely referential singular term; the semantic correlate of an object. Hence (Q').

Failing to distinguish between a term which purely refers to an object and one which designates the object will of course fit the criterion to contexts which are insensitive to this distinction. Namely, contexts which are extensional. However once such a context is placed within the scope of a modal operator, the difference between a purely referential term and one which designates becomes crucial. Substitutivity will fail only in the later case. For what may have been intended in a non-modal context as designation by an accidental property, becomes in modal context an essential trait.

(Q'), however, although a corrective to a clearly extensional bias, misses what we take to be the crucial factor in correct substitution. Namely, the relativity of substitution to the contexts in which the singular term occurs. What appears to be relevant to truth preservation in substitution, is the *acceptance of the identity in that context*. Thus for example, in belief contexts the believer must *accept*⁵ the identity. And in modal contexts the identity must be necessary.

That is why substitution in modal contexts by rigid designators which are not names, will work just as well as names. E.g., the soundness of:

- (C) 1. $\Box 5 > 1$
 2. $5 = \text{the \# between 4 and 6}$
 \therefore 3. $\Box \text{the number between 4 and 6} > 1.$

is forthcoming, for not only is 2. true but so is $\Box 2$. It is the kind of identity required in modal contexts.

In belief contexts, 'a' and 'b' are intersubstitutable if and only if it is true of the believer *j* that:

⁵Accept = internalize?

(Bel) $(B_jFa \cdot B_ja = b) \supset B_jFb$ (where ' B_i ' is an indexed belief operator).⁶

Given (Bel) it will follow that:

- (D) 1. B_jFa
 2. $B_ja = b$
 \therefore 3. B_jFb

is valid.

We thus contend that although (Q') is a corrective to (Q), it misleads. For we are led to believe: one, that referential transparency and substitutivity are correlative; and two, it disregards context in permissible substitution. We have seen on the contrary that while referential transparency implies substitutivity the converse is not true. And we maintain that the key to permissible substitution is context of substitution.

Extensional contexts demand that the singular terms name the same object, modal contexts that the identity be necessary; and intensional contexts in general that the intensional identity "carry over" in that context.

An appropriate meta-principle of substitution may thus be expressed as:

(Subst) $(\alpha)(\beta)$ [If α and β are singular closed terms and ' $\varphi F\alpha$ ' and ' $\varphi * \alpha = \beta$ ' are true, then so is ' $\varphi F\beta$ ']

where φ gives the relevant context of the sentence, ' $\varphi * \alpha = \beta$ ' indicates that the identity is recognized as such in the indicated context φ ,⁷ and "" are corner quasi-quotes.

⁶Belief contexts and perhaps no intensional context *needs* to be opaque. For the being who has the intension may be infallible.

⁷Thus for example: A doubts that Fa
 A doubts that $a = b$
 $\therefore A$ doubts that Fb

is of course not valid, for the identity here is not recognized but questioned. The appropriate statement of identity would here have to be " A has no doubt that $a = b$ is in fact the case".

In extensional contexts φ may be replaced by a truth operator, in modal contexts by ' \square ' and in other intensional contexts by the relevant intensional operators, as say, in (Bel) above. φ may be selective, as in theoretical explanation. It may assign a truth operator to the non-identities and to the identities a restriction that they be so recognized by the theory.

Bar-Ilan University
e-mail: blumal@mail.biu.ac.il

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