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QUANTIFICATION AND PREDICATION IN MODAL PREDICATIVE PROPOSITIONAL LOGIC

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Abstract

The main objective of this paper is to enrich first order propositional predicative logic by dealing all together with intensional attributes, quantification, logical and historic modalities and ramified time. Predicative propositional logic advocates a finer analysis in terms of predication of the logical form of propositions. Like Church's logic of sense and denotation (Church 1951), my new predicative approach of quantification[1] is based on Frege's theory of indirect reference (Frege 1892). However in my approach, like in algebraic intensional logic, generalized propositions predicate first order generalizations of attributes. In the first section, I will analyze in terms of predication the logical form of elementary propositions with all kinds of attributes (whether intensional or extensional) and of generalized, modal and temporal propositions.[2] Next I will define the ideographic object-language of my logic. Its formulas can express different propositions in different contexts of utterance. In the third section, I will define the structure of a standard model for my ideography. In the last section, I will enumerate new valid laws of my logic.

Keywords

Predicative logic · intensional logic · sense and denotation · model-theoretic semantics · modalities · propositions · quantification · predication.

1. The logical form of propositions

Propositions with the same truth conditions are not substitutable *salva felicitate* within the scope of illocutionary forces and psychological modes. We can assert (and believe) that Rome is a city without *eo ipso* asserting (and believing) that it is a city and not a hypotenuse. We need a stronger criterion of propositional identity in logic. Carnap (1956) was wrong in identifying each proposition with the set of possible circumstances in which it is true.

[1] I am very grateful to Nuel Belnap, Paul Gochet and Philippe de Rouilhan for their critical remarks.

[2] The present predicative analysis of quantification is more sophisticated than my previous analysis in 1997. It offers a unified treatment of indirect reference, predication and generalization.

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On the basis of speech act theory, I have been advocating since **Meaning and Speech Acts** Vanderveken (1990-91) a finer analysis of the logical type of propositions that takes into account the fact that they are both units of sense and contents of illocutions and attitudes. My propositional logic is *predicative* in the general sense that it considers acts of predication that we make in expressing and understanding propositions. Here are basic principles of my logic of sense and denotation (Vanderveken 2009):

Propositions have a *structure of constituents*. In expressing them we *predicate* in a certain order *attributes* (properties or relations) *of* objects to which we *refer*. Propositional contents are then composed from *elementary propositions* corresponding to *acts of predication*. As Frege pointed out, we always *refer to* objects by subsuming them under senses. We cannot directly have in mind *individuals* like material bodies and persons. We rather have in mind *concepts* of such individuals and we *indirectly* refer to them through these concepts. So individual objects towards which our thoughts are directed are *individuals under a concept* (called an *individual concept*) rather than pure denotations. Concepts through which we refer can be deprived of denotation. By recognizing the indispensable role of concepts in reference, predicative logic accounts for thoughts whose individual concepts do not apply to a single object. It also explains well known failures of the law of extensionality. Many properties that we predicate of individual objects are *intensional*. They can be possessed by an individual under a concept without being possessed by the same individual under other concepts. In predicative logic, the denotation of a first order property of individuals in a circumstance is a set of individuals under a concept rather than a set of individual objects.

From a type-theoretical point of view, what I call an *individual under a concept* is an individual concept. Frege's idea that propositional constituents are senses and not pure denotations clearly explains the difference in cognitive value between the two propositions that the morning star is the morning star and that the morning star is the evening star. It moreover preserves the *minimal rationality* of speakers (Cherniak 1986). We can be mistaken and wrongly believe that the morning star is not the evening star. But we could never believe the obvious contradictory propositions that the morning star is not the morning star. Our human reason prevents us to be totally irrational. So logic has to reject the theory of *direct reference* and *externalism*; there are no *singular propositions* in an adequate logic of action. Like Frege, I advocate that any *object of reference* is subsumed *under a concept*. We, human beings, have restricted cognitive abilities. We can only refer to a finite number of different objects and predicate of them a finite number of attributes. Consequently, propositions which are senses of sentences have a finite positive number of propositional constituents. Furthermore, one must reject standard objectual and substitutional analyses of quantification. Given the indispensability of concepts in reference, semantic values of individual variables are individual concepts rather than individual objects. Like in Church's intensional logic, traditional laws of universal instantiation and of existential generalization are valid in my approach. Whoever is pursuing the fountain of youth is pursuing something even if there is no such fountain. He or she is directed at an intentional object. Many individuals to which we refer do not exist when we think of them. Others do not or even could not exist at any moment. In order to account for the inexistence of certain individuals, my predicative logic contains in its lexicon an *existence predicate* (like Kripke's logic (Kripke 1963)). Whoever makes a generalization does not and even could not refer under all individual concepts. Generalized propositions are not composed of all elementary propositions of their substitution instances. Contrary to what Wittgenstein (1961) said, universal (existential) generalizations are not conjunctions (disjunctions) of their substitution instances.

Frege (1879) was the first to conceive generalization as a kind of predication. In his *Begriffschrift*, quantifiers over individual objects are introduced as second order predicates. In his view, to generalize universally that all objects have a certain property is not to predicate the generalized property of each of them in particular. It is rather to predicate of the property in question the second order logical property that its denotation is universal (that it is possessed by every object).[3] Thus to generalize that all individual objects are identical with themselves is just to predicate of the property of being identical with itself the second order property of being universal. Church (1951) and Montague (1974) follow Frege. By conceiving generalization as a higher order predication, Frege was committed to admitting second order properties in his formal ontology. However, as algebraic logic showed (Craig 1974), we need no second order predication in order to account for generalizations over individuals. We can remain in a simpler first order ontology. First order generalized propositions predicate first order attributes which are *generalizations* of others. To think that all objects are such that God knows them is just to predicate of God the property of omniscience which is a universal generalization of the knowledge relation. To be omniscient is to know everything. In my view, like in algebraic intensional logic (Bealer 1982), generalized propositions are composed of new elementary propositions predicating generalizations of attributes of their arguments.

Concepts serve to refer, and attributes to predicate. Moreover, as Frege pointed out, there is no reference without predication. In logic, attributes of individuals of degree n are *senses* of n-ary *predicates*, while individual concepts are *senses of individual terms*. So the domain of any possible

[3] Similarly, to generalize existentially that some individual object has a property is to predicate of the property in question the second order property that its denotation is not empty.

interpretation of predicative logic contains a non empty set *Individuals* of individuals and the two non empty sets *Concepts* of individual concepts and *Attributes* of attributes of individuals. *Properties* are attributes of degree one. In the Frege - Church approach, there is a fundamental logical *relation of correspondence* between senses and denotations underlying the relation of correspondence between words and things. To propositional constituents *correspond real denotations* of certain types in *possible circumstances*. To each individual concept corresponds in each circumstance the single individual object which *really* falls under that concept in that circumstance whenever there is such an object. Otherwise that concept is deprived of denotation in that circumstance. And to each property of individuals corresponds in each circumstance the set of objects under concepts which *really* possess that property in that circumstance. Individuals change during their existence. Thus different sets of objects under concepts can correspond to a property in different circumstances. Moreover individuals have certain unique real essential properties in all possible circumstances. So are the properties for each human being to have his or her parents. We do not know all *essential properties* of individual objects. Some of us ignore the identity of their parents. Others are wrong about their identity. They then have necessarily false beliefs. However when we refer to one object under a concept and predicate of it properties we *conceive* that concept and properties. We have identity criteria for that object of reference and we also understand the nature of predicated properties that are determined by meaning.

Our knowledge of the world is incomplete. We do not know real denotations of most propositional constituents in possible circumstances. We often *refer* to an object under a concept without being able to identify that object. The police officer pursuing Smith's murderer can just refer to whoever in the world is that murderer. His concept gives *identity criteria* for the object of reference (to have killed Smith) which do not enable him to *identify* that object. Some of our beliefs are false. The presumed murderer is sometimes innocent. In that case the *object* to which we refer is not the *denotation* of the concept that we have in mind. It also happens that no object satisfies the identity criteria. Suppose that Smith's death was accidental. In the traditional theory of indirect reference, whoever refers to an object under a concept *presupposes* that a single object falls under that concept. However the fact that a single object falls under an individual concept does not imply its existence. Moreover we often predicate properties of objects without presupposing their existence. Whoever denies that Santa Claus exists does not presuppose his existence. He just refers to him. In my approach, one can refer and presuppose that a single object falls under a concept without presupposing its existence.

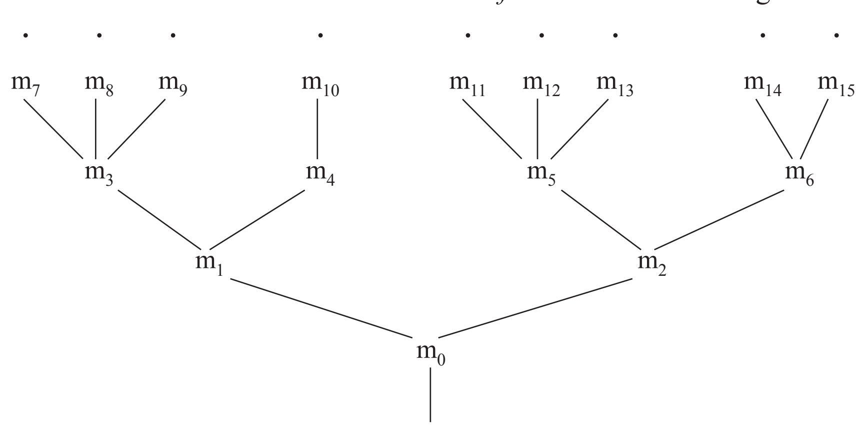
The chief of police can ignore who has killed Smith. But he can think of persons who could have committed the crime. Whoever *conceives* attributes can in principle assign to them possible denotations of appropriate type.

So in any use and interpretation of language, there are many *possible denotation assignments* to attributes and concepts in addition to the standard *real denotation assignment* of classical logic that associates with senses their real denotation in each circumstance. All possible denotation assignments to senses are functions of the same type: they associate with individual concepts one or no individual object at all and with attributes of degree n a set of n-ary sequences of individual concepts in possible circumstances. According to the real denotation assignment, Smith's murderer is the person who really killed Smith when there is such a unique person. According to other possible denotation assignments, Smith's murderer is someone else or nobody. In spite of such differences, all possible denotation assignments respect *meaning postulates*. According to any, a murderer is a killer. We ignore how things are in actual and possible (especially future) circumstances of the reality. So we cannot determine which possible denotations are the *real ones*. But we can in principle think of denotations that attributes could have. Moreover, when we have in mind concepts and attributes, only some possible denotation assignments to these senses *are then compatible with* our beliefs. Suppose that according to the chief of police Smith's murderer is a woman. In that case, possible denotation assignments according to which Smith was killed by a man are then incompatible with that policeman's beliefs. In my approach (Vanderveken 2014), possible denotation assignments rather than possible circumstances are therefore compatible with the truth of agents' attitudes.

In predicative propositional logic truth is defined with respect to both possible circumstances and denotation assignments. In understanding propositions we in general do not know whether they are true or false. We just know that their truth in a circumstance is compatible with certain possible denotation assignments to their concepts and attributes, and incompatible with all others. An elementary proposition predicating a property of an object under a concept is true in a circumstance according to a denotation assignment when according to that assignment the individual under that concept has that property in that circumstance. Otherwise, it is false in that circumstance according to that assignment. Most propositions have therefore *a lot of possible truth conditions*. Of course, as Aristotle pointed out, truth is based on *correspondence* with reality. In order to be *true in a circumstance* a proposition has to be *true in that circumstance according to the real denotation assignment*. So among all possible truth conditions of a proposition, there are its *real Carnapian truth conditions* which correspond to the set of possible circumstances where it is true according to the real denotation assignment.

In my approach, propositions are *identical* when they contain the same elementary propositions and they are true in the same circumstances according to the same possible denotation assignments. Such a finer criterion of propositional identity explains why many strictly equivalent propositions have a different cognitive value. Propositions whose expression requires different predications have a different structure of constituents. We do not express them at the same moments. My identity criterion also distinguishes propositions that we do not understand to be true in the same conditions: they are not true according to the same possible denotation assignments to their constituent senses. Few necessarily true propositions are *obvious tautologies* that we know *a priori* by virtue of competence. A proposition is *necessarily true* when it is true in every possible circumstance according to the real denotation assignment. In order to be *obviously tautological*, a proposition must be true in every circumstance according to every possible denotation assignment to its constituents. Unlike the proposition that whales are whales, the necessarily true proposition that they are mammals is not an obvious tautology. Logic can now distinguish subjective and objective possibilities. A proposition is *subjectively possible* when it is true in a possible circumstance according to a possible denotation assignment. In order to be *objectively possible* a proposition has to be true in a circumstance according to the *real* denotation assignment. Many subjective possibilities are not objective. Some possible denotation assignments compatible with the truth of our beliefs violate essential properties. So we are *not perfectly rational* and sometimes *inconsistent*. However all possible denotation assignments respect meaning postulates. This is why we are at least *minimally rational*.

We need in the logic of action a *ramified* conception of time (Prior 1967) compatible with indeterminism and the apparent liberty of human agents. In branching time, a *moment* is a complete possible state of the actual world at a certain instant and the *temporal relation of anteriority* between moments is partial rather than linear. There is a single causal route to the past. However, there are multiple future routes: several incompatible moments might be directly posterior to a given moment. For facts, events or actions can have incompatible future effects. Consequently, the set *Time* of moments of time has the formal structure of a *tree-like frame* of the following form:



A maximal chain h of moments of time is called a *history*. It represents a *possible course of history* of our world. When a history has a first and a last moment, the world has according to it a beginning and an end. As Belnap *et al.* (2001) pointed out, a *possible circumstance* is a pair of a moment m and of a history h to which that moment belongs.

Thanks to histories temporal logic can analyze important modal notions like settled truth and historic necessity and possibility. Certain propositions are true at a moment according to all histories. Their truth is then *settled at that moment* no matter how the world continues. So are past propositions because the past is unique. Their truth does not depend at all on histories. Contrary to the past, the future is open. The world can continue in various ways after indeterminist moments. Thus the truth of future propositions is not settled at such moments. It depends on which historical continuation of that moment is under consideration. When there are different possible historic continuations of a moment, its actual future continuation is not then determined. Two moments of time are *coinstantaneous* when they belong to the same instant. Coinstantaneous moments m and m' *represent* two complete possible states of the world in which things could then be. They are on the same horizontal line in each tree-like frame. One can analyze *historic necessity* by quantifying over coinstantaneous moments. The proposition that P is then necessary (in symbols $\Box P$) – in the sense that it is then inevitable that P- is true at a moment in a history when P is true at all coinstantaneous moments according to all histories. Logical necessity is stronger than historic necessity. It is logically necessary that P (in symbols: $\blacksquare P$) when proposition P is true in all possible circumstance.

We respect *meaning postulates* in assigning possible denotations of appropriate type to senses and truth conditions to propositions. For we apprehend the logical form of propositions in understanding them. Possible denotation assignments associate with each individual concept c_e and possible circumstance a single individual object or no individual at all. Thus $val(c_e, m/h) \in Individuals$ or no individual falls according to val under the concept c_e in circumstance m/h. Like Carnap, I will for the sake of simplicity identify $val(c_e, m/h)$ with the empty set $\varnothing$ in the case of lack of denotation. Possible denotation assignments moreover associate with each attribute R_n of degree n of individuals and possible circumstance a set of n-ary sequences of individual concepts. So $val(R_n, m/h) \in \mathcal{P}(Concepts^n)$. The denotation in each circumstance of the *binary relation of identity between individuals* according to each possible denotation assignment is the set of all pairs of individual concepts that apply to the same individual in that circumstance according to that assignment. The denotation of the *existence property* according to a denotation assignment val in circumstance m/h is the set of individual concepts that apply according to val to one individual existing at moment m.

Until now I have mainly analyzed *elementary propositions* which predicate simple attributes. What is the structure of constituents of truth functions and modal, temporal and generalized propositions? As Wittgenstein pointed out in the *Tractatus*, truth connectives do not serve to make new acts of reference or predication. Their meaning just contributes to determining truth conditions. Truth functions are then composed from all and only the elementary propositions of their arguments. Unlike truth connectives, quantifiers, modal and temporal connectives serve to predicate new *generalized, modal and temporal attributes*. In quantifying we predicate generalizations of attributes. Whoever thinks that God knows all objects predicates of God the property of omniscience which is a universal generalization of the knowledge relation. Similarly, in thinking that it is impossible that God makes mistakes we predicate of Him the modal property of infallibility, namely that He does not make a mistake in any possible circumstance. Infallibility is the logical necessitation of the property of not making mistakes. The same holds for temporal propositions. Whoever thinks that God created the world predicates of God the past property of having earlier created the world. There is no need of ramified types of propositions in order to analyze generalized, modal, and temporal propositions. Generalized propositions that quantify over every (or over at least one) individual, predicate first order universal (or existential) generalizations of attributes. In algebraic intensional logic, operations of *universal* and *existential generalization* associate to each n-ary attribute R_n and place k such that $0 < k \leq n$, two (n-1)-ary attributes $(\forall k)R_n$ and $(\exists k)R_n$ that are respectively called the *universal* and the *existential generalization* at the k-th place of attribute R_n. A (n-1)-ary sequence of individuals under concepts satisfies attribute $(\forall k)R_n$ in a circumstance according to a denotation assignment *val* when, for every individual concept c_e, sequence $c_e^1, \ldots, c_e^{k-1}, c_e, c_e^{k+1}, \ldots, c_e^n$ satisfies attribute R_n in that circumstance according to *val*.[4] The new attributes of modal propositions according to which it is then necessary that P (in symbols: $\Box P$) or it is then possible that P (in symbols $\Diamond P$) are modal attributes of the first order that are the *historical necessitation* $\Box R_n$ and the *historical possibilitation* $\Diamond R_n$ of attributes R_n of that proposition. An object under concept possesses the *historical necessitation* (or the *historical possibilitation*) of a property in a circumstance according to a denotation assignment *val* when it possesses that property in every (or in at least one) coinstantaneous circumstance according to *val*. In my symbolism, $\blacksquare R_n$ and $\blacklozenge R_n$ are respectively the *logical necessitation* and the *logical possibilitation* of attribute R_n and

[4] Similarly a (n-1)-ary sequence of individuals under concepts $c_e^1, \ldots, c_e^{n-1}$ satisfies the existential generalization $(\exists k)R_n$ at the k-th place of n-ary attribute R_n in a circumstance according to a denotation assignment *val* when for at least one individual concept c_e, sequence $c_e^1, \ldots, c_e^{k-1}, c_e, c_e^{k+1}, \ldots, c_e^n$ satisfies attribute R_n in that circumstance according to *val*.

WillR_n and *Was*R_n are respectively the futurization and the pastization of attribute R_n. An object under concept possesses the futurization of a property in a circumstance *m* / *h* according to a denotation assignment *val* when it possesses that property according to *val* in a future circumstance *m′* / *h* whose moment *m′* is posterior to *m*. And similarly for the pastization except that moment *m′* is anterior to *m*. One can define in predicative logic a new *strong propositional implication* much finer than C.I. Lewis' *strict implication* that is important for the analysis of agents' commitments. A proposition *strongly implies* another when whoever expresses it can express the other and it cannot be true in a circumstance according to a denotation assignment unless the other proposition is also true in that circumstance according to that assignment. Strong implication is finite, tautological, paraconsistent, decidable and *a priori known.*

2. The ideal object language

The formal ontology of the ideographic object language $\mathcal{L}$ of my predicative logic of attributes is simple. It only contains individual concepts, first order attributes of individuals and propositions containing such propositional constituents. Like the object language of Church's logic of sense and denotation, my ideal object language $\mathcal{L}$ does not contain any individual term whose semantic value is an individual object. All its individual terms *express* in each context an individual concept. Moreover they need not denote an individual object and some can denote inexistent individuals. Like combinatory logic, my predicative logic needs no individual variables. Its predicates *express* intensional attributes satisfied by sequences of individual concepts rather than sequences of individuals. The primitive types of my ideal object language are the type #e of concepts of individual objects, the type *r* of *attributes* of individuals under concepts and the type *p* of *propositions*. All attributes like in D. Lewis (Lewis 1972) are of the first order.

2.1. *Vocabulary of $\mathcal{L}$*

The vocabulary of $\mathcal{L}$ contains:

- A series of **individual constants** c, c′, c″, ... of type #e expressing *concepts of individual objects*.
- And, for each natural number n, a series of **predicates of degree** n: r_n, r_n', r_n'', r_n''', ... expressing n-ary *attributes* or *relations* between individuals including the binary identity predicate $=_2$ and the unary predicate E_1 of existence.
- The **syncategorematic symbols** of $\mathcal{L}$ are: $\neg$, $\wedge$, $\forall$, }, $\Box$, *Was, Will, Settled, Tautological, Refl*, x, ′, [, (,] and).

2.2. *Rules of formation of predicates*

All predicates of the lexicon are predicates of $\mathcal{L}$. If R_n is a predicate of degree n of $\mathcal{L}$ then $\neg R_n$, $\Box R_n$, *Will*R_n,*Was*R_n, *Settled*R_n, and *Tautological*R_n are new complex predicates of degree n of $\mathcal{L}$. If R_n is a predicate of degree n of $\mathcal{L}$ and k and m are natural numbers such that $0 < k < m \leq n$ then *Refl*(k,m)R_n and $\forall kR_n$ are new predicates of $\mathcal{L}$ of degree n – 1. If R_n and R_m are predicates of degree n and m respectively, (R_n } R_m) is a new predicate of degree 0 and ($R_n \wedge R_m$), a new predicate of degree (n + m) of $\mathcal{L}$.

A complex predicate of the form $\neg R_n$ expresses the n-ary attribute which is the *truth functional negation* of the attribute expressed by R_n. A sequence of n individual concepts satisfies the *negation* of a n-ary attribute in a circumstance iff it does not satisfy that attribute in that circumstance.

A complex predicate of the form *Settled*R_n expresses the n-ary attribute whose denotation in each circumstance m/h is the set of all sequences of n individual concepts that satisfy that attribute in every possible circumstance whose history contains the moment *m*. I will call such an attribute the *settlednization* of the attribute expressed by R_n.

A complex predicate of the form $\Box R_n$ expresses the n-ary attribute which is the *historical necessitation* of the attribute expressed by R_n. A sequence of n individual concepts satisfies the *historical necessitation* of a n-ary attribute in a circumstance iff it satisfies that attribute in all possible circumstances that are coinstantaneous with that circumstance.

A complex predicate of the form *Tautological*R_n expresses the n-ary attribute which is the obvious tautologization of the attribute expressed by R_n. A sequence of n individual concepts satisfies the *obvious tautologization* of an attribute in a circumstance iff it satisfies that attribute in all possible circumstances according to all possible denotation assignments.

A complex predicate of the form *Will*R_n expresses the n-ary attribute which is the *futurization* of the attribute expressed by R_n. A sequence of n individual concepts satisfies the *futurization* of an attribute in a circumstance m/h iff it satisfies that attribute in at least one possible circumstance m'/h whose moment is posterior to *m*. And similarly for complex predicates of the form *Was*R_n except that moment m' is anterior to *m*.

A complex predicate of the form *Refl*(k,m)R_n expresses the attribute of degree (n – 1) which is the *reflexivization* at the k-th and m-th places of the attribute expressed by R_n. For example, *Refl*(1,2)$=_2$ expresses the property of being identical with itself. By definition, a sequence of n-1 individual concepts $c_e^1,\dots,c_e^{n-1}$ satisfies the attribute *Refl*(k,m)R_n in a circumstance iff the n-ary sequence $c_e^1,\dots,c_e^{m-1}, c_e^k, c_e^{m+1},\dots,c_e^n$ satisfies the attribute R_n in that circumstance.

A complex predicate of the form $\forall_k R_n$ expresses the attribute of degree n – 1 which is the *universal generalization* at the k-th place of the attribute expressed by R_n. For example, $\forall_2 =_2$ expresses the property of being identical with everything. A sequence of n-1 individual concepts $c_e^1,\ldots,c_e^{n-1}$ satisfies the attribute $\forall_k R_n$ in a circumstance iff all n-ary sequences of the form $c_e^1,\ldots,c_e^{k-1}, c_e, c_e^{k+1},\ldots,c_e^n$ satisfies the attribute R_n in that circumstance, for any individual concept c_e.

A predicate of the form $(R_n \wedge R_m)$ expresses the attribute of degree n + m which is the *truth functional conjunction* of attributes expressed by R_n and R_m. A sequence of (n + m) individual concepts $c_e^1,\ldots,c_e^{n+m}$ satisfies the truth functional conjunction of two attributes iff the n-ary sequence $c_e^1,\ldots,c_e^n$ of the first n individual concepts satisfies the first attribute and the m-ary sequence $c_e^{n+1},\ldots, c_e^{n+m}$ of the last m individual concepts satisfies the second attribute in that circumstance.

Finally a predicate of the form $(R_n \} R_m)$ expresses the attribute of degree 0 which is satisfied in a circumstance iff the attribute expressed by predicate R_m is a component of or is identical with the attribute expressed by R_n. Any predicate that occurs in another predicate expresses an attribute that is a component of the attribute expressed by that other predicate.

2.3. *Rules of formation of propositional formulas*

If R_n is a predicate of degree n of the lexicon and $t_1,\ldots,t_n$ is a sequence of n individual terms of $\mathcal{L}$, then $[R_n t_1,\ldots,t_n]$ is a propositional formula of $\mathcal{L}$ of type *p* expressing the *proposition* that predicates the attribute expressed by R_n of the n individuals under concepts expressed by $t_1,\ldots,t_n$ in that order. Thus $[E_1 t]$ means that an existent individual object falls under the concept expressed by t. $[=_2 t_1 t_2]$ means that the same individual object falls under the concepts expressed by t_1 and t_2. The proposition expressed by $[=_2 t_1 t_2]$ in a context is then false when t_1 or t_2 is deprived of denotation.

One can introduce by rules of abbreviation thanks to our operators on predicates usual truth, modal and temporal connectives in my predicative logic of attributes. Thanks to the syncategorematic symbols x and ‘ of $\mathcal{L}$ one can introduce also free occurrences of individual variables x, x′, x″, … of type #e in propositional formulas of $\mathcal{L}$ and give abbreviations for usual propositional formulas $\forall v A_p$ and $\exists v A_p$ without any free variable that express generalized propositions that are obtained by quantifying universally and existentially over all free occurrences of variable v in A_p.

2.4. *Rules of abbreviation*

I will use ordinary rules of abbreviation for *parentheses*, *disjunction* $\vee$, *material implication* $\Rightarrow$, *material equivalence* $\Leftrightarrow$, *historical possibility* $\Diamond$ and *definite descriptions*.

Identity of individual concepts:

$[\hat{}t_1 = \hat{}t_2] =_{\text{def}} [(\mathit{Refl}(1,2) =_2 t_1) \} (\mathit{Refl}(1,2) =_2 t_2]$

Propositional negation:

$\neg[R_n t_1 \ldots t_n] =_{\text{def}} [\neg R_n t_1 \ldots t_n]$

And similarly for $\Box[R_n t_1 \ldots t_n]$, $\mathit{Settled}[R_n t_1 \ldots t_n]$, $\mathit{Was}[R_n t_1 \ldots t_n]$, $\mathit{Will}[R_n t_1 \ldots t_n]$ and $\mathit{Tautological}[R_n t_1 \ldots t_n]$

Propositional conjunction:

$([R_n t_1 \ldots t_n] \wedge [R_m d_1 \ldots d_n]) =_{\text{def}} [(R_n \wedge R_m) t_1 \ldots t_n d_1 \ldots d_m]$

Identity of attributes:

$[\hat{}R_n = \hat{}R_n^k] =_{def} [R_n \} R_n^k] \wedge [R_n^k \} R_n] \wedge [\mathit{Tautological} \forall_1 \ldots \forall_n (R_n \Leftrightarrow R_n^k)]$

Inclusion of predications:

$[R_n t_1 \ldots t_n] \} [R_m d_1 \ldots d_m] =_{\text{def}} (([R_n \} R_m]) \wedge ([\hat{}d_1 = \hat{}t_1] \vee \ldots \vee [\hat{}d_1 = \hat{}t_n] \wedge \ldots \wedge ([\hat{}d_m = \hat{}t_1] \vee \ldots \vee [\hat{}d_m = \hat{}t_n]))$

Same structure of constituents:

$[R_n t_1 \ldots t_n] \}\{ [R_m d_1 \ldots d_m] =_{\text{def}} [R_n t_1 \ldots t_n] \} [R_m d_1 \ldots d_m] \wedge ([R_m d_1 \ldots d_m] \} [R_n t_1 \ldots t_n])$

Propositional identity:

$[R_n t_1 \ldots t_n] = [R_n^k d_1 \ldots d_m] =_{\text{def}} [R_n t_1 \ldots t_n] \}\{ [R_m d_1 \ldots d_m] \wedge \mathit{Tautological}([R_n t_1 \ldots t_n] \Leftrightarrow [R_m d_1 \ldots d_m])$.

$\mathit{Sometimes}\,A =_{\text{def}} (\mathit{Was}\,A) \vee A \vee (\mathit{Will}\,A)$

Universal generalization at the k-th place in R_n over existing individuals:

$(\forall_k E) R_n =_{\text{def}} \forall_1 \mathit{Refl}(1,k+1) (\mathit{Sometimes}\,E_1 \Rightarrow R_n)$

Existential generalization at the k-th place in R_n: $\exists_k R_n =_{\text{def}} \neg\forall_k \neg R_n$

Existential generalization at the k-th place in R_n over existing individuals:

$(\exists_k E) R_n =_{\text{def}} \neg\forall_1 \mathit{Refl}(1,k+1) (\mathit{Sometimes}\,E_1 \wedge \neg R_n)$

Reflexivization at several places:

$\mathit{Refl}(k_1, \ldots, k_m) R_n =_{\text{def}} \mathit{Refl}(k_1, k_2), \ldots, \mathit{Refl}(k_{m-1}, k_m) R_n$
where $k_1 < k_2, \ldots,$ and $k_{m-1} < k_m$.

Universal propositional quantification:

$(\forall v)[R_n t_1 \ldots t_n] =_{\text{def}} [\forall_{k_1} \mathit{Refl}(k_1, \ldots, k_r) R_n d_1 \ldots d_m]$
when v occurs free successively at the k_1th,..., k_rth place in $[R_n t_1 \ldots t_n]$ and $d_1, \ldots, d_m$ are the first,..., and the last individual terms in $[R_n t_1 \ldots t_n]$ different from v.

Universal propositional quantification over existing individuals:

$\forall v(\textit{Sometimes}\mathrm{E}v \Rightarrow A_p)$

Existential propositional quantification:

$\exists v A_p =_{def} \neg\forall v \neg A_p$

Existential propositional quantification over existing individuals:

$\mathrm{E}v A_p =_{def} \exists v(\textit{Sometimes}\mathrm{E}v \wedge A_p)$

To have a denotation:

$\textit{Denotes}(t) =_{def} [\exists_1 =_2 t]$

It has always been the case:

$\textit{WasAlways}A =_{def} \neg\textit{Was}\neg A$

It will always be the case:

$\textit{WillAlways}A =_{def} \neg\textit{Will}\neg A$

$\textit{Always}A =_{def} \textit{WasAlways}A \wedge A \wedge \textit{WillAlways}A$
Universal Necessity:

$\blacksquare A =_{def} \textit{Always}\Box A \wedge \Box\textit{Always}A$

Strict implication (Lewis and Langford 1959): $(A \prec B) =_{def} \blacksquare(A \Rightarrow B)$.
Universal Possibility: $\blacklozenge A = \text{def } \neg\blacksquare\neg A$
Strict equivalence: $(A \prec\succ B) =_{def} (A \prec B) \wedge (B \prec A)$
Analytic implication (Parry 1972):

$(A_p \rightarrow B_p) =_{def} (A_p \} B_p) \wedge (A_p \prec B_p)$

Strong implication:

$(A_p \mapsto B_p) =_{def} (\Box A_p \} B_p) \wedge (\textit{Tautological}(A_p \Rightarrow B_p)$[5]

3. The structure of a semantic interpretation

The formal semantics for my logic is model-theoretical: it specifies how meanings can be assigned to formulas of its ideographical object language in arbitrary *possible interpretations* for that language. A possible interpretation of a language is a *model* of all the sentences that are true in all circumstances of that interpretation.

[5] The analysis of quantification and modalities requires a larger notion of strong implication. Whoever predicates an attribute is able to predicate any generalisation and the necessitation of that attribute. So the law of existential generalization has to be a valid law of strong implication. And similarly for possibility.

Formally, a *standard possible interpretation* or *model* for $\mathcal{L}$ is a structure $\mathcal{M} = \langle Time, Individuals, Concepts, Attributes, Val, Context, \mathcal{U}, \|\ \| \rangle$ where *Time, Individuals, Concepts, Attributes* and *Context* are disjoint non-empty sets and *Val*, $\mathcal{U}$ and $\|\ \|$ are functions which satisfy the following meaning postulates.

(1) *Time* is a non-empty set of *moments* that represent *complete possible states* of the world at certain instants. There is a partial order $\leq$ on the set *Time* representing the temporal relation of *anteriority/posteriority*. The future can be open. Several incompatible moments might directly follow upon a given moment. But the past is unique. Any two moments have a common historical ancestor and there is *no backward branching*. Thus (*Time*, $\leq$) is a *tree-like frame* of the kind illustrated above. A maximal chain h of moments of *Time*, called a *history*, represents a *possible course of history* of our world in the model $\mathcal{M}$. Let *History* be the set of all histories of $\mathcal{M}$. *Instant* is a partition of the set *Time* that satisfies *unique intersection* and *order preservation*. Two moments m and m' are *coinstantaneous* when they belong to the same instant.

(2) The set *Circumstance* of all *possible circumstances* is the subset of the Cartesian product *Time* $\times$ *History* that contains all pairs of the form m/h such that $m \in h$.

(3) *Individuals* is a non empty set of *individual objects*. For any moment m, $Individuals_m$ is the set of individuals existing at that moment according to the model $\mathcal{M}$.

(4) *Concepts* is the non empty set of *individual concepts* and *Attributes* is the set of *attributes* of individuals under concepts considered in model $\mathcal{M}$. For each natural number n, *Attributes*(n) is the non empty subset of *Attributes* containing all *relations* of degree n. The set of attributes contains the unary property of existence and the binary relation of identity and it is closed under the logical operations (truth functional negation, reflexivisation, historic necessitation and possibilitation, generalizations, etc.) defined above. The nature of complex attributes depends on the nature of their arguments. Attributes obtained by applying operations to different attributes are different.

(5) *Context* is the arbitrary non-empty set of *possible contexts of utterance* c, c′, c″, etc. considered in possible interpretation $\mathcal{M}$.

(6) The set *Val* of all possible *denotation assignments to propositional constituents* of the model $\mathcal{M}$ is a proper subset of $((Concepts \times Circumstances) \to (Individuals \cup \{\varnothing\})) \cup \bigcup_n ((Attributes(\mathrm{n}) \times Circumstances) \to \mathcal{P}(Concepts^{\mathrm{n}}))$.

For any individual concept c_e and possible circumstance m/h, $val(c_e, m/h) \in$ *Individuals* when an object falls under individual concept c_e in circumstance m/h according to denotation assignment *val*. Otherwise, $val(c_e, m/h) = \varnothing$.

For any attribute R_n of degree n, $val(R_n, m/h) \in \mathcal{P}(Concepts^n)$. $val(R_n, m/h)$ is the set of n-uples of individual concepts that satisfy according to *val* the attribute R_n in circumstance m/h. The set *Val* contains a *real valuation* $val\mathcal{M}$ that assigns to concepts and attributes their *real denotation* in each possible circumstance according to the model $\mathcal{M}$. In any circumstance at least one individual falls under a concept according to $\mathcal{M}$. Possible denotation assignments respect meaning postulates governing the existence property, the identity relation and logical operations on attributes like truth functional negation, historical necessitation, tautologization, reflexivisation and universal generalization. See later.

(7) The set *Predications* is the subset of $\mathcal{P}(Attributes \cup Concepts)$ containing all sets of propositional constituents with which one can make predications in the language $\mathcal{L}$. Such sets are of the form $\{R_n, c_e^1, \ldots, c_e^k\}$; they contain a single attribute R_n of degree n and a positive number k, $(k \leq n)$, of individual concepts $c_e^1, \ldots, c_e^k$ when n is positive.

Given logical operations on attributes, the power set $\mathcal{P}Predications$ is closed in each model under union and a modal and temporal unary operation *. By definition, for any set $\Gamma \subseteq Predications$, $\cup^*\Gamma$ contains all modal, temporal and generalized predications that one can make from the predications of $\cup\Gamma$. Thus if $\{R_n, c_e^1, \ldots, c_e^n\} \in \cup\Gamma$ then $\{\Box R_n, c_e^1, \ldots, c_e^n\}$, $\{\Box\neg R_n, c_e^1, \ldots, c_e^n\}$, $\{\neg\Box R_n, c_e^1, \ldots, c_e^n\}$, and $\{\neg\Box\neg R_n, c_e^1, \ldots, c_e^n\} \in \cup^*\Gamma$ and similarly for other modal and temporal operations. Moreover for any k such that $1 \leq k \leq n$, $\{\forall_k R_n, c_e^1, \ldots, c_e^{k-1}, c_e, c_e^{k+1}, \ldots, c_e^n\}$, $\{\forall_k \neg R_n, c_e^1, \ldots, c_e^{k-1}, c_e, c_e^{k+1}, \ldots, c_e^n\}$, $\{\neg\forall_k R_n, c_e^1, \ldots, c_e^{k-1}, c_e, c_e^{k+1}, \ldots, c_e^n\}$, and $\{\neg\forall_k \neg R_n, c_e^1, \ldots, c_e^{k-1}, c_e, c_e^{k+1}, \ldots, c_e^n\} \in \cup^*\Gamma$. The set $\cup^*\Gamma$ is also closed under reflexivisation and conjunction. Thus when $\{R_n, c_e^1, \ldots, c_e^n\} \in \cup^*\Gamma$ and $0 < k < m \leq n$, the set $\{Refl(k, m)\ R_n, c_e^1, \ldots, c_e^{m-1}, c_e, c_e^{m+1}, \ldots, c_e^n\}\} \in \cup^*\Gamma$. When $\{R_n, c_e^1, \ldots, c_e^n\} \in \cup^*\Gamma$ and $\{R_n, c_e^{n+1}, \ldots, c_e^{m+n}\} \in \cup^*\Gamma$, $\{(R_n \wedge R_n), c_e^1, \ldots, c_e^n, c_e^{n+1}, \ldots, c_e^{m+n}\} \in \cup^*\Gamma$. As usual, modal and temporal unary operation * obey the following postulates: For any sets Γ, Γ_1 and $\Gamma_2 \in \mathcal{P}Predications$, $\Gamma \subseteq {}^*\Gamma$. Moreover, $^*(\Gamma_1 \cup \Gamma_2) = {}^*\Gamma_1 \cup {}^*\Gamma_2$ and $^{**}\Gamma = {}^*\Gamma$.

(8) $\mathcal{U}$ is a function that associates with each type α the set $\mathcal{U}_\alpha$ of all entities that are possible semantic values of formulas of that type in the model $\mathcal{M}$. By definition, $U_{\#e} = Concepts$; $U_r = Attributes$ and $U_p \subseteq \mathcal{P}[Predications] \times (Circumstances \rightarrow \mathcal{P}(Val))$. The union of all such sets U_α is the *domain* of possible interpretation $\mathcal{M}$.

Explanations. U_p is the set of all *first order propositions* which are considered in the model $\mathcal{M}$. The first term id_1P of each proposition P represents its *structure of constituents*. When an attribute R_n is predicated in a certain order of n individuals under concepts $c_e^1, \ldots, c_e^k$ in a proposition P, the set $\{R_n, c_e^1, \ldots, c_e^k\} \in id_1P$. The set of all propositional constituents of a proposition P is then the union $\bigcup id_1P$. The second term id_2P of a proposition P

is the family of all sets of possible denotation assignments to propositional constituents that are compatible with the truth of that proposition in each circumstance. A proposition P is true in a circumstance m/h according to a possible denotation assignment *val* in possible interpretation $\mathcal{M}$ when $val \in id_2P(m/h)$.

(10) Finally, $|| \ ||$ is a function that associates with each term, predicate or formula A of $\mathcal{L}$ and context c the *sense* $||A||_c$ that A expresses in that context in the interpretation $\mathcal{M}$.

For any individual constant c of the lexicon, $||c||_c \in Concepts$.

For any predicate Rn of degree n of the lexicon, $||R_n||_c \in Attributes(n)$.

- In particular, $c_e \in val(||E_1||_c, (m/h))$ iff $val(c_e, m/h) \in Individuals_m$ and $<c_e^1, c_e^2> \in val(||{=_2}||_c, (m/h))$ iff $val(c_e^1, (m/h)) \in Individuals$ and $val(c_e^1, (m/h)) = val(c_e^2, (m/h))$.
- $||\neg R_n||_c$ is the *truth-functional negation* of n-ary attribute $||R_n||_c$. Thus $<c_e^1, \ldots, c_e^n> \in val(||\neg R_n||_c, m/h)$ iff $<c_e^1, \ldots, c_e^n> \notin val(||R_n||_c, m/h)$.
- $||Settled R_n||_c$ is the *settlednization* of n-ary attribute $||R_n||_c$. Thus $<c_e^1, \ldots, c_e^n> \in val(||Settled R_n||_c, m/h)$ iff for any h' such that $m \in h'$, $<c_e^1, \ldots, c_e^n> \in val(||R_n||_c, m/h')$.
- $||\Box R_n||_c$ is the *historical necessitation* of n-ary attribute $||R_n||_c$. $<c_e^1, \ldots, c_e^n> \in val(||\Box R_n||_c, m/h)$ iff for any possible circumstance m'/h' whose moment m' coinstantaneous with m, $<c_e^1, \ldots, c_e^n> \in val(||R_n||_c, m'/h')$.
- $||Tautological R_n||_c$ is the *obvious tautologization* of n-ary attribute $||R_n||_c$. $<c_e^1, \ldots, c_e^n> \in val(||Tautological R_n||_c, m/h)$ iff for any denotation assignement val' and any circumstance m'/h', $<c_e^1, \ldots, c_e^n> \in val'(||R_n||_c, m'/h')$.
- $||Will R_n||_c$ is the *futurization* of n-ary attribute $||R_n||_c$. Thus $<c_e^1, \ldots, c_e^n> \in val(||Will R_n||_c, m/h)$ iff for some moment $m' > m$, $<c_e^1, \ldots, c_e^n> \in val(||R_n||_c, m'/h)$. And similarly for $||Was R_n||_c$ except that $m' < m$.
- $||Refl(k,m) R_n||_c$ is the *reflexivization* at the k-th and m-th places of attribute $||R_n||_c$. $<c_e^1, \ldots, c_e^n> \in val(||Refl(k,m) R_n||_c, m/h)$ iff $<c_e^1, \ldots, c_e^{m-1}, c_e^k, c_e^{m+1}, \ldots, c_e^n> \in val(||R_n||_c, m/h)$.
- $||\forall_k R_n||_c$ is the *universal generalization* at the k-th place of n-ary attribute $||R_n||_c$, that is to say the attribute of degree $n-1$ such that $<c_e^1, \ldots, c_e^{n-1}> \in val(||\forall_k R_n||_c, m/h)$ iff $<c_e^1, \ldots, c_e^{k-1}, c_e, c_e^{k+1}, \ldots, c_e^n> \in val(||R_n||_c^\sigma, m/h)$ for any individual concept c_e.
- $val(||R_n \} R_m||_c, m/h)$ is the singleton containing the empty sequence iff for any predicate R_k occurring in R_m there is a predicate R'_k in R_n such that $||R'_k||_c = ||R_k||_c$.[6]

[6] As usual, the empty sequence is identified with *the true*. Thus an attribute R_0 of degree 0 is satisfied in a circumstance m/h according to *val* when the empty sequence belongs to $val(R_0, m/h)$.

– $||R_n \wedge R_m||_c$ is the *truth-functional conjunction* of attributes $||R_n||_c$ and $||R_m||_c$. Thus $<c_e^1, \ldots, c_e^n, c_e^{n+1}, \ldots, c_e^{n+m}> \in val\,||(R_n \wedge R_m)||_c, m/h)$ iff $<c_e^1, \ldots, c_e^n> \in val\,||R_n||_c, m/h)$ and $<c_e^{n+1}, \ldots, c_e^{n-m}> \in val\,||R_m||_c, m/h)$.

For each propositional formula A in each context c, the first order proposition $||A||_c \in U_p$ expressed by A in the context c according to model $\mathcal{M}$ is defined as follows:

– When R_n is a predicate of degree n of the lexicon, $id_1\,||[R_n t_1 \ldots t_n]||_c = \{||R_n||_c, ||t_1||_c, \ldots, ||t_n||_c\}$.
– When R_n is of the form $\neg R'_n$, $id_1\,||[R_n t_1 \ldots t_n]||_c = id_1\,||[R'_n t_1 \ldots t_n]||_c$
– When R_n is of the form $(R_n \wedge R_m)$, $id_1\,||[(R_n \wedge R_m) t_1 \ldots t_{n+m}]||_c = id_1\,||[R_n t_1 \ldots t_n]||_c \cup id_1\,||[R_m t_{n+1} \ldots t_{n+m}]||_c$.
– When R_n is of the form $\Box R'_n$, *Settled*R'_n, *Will*R'_n, *Was*R'_n or *Tautological*R'_n, $id_1\,||[R_n t_1 \ldots t_n]||_c = {}^*id_1\,||[R'_n t_1 \ldots t_n]||_c$.
– When R_n is of the form $(Refl(k,m)R'_n$, $id_1\,||[(Refl(k,m)R'_n t_1 \ldots t_{n-1}]||_c = \{||R'_n||_c, ||t_1||_c, \ldots, ||t_{n-1}||_c\}$.
– When R_n is of the form $\forall_k R_n$, $id_1\,||[\forall_k R_n t_1 \ldots t_{n-1}]||_c = {}^*\{||R_n||_c^\sigma, ||t_1||_c, \ldots, ||t_{n-1}||_c\}$.
– When R_n is of the form $(R_n \} R_m)$, $id_1\,||[(R_n \} R_m)]||_c = {}^*\{||(R_n \} R_m)||_c\}$.
– Furthermore, for any simple or complex predicate R_n, $id_2\,||[R_n t_1 \ldots t_n]||_c\,(m/h) = <||t_1||_c, \ldots, ||t_n||_c> \in val\,(||R_n||_c\,(m/h))$.

Definition of truth and validity

A propositional formula A_p of $\mathcal{L}$ is true in a circumstance m/h in a standard model $\mathcal{M}$ iff $||A_p||$ is true in m/h according to $val\mathcal{M}$. The formula A_p is valid (in symbols: $\models A_p$) when it is true in all possible circumstances according to all standard models.

4. Valid laws

As one can expect, all instances in language $\mathcal{L}$ of classical axiom schemas of the first order predicate calculus without identity, S5 modal logic for settled truth, historic and universal necessities and branching temporal logic are valid formulas. Here is a list of important fundamental valid laws. I will mention a few laws which are not valid.

Valid schemas for obvious tautologies

(T1) $\models$ *Tautological*$A_p \Rightarrow \blacksquare A_p$. Notice that $\not\models \blacksquare A_p \Rightarrow$ *Tautological*A_p.
(T2) $\models$ *Tautological*$A_p \Rightarrow$ *Tautological Tautological*A_p

(T3) $\vDash \neg \textit{Tautological}\, A_p \Rightarrow \textit{Tautological}\ \neg \textit{Tautological}\, A_p$

(T4) $\vDash \textit{Tautological}\, A_p \Rightarrow (\textit{Tautological}(A_p \Rightarrow B_p) \Rightarrow \textit{Tautological}\, B_p)$

(T5) $\vDash \textit{Tautological}\, B_p) \Rightarrow (\textit{Tautological}(A_p \Rightarrow B_p) \Rightarrow \textit{Tautological}\, A_p)$

(T6) $\vDash (A_p \} B_p) \Rightarrow \textit{Tautological}(A_p \} B_p)$

(T7) $\vDash \textit{Tautological}\, A_p \Rightarrow\ \forall v A_p$

(T8) $\vDash \neg(A_p \} B_p) \Rightarrow \textit{Tautological}\neg(A_p \} B_p)$

Valid schemas for generalization

($\forall$1) The law of universal instantiation. $\vDash [\forall_k R_n t_1 \dots t_{n-1}] \Rightarrow [R_n t_1 \dots t_{k-1}\ t\ t_k \dots t_{n-1}]$ for any individual constant t. Similarly $\vDash [R_n t_1 \dots t_{k-1}\ t\ t_k \dots t_{n-1}] \Rightarrow [\exists_k R_n t_1 \dots t_{n-1}]$ for any constant t.
However $\nvDash [R_n t_1 \dots t_n] \Rightarrow \textit{Denotes}(t_k)$ and $\nvDash [R_n t_1 \dots t_n] \Rightarrow [E_1 t_k]$ for $0 \leq k \leq n$.

($\forall$2) When v occurs successively at the k_1th, $\dots$, k_mth place in $[R_n t_1 \dots t_n]$ and $d_1, \dots, d_m$ are the first, …, and the last individual term in $[R_n t_1 \dots t_n]$ different from v, $\vDash \textit{Tautological}\ (\forall v[R_n t_1 \dots t_n] \Leftrightarrow [\forall k_1(\textit{Refl}(k_1, \dots, k_m)) R_n\ d_1, \dots, d_m])$
In particular, $\vDash \textit{Tautological}(\forall v[R_n t_1 \dots t_n] \Leftrightarrow [\forall k R_n t_1 \dots t_{k-1}\ t_{k+1} \dots t_{n-1}])$, when v is only the kth individual term of $[R_n t_1 \dots t_n]$.

($\forall$3) $\vDash (\forall v \blacksquare A_p) \Leftrightarrow (\blacksquare \forall v A_p)$. Barcan formulas and their converse are valid for logical necessity and other modal connectives. However, $\nvDash (\blacksquare \exists v A_p) \Rightarrow (\exists v \blacksquare A_p)$.

Valid schemas for propositional composition

(C1) $\vDash A_p \} A_p$

(C2) $\vDash (A_p \} B_p) \Rightarrow ((B_p \} C_p) \Rightarrow (A_p \} C_p))$

(C3) $\vDash (A_p \wedge B_p) \} A_p$

(C4) $\vDash (A_p \wedge B_p) \} B_p$

(C5) $\vDash \Box A_p \} A_p$

(C6) $\vDash ((C_p \} A_p) \wedge (C_p \} B_p)) \Rightarrow C_p \} (A_p \wedge B_p)$

(C7) $\vDash A_p \}\{ \neg A_p$

(C8) $\vDash \Box A_p \}\{ \textit{Tautological}\, A_p$. Similarly for $\textit{Will}\, A_p$, $\textit{Was}\, A_p$, $\textit{Settled}\, A_p$ and $\forall v A_p$

(C9) $\vDash (A_p \} B_p) \}\{ \Box(A_p \wedge B_p)$

(C10) $\vDash [R_n t_1 \dots t_{k-1}\ t\ t_k \dots t_{n-1}] \} [\exists_k R_n t_1 \dots t_{n-1}]$

(C11) $\vDash \Box\neg A_p \}\{ \Box A_p$

(C12) $\vDash \Box(A_p \wedge B_p) \}\{ (\Box A_p \wedge \Box B_p)$

(C13) $\vDash \Box\Box A_p \}\{ \Box A_p$

Valid schemas for strong implication

(S1) $\models (A_p \mapsto B_p) \Rightarrow (A_p \} B_p)$. Consequently $\models [R_n t_1 \ldots t_{k-1}\ tt_k \ldots t_{n-1}] \mapsto [\exists_k R_n t_1 \ldots t_{n-1}]$;
$\models [R_n t_1 \ldots t_{k-1}\ tt_k \ldots t_{n-1}] \mapsto [\blacklozenge R_n t_1 \ldots t_{n-1}]$.
But $\not\models (\forall v A_p \mapsto [t/v] A_p)$. For $\not\models (\forall v A_p \} [t/v] A_p)$.

(S2) $\models (A_p \mapsto B_p) \Rightarrow$ *Tautological* $(A_p \Rightarrow B_p)$.
Consequently strong implication is finer than analytic and strict implications.
For $\not\models (A \multimap B) \Rightarrow$ *Tautological* $(A_p \Rightarrow B_p)$.

(S3) $\models (A_p \mapsto B_p) \wedge (B_p \mapsto A_p) \Leftrightarrow (A_p = B_p)$. Strong implication is an equivalence relation.

Valid schemas for propositional identity

(I1) $\models A_p = A_p$

(I2) $\models (A_p = B_p) \Rightarrow (C \Rightarrow C^*)$ where C* and C are propositional formulas which differ at most by the fact that an occurrence of B_p in C* replaces an occurrence of A_p in C.

(I3) $\models (A_p = B_p) \Rightarrow$ *Tautological* $(A_p = B_p)$

(I4) $\models \neg(A_p = B_p) \Rightarrow$ *Tautological* $\neg(A_p = B_p)$

(I5) $\models [\mathit{Refl}(k, m) R_n t_1 \ldots t_n] = [R_n t_1 \ldots t_{m-1}, t_k, t_{m+1}, \ldots, t_{n-1}]$

(I6) $\models ([R_n t_1 \ldots t_n] = [R_n d_1 \ldots d_m]) \Rightarrow ([\hat{} R_n = \hat{} R_m]) \wedge (([\hat{} t_1 = \hat{} d_1] \vee \ldots \vee [\hat{} t_1 = \hat{} d_m]) \wedge \ldots \wedge ([\hat{} t_n = \hat{} d_1] \vee \ldots \vee [\hat{} t_n = \hat{} d_m])) \wedge (([\hat{} d_1 = \hat{} t_1] \vee \ldots \vee [\hat{} d_1 = \hat{} t_n]) \wedge \ldots \wedge ([\hat{} d_m = \hat{} t_1] \vee \ldots \vee [\hat{} d_m = \hat{} t_n]) \wedge (\mathit{Tautological}([R_n t_1 \ldots t_n] \Leftrightarrow [R_m d_1 \ldots d_m]))$.

All the classical *Boolean laws of idempotence, commutativity, associativity* and *distributivity* are valid laws of propositional identity: $\models A_p = (A_p \wedge A_p)$; $\models (A_p \wedge B_p) = (B_p \wedge A_p)$ and $\models \blacksquare(A_p \wedge B_p) = (\blacksquare A_p \wedge \blacksquare B_p)$. Classical laws of *reduction* are also valid: $\models \neg\neg A_p = A_p$. Identical propositions need not be *intensionally isomorphic*. The order and number of applications of propositional operations does not always affect the logical form. Unlike hyperintensional logic (Cresswell 1975), my predicative logic does not require intensional isomorphism. But it requires more than the *co-entailment* advocated in the logic of relevance (Anderson, Belnap & Dunn 1992). As M. Dunn (1992) pointed out, it is unfortunate that A_p and $(A_p \wedge (A_p \vee B_p))$ co-entail each other because it allows for the introduction of new senses. $\not\models A_p \mapsto (A_p \wedge (A_p \vee B_p))$ because $\not\models (A_p \} B_p)$.

Valid schemas for identity between individuals

(=1) The identity relation between individuals is symmetric and transitive.

(=2) But it is not reflexive. $\nvDash [=_2 \text{ t t}]$. For $\vDash [=_2 \text{ t t}] \Rightarrow$ *Denotes*(t). The same holds for definite descriptions. For $\nvDash [R_n t_1 \ldots t_n] \Rightarrow$ *Denotes*$(t_k) \wedge (([=_2 t_k t_1] \wedge [=_2 t_k t_2] \Rightarrow [=_2 t_k t_2]))$.

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