TRANSITIVITY AND CONDITIONALS

Richard SHARVY

A. J. Dale attempted to show that two claims by Strawson (and others) about hypothetical statements conflict. The claims in question were (i) that a hypothetical is correct only if there is some relevance or connection between antecedent and consequent, and (ii) that 'if *p* then *q*', and 'if *q* then *r*' entail 'if *p* then *r*': «transitivity» (1).

Dale gives two counterexamples. I agree that Dale's counterexamples succeed, and wish to offer an explanation of why they work. I will suggest that there are at least two sorts of conditionals, and that «mixed transitivity» is actually what fails.

Dale's examples are essentially:

(A)
(1) If I knock this typewriter off the desk, then it will fall.
(2) If it falls, then it is heavier than air.
∴ (3) If I knock this typewriter off the desk, then it is heavier than air.

(B)
(1) If Jones passes his maths degree, then he knows that $2 + 2 = 4$.
(2) If Jones knows that $2 + 2 = 4$, then $2 + 2 = 4$.
∴ (3) If Jones passes his maths degree, then $2 + 2 = 4$.

In the premisses of each example, the antecedent and consequent have a connection of relevance, which is absent in the conclusion.

I do not dispute that Dale has shown what he claims (although John Bryant has raised some weak quibbles with Dale's

(1) It's not quite proper to call this property of the conditional «transitivity», since transitivity is a property attributable to relations, and 'if *p* then *q*' is a schema for a *connective*, with '*p*' and '*q*' as schematic letters whose substituends are sentences.

examples). I would like to suggest an explanation.

Consider a number of pairs of conditionals of the forms 'if *p* then *q*' and '*p* only if *q*':

(IS) If I push this button, then that light flashes
(1N) I push this buttom only if that light flashes
(2S) If the king says something, then it is true
(2N) The king says something only if it's true
(3S) If it falls, then it's heavier than air
(3N) It falls only if it's heavier than air
(4S) If John knows that $2 + 2 = 4$, then $2 + 2 = 4$,
(4N) John knows that $2 + 2 = 4$ only if $2 + 2 = 4$.

Notice that within each pair, each sentence has the same antecedent and same consequent, and would be symbolized as the same material conditional. Yet there is a definite difference in sense within each pair.

And the difference is systematic. The 'if...then' sentences are forward-looking in temporal or logical priority, and the 'only if' sentences are backward-looking. (1S) suggests that the button-pushing occurs before the light's flashing; that the button-pushing is a condition for the light's flashing. (1N) suggests that the light's flashing occurs before the button-pushing — that I sit and wait for flashes, and don't push except when one occurs; that from my button-pushing we could infer that the light *has* flashed, not that it *will* flash; that the light's flashing is a condition for the button-pushing.

I have labelled the pairs 'N' and 'S' to indicate what I feel is a regularity:

> In 'if *p*, then *q*' *p* is expressed as a sufficient condition for *q*

and

> In '*p* only if *q*' q is expressed as a necessary condition for *p*.

David Sanford has discussed many differences between A's being a sufficient condition for B and B's being a necessary condition for A, and finally rejects any equivalence between these (p. 197 ff). I shall not repeat Sanford's reasons here. Let me simply recommend his paper and go on.

S (2L) suggests that the king's saying something somehow makes it true; (2N) suggests that the king is very careful about what he says.

(3S) (which is (A)(2) above) suggests that the typewriter's falling makes it heavier than air; (3N) suggests that its being heavier than air permits it to fall.

Consider (4S) (which is (B)(2) above) and (4N). I have had a roomful of beginning philosophy students balk at «if S knows p, then p is true», while being quite willing to accept «S knows p only if p is true». We might even consider calling statements like (4S) false, on the grounds that a person's knowing something is not a condition for its truth; rather, something's truth is a condition for someone's knowing it.

Given the systematic differences, there is good reason to think that we should try to develop a logic of conditionals with *two* conditional connectives:

'$p \rightarrow q$' for 'if p then q'

and

'$p \succ\!\!- q$' for 'p only if q'.

We could then have principles of unmixed transitivity:

from '$p \rightarrow q$ & $q \rightarrow r$' infer '$p \rightarrow r$'

and

from '$p \succ\!\!- q$ & $q \succ\!\!- r$' infer '$p \succ\!\!- r$'

At the same time, our system would reject inferences of mixed transitivity (of which there are six) such as:

from '$p \rightarrow q$ & $q \succ\!\!- r$' infer '$p \rightarrow r$'.

The same considerations which led us to distinguish 'if p then q' from 'p only if q' suggest some modifications of the principle of *contraposition.* (1N) seems equivalent to:

(1NC) If that light doesn't flash, then I don't push the button.

but not to:

That light fails to flash only if I don't push this button.

This suggests that '$p \succ q$' is equivalent to '$\sim q \rightarrow \sim p$', but not to '$\sim q \succ \sim p$'. Let us then recommend a principle of *dual contraposition:*

(DC) '$\mathrm{p} \succ \mathrm{q}$' just if '$\sim \mathrm{q} \; \sim \mathrm{p}$'.

This will imply (via easy substitutions and double negation) its twin:

'$\mathrm{p} \rightarrow \mathrm{q}$' just if '$\sim \mathrm{q} \succ \sim \mathrm{p}$'.

Given this principle of dual contraposition, the reader can easily verify that any principle of mixed transitivity implies the other five mixed ones.

Now consider Dale's examples. I have agreed that they establish his point. But I would like to point out that they can both be viewed as of the form:

$$\begin{array}{ll} & p \rightarrow q \\ & q \succ r \\ \therefore & p \rightarrow r, \end{array}$$

because the second premiss in each would really be expressed better using the necessary conditional 'only if' rather than the sufficient conditional 'if... then'. Hence Dale's examples would not be counter to a logic with two conditionals. We might hesitate at this inference:

If Jones shouts, then I push this button.
I push this button only if that light flashes.
∴ If Jones shouts, then that light flashes.

But if the second premiss is changed to 'If I push this button then that light flashes', the argument seems fine. Dale's examples then actually show the value of developing a logic which makes a distinction between the sufficient conditional 'if... then' and the necessary conditional 'only if'.

Philosophy Department
University of Auckland
New Zealand

Richard SHARVY

REFERENCES

BRYANT, John, «Dale on the Transitivity of 'if-then'», *Logique et Analyse* 20 (1977): 348-50. 160.5/L83.

DALE, A. J., «The Transitivity of 'if, then'», *Logique et Analyse* 15 (1972): 439-41. 160.5/L83.

SANFORD, David «The Direction of Causation and the Direction of Conditionship», *Journal of Philosophy* 73 (1976): 193-207. 105/J86p.

SHARVY, Richard, «Conditionals», forthcoming.