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MATHEME AND MATHEMATICS

On the Main Concepts of the Philosophy of Alain Badiou

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ABSTRACT

In this paper, I present a critical discussion of mathematical arguments employed in the philosophy of event of Alain Badiou. On the basis of "Being and Event" as well as his other writings, I analyze the main notions of his philosophy such as the indiscernible, the undecidable, and the unnameable. The focus of my analysis is both on their mathematical consistency, and their philosophical consequences. I argue that the mathematical approach developed by Badiou is seriously defective, and, as a result, that it cannot serve as an ontological basis for the concept of event as presented in "Being and Event".

1. Introduction

"Being and Event" is an ontological theory of the effect of *event*, it is a science of the possibility of novelty that breaks with being's determinants. Its existence can never be decided once and for all, because the occurrence of an event is always an *intervention*: its ultimate source lies in a decision of the subject who is constituted in this act. As a result, a post-evental *truth* enters being. It is *udecidable* and *indiscernible*, which means that it cannot be grounded nor described in terms of authoritative *knowledge*. It modifies the situation by overruling some elements of knowledge, and making other *veridical*. Even though a truth cannot be accessed in its entirety, the subject supporting a *procedure of truth* can decide that something belongs to its realm. These *gestures of fidelity* give rise to the *subject language*, which *forces* statements describing the state of affairs after the occurrence of an event.

These are the core concepts of Badiou's philosophy of event in a nutshell. However — and that is perhaps the most striking feature of "Being and Event" — all of its fundamental categories have a mathematical character, and the main theses of this book derive from subtle and quite technical considerations based on the area of mathematics called set theory. Eventually, the science of event turns out to be the result of an investigation into set theory, understood as ontology, or rather proved to be ontology, i.e. the theory of "being qua being".

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In this paper, I am mostly concerned with the consistency of mathematical aspects of Badiou's analysis, as well as some of its philosophical consequences in certain strictly defined key aspects. It is an important reservation, as a large part of Badiou's thought can be presented without referring to any technical terms. Many of its categories — such as event itself, subject, fidelity, the void or excess — are well rooted in the language of contemporary philosophy.

Taking a broader perspective, I am interested in the postulated deep ontological structure behind four fundamental domains of truth, which are science, politics, art and love. Of course, we do not need any special doctrine in order to recognize the groundbreaking character of basic examples of event discussed by Badiou — Mallarme's poetry, relativistic physics, the communist revolution or dodecaphony, as well as some similarities between them — a break involved in their occurrence, the resulting new paradigm, a peculiar indeterminateness of their status. On the other hand, some of these examples do not seem to have that much in common with others, at least on the surface of things. In order to acknowledge the relevance of Badiou's position, more is needed than a simple rephrasing of well known ideas in a new language: what is required is a convincing argument showing that the ontological theory of event is an autonomous interpretative tool, independent of specific subject matters. Moreover, it is rather hard to imagine an argument of this kind that would not contain a proof that the mathematical part of Badiou's thought is consistent and agrees with his fundamental philosophical aims. The formalized doctrine presented by Badiou not only allows for a realization of this postulate but it explicitly requires it.

Finally, one could describe this article as a contribution to the reflection on the concept of matheme. In "Being and Event", a matheme is understood as a philosophical idea subjected to rigors of deduction, and opposed to the pre-platonic poem. However, the very term 'matheme' comes from late writings of Lacan, which are an important reference point for Badiou. There it denotes mathematical objects — such as the Boromean knot or the Klein bottle — allowing to grasp the order of the real extending beyond the reach of language. According to this understanding, a matheme would be a place in philosophy, where mathematics attains an autonomous status, completing philosophical discourse, and generating statements that are binding for it.

My assessment of the method employed in "Being and Event" is negative. After providing the first part of Badiou's doctrine, that is the identification of ontology with set theory, and outlining mathematical foundations of the theory of event, I present a critical discussion of four key concepts of this theory: the indiscernible, the undecidable, the unnameable and the evental site. The conclusion is that their mathematical structure and its implications for Badiou's philosophy turn out not to meet expectations. They are ill-designed, and this leads to mathematical inconsistencies as well as philosophical consequences that contradict Badiou's intentions. Far from rejecting the philosophical substance of these categories, I claim that their mathematical side may serve as an inspiring metaphor or analogy, but it has clearly defined bounds of meaningful interpretation.

2. Mathematics as ontology

The edifice of Badiou's philosophy of "being qua being" is founded on a thesis establishing the identity of mathematics and ontology. In order to accomplish this task, Badiou, in the very first words of "Being and Event" goes back to the old Parmenidean dispute on the status of the one and the multiple. He decides it by giving priority to the multiple, while considering the one as an effect of operation of imposing structure upon being, organizing the 'primordial' *inconsistent multiplicity*. Inconsistent means here exactly this: "without any unifying principle".

Being such as it presents itself — and "no access to being is offered to us except presentations" [BE, p. 27] — is a *consistent multiplicity*. It is a multiplicity because "if the one is not reciprocal with being, the multiple, however, is reciprocal with presentation." [BE, s. 28]; also, it is consistent because a presented being is always the result of certain organizing principle, certain *operation of counting-as-one*. The inconsistent multiplicity is a *subtractive* basis of an already structured presentation — it is possible to be discerned only retroactively, as that 'something' on which a count-as-one operated [BE, p. 25]. From the perspective of presentation it is only the void because the effect of structure encompasses everything without exception. In this manner, the inconsistent multiplicity "sutures presentation to being" [BE, p. 55]; the void is the name of being [BE, p. 56]. This establishment of a consistent multiplicity resulting from some operation of counting-as-one, a combination of the earlier inconsistent and later consistent multiplicity, is called a *situation*.

It is the inconsistent multiplicity that forms the domain of ontology. "Ontology can be solely the theory of inconsistent multiplicities as such. 'As such' means that what is presented in the ontological situation is the multiple without any other predicate than its multiplicity." [BE, p. 28] How is the presentation of being regarded as multiplicity — the presentation of presentation — possible if "being has no structure" [BE, p. 27]? What requirements does such a situation need to satisfy? Firstly, a multiple without one is a multiple that consists only of multiplicities. Secondly, the operation of counting-as-one of the ontological situation cannot be anything more than a collection of conditions through which the multiple can be recognized as multiple. Finally, ontology must be a theory of "the suture of presentation to being”, that is, a theory of the subtractive void of presentation. In other words, it should derive the existence of its multiplicities only out of the void.

The science satisfying these postulates is set theory — a mathematical theory of the relation of belonging $\in$, together with appropriate axioms: the axiom of extensionality, regularity, pairing, union, infinity, power set, and the axiom schema of comprehension and replacement. First of all, the universe of set theory contains only sets, so set theory presents only multiplicities consisting of multiplicities. Moreover, set theory is an axiomatic theory, and axioms are formal rules that do not explicitly define objects they refer to. Badiou says: “an axiomatic presentation consists, on the basis of non-defined terms, in prescribing the rule for their manipulation.” [BE, p. 29] Sets *de facto* satisfy axioms of set theory, however the axioms themselves do not form the criteria of being a set. In other words, what axioms determine about sets is that they are multiplicities, nothing more. Finally, the unpresented subtractive void of situation appears in set theory in the form of the empty set, which is a basic building block of all sets. It must be a set because every element presented in the ontological situation is the effect of its operation of counting-as-one, however in fact, it is a “‘multiple’ which is neither one nor multiple, being the multiple of nothing, and therefore, as far as it is concerned, presenting nothing in the form of the multiple, no more than in the form of the one. This way ontology states that presentation is certainly multiple, but that the being of presentation — that which is presented — being void, is subtracted from the one/multiple dialectic.” [BE, p. 59]

3. Belonging, inclusion, and the impasse of being

Let us take a closer look at the mathematical structure of key categories of “Being and Event”. Ontologically, situations — as it has been already said — take the form of sets: multiplicities whose one is nothing more than the unity of the elements they consist of. The two basic types of relations between sets are *belonging*, that is being an element of a set, being *presented in a situation*, and *inclusion*, that is, being a subset of a set, or being *represented in a situation*. Even though the latter relation is formally reducible to the former one — y is a subset of x if and only if every element of y is an element of x — their properties differ considerably. The tension between belonging and inclusion is fundamental in so far as this is where the “impasse of being” arises, opening up a situation to the interventional occurrence of an event. More light can be shed on this tension with the help of the concept of power set.

The power set $p(x)$ of x — *the state of situation* x or its *metastructure* [BE, p. 94] — is defined as the set of all subsets of x. Now, basic relations between sets can be expressed as the following relations between sets and their power sets. If for some x, every element of x is also a subset of x, then x is a subset of $p(x)$, and x can be reduced to its power set. Conversely, if every subset of x is an element of x, then $p(x)$ is a subset of x, and the power set $p(x)$ can be reduced to x. Sets that satisfy the first condition are called *transitive*. For obvious reasons the empty set is transitive; other examples of transitive sets can also be easily found. However, the second relation never holds. The mathematician Georg Cantor proved that not only $p(x)$ can never be a subset of x, but in some fundamental sense it is strictly larger than x. On the other hand, axioms of set theory do not determine the extent of this difference. Badiou says that it is an "excess of being", an excess that at the same time is its impasse.

In order to explain the mathematical sense of this statement, one needs to recall the notion of *cardinality*, which clarifies and generalizes the common understanding of quantity. We say that two sets x and y have the same cardinality if there exists a function defining a one-to-one correspondence between elements of x and elements of y. For finite sets, this definition agrees with common intuitions: if a finite set y has more elements than a finite set x (say, y has 10 elements and x has 7 elements), then regardless of how elements of x are assigned to elements of y, something (some 3 elements) will be left over in y — precisely because it is larger. In particular, if y contains x *and* some other elements, then y does not have the same cardinality as x. This seemingly trivial fact is not always true outside of the domain of finite sets. To give a simple example, the set of all natural numbers $\mathbb{N}$ contains quadratic numbers, that is, numbers of the form n^2, as well as some other numbers but the set of all natural numbers, and the set of quadratic numbers have the same cardinality. The correspondence witnessing this fact assigns to every number n a unique quadratic number, namely n^2.

Counting finite sets has always been done via natural numbers 0, 1, 2,… In set theory, the concept of such a canonical measure can be extended to infinite sets, using the notion of *cardinal numbers*. Without getting into details of their definition, let us say that the series of cardinal numbers begins with natural numbers, which are directly followed by the number ω_0, that is, the size of the set of all natural numbers $\mathbb{N}$, then by ω_1, the first uncountable cardinal numbers, etc. The hierarchy of cardinal numbers has the property that every set x, finite or infinite, has cardinality (i.e. size) equal to exactly one cardinal number κ. We say then that κ is the cardinality of x.

It is not hard to prove that the cardinality of the power set $p(x)$ is 2^n for every finite set x of cardinality n. However, something quite paradoxical happens when infinite sets are considered. Even though Cantor's theorem does state that the cardinality of $p(x)$ is always larger than x — similarly as in the case of finite sets — axioms of set theory *never* determine the exact cardinality of $p(x)$. Moreover, one can formally prove that there exists no proof determining the cardinality of the power sets of any given infinite set.

There is a general method of building models of set theory, discovered by the mathematician Paul Cohen, and called *forcing*, that yields models, where — depending on construction details — cardinalities of infinite power sets can take different values. Consequently, quantity — "a fetish of objectivity" [BE, p. 83] as Badiou calls it — does not define a measure of being but it leads to its impasse instead. It reveals an undetermined gap [BE, p. 83], where an event can occur — "that-which-is-not being-qua-being" [BE, p. 184].

4. Forcing, truth and the place of the subject

In order to make the exposition more accessible, let us consider only the power set of the set $\mathbb{N}$ of all natural numbers, which is the smallest infinite set — the countable infinity. Simplifying things slightly, the argument proceeds as follows. By a model of set theory we understand a set in which — if we restrict ourselves to its elements only — all axioms of set theory are satisfied. It follows from Gödel's completeness theorem that as long as set theory is consistent, no statement which is true in some model of set theory can contradict logical consequences of its axioms. If the cardinality of $p(\mathbb{N})$ was such a consequence, there would exist a cardinal number κ such that the sentence 'the cardinality of $p(\mathbb{N})$ is κ' would be true in all the models. However, for every cardinal κ the technique of forcing allows for finding a model M where the cardinality of $p(\mathbb{N})$ is not equal to κ. Thus, for no κ, the sentence 'the cardinality of $p(\mathbb{N})$ is κ' is a consequence of the axioms of set theory, i.e. they do not decide the cardinality of $p(\mathbb{N})$.

The starting point of forcing is a model M of set theory — called the *ground model* — which is countably infinite and transitive (this is a crucial assumption). As a matter of fact, the existence of such a model cannot be proved but it is known that there exists a countable and transitive model for every *finite* subset of axioms. As far as the logic of the construction is concerned, in particular the decidability of sentences obtained by forcing, this difference does not play any role.

A characteristic subtlety can be observed here. From the perspective of 'an inhabitant of the universe', that is, if all the sets are considered, the model M is only a small part of this universe. It is deficient in almost every respect; for example all of its elements are countable, even though the existence of uncountable sets is a consequence of the axioms of set theory. However, from the point of view of an 'inhabitant of M', that is, if elements outside of M are disregarded, everything is in order. Some of the sets that are countable in the universe, are actually uncountable in M because in this model there are no functions establishing a one-to-one correspondence between them and ω_0. One could say that M "simulates" the properties of the whole universe.

The main objective of forcing is to build a new model $M[G]$ based on M, which contains M, and satisfies certain additional properties. The model $M[G]$ is called the *generic extension* of M. In order to accomplish this goal, a particular set is distinguished in M — its elements are referred to as conditions — which will be used to determine basic properties of the generic extension. In case of the forcing that proves the undecidability of the cardinality of $p(\mathbb{N})$, the set of conditions codes finite fragments of a function witnessing the correspondence between $p(\mathbb{N})$ and a fixed cardinal κ.

In the next step, an appropriately chosen set G is added to M as well as other sets that are indispensable in order for $M[G]$ to satisfy the axioms of set theory. This set — called *generic* — is a subset of the set of conditions that always lays outside of M. The construction of $M[G]$ is exceptional in the sense that its key properties can be described and proved using M only, or just the conditions, thus, without referring to the generic set. This is possible for three reasons. First of all, every element x of $M[G]$ has a "name" existing already in M (that is, an element in M that codes x in some particular way). Secondly, based on these names, one can design a language called the *forcing language* or — as Badiou terms it — the *subject language* that is powerful enough to express every sentence of set theory referring to the generic extension. Finally, it turns out that the validity of sentences of the forcing language in the extension $M[G]$ depends on the set of conditions: the conditions *force* validity of sentences of the forcing language in a precisely specified sense. As it has already been said, the generic set G consists of some of the conditions, so even though G is outside of M, its elements are in M. Recognizing which of them will end up in G is not possible for an inhabitant of M, however in some cases the following can be proved: provided that the condition p is an element of G, the sentence S is true in the generic extension constructed using this generic set G. We say then that p forces S.

In this way, with an aid of the forcing language, one can prove that every generic set of the Cohen forcing codes an entire function defining a one-to-one correspondence between elements of $p(\mathbb{N})$ and a fixed cardinal number — it turns out that *all* the conditions force the sentence stating this property of G, so regardless of which conditions end up in the generic set, it is always true in the generic extension. On the other hand, the existence of a generic set in the model M cannot follow from axioms of set theory, otherwise they would decide the cardinality of $p(\mathbb{N})$.

The method of forcing is of fundamental importance for Badiou's philosophy. The event escapes ontology; it is "that-which-is-not-being-qua-being", so it has no place in set theory or the forcing construction. However, the post-evental truth that enters, and modifies the situation, is presented by forcing in the form of a generic set leading to an extension of the ground model. In other words, the situation, understood as the ground model M, is transformed by a post-evental truth identified with a generic set G, and becomes the generic model $M[G]$. Moreover, the knowledge of the situation is interpreted as the language of set theory, serving to discern elements of the situation; and as axioms of set theory, deciding validity of statements about the situation. Knowledge, understood in this way, does not decide the existence of a generic set in the situation nor can it point to its elements (this property of truth will be thoroughly discussed later). A generic set is always *undecidable* and *indiscernible*.

Therefore, from the perspective of knowledge, it is not possible to establish, whether a situation is still the ground-model or it has undergone a generic extension resulting from the occurrence of an event; only the subject can interventionally decide this. And it is only the subject who decides about the belonging of particular elements to the generic set (i.e. the truth). A *procedure of truth* or *procedure of fidelity* [BE, p. 329] supported in this way gives rise to the subject language. It consists of sentences of set theory, so in this respect it is a part of knowledge, although the *veridicity* of the subject language originates from decisions of the faithful subject. Consequently, a procedure of fidelity forces statements about the situation as it is after being extended, and modified by the operation of truth.

5. Mathemes of the undecidable and of the evental site

According to Badiou, the undecidable truth is located beyond the boundaries of authoritative claims of knowledge. At the same time, undecidability indicates that truth has a post-evental character: "the heart of the truth is that the event in which it originates is undecidable" [BE, p. 221]. Badiou explains that, in terms of forcing, undecidability means that the conditions belonging to the generic set force sentences that are not consequences of axioms of set theory. However, one also needs to answer the question about the role played by axioms in the structure of *historical* situations. If in the domains of specific languages (of politics, science, art or love) the effects of event are not visible, the content of "Being and Event" is an empty exercise in abstraction: even science — perhaps excluding some entirely formalized areas of theoretical physics — let alone art or love — cannot for obvious reasons be exhaustively described solely in terms of the relation of belonging. Anyway, it is doubtful that — to consider just one example — the status of the French revolution is different from the status of the absolute monarchy preceding it as far as the axioms of set theory are concerned. In other words, most likely either all historical facts are decidable or none of them is. Both possibilities lead to a trivial notion of the undecidable.

Judging by numerous examples discussed by Badiou, it seems that he distances himself form such a narrow interpretation of the function played by axioms. He rather regards them as collections of basic convictions that organize situations, the conceptual or ideological framework of a historical situation. For example, the nature of politics in Rousseau's writings is formulated in the following way: "The major axiom is that in order to definitely have the expression of the general will, [there must] be no partial society in the State" [BE, p. 348]. This approach is also indicated in the only part of "Being and Event" which considers that issue in general terms: "Let us agree that a proposition is singular (…) if, within a historically structured mathematical situation, it implies many other significant propositions, yet it cannot itself be deduced from the axioms which organize the situation. (…) Say that A is this proposition. (…) An event, named by an intervention, is then, at the theoretical site indexed by the proposition A, a new apparatus, demonstrative or axiomatic, such that A is henceforth clearly admissible as a proposition of the situation." [BE, p. 246] Accordingly, the undecidability of a truth would consist in transcending the theoretical framework of a historical situation or even breaking with it in the sense that the faithful subject accepts beliefs that are impossible to reconcile with the old mode of thinking.

A clear illustration of the effect of event which in Badiou's opinion results in breaking with the determinants of the old paradigm, rather than just in moving beyond them, is the birth of relativistic physics: "After Einstein's texts of 1905, if I am faithful to their radical novelty, I cannot continue to practice physics within its classical framework" [Ethics, p. 42]. The novelty of relativistic physics cannot be reduced to a mere substitution of certain equations with different, more precise ones, because it gives rise to a completely new understanding of fundamental physical categories such as space, time, reference point or motion. For profound reasons classical mechanics rules out — instead of simply not deciding it — the very possibility of the theory of relativity emerging within its own conceptual framework. Similarly, the French revolution and communism — essential examples of the effect of event — violently rupture the historical order preceding them. The French revolution overthrew the king and established the sovereignty of the people. And if something can be said with certainty about any real or imaginary realization of the communist idea, it is that it definitely abolishes the "capital-parliamentarism".

However, if one consequently identifies the effect of event with the structure of the generic extension, they need to conclude that these historical situations are by no means the effects of event. This is because a crucial property of every generic extension is that axioms of set theory remain valid within it. It is the very core of the method of forcing, stated in the Theorem of the Generic Model [Jech, Th. 14.5]. Without this assumption, Cohen's original construction would have *no raison d'etre* because it would not establish the undecidability of the cardinality of infinite power sets. Let us say this once more: every generic extension satisfies axioms of set theory. In reference to historical situations, it must be conceded that a procedure of fidelity may modify a situation by forcing undecidable sentences, nonetheless it never overrules its organizing principles.

From the point of view of the generic theory of truth, some hypothetical type of social democracy might be considered as the effect of event. It would abolish chaos and inequalities, resulting from mechanisms of democratically controlled market economy, by the operation of a new idea transfiguring the nature of these mechanisms from within. As a religious event, transgressing the Law without literally breaching it, one could probably point to Messianic Judaism or Protestantism. Another interesting case is a theory of the Danish astronomer Tycho de Brahe who in the 16th century proposed a solution that allowed for keeping the empirical advantages of the heliocentric model, while letting the Earth stay in the center of the Universe. In this conception, all the planets revolve around the Sun, except for Earth, which is encircled by the Sun. In terms of kinetics, that is, if the force of gravity — unknown at that time — is disregarded, de Brahe's model is entirely equivalent to the Copernican one.

Another notion which cannot be located within the generic theory of truth without extreme consequences is *evental site*. An evental site — an element "on the edge of the void" [BE, p. 175] — opens up a situation to the possibility of an event [BE, p. 179]. Ontologically, it is defined as "a multiple such that none of its elements are presented in the situation" [BE, p. 175]. In other words, it is a set such that neither itself nor any of its subsets are elements of the state of the situation. As the double meaning of this word indicates, the state (*état*) in the context of historical situations takes the shape of the State (*État*) [BE, p. 104]. A paradigmatic example of a historical evental site is the proletariat — "entirely dispossessed, and absent from the political stage" [Ethics, p. 69].

The existence of an evental site in a situation is a necessary requirement for an event to occur. Badiou is very strict about this point: "we shall posit once and for all that there are no natural events, nor are there neutral events" [BE, p. 178] — and it should be clarified that situations are divided into natural, neutral, and those that contain an evental site. The very matheme of event — its formal definition is of no importance here — is based on the evental site [Ethics, p. 179]. The event raises the evental site to the surface, making it represented on the level of the state of the situation. Moreover, a novelty that has the structure of the generic set but it does not emerge from the void of an evental site, leads to a *simulacrum of truth* [Ethics, p. 72], which is one of the figures of Evil [Ethics, p. 87]. An example of utterly destructive effects of a *simulacrum of truth* is the Nazi revolution whose source was the "plenitude" of the German people [Ethics, p. 73].

However, if one takes the mathematical framework of Badiou's concept of event seriously, it turns out that there is no place for the evental site there — it is forbidden by the assumption of transitivity of the ground model M. This ingredient plays a fundamental role in forcing, and its removal would ruin the whole construction of the generic extension. As it has already been mentioned, transitivity means that if a set belongs to M, all its elements also belong to M. However, an evental site is a set none of whose elements belongs to M. Therefore, contrary to Badiou's intentions, there cannot exist evental sites in the ground model. Using Badiou's terminology one can say that forcing may only be the theory of the simulacrum of truth.

6. The mathemes of the indiscernible and the unnameable

"Thought is nothing other than the desire to finish with the exorbitant excess of the state" [BE, p. 282]. Since Cantor's theorem implies that this excess cannot be removed or reduced to the situation itself, the only way left is to take control of it. A basic, paradigmatic strategy for achieving this goal is to subject the excess to the power of language. Its essence has been expressed by Leibniz in the form of the principle of indiscernibles: there cannot exist two things whose difference cannot be marked by a describable property [BE, p. 283]. In this manner, language assumes the role of a "law of being" [BE, p. 283], postulating identity, where it cannot find a difference. Meanwhile — according to Badiou — the generic truth is indiscernible: there is no property expressible in the language of set theory that characterizes elements of the generic set. Truth is beyond the power of knowledge, only the subject can support a procedure of fidelity by deciding what belongs to a truth. This key thesis is established using purely formal means, so it should be regarded as one of the peak moments of the mathematical method employed by Badiou. In order to assess its grounding and possible limitations, one needs to analyze the matheme of the indiscernible as closely as possible.

To the reader's surprise, Badiou composes the indiscernible out of as many as three different mathematical notions. First of all, he decides that it corresponds to the concept of the inconstructible [BE, p. 355]. Later, however, he writes that "a set δ is discernible (…) if there exists (…) an explicit formula $\lambda(x)$ (…) such that 'belong to δ' and 'have the property expressed by $\lambda(x)$' coincide" [BE, p. 367]. Finally, at the outset of the argument designed to demonstrate the indiscernibility of truth, he brings in yet another definition: "let us suppose the contrary: the discernibility of G. A formula thus exists $\lambda(x, a_1, \ldots, a_n)$ with parameters $a_1, \ldots, a_n$ belonging to $M[G]$ such that for an inhabitant of $M[G]$ it defines the multiple G" [BE, p. 386]. In short, discernibility is understood as:

1. constructibility
2. definability by a formula $F(y)$ with one free variable and no parameters. In this approach, a set a is definable if there exists a formula $F(y)$ such that b is an element of a if and only if $F(b)$ holds.
3. definability by a formula $F(y, z_1, \dots, z_n)$ with parameters. This time, a set a is definable if there exists a formula $F(y, z_1, \dots, z_n)$ and sets $a_1, \dots, a_n$ such that after substituting $z_1 = a_1, \dots, z_n = a_n$, an element b belongs to a if and only if $F(b, a_1, \dots, a_n)$ holds.

Even though in “Being and Event” Badiou does not explain the reasons for this variation, it clearly follows from his other writings (such as [Conditions, p. 135]) that he is convinced that these notions are equivalent. It should be emphasized then that this is not true: a set may be discernible in one sense, but indiscernible in another. First of all, the last definition has been included probably by mistake because it is trivial. Every set in $M[G]$ is discernible in this sense because for every set a the formula $F(y, x)$ defined as ‘y belongs to x’ defines a after substituting $x = a$. Accepting this version of indiscernibility would lead to the conclusion that truth is always discernible, while Badiou claims that it is not so. In particular, the proof of the indiscernibility of truth presented by Badiou on page 386 of “Being and Event”, based on this definition of indiscernibility, is incorrect[1].

Is it not possible to choose the second option and identify discernibility with definability by a formula with no parameters? After all, this notion is most similar to the original idea of Leibniz — intuitively, the formula $F(y)$ expresses a property characterizing elements of the set defined by it. Unfortunately, this solution does not warrant indiscernibility of the generic set either. There are examples of generic extensions $M[G]$, where the generic set is definable, or even definable in $M[G]$, by a formula with no parameters (see [Enayat], and [Fuchs].) Therefore, in this approach truth may be seized by knowledge replacing the intervening subject.

As a matter of fact, assuming that in ontology, that is, in set theory, discernibility corresponds to constructibility, Badiou is right that the generic set is necessarily indiscernible. However, constructibility is a highly technical notion, and its philosophical interpretation seems very problematic. Let us take a closer look at it.

The class of constructible sets — usually denoted by the letter L — forms a hierarchy indexed or ‘numbered’ by ordinal numbers. Without getting into details of the definition of the ordinal number — closely related

[1] There are several flaws in the proof but the most important one is that Badiou wrongly assumes that an element defined by a formula with parameters in the ground model must be an element of the ground model as well. This is not true as the following discussion will show.

to that of cardinal number — the inductive procedure of constructing the constructible hierarchy goes as follows. The lowest level L_0 is simply the empty set. Assuming that some level — let us denote it by L_α — has already been constructed, the next level $L_{\alpha+1}$ is constructed by choosing all subsets of L_α that can be defined by a formula (possibly with parameters) *bounded* to the lower level L_α.

Bounding a formula to L_α means that its parameters must belong to L_α and that its quantifiers are restricted to elements of L_α. For instance, the formula 'there exists z such that z is in y' simply says that y is not empty. After bounding it to L_α this formula takes the form 'there exists z in L_α such that z is in y', so it says that y is not empty, and some element from L_α witnesses it. Accordingly, the set defined by it consists of precisely those sets that contain an element from L_α.

After constructing an infinite sequence of levels (or — strictly speaking — a limit sequence of levels) the level directly above them all is simply the set of all elements constructed so far. For example, the first infinite level L_ω consists of all elements constructed on levels L_0, L_1, L_2, ...

As a result of applying this inductive definition, on each level of the hierarchy all the formulas are used, so that two distinct sets may be defined by the same formula. On the other hand, only bounded formulas take part in the construction. The definition of constructibility offers too little (because only bounded formulas are accepted) and too much at the same time (because many sets can be defined by one formula). This technical notion resembles the Leibnizian discernibility only in so far as it refers to formulas. In set theory there are more notions of this type though.

To realize difficulties involved in attempts to philosophically interpret constructibility, one may consider a slight, purely technical, extension of it. Let us also accept sets that can be defined by a formula $F(y, z_1, \ldots, z_n)$ with constructible parameters, that is, parameters coming from L. Such a step does not lead further away from the common understanding of Leibniz's principle than constructibility itself: if parameters coming from lower levels of the hierarchy are admissible when constructing a new set, why not admit others as well, especially since this condition has no philosophical justification?

Actually, one can accept parameters coming from an even more restricted class, e.g., the class of ordinal numbers. Then we will obtain the notion of *definability from ordinal numbers* (*OD*). This minor modification of the concept of constructibility — a relaxation of the requirement that the procedure of construction has to be restricted to lower levels of the hierarchy — results in drastic consequences. Kenneth McAloon ([McAloon]) proved in 1972 that the generic set as well as all other elements of the generic extension may be definable from ordinal numbers. Therefore, replacing constructibility with a concept very similar to it — essentially identical from the point of view of

Badiou's philosophical motivations — leads to an ontology allowing for the discernibility of truth.

Another example of such a heterogeneous composition is the matheme of *unnameable*. In "Being and Event" this notion functions as a synonym of the indiscernible, however in Badiou's later writings — especially "Conditions" and "Ethics" — it becomes a distinct, autonomous concept. Referring to Lacanian psychoanalysis, Badiou says: "in the field determined by a situation and the generic becoming of its truth, a real is attested to by a term, a point, and only one, at which the power of truth is suspended. There is only one term in relation to which no anticipatory hypothesis on the generic subset permits us to force a judgment, (…) no naming is appropriate for this term. That is why I call it unnameable" [Conditions, p. 141]. Also, the unnameable understood in the same manner — as a "point that the truth cannot force" [Ethics, p. 85] — witnesses the "powerlessness of truth" in Badiou's ethical thought. A totally powerful truth — its subject language reaching all the elements of the situation — leads to a "disaster", which is another figure of Evil, along with the simulacrum of truth [Ethics, p. 85].

Thus, in set theory, the unnameable has a precisely defined form: it is the *unique* element of the generic extension such that no condition forces a sentence of the forcing language referring to this element. Unfortunately, the range of this definition turns out to be empty — forcing unequivocally rules out the existence of the unnameable. This is because one of the basic features of the forcing language is ([Jech, Th. 14. 7 ii) a)]) that if no condition forces a given sentence S, then every condition forces the negation of S. In the generic extension, the unnameable does not exist.

Yet Badiou's position is that mathematics indicates a possibility of the unnameable. He points to a construction due to Furkhen "in which one term exists, and only one, which cannot receive a name in the sense that it cannot be identified by a formula in the language" [Conditions, p. 143]. A familiar substitution can be recognized here. Initially, the unnameable is defined in terms of the forcing language, however Furkhen's construction refers to definability by a formula. It should be emphasized that this object is not even a model of set theory [Conditions, p. 120]; it requires a richer language, and it does not satisfy the axioms of set theory. Therefore it cannot serve as the ground model or a generic extension — forcing has simply no use in this framework. Moreover, the unnameable becomes the indiscernible again with an additional requirement of uniqueness. Is a truth a real of the situation then? Such a suggestion never appears in Badiou's writings. Otherwise, there always exist at least two indiscernible elements — a truth and a real of the situation — even though the unnameable is supposed to be unique. In presence of these overwhelming formal and interpretative difficulties, one should rather conclude that the matheme of unnameable is an ill-conceived concept.

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The mathematical form of the undecidable implies that a truth never overrules the fundamental conceptual framework of a situation from before the occurrence of an event. This consequence remains in sharp contrast with basic examples of the effect of event considered by Badiou. An evental site cannot open up a situation to an evental truth because forcing rules out the very existence of an evental site in ground models. As a result, the generic set may possibly describe the structure of the simulacrum of truth — using Badiou's terminology — but not the structure of truth itself. Two two remaining mathemes — the indiscernible and the unnameable — are built out of mathematical concepts that do not fit together; moreover, their properties are not as Badiou claims.

This list of problems can be extended. The "name of the event" is not "prohibited by being" [BE, p. 184] because the axiom of foundation may be consistently replaced with its negation — for example, the Aczel anti-foundation axiom is consistent with the remaining axioms of set theory. Finite sets are not necessarily definable, even though Badiou tries to prove the opposite [Conditions, p. 137], mistakenly identifying finite sets with numerals. In the so called nonstandard models of set theory the standard omega is finite and non definable. Generic sets are not coextensive with unconstructible sets [Conditions, p. 135] — for example, under the Ground Axiom. It seems though that the discussion of the mathemes of indiscernible, undecidable, unnameable, and evental site is sufficient to defend the main thesis of this paper. Mathematics — forced to accept compromises going definitely too far — responds with outcomes which are hostile to fundamental philosophical motivations of Badiou's doctrine. Despite some points of convergence, his generic theory of truth and his philosophy of event can coexist only at a price of selective and instrumental interpretation of the mathematical component. Therefore one has to conclude that "Being and Event" provides no grounding for a deep ontological structure behind the realms of science, art, love and politics, and that the mathematical formulation of the theory of event has no positive content.

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