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NOTE ON THE GEOMETRY OF SOLIDS

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In memory of B. Sobocinski

In [1], Tarski showed that 3-dimensional Euclidean Geometry could be constructed on the single primitive notion of sphere. His method is based on the ability to define the notions of point and equidistance in terms of sphere.

In this note we simplify his work by eliminating two auxiliary definitions. This is done by noting that "spheres A and B are externally diametrical to sphere C" can be defined without the notion of "spheres A and B are externally tangent". And also that the notion of internally diametrical can be defined without the notion of internally tangent. We do so in ordinary geometrical language.

We start with primitive name 'sphere' (open solid sphere). All Capital letters represent spheres.

Defn 1. A and B are DISJOINT if they have no sphere in common. Figure 1.

Figure 1

Intuitively, the next definition says that A and B are both disjoint from C and tangent to C at opposite ends of a diameter. But the word "tangent" is not used.

Defn 2. A and B are EXTERNALLY DIAMETRICAL to C if and only if A and B are both DISJOINT from C, and for all X,Y, if X and Y are both DISJOINT from C, A is contained in X, and B is contained in Y, then X is DISJOINT from Y. Figure 2.

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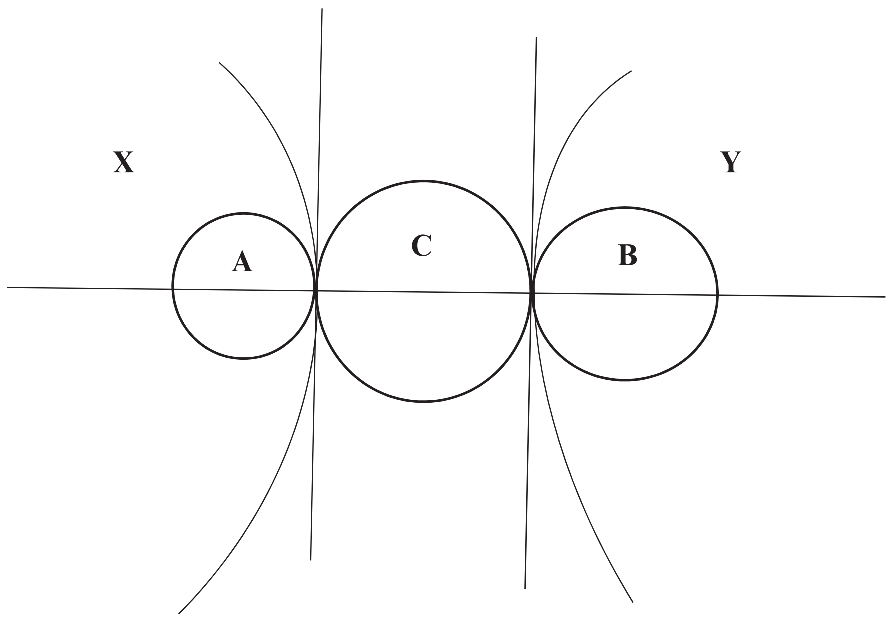

Figure 2

Intuitively, the next definition says that A and B are both contained in C and tangent to C at opposite ends of a diameter. Again, "tangent" is not used.

Defn 3. A and B are INTERNALLY DIAMETRICAL to C if and only if A and B are both contained in C and there exist D,E,F, such that D and E are EXTERNALLY DIAMETRICAL to A, D and F are EXTERNALLY DIAMETRICAL to B, and E and F are EXTERNALLY DIAMETRICAL to C. Figure 3

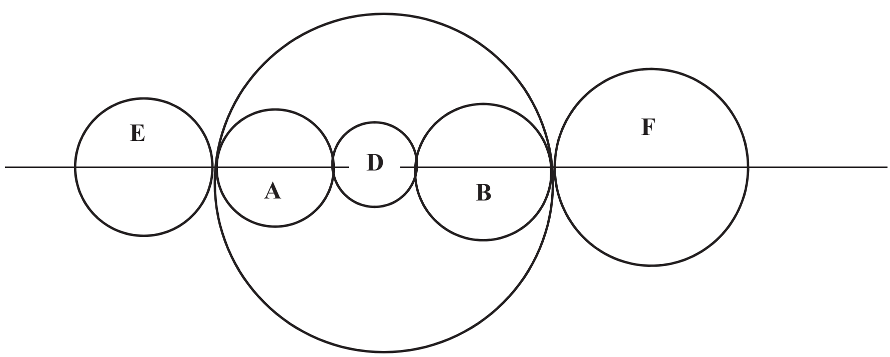

Figure 3

Note that Defn 3 is more restrictive than Tarski's definition of internally diametrical. InTarski's definition spheres A and B are allowed to overlap, whereas in Defn 3, they must be exterior to each other. However, in their applications in the definition of concentric below, the two definitions are of equal strength.

Before defining concentric spheres we note that if A is contained in B and they are not concentric, there is always just one pair C and D for which C and D are EXTERNALLY DIAMETRICAL to A and INTERNALLY DIAMETRICAL to B. Both C and D must lie along the line joining the centers of A and B. Figure 4.

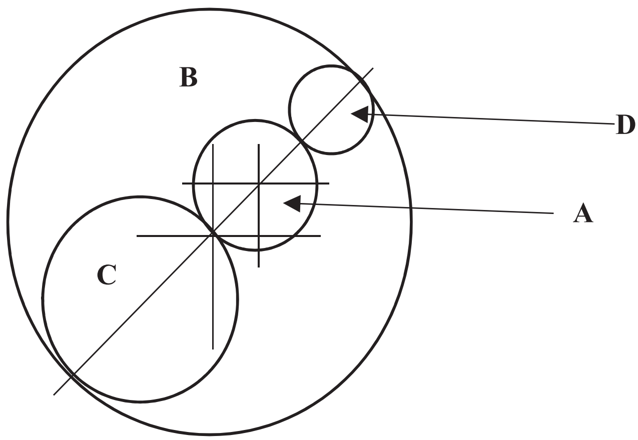

Figure 4

Note that for concentric spheres there are infinitely many such pairs. However, in order to define concentric it suffices to claim two such pairs.

Defn 4. A is concentric to B iff 1) A = B or 2) A is contained in B and there exist two distinct pairs D,E and F,G such that D and E are EXTERNALLY DIAMETRICAL TO A and INTERNALLY DIAMETRICAL TO B, and F and G are EXTERNALLY DIAMETRICAL to A and INTERNALLY DIAMETRICAL TO B or 3) B is contained in A and there exist two distinct pairs D,E and F,G such that D and E are EXTERNALLY DIAMETRICAL TO B and INTERNALLY DIAMETRICAL TO A, and F and G are EXTERNALLY DIAMETRICAL to B and INTERNALLY DIAMETRICAL TO A. Figure 5 is for condition 2).

From this point on, the development follows exactly as in [1].

Tarski's work shows that the notion externally diametrical can be defined in terms of the notion of externally tangent. The following definition shows that the reverse is also true. Thus the two notions are interdefinable.

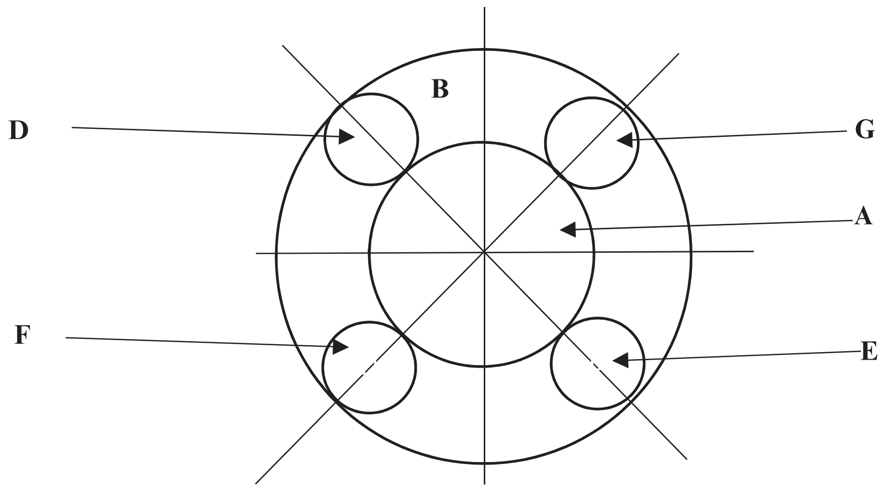

Figure 5

Defn. 5. A and B are externally tangent iff there is C such that C and B are externally diametrical to A or C and A are externally diametrical to B.

Bibliography

[1] Tarski, Alfred. *Foundations of the geometry of solids*, Logic, Semantics, Mathematics. Clarendon Press at Oxford (1956).

[2] Jaskowski, S. *Une modification des definitions fondamentales de la geometrie des M. A Tarski*, Annales Polonici Mathematici, vol. 21 (1948), pp. 298–301.