

ONE DOGMA OF ANALYTICISM

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ABSTRACT

According to one view on analyticity in formal languages, a definition of ‘analytic’ can be given by semantic notions alone. In this contribution we are going to show that a purely semantic conception of analyticity is inadequate. To do so, we provide a method for transforming theories with a synthetic empirical basis into logical equivalent theories with an analytic “empirical” basis. We draw the conclusion that a definition of analyticity is adequate only if it is a pragmatic one.

Keywords: empiricism, analytic-synthetic distinction, definability, logicity

1. Introduction

Logical positivism was the position in philosophy of science that claimed (i) the consequences of empirical theories can be distinguished into analytic and synthetic consequences, that (ii) the synthetic ones are reducible to a set of observational statements and that (iii) the reduction consists of verification in a strong sense. Since the programme of verification in a strong sense was recognized very early as unrealistic, the latter claim was weakened and *logical empiricism* arose, consisting mainly of (i) and (ii). It was Willard Van Orman Quine who most prominently criticised these “two dogmas”, where we may call the first one the ‘dogma of analyticism’ and the second ‘dogma of reductionism’. In this paper we are going to concentrate on the first dogma. Whereas Quine’s first criticism is regarded as a problem of defining the notion of analyticity for natural languages and its correct application within artificial languages, we are concerned here with a critique of the notion of analyticity for artificial languages only. Quine himself excludes in his circularity argument against a definition of ‘analyticity’ via ‘synonymy’ by ‘definability’ purely formal considerations of definitions in artificial languages. He claims that “in formal and informal work alike, thus, we find that definition – except in the extreme case of the explicitly conventional introduction of new notations – hinges on prior relations of synonymy” (Quine 1951, p. 27).

In particular, we are going to criticise the claim that the notion of analyticity for artificial languages can be defined in syntactic and semantic terms

alone. The semantic view on analyticity can be found, e.g., already in Gottlob Frege's writings, when he claims:

“[The task of justifying is] that of finding the proof of the proposition, and of following it up right back to the primitive truths. If, in carrying out this process, we come only on general logical laws and on definitions, then the truth is an analytic one [...] If, however, it is impossible to give the proof without making use of truths which are not of a general logical nature, but belong to the sphere of some special science, then the proposition is a synthetic one.” (cf. Frege 1960, §3)

Regarding the semantic status of the adequacy of definitions he states: “This much everyone would allow, that any enquiry into the cogency of a proof or the justification of a definition must be a matter of logic” (cf. Frege 1960, p. xxi). Contrary to Frege, Kazimierz Ajdukiewicz, e.g., explicitly considered and criticised such a semantic characterisation:

“We shall say that a sentence *S* is analytic in the language *L* in the semantic sense if it is a postulate of *L* or a logical consequence of the postulates of *L*. The concept of a postulate used in this definition is semantic, i.e. its definition would be in terms of terminological conventions which have a semantic character. The concept of an analytic sentence so defined we shall call the semantic concept of an analytic sentence.” (Ajdukiewicz 1978, p. 256)

Ajdukiewicz criticises on this conception of analyticity, that just referring to conventions does not suffice for guaranteeing existential assumptions needed in analytical propositions (cf. Ajdukiewicz 1978, p. 258). As we will see below, our explication of this conception puts some constraints on conventions such that certain existential assumptions have to be guaranteed by setting up a convention. Our aim is to criticise this conception by showing that also such constraints on conventions are not sufficient for ruling out inadequate results.

Our ascription of analyticity as a semantic notion to Frege is in accordance with that of Paul Boghossian who even labels *a* semantic conception of analyticity as ‘Frege-analyticity’. However, strictly speaking Boghossian ascribes to Frege just the characterisation of ‘analytical truth’ as “transformable into a logical truth by the substitution of synonyms for synonyms” (cf. Boghossian 1996, p. 366). It is clear that Frege sometimes talks about analyticity in this sense. But as can be seen by the citation above, he also had this broader notion of analyticity in mind and it is this we are concentrating on – and wherein one can skip the whole debate about an adequate characterisation of ‘synonymy’ for both, natural as well as artificial languages. The label ‘Carnap-analyticity’ fits better for the conception we are criticising in the sense that analytical truths might be characterised as “*implicit definitions* of their ingredient terms” and inasmuch such a position can arguably be ascribed to Carnap (cf. Boghossian 1996, p. 368 and the discussion in III).

In our investigation we will first present the semantic characterisation of analyticity (section 2) and provide an argument in favour of this characterisation by hinting at some results on unifying the notion of analyticity (section 3). Then we will show that such a characterisation of analyticity fails since empirical statements can be constructed analytically therein (section 4). Finally we will give a short summary of the main line of our argumentation (section 5).

The language we are dealing with is a minimal artificial elementary language \mathcal{L} with—for all $n, m \in \mathbb{N}$ —: $\neg, \&, \forall, P_2^2, (,)$, x_n as logical and c_n, P_n^m (for not $n = m = 2$) as descriptive vocabulary. Note that P_2^2 serves as the logical identity symbol here. Terms t are all x_n, c_n . Formulæ of \mathcal{L} are all $P_n^m(t_1, \dots, t_m)$ (for at least $o_1, \dots, o_m \in \mathbb{N}$ such that t_1 equals c_{o_1} or x_{o_1}, \dots, t_m equals c_{o_m} or x_{o_m}). Furthermore, if φ, ψ are formulæ of \mathcal{L} , then $\neg\varphi, (\varphi\&\psi)$, and $\forall x_n\varphi$ are also. The other common connectives $\vee, \rightarrow, \leftrightarrow$ are assumed to be meta-linguistic abbreviations: $\lceil(\varphi \vee \psi)\rceil$ for $\lceil\neg(\neg\varphi\&\neg\psi)\rceil$, $\lceil(\varphi \rightarrow \psi)\rceil$ for $\lceil\neg(\varphi\&\neg\psi)\rceil$, and $\lceil(\varphi \leftrightarrow \psi)\rceil$ for $\lceil\neg(\varphi\&\neg\psi)\&\neg(\psi\&\neg\varphi)\rceil$. Also $\lceil\exists x_n\rceil$ is used as a meta-linguistic abbreviation for $\lceil\neg\forall x_n\neg\rceil$. We will use further meta-linguistic abbreviations within our examples (e.g. \in for some P_m^n , not $n = 2 = m$, and also function-symbols for their complex corresponding relational-formulation).

2. Analyticity and Definability

As is well known, the analytic-synthetic distinction traces back to Immanuel Kant and was first formally spelled out by Frege. Without going into much historical detail, analyticity is according to the proposal of Frege and the discussion initiated later on by Quine, applied to formulæ of an artificial language that are derivable by the laws of the logic of the language and definitional conventions of the language alone. The expression ‘analytic falsity’ may then be defined as contradicting an analytic truth. And ‘synthetic truth’ as being true, but not analytically true; ‘synthetic falsity’ as being false, but not analytically false. The latter two categories make up *in toto* the non-definitional contingent truths of the language under consideration:

Definition 1.

- 1.1 φ of \mathcal{L} is analytically true iff there is a set Ψ of definitions of expressions in \mathcal{L} such that $\Psi \models \varphi$.
- 1.2 φ of \mathcal{L} is analytically false iff there is a ψ of \mathcal{L} such that ψ is analytically true in \mathcal{L} and $\models \varphi \leftrightarrow \neg\psi$.
- 1.3 φ of \mathcal{L} is synthetically true iff φ is true, but not analytically true in \mathcal{L} (the former part expresses the condition that for *the* standard interpretation \mathcal{I} of \mathcal{L} it holds that $\models_{\mathcal{I}} \varphi$).

- 1.4 φ of \mathcal{L} is synthetically false iff φ is false, but not analytically false in \mathcal{L} (the former part expresses the condition that for *the* standard interpretation \mathcal{I} of \mathcal{L} it holds that $\models_{\mathcal{I}} \neg\varphi$).

This distinction can be quite similarly extended to a syntactic consideration of an artificial language: instead of ‘analytic truth’ one may speak similarly and for the elementary case equivalently of ‘definitional derivability’. Instead of ‘analytic falsity’ one may speak of ‘definitional contradictability’. Only the category of synthetically true or false statements cannot be distinguished in a pure syntactic way for obvious reasons (not always *the* standard interpretation \mathcal{I} of all formulæ of a language can be “codified” by syntactic means alone).

One can also expand the terminology from the level of formulæ to the level of theories—we do so by disregarding truth and falsity of theories and defining a theory (logically closed set of some formulæ) to be analytic iff all its consequences are analytically true or some are analytically false; and to be synthetic otherwise.

We clearly see now that the notion of analyticity for artificial languages is defined via logicality and definability—that is what we called the ‘dogma of analyticism’ in the introductory section. Let us first consider the notion of definability. Definability is a property of constants of theories with respect to some antecedent theories. So, e.g., ‘inclusion’ \subseteq and ‘power set’ \wp are definable with respect to naïve set theory (i.e. extensionality plus naïve comprehension) in naïve set theory plus the standard definitions of \subseteq and \wp :

$$\text{NST1 } \forall x_1 \forall x_2 (\forall x_3 (x_3 \in x_1 \leftrightarrow x_3 \in x_2) \rightarrow P_2^2(x_1, x_2)) \quad (\text{Extensionality})$$

$$\text{NST2 } \exists x_1 \forall x_2 (x_2 \in x_1 \leftrightarrow \varphi[x_2]) \quad (\text{Naïve Comprehension})$$

$$\text{NST3 } \forall x_1 \forall x_2 (x_1 \subseteq x_2 \leftrightarrow \forall x_3 (x_3 \in x_1 \rightarrow x_3 \in x_2)) \quad (\text{Definition of } \subseteq)$$

$$\text{NST4 } \forall x_1 (P_2^2(\wp(x_2), x_1) \leftrightarrow \forall x_3 (x_3 \in x_1 \leftrightarrow x_3 \subseteq x_2)) \quad (\text{Definition of } \wp)$$

Explicit definitions like the one given above satisfy specific conventional and formal constraints. For our argument we only need to consider so-called “equivalence definitions”:

Definition 2. φ is an explicit equivalence definition of a descriptive symbol s in a theory T' with respect to a theory T iff the following conditions hold:

- 2.1 $T' \models T \cup \{\varphi\}$, i.e.: T' is an extension of T by φ
- 2.2 φ is of the form $P_m^n(t_1, \dots, t_n) \leftrightarrow \psi$, for some $n, m \in \mathbb{N}$ and ψ of T
- 2.3 t_1, \dots, t_n are pairwise disjoint and only individual variables x_1, \dots or s and ψ contains exactly the same free variables as those in t_1, \dots, t_n
- 2.4 s occurs in no axioms of T and either (i) $s = P_m^n$ or (ii) $s = t_1$

2.5 In case of (ii), $n = 2 = m$ and $T \models \exists x(\psi[t_2/x] \& \forall y(\psi[t_2/y] \rightarrow P_2^2(x, y)))$, for some x, y not occurring in ψ

$P_m^n(t_1, \dots, t_n)$ is called ‘the definiendum of φ ’ and ψ is called ‘the definiens of φ ’. E.g., NST3 is an explicit equivalence definition of \subseteq in $\{\text{NST1}, \text{NST2}, \text{NST3}\}$ with respect to $\{\text{NST1}, \text{NST2}\}$ and NST4 is an explicit equivalence definition of \wp in $\{\text{NST1}, \text{NST2}, \text{NST3}, \text{NST4}\}$ with respect to $\{\text{NST1}, \text{NST2}, \text{NST3}\}$ because existence of a power set of a set follows by NST2 and uniqueness of *the* power set of a set follows by NST1.

Explicit equivalence definitions have two very important features: First, they are conservative in the sense that no new consequences about the domain of discourse are introduced. Second, they allow for complete eliminability of the new symbol. Here are the details for conservativity:

Definition 3. s is conservatively introduced into a theory T' with respect to T iff

3.1 $T' \models T$, and:

3.2 No axiom of T contains s and T' contains at least one axiom with s , and:

3.3 for all formulæ φ not containing s it holds: If $T' \models \varphi$, then $T \models \varphi$.

So, conservativity guarantees that no new claims formulated in the former vocabulary of a theory are derivable by introducing a new expression. Eliminability is about statements that use the newly introduced expression. Here are the details:

Definition 4. s is eliminable in a theory T' iff

4.1 for all formulæ φ containing s there is a formula ψ not containing s such that $T' \models (\varphi \leftrightarrow \psi)$.

So, eliminability guarantees that every claim with the eliminable expression can be stated equivalently with a claim not containing the expression. Both criteria are independent of each other and epistemically seen both are of equal importance: Eliminability allows one to translate claims with new vocabulary equivalently into claims using only the old vocabulary, which is already well understood. And conservativity guarantees that the already well-known meaning of the old expressions stays unchanged (‘meaning’, ‘knowing’, and ‘understanding’ is to be understood here extensionally: to be about the construction of models). Note that a unary relation/property of eliminability and conservativity as defined, e.g., in Kleinknecht (1981) by just existentially quantifying over T' makes eliminability and conservativity depended of each other—such a property of eliminability guarantees also such a property of conservativity; but both notions are not taking into account and fixing the underlying theory a symbol s is introduced to and

by this the dependence just states that if s is introduced into some theory T' in an eliminable way, then s is also conservatively introduced into some theory T'' ; for a further critique of these unary notions of eliminability and conservativity cf. Peppinghaus and Schirn (1983). Independently of the exact relation between conservativity and eliminability it can be shown that the two criteria, i.e. the conditions in definitions 3 and 4, are satisfied in introducing new descriptive symbols into a theory exactly if one also satisfies the rules for explicit equivalence definitions of the form given above (cf. definition 2):

Observation 5. *s is conservatively and eliminable introduced into T' with respect to T iff there is an explicit equivalence definition φ of s in T' with respect to T .*

Since the motivation for the formal characterisation of definitions hinges exactly on these two criteria, this may be regarded as one main result of the formal theory of definition. For this reason we are also going to provide a...

Sketch of the proof. For simplicity reasons take T' to be the closure of $T \cup \{\varphi\}$ —it is easy to see that conservativity and eliminability are preserved under logical equivalence. (\Leftarrow): Induction over the complexity of formulæ of T' guarantees for the basis case of atomic formulæ eliminability by help of an explicit equivalence definition directly (all relevant sub-cases are covered by the variable condition 2.3). Since T' -equivalences can be substituted in all complex cases ($\neg\varphi$, $(\varphi \& \chi)$, etc.), also the induction step is guaranteed to preserve eliminability. Regarding conservativity, the key idea is that every formula in a T' -proof containing the new symbol can be substituted (via eliminability) T' -equivalently by a formula not containing the new symbol. Since the only axiom used in such a proof that is not already an axiom of T is the explicit equivalence definition itself, the previously described substitutions are logically valid formulas (for this reason condition 2.5 has to be satisfied, otherwise the substitution would not turn out to be logically valid) and can be skipped. By this a T' -proof can be transformed into a T -proof.

(\Rightarrow): With conservativity one gets $T' \models T$. By eliminability one gets $T' \models P_m^n(t_1, \dots, t_n) \leftrightarrow \psi$, where $n, m \in \mathbb{N}$ and conditions 2.2–2.4 are satisfied for ψ . Let φ be $P_m^n(t_1, \dots, t_n) \leftrightarrow \psi$. So $T' \models T \cup \{\varphi\}$. With conservativity one also gets 2.5 for φ . So only $T \cup \{\varphi\} \models T'$ has to be shown. Assume $T' \models \chi$. If the eliminable symbol does not occur in χ , then by conservativity we also get $T \models \chi$. If it occurs in χ , then we have to distinguish between two cases: (i) for the, with respect to T new expression, s in φ it holds that $s = P_m^n$. It is clear that $\varphi \models \chi \leftrightarrow \chi[P_m^n(t_1, \dots, t_n)/\psi]$. But then, by conservativity we get $T' \models \chi[P_m^n(t_1, \dots, t_n)/\psi]$ iff $T \models \chi[P_m^n(t_1, \dots, t_n)/\psi]$ and since $T' \models \chi$ we get $T \cup \{\varphi\} \models \chi$. The other case is (ii) $s = t_1$. Then it is clear (by substitutivity of identicals) that $\varphi \models \psi \rightarrow (\chi \leftrightarrow \chi[t_1/t_2])$. Since by conservativity $T' \models \psi$ and $T' \models \chi[t_1/t_2]$ iff $T \models \chi[t_1/t_2]$ and since $T' \models \chi$ we get again

$T \cup \{\varphi\} \models \chi$. Hence 2.1 holds for φ . And hence φ is an explicit equivalence definition of the eliminable symbol of T' in T' with respect to T .

Just to mention a historical detail: These criteria are already implicitly discussed by Blaise Pascal (cf., e.g., Pascal 2000a, pp.112f). According to Patrick Suppes, Stanislaw Leśniewski was the first who discussed definitions with respect to the two criteria of conservativity and eliminability explicitly (cf. the footnote in Suppes 1957, p.153). Although it is clear that Leśniewski formulated syntactic rules for definitions (cf. Lesniewski 1967), it is only a myth that he made the criteria explicit (cf. Urbaniak and Severi Hämäri 2012 and Hodges 2008, p.105).

Back to our application of these results on definability: It is also generally assumed that the notion of analyticity allows for partial eliminability of expressions—so, e.g., in mathematics quite often only partial characterisations of expressions are given and accepted (think, e.g., of the division operation). In such a case, explicit equivalence definitions are weakened to conditional ones:

Definition 6. φ is a partial or conditional equivalence definition of a descriptive symbol s in a theory T' with respect to a theory T iff conditions 2.1–2.5 hold, with the modification:

ad 2.2 The form of φ is $\chi \rightarrow (P_m^n(t_1, \dots, t_n) \leftrightarrow \psi)$ with χ of T

ad 2.5 the existence and uniqueness condition is conditioned on χ : $T \models \chi \rightarrow \exists x(\psi[t_2/x] \& \forall y(\psi[t_2/y] \rightarrow P_2^2(x, y)))$, for some x, y not occurring in ψ

Since partial equivalence definitions are logically weaker than explicit or unconditioned ones, they also guarantee conservativity. However, they allow for eliminability only conditioned on χ : If s is introduced into T' with respect to T by a partial definition (conditioned on χ), then for all formulæ φ containing s there is a formula ψ not containing s such that $T' \models \chi \rightarrow (\varphi \leftrightarrow \psi)$. So, assuming χ , s is eliminable in such a theory T' , but if χ does not hold, it is not.

In the following we will assume that analyticism, i.e. the dogma of analyticism, also allows for such weak definability by partial equivalence definitions. Furthermore, definitions can be concatenated in order to introduce more and more complex notions into theories. One can easily expand the notion of a partial or explicit equivalence definition above to such chains:

Definition 7. s is introduced into T' with respect to T by help of partial/explicit equivalence definitions iff there are T_1, \dots, T_n and $\varphi_2, \dots, \varphi_n$ and pairwise disjoint s_2, \dots, s_n , $n \in \mathbb{N}$, such that:

7.1 $T = T_1$, $T' = T_n$, and $s = s_n$

7.2 φ_i is a partial/explicit equivalence definition of s_i in T_i with respect to T_{i-1} , $1 < i \leq n$

We can now state directly that \wp is introduced into $\{\text{NST1}, \text{NST2}, \text{NST3}, \text{NST4}\}$ with respect to naïve set theory $\{\text{NST1}, \text{NST2}\}$ by help of explicit equivalence definitions without mentioning the intermediate step NST3. Such definitional chains preserve also conservativity and partial/full eliminability, since eliminability passes on to logically stronger theories and conservativity passes on to logically weaker theories.

Finally, the last ingredient we need in our main argument, and which seems even more acceptable than the introduction of expressions by means of partial equivalence definitions, is the multiple introduction of an expression by means of partial equivalence definitions. The idea here is the following: The multiple explicit equivalence definition of, e.g., P_m^n does no harm to eliminability, but may be harmful for conservativity. So, e.g., by characterising P_1^1 as $P_1^1(x_1) \leftrightarrow \psi[x_1]$ and $P_1^1(x_1) \leftrightarrow \neg\psi[x_1]$ one reaches the worst case of introducing inconsistency into a theory. But what is harmful for explicit definitions might be useful for partial definitions inasmuch as one might preserve conservativity by increasing eliminability. So, e.g., multiple partial equivalence definition of P_1^1 by $\chi_1 \rightarrow (P_1^1(x_1) \leftrightarrow \psi_1[x_1])$ and $\chi_2 \rightarrow (P_1^1(x_2) \leftrightarrow \psi_2[x_2])$ with respect to an antecedent theory T does no harm to conservativity with respect to T and even increases eliminability of P_1^1 in the multiple partial definitional extension iff $T \models \chi_1 \& \chi_2 \rightarrow (\psi_1[x_1] \leftrightarrow \psi_2[x_2])$ and $\chi_1 \vee \chi_2$ is logically weaker than both, χ_1 as well as χ_2 . The former guarantees that the possibly creative part of the extension is already a consequence of the antecedent theory T . The latter guarantees that eliminability increases since then for all formulæ φ_1 of the extension there is a formula φ_2 of the antecedent theory T such that $(\chi_1 \vee \chi_2) \rightarrow (\varphi_1 \leftrightarrow \varphi_2)$ is a consequence of the extension. So we can say that:

Definition 8. s is introduced into T' with respect to T by help of possibly multiple partial/explicit equivalence definitions iff there are T_1, \dots, T_n and $\varphi_2, \dots, \varphi_n$ and (not necessarily pairwise disjoint) s_2, \dots, s_n , $n \in \mathbb{N}$, such that:

- 8.1 As condition 7.1 above and
- 8.2 As condition 7.2 above with the specification:
- 8.3 For all $1 < i < j \leq n$: If $s_{i-1} = s_j$, then φ_j is a *partial* equivalence definition of s_j in T_j with respect to T_{i-1} ; furthermore, let χ_i be the antecedence of φ_i and χ_j that of φ_j ; let ψ_i be the definiens of φ_i and ψ_j that of φ_j ; then it holds:
 - $T_{i-1} \models \chi_i \& \chi_j \rightarrow (\psi_i \leftrightarrow \psi_j)$
 - $\chi_i \vee \chi_j$ is logically weaker than both, χ_i as well as χ_j

The idea of definition 8 is that multiple partial equivalence definitions can be brought into a successive order and that the non-creativity constraint (8.3) is satisfied pairwise by the multiple partial equivalence definitions.

We are now able to explicitly state the dogma of analyticism under consideration:

Analyticism. Analyticity can be adequately characterised semantically as follows: A theory T is analytic iff either T is inconsistent or there is an s , T' , T'' such that $T'' \models T$, T' is purely logical (i.e. the closure of \emptyset) and s is introduced into T'' with respect to T' by help of possibly multiple partial/explicit equivalence definitions.

In other words, a theory T is analytic iff it is inconsistent or all its consequences can be derived by logical and definitional means alone. E.g., it was an effort of Frege's *logicistic approach* to reconstruct mathematical notions by logical (including, e.g., class abstraction) and definitional (including, e.g., recursive) means alone. And in fact, it turned out that Frege's reconstruction was analytic; the only problem was that it was not analytic in the bright way of being at least not proven to be inconsistent.

So much for definability in the context of analyticity. But what about logicity? We will shortly explore this notion in the next section.

3. Reducibility of Logicity to Definability

It is well known that our definitions of logicity depend heavily on our distinction of *logical* from *descriptive* symbols within a language under consideration. In the literature about this topic, three main criteria for such a distinction are discussed. It was Quine who prominently proposed a criterion for distinguishing the logical from the descriptive symbols of a language by their so-called feature of being *substitutable salva congruitate* (cf. Quine 1986, chpt.2). The idea behind this criterion is that the set of logical symbols of a language is the smallest set of symbols that can be substituted pairwise in such a way that the substitution result is of the same category (formula/non-formula) as the original expression. Of course, the adequacy of this definition hinges on a quite arbitrary interpretation of what counts as 'smallest' and also on our assumptions about categories. Why, e.g., form $(\varphi \& \psi)$ and $(\varphi P_1^1 \psi)$ expressions of different categories, whereas $(\varphi \& \psi)$ and $(\varphi \vee \psi)$ form expressions of the same category, namely formulæ?

For this reason logicians came up with two further different criteria of separating logical from descriptive symbols of a language. Very promising, at least at first glance, is the so-called permutation-criterion of Alfred Tarski—later on spelled out as criterion of invariance under isomorphisms (cf. Sher 1991, chpt.3; Sher recently also expanded the interpretation of the invariance under isomorphism criterion to the grounding debate and its ontological implications: cf. Sher 2013, pp.174–182). According to Tarski, logical operations on a domain are those operations on the domain whose

results guarantee invariance under permutation within the domain. As a feature of this criterion one may note that it is a nice explication of ‘topic-neutrality’ in a way that those operations which are absolutely neutral regarding any topic or any elements of a domain, are logical operations. The main problem with this criterion is its incapability of dealing with different sizes of domains—logical operations do not guarantee invariance under domain-size-transformation. Other problems or features of the approach are still controversial. So, e.g., Vann McGee showed an adequacy result by demonstrating that only the logical operations (including infinite disjunction and infinite existential quantification) are invariant under permutations (cf. McGee 1996). However, he also gave an example of strange connectives like *unicorn negation* \mathcal{U} —defined as $\lceil \mathcal{U}\varphi \rceil$ iff $\lceil \neg\varphi \rceil$ and there are no unicorns—which, due to its coextensionality with the \neg -operation, counts according to the permutation-criterion also as logical operation. Whatever stance one may take here, it is the following approach on logical symbols that fits best into our line of argumentation for considering analyticity to be just the *conventional* definability.

The third criterion proposed in the literature by, e.g., Nuel Belnap is the so-called *definability-criterion* for separating logical constants (arguably also Carnap held such a view—(cf. Boghossian 1996, section III)): Those constants whose role in reasoning can be explicated in a definitional way alone are assumed to be logical. It is exactly this criterion which indicates how the semantic conception of ‘analyticity’ can be reduced to that of definability. It can be shown that the usual introduction and elimination rules of natural deduction for \neg , $\&$, \forall , and P_2^2 satisfy the condition of conservativity and also a condition of partial eliminability: Starting with just the structural rules of argumentation or logical consequence, i.e. assuming that for all formulæ $\varphi, \varphi_1, \dots, \varphi_n, \psi, \psi_1, \dots, \psi_m, \chi, \rho$:

- Reflexivity: $\varphi: \varphi \vDash \varphi$
- Transitivity: If $\varphi_1, \dots, \varphi_n \vDash \chi$ and $\psi_1, \dots, \psi_m, \chi \vDash \rho$,
then $\varphi_1, \dots, \varphi_n, \psi_1, \dots, \psi_m \vDash \rho$
- Permutation: If $\varphi_1, \dots, \varphi_n, \psi_1, \dots, \psi_m \vDash \chi$,
then $\varphi_1, \dots, \varphi_{n-1}, \psi_1, \varphi_n, \psi_2, \dots, \psi_m \vDash \chi$
- Contraction: If $\varphi_1, \dots, \varphi_n, \varphi_n \vDash \psi$, then $\varphi_1, \dots, \varphi_n \vDash \psi$
- Weakening: If $\varphi_1, \dots, \varphi_n \vDash \psi$, then $\varphi_1, \dots, \varphi_n, \chi \vDash \psi$

One can introduce by the usual rules of natural deduction all logical symbols in a conservative and partial eliminable manner. So, e.g., the introduction rule for conjunction allows for the introduction of $\&$ in the following way: ($\&I$): $\varphi, \psi \vDash \varphi \& \psi$. On the other side, $\&$ can be eliminated by

simplification ($\&E$): $\varphi \& \psi \vDash \varphi$ and $\varphi \& \psi \vDash \psi$ in this specific context. More generally, $\&$ can be eliminated in all contexts by:

- $\varphi \& \psi \vDash \chi$ iff $\varphi, \psi \vDash \chi$
- $\chi \vDash \varphi \& \psi$ iff $\chi \vDash \varphi$ and $\chi \vDash \psi$

$\&$ can be completely eliminated in an inference, since by ($\&E$), transitivity, and permutation one gets rid of it in the consequence, and by ($\&I$), transitivity, and permutation it can be always introduced into the consequence. If it occurs in the premises, then one needs to recursively apply ($\&I$), ($\&E$), transitivity, and permutation. So, e.g., for the basis case where we have only one premise, the premise is $\varphi \& \psi$; and this case is similar to the one above. With two $\&$ -premises: $\varphi_1 \& \psi_1, \varphi_2 \& \psi_2 \vDash \chi$ one first shows $\varphi_1, \psi_1, \varphi_2 \& \psi_2 \vDash \chi$ as in the basis case; then $\varphi_1, \psi_1, \varphi_2, \psi_2 \vDash \chi$ as in the basis case; for further premises just along this line.

One can also introduce negation with the usual introduction-and-elimination rules of reductio; it turns out that such an introduction is a conservative extension of the structural rules above. But it also turns out to be not completely eliminable ($\neg I$): If $\varphi \vDash \psi$ and $\varphi \vDash \neg\psi$, then $\vDash \neg\varphi$, and similar ($\neg E$): If $\neg\varphi \vDash \psi$ and $\neg\varphi \vDash \neg\psi$, then $\vDash \varphi$. The problem here is that one needs to start with a negation-statement in order to introduce \neg . As the following characterization shows, more generally one might introduce and eliminate \neg in the consequence, but one cannot introduce and eliminate it from scratch in the premises (note that this characterisation would also not allow for a characterisation of finite proofs):

- $\neg\varphi \vDash \psi$ iff $\varphi, \neg\psi \vDash \chi$ for all χ
- $\varphi \vDash \neg\psi$ iff $\varphi, \psi \vDash \chi$ for all χ

Similar for the quantifiers: By the ordinary rules ($\forall I$) and ($\forall E$) one can eliminate the quantifiers in case the variable condition is satisfied (x does not occur in ψ , i.e. $\psi[x/y] = \psi$ for some $y \neq x$):

- $\forall x \varphi[x] \vDash \psi$ iff $\varphi[x] \vDash \psi$
- $\psi \vDash \forall x \varphi[x]$ iff $\psi \vDash \varphi[x]$

But the eliminability is of course only partial, since, e.g., in the context $\neg\forall x \varphi[x]$ we cannot easily skip $\forall x$, for the former means that not every x has property φ , whereas the latter ($\neg\varphi[x]$) would mean that every x has not this property.

Also the identity-symbol can be introduced and eliminated only in the contexts provided by the corresponding rules of natural deduction: (P_2^2I) $\vDash P_2^2(x, x)$ and (P_2^2E): $P_2^2(x, y), \varphi[x] \vDash \varphi[x/y]$.

If one allows a weak form of definability for analyticity in the sense that only the conservativity criterion has to be fully satisfied, whereas the eliminability criterion may be only partially satisfied, then the discussion above shows that the non-structural rules of classical logic can be expanded purely conservatively and partially eliminative by introduction-and elimination rules for the logical vocabulary. One may wonder then how to argue for the structural rules, i.e., how to introduce \models ? Here it can be shown that even the structural rules can be introduced from scratch by partial definitions, except for transitivity which needs a circular characterisation and by this does not even allow for partial eliminability (cf. Feldbacher-Escamilla 2015). So, more generally it holds that all classical logical notions \models , \neg , $\&$, \forall, P_2^2 can be introduced from scratch in a way fully satisfying the conservativity criterion and satisfying the eliminability criterion more or less well. In this sense one may claim that analyticity consists mainly in definability. Logicity is then just introduced as a special case of definability.

Note that this does not show, of course, that only the principles of classical logic can be introduced by definitional means alone. A consequence relation \approx could be also introduced in a similar way as above by stating reflexivity, transitivity, permutation, and contraction, but without stating weakening. Such a non-monotonic consequence relation would also satisfy the same constraints as above and a logic based on it would therefore also count as analytical in the sense of being reducible to definability as indicated here.

4. Conventionalism about Empiricality

In the preceding section we argued for the claim that the dogma of analyticism, namely that for artificial languages one can provide an adequate definition of ‘analyticity’ in syntactic and semantic terms only, is supported by the fact that even the notions of logic can be introduced with the help of definitional methods alone. In this section we are going to challenge this dogma by showing that a purely syntactic and semantic definition fails to distinguish analytic from synthetic claims in very important cases.

For this purpose we begin with splitting up the descriptive vocabulary of our language \mathcal{L} into two parts, the set of non-logical odd-numbered predicates $\mathcal{O} = \{P_1^n, P_3^n, \dots\}$ and the set of non-logical and even-numbered predicates $\mathcal{E} = \{P_2^{n \neq 2}, P_4^{n \neq 2}, \dots\}$ (for all $n \in \mathbb{N}$). Below we will interpret \mathcal{E} as the set of empirical or observational predicates. From the set of formulæ of \mathcal{L} we can also pick out a specific set that will be later on interpreted as the set of empirical or observational statements $\mathcal{E}_{\mathcal{F}}$. We do so by the classical empiricist characterisation of the formal structure of such statements: $\{P_m^n(c_{o_1}, \dots, c_{o_n}) : \frac{m}{2}, n \in \mathbb{N} \text{ and at least } o_1, \dots, o_n \in \mathbb{N}\}$ as well as their negations $\{\neg P_m^n(c_{o_1}, \dots, c_{o_n}) : \frac{m}{2}, n \in \mathbb{N} \text{ and at least } o_1, \dots, o_n \in \mathbb{N}\}$ are observational

statements, i.e. in $\mathcal{E}_{\mathcal{F}}$. Furthermore, if φ, ψ are observational statements whose conjunction is logically contingent (i.e.: $\not\models \neg(\varphi \& \psi)$), then $(\varphi \& \psi)$ is an observational statement, i.e. in $\mathcal{E}_{\mathcal{F}}$. Nothing else is to be considered as an observational statement.

With the help of this distinction we can characterize empirical bases of theories: An empirical or observational basis T' of a theory T is a (possibly infinite) axiomatisation of those consequences of T which are observational statements, i.e.:

Definition 9. T' is the empirical or observational basis of T iff T' is the logical closure of $\mathcal{E}_{\mathcal{F}} \cap T$.

Now let us come to our main objection against the dogma of analyticism: It can be shown that according to this dogma empirical or observational bases are analytic:

Observation 10. *The empirical or observational basis T' of a theory T is analytic.*

Proof. In the case that T is inconsistent, the empirical basis T' of T is also inconsistent and by this analytic. If T is consistent, let T' be the empirical or observational basis of consistent T , i.e. the closure of $\mathcal{E}_{\mathcal{F}} \cap T$. Then, since $\mathcal{E}_{\mathcal{F}}$ contains only observation sentences, i.e. atomic formulæ, negations of atomic formulæ, and conjunctions of such formulæ, T' can be also (possibly infinitely) axiomatised by atomic formulæ and negations of atomic formulæ. Let $\mathcal{A}_{P_m^n}$ be the set of sequences of individual constants c_{o_1}, \dots, c_{o_n} such that $T' \models P_m^n(c_{o_1}, \dots, c_{o_n})$ and let $\mathcal{B}_{P_m^n}$ be the set of sequences of individual constants c_{o_1}, \dots, c_{o_n} such that $T' \models \neg P_m^n(c_{o_1}, \dots, c_{o_n})$ ($n, m \in \mathbb{N}$ and at least $o_1, \dots, o_n \in \mathbb{N}$); so $\mathcal{A}_{P_m^n}$ represents the extension of P_m^n , whereas $\mathcal{B}_{P_m^n}$ represents the anti-extension of P_m^n . Since T and by this also T' is consistent $\mathcal{A}_{P_m^n}$ and $\mathcal{B}_{P_m^n}$ are disjoint for every $n, m \in \mathbb{N}$. Now it is easy to see that T' can be constructed by purely definitional means alone: Let T'' be the closure of the following formulæ:

- The trivial, i.e. purely logical, introduction of all individual constants:
 $P_2^2(c_n, c_n), n \in \mathbb{N}$
- The inclusive possibly multiple partial equivalence definition of the P_m^n s: For all $n, m \in \mathbb{N}$ and $o_1, \dots, o_n \in \mathbb{N}$ and $c_{o_1}, \dots, c_{o_n} \in \mathcal{A}_{P_m^n}$:
 $(P_2^2(x_1, c_{o_1}) \& \dots \& P_2^2(x_n, c_{o_n})) \rightarrow$
 $(P_m^n(x_1, \dots, x_n) \leftrightarrow (P_2^2(x_1, c_{o_1}) \& \dots \& P_2^2(x_n, c_{o_n})))$
- The exclusive possibly multiple partial equivalence definition of the P_m^n s: For all $n, m \in \mathbb{N}$ and $o_1, \dots, o_n \in \mathbb{N}$ and $c_{o_1}, \dots, c_{o_n} \in \mathcal{B}_{P_m^n}$:
 $(P_2^2(x_1, c_{o_1}) \& \dots \& P_2^2(x_n, c_{o_n})) \rightarrow$
 $(P_m^n(x_1, \dots, x_n) \leftrightarrow (\neg P_2^2(x_1, c_{o_1}) \& \dots \& \neg P_2^2(x_n, c_{o_n})))$

Then $T'' = T'$ since every such formula is equivalent with the respective atomic formula $P_m^n(c_{o_1}, \dots, c_{o_n})$ or the negation $\neg P_m^n(c_{o_1}, \dots, c_{o_n})$. Furthermore, all the predicates P_m^n are introduced by multiple partial equivalence definitions, since all the conditions of definition 6 are satisfied, and the chaining is vacuously correct (there is no chaining of expressions); also the multiple definitions preserve conservativity since each conjunction of the definitions' antecedences logically implies the respective equivalence of the definienda. E.g.: $P_2^2(x_1 \neq c_1) \& P_2^2(x_2, c_2)$ logically implies $P_2^2(x_1 \neq c_1) \leftrightarrow P_2^2(x_2, c_2)$. \square

Let us illustrate this result by help of an example and an application: Take, e.g., an ornithologist claiming that all swans are white. So, assume her simple theory to be just the closure of:

$$\begin{array}{ll} \forall x(P_2^1(x) \rightarrow P_4^1(x)) & P_2^1(x) \dots x \text{ is a swan.} \\ & P_4^1(x) \dots x \text{ is white.} \end{array}$$

Then she might try hard to falsify or undermine her theory by studying outdoor swan's plumage colour. But she could also switch to semantically equivalently considering her claims to be partial definitions of 'white':

$$\forall x(P_2^1(x) \rightarrow P_4^1(x) \leftrightarrow P_2^1(x))$$

Of course, nobody seriously interested in empirical claims would make the move to such an *indoor ornithology*. Nevertheless, the semantic criterion of distinguishing analytic from synthetic statements alone does not prevent such a move.

For the application let T be an empirical theory (i.e. $\mathcal{E}_{\mathcal{F}} \cap T \neq \emptyset$). Then there are two kinds of tests one can perform: theoretical tests checking, e.g., internal consistency, consistency with other established theories, categorical equivalence with other or parts of other established theories etc.; and empirical tests by help of experiments. One main application of an analytic-synthetic distinction in classical empiricism would be to figure out by help of such a distinction which consequences of T have to be tested theoretically, and which ones empirically. According to the second dogma of empiricism, all synthetic consequences of T should be somehow reducible to *a posteriori* truths of T . So, in the simplest case (i.e. a case of complete reduction), according to the second dogma one would expect that exactly those consequences of T are synthetic, that are in the empirical basis of T . But, as it turns out by distinguishing analytic from synthetic consequences of T according to the semantic conception of analyticity, this is not the case. One can always construct (semantically by applying an expanded version of the so-called *Padoa-method* or syntactically by providing definitional chains as characterised above) T in such a way that the empirical basis of T , i.e. $\mathcal{E}_{\mathcal{F}} \cap T$, turns out to be analytic. For this reason it seems to be necessary to take some further restrictions of the definition of 'analyticity' into account. Especially, one may prevent the definition of

empirical or observational predicates. But then, since a justified distinction of the descriptive vocabulary into theoretical and empirical or observational predicates is not semantic, but pragmatic, also the dogma of analyticism turns out to be incorrect.

5. Conclusion

We saw that logical empiricism is considered to contain two dogmas, namely the dogma of analyticism, stating that there can be drawn a distinction between analytic and synthetic statements, and the dogma of reductionism, stating that all synthetic consequences of a theory can be reduced to a set of observational statements. It is important to note that the first dogma is about such a distinction for artificial languages, and not for natural languages (Quine uncovered several theoretical flaws for a definition of ‘analytic’ in natural languages). Furthermore, the semantic characterisation of ‘analytic’ states that the consequences of a theory are analytic which can be derived by purely logical and definitional means.

As was hinted at in section 3, the semantic characterisation can be even unified to definitional means alone, since logical principles and rules can be justified also by definitional means alone. So analyticity consists, according to the semantic characterisation, of being justifiable by definitional means alone.

However, in the foregoing paragraph we have shown that the empirical or observational basis of a theory, which is usually seen as the synthetic core of a theory, turns out to be analytic according to the semantic characterisation. This is due to the fact that such a basis of a theory can always be constructed by definitional means alone.

For this reason the semantic characterisation, and by this also the dogma of analyticism, is inadequate. There is a need to characterise ‘analyticity’ by help of pragmatic notions, as, e.g., the notion of being observable, being an observational predicate, and being an observational statement: Thus modified a pragmatic characterisation of ‘analytic’ states that a consequence of a theory is analytic iff it can be derived by logical and definitional means alone, where the definitional means explicitly prohibit a conventional characterisation of observational predicates.

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