# FILLING A GAP IN PROFESSOR VON KUTSCHERA'S DECISION PROCEDURE FOR DEONTIC LOGIC*

Jules SPELLER

The decision procedure proposed by Professor von Kutschera in his introduction to deontic logic(1) might be described as follows:

Let S be a deontic formula and S′ the same formula where the occurrences of the deontic operators P and F have been replaced by their equivalents in terms of the O-operator.
Then:
S′ and therefore its equivalent S are deontologically true (D-true) if, and only if, the disjunctive normal form of $\neg S'$, $\Phi(\neg S')$, is deontically unsatisfiable (d-unsatisfiable) or else: if, and only if, each member M of the disjunction $\Phi(\neg S')$ is d-unsatisfiable.
Now:
For every M, where $O(B_1)$, $O(B_2)$, ..., $O(B_n)$ are the formulas of the form O(A) that are conjuncts of M, and $\neg O(C_1)$, $\neg O(C_2)$, ..., $\neg O(C_m)$ the formulas of the form $\neg O(A)$ that are conjuncts of M, M is d-unsatisfiable if at least one of the following conditions is fulfilled:

$C_1$: M is closed, i.e. M contains a formula (deontic or not) and its denial.
$C_2$: Some $\Psi_i(M)$ of the form $(B_1 \wedge B_2 \wedge \ldots \wedge B_n) \supset C_i$ is a two-valued logical truth.

This procedure runs, however, into difficulty with formulas such as:

$$S_1 = ((O(p) \supset O(q)) \wedge O(\neg q)) \supset \neg O(p)$$

and $$S_2 = ((\neg O(p) \wedge \neg q) \wedge (O(r \vee \neg r) \supset q)) \supset O(s).$$

The reason why seems clear. The corresponding normal forms

(*) I wish to thank my colleagues Dr. Ginette Kremer-West, Professor Louis Vax and Professor Jean-Paul Harpes for much helpful criticism.

(1) See: Franz von Kutschera, Einführung in die Logik der Normen, Werte und

$$\Phi(\neg S_1') = (\neg O(p) \wedge O(\neg q) \wedge O(p)) \vee (O(q) \wedge O(\neg q) \wedge O(p)) \text{ and}$$
$$\Phi(\neg S_2') = (\neg O(p) \wedge \neg q \wedge \neg O(r \vee \neg r) \wedge \neg O(s)) \vee (\neg O(p) \wedge \neg q \wedge q \wedge \neg O(s))$$

contain each at least one unclosed M lacking either conjuncts of the form O(A) or conjuncts of the form $\neg O(A)$. This means that for lack of either antecedent or consequent no $\Psi_i(M)$ (corresponding to those M) is construable.

As our examples show, the ranges of the two indices n and m have to be represented by $n \geq 0$ and $m \geq 0$, and, therefore, are larger than those apparently supposed by Professor von Kutschera.

*
**

One simple way to restore the applicability of the procedure would consist in replacing criterion $C_2$ by:

$C_2'$: Some $X_i(M)$ of the form $B_1 \wedge B_2 \wedge \ldots \wedge B_n \wedge \neg C_i$ is a two-valued logical falsehood.

The correctness of criterion $C_2'$ already results from the correctness of the original procedure (as proven by Professor von Kutschera(2)) together with the equivalence of $V(p \supset q) = 1$ and $V(p \wedge \neg q) = 0$.

But it also can be shown independently by deriving

(1) If some $X_i(M)$ is a two-valued logical falsehood, then M is d-unsatisfiable,

from a fundamental principle of deontic logic which D. Føllesdal and R. Hilpinen state as follows(3):

(E1) If a set of sentences A is consistent and $\{Of_1, Of_2, \ldots, Of_n, Pg\} \subseteq A$, then $\{f_1, f_2, \ldots, f_n, g\}$ is consistent,

together with some quite obvious truths.

Entscheidungen, Freiburg/München, 1973, pp. 61-66.

(2) See op. cit. pp. 64-65.

(3) See: Dagfinn Føllesdal and Risto Hilpinen, Deontic Logic, An Introduction, in: Risto Hilpinen (ed.), Deontic Logic, Introductory and Systematic Readings, Dordrecht, 1971, pp. 1-35 (p. 16).

This can easily be done in the following way:

Let M be $(O(B_1) \wedge O(B_2) \wedge \ldots \wedge O(B_n) \wedge \neg O(C_1) \wedge \neg O(C_2) \wedge \ldots \wedge \neg O(C_m) \wedge D_1 \wedge D_2 \wedge \ldots \wedge D_l)$ with $n \geqslant 0$, $m \geqslant 0$ and $l \geqslant 0$, where $D_1, D_2, \ldots, D_l$ are the non-deontic conjuncts of M.

Let A be the set $\{O(B_1), O(B_2), \ldots, O(B_n), \neg O(C_1), \neg O(C_2), \ldots, \neg O(C_m), D_1, D_2, \ldots, D_l\}$.

Let the conjunctions of the form $B_1 \wedge B_2 \wedge \ldots \wedge B_n \wedge \neg C_i$ be the different $X_i(M)$ $(i = 0, 1, 2, \ldots, m)$.
It will be granted at once that:

(2) If some $X_i(M)$ is a two-valued logical falsehood, then $\{B_1, B_2, \ldots, B_n, \neg C_i\}$ is inconsistent,

and that:

(3) If A is inconsistent, then M is d-unsatisfiable.

From (E1) we get (by substituting $f_1/B_1$, $f_2/B_2$, ..., $f_n/B_n$, $g/\neg C_i$, then by replacing $P(\neg C_i)$ by $\neg O(C_i)$, according to the definition $P(A) := \neg O(\neg A)$ and Double Negation, and finally by Commutation, Exportation and Transposition):

(4) If $\{O(B_1), O(B_2), \ldots, O(B_n), \neg O(C_i)\} \subseteq A$, then if $\{B_1, B_2, \ldots, B_n, \neg C_i\}$ is inconsistent, then A is inconsistent.

Now (by simple inspection):

(5) $\{O(B_1), O(B_2), \ldots, O(B_n), \neg O(C_i)\} \subseteq A$

Therefore (from (4) and (5) by detachment):

(6) If $\{B_1, B_2, \ldots, B_n, \neg C_i\}$ is inconsistent, then A is inconsistent,

and from (2), (6), and (3) (by repeated use of Hypothetical Syllogism):

(1) If some $X_i(M)$ is a two-valued logical falsehood, then M is d-unsatisfiable. (Q E D)

As will by now be obvious, the applicability of the procedure is restored by criterion $C_2'$, for the latter works even where $C_2$ fails, namely in those cases where $n = 0$ or $m = 0$ (though with $n = 0$ the m X(M) will all be single-membered).

We may conclude by noting that for some M, such as the second M of $\Phi(\neg S_1')$, one can reach the decision without actually constructing and testing any $X_i(M)$. In fact, as soon as M is of the form $\ldots \wedge O(A) \wedge \ldots \wedge O(\neg A) \wedge \ldots$, i.e. contains what might be called an "open" deontic contradiction, one can at once be sure that it is d-unsatisfiable, for the corresponding $X_i(M)$ would all contain as conjuncts both A and $\neg A$, i.e. they would all be two-valued logical falsehoods.

*Centre Universitaire de Luxembourg* Jules SPELLER
162A, avenue de la Faïencerie
L-1511 Luxembourg