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ON ANALYTIC *A POSTERIORI* STATEMENTS: ARE THEY POSSIBLE?

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ABSTRACT

Traditionally, the notions of analyticity, aprioricity and necessity have been considered coextensive, and also their counterparts, namely, syntheticity, aposterioricity and contingency. Such coextensiveness has been questioned by philosophers like Kant and Husserl who, on the basis of very different definitions of analyticity, postulated the existence of synthetic a priori statements and, on the other hand, by Kripke, who argued for the existence of contingent a priori and necessary a posteriori statements. In this paper, on the basis of a new definition of analyticity that can be seen as a refinement of Husserl's, it is argued for the existence of analytic a posteriori instantiations of analytic laws.

KEYWORDS

Saul Kripke · Edmund Husserl · necessity · analyticity · aprioricity.

1. Preliminaries

Traditionally, the notions of necessity and aprioricity, on the one hand, and the notions of contingency and aposterioricity, on the other, were considered to have the same extension. In his seminal papers 'Naming and Necessity' (Kripke 1972) and 'Identity and Necessity' (Kripke 1977), Saul Kripke challenged the received view and distinguished between the metaphysical notions of necessity and contingency, on the one hand, and the epistemological notions of *a priori* and *a posteriori*, on the other. Kripke attempted to offer examples both of contingent *a priori* and of necessary *a posteriori* statements, in the last case basing his examples on the questionable contention that strict proper names are rigid designators. Notwithstanding the fragility of Kripke's examples, the fact of the matter is that the distinction between the two pairs of notions retains its importance irrespectively.

A third pair of related notions has traditionally been related to the two former ones, namely, that of analyticity and syntheticity. It has usually been considered that the notion of analyticity has the same extension as that of aprioricity and, thus, *a fortiori* of that of necessity. And though three of the

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greatest philosophers ever, namely, Kant, Frege and Husserl have questioned the identification of syntheticity with contingency and aposterioricity, arguing for the existence of synthetic *a priori* statements, especially in empiricist circles the three notions have been considered as being at least extensionally equivalent. But the notions of analyticity and syntheticity are neither epistemological nor metaphysical, but semantic. Hence, an argument needs to be offered to establish the extensional equivalence of the semantic notions be it with the metaphysical or with the epistemological notions. In this paper, however, it will be shown that the semantic notions of analyticity and syntheticity are extensionally equivalent neither to the metaphysical notions of necessity and contingency nor to the epistemological notions of aprioricity and aposterioricity.

2. On Analyticity

The task of defining analyticity has been a very hard one. Since Kant characterized a statement as analytic when the concept of its predicate is included in the concept of its subject,[1] but in the same work later characterized analytic statements as those derivable from the Principle of Non-Contradiction,[2] and, thus, really offered two non-equivalent characterizations of analyticity, there have been multiple attempts to define that elusive notion, as attested by the list of more than sixty enumerated by Jan Wolenski in his 'Analytic vs. Synthetic and A Priori vs. A Posteriori' (Wolenski 2004).

In any case, the two definitions of analyticity best known in analytic circles are those of Frege and Carnap, both of which are refinements of Kant's two different notions of analyticity. Carnap's characterization of analytic statements as those whose truth could be known by the mere analysis of the concepts involved[3] is inspired by Kant's first characterization, and seems vulnerable both with respect to Quine's objections in his famous 'Two Dogmas of Empiricism' (Quine 1953) and to the objection that the truth of statements like 'All bachelors are not married' is dependent on the historical and, thus, empirical evolution of language. Frege's characterization of analyticity in *Die Grundlagen der Arithmetik* (Frege 1984), according to which a statement is analytic if it can be derived from the logical principles and definitions is clearly a refinement of Kant's second characterization and, though it is immune to Quine's criticism, it faces the difficulties resulting from the collapse of logicism, namely, that since arithmetic and the whole

[1] *Kritik der reinen Vernunft*, (Kant 1990, A7-8, B11-12).
[2] Ibid., A150-153, B190-193.
[3] See Carnap (1947, enlarged edition 1956), and, especially, his paper 'Meaning Postulates' of 1952, included as Appendix B in the enlarged edition.

analysis cannot be derived exclusively from logical laws and definitions, on the basis of Frege's notion of analyticity, arithmetical statements and statements of mathematical analysis would have to be considered as synthetic.

A less well-known but more solid definition of analyticity is that of Husserl, according to which a statement is analytic if it is true and its truth can be completely formalized *salva veritate*, that is, without its truth being affected. Husserl's definition is immune both to Quine's criticism and to the demise of logicism. Nonetheless, it seems more adequate as a definition of logical truth[4] and would face the problem that concrete number-theoretic truths, like $1^3+2^3+3^3+4^3=100$ or even more trivial ones like '2 is both even and prime' would turn out to be non-analytic, since they cannot be completely formalized *salva veritate*. In any case, though Husserl's definition based on logical form is on the right track, it seems to be a syntactical definition of a semantic notion. Thus, the task is to offer a definition of analyticity based not on syntactical but on semantic form.

In my papers 'Husserl on Analyticity and Beyond' (Rosado Haddock 2008) and 'Some Uses of Logic in Rigorous Philosophy' (Rosado Haddock 2010)[5] I have offered a new definition of analyticity based on the semantic form of statements, in fact a model-theoretical definition, namely: A statement is analytic if it is true in a model M and when true in a model M, it is true in any model M^* isomorphic to M. In other words, a statement σ is analytic whenever (i) $\{\sigma\}$ has a model and (ii) if $\{\sigma\}$ has a model M, then any structure M^* isomorphic to M is also a model of $\{\sigma\}$, that is $MOD\{\sigma\}$ is closed under isomorphisms. Such a definition was intended to capture a semantic property of mathematical (and also logical) statements, not shared with any other sort of statement. However, I now acknowledge that the definition is too wide and would admit as analytic statements like, e.g. 'Two colours cannot cover the same surface at the same time', which clearly have material content, though they seem to be true in any physical world. Moreover, one could also argue that the laws of physics are supposed to be invariant under isomorphisms, though they are certainly not true in any physical world. In any case, it seems pertinent to introduce an additional clause that can serve to exclude exactly those two sorts of statements without excluding any mathematical statement.[6] One could try to add a third clause excluding analytic statements from having empirical content.

[4] In the second volume (2005) of his masterful *Logical Forms*, Oswaldo Chateaubriand (Chateaubriand 2001) characterized logical truth in a similar way to Husserl's definition of analyticity. I side with Chateaubriand against Husserl on this point.

[5] See also my brief treatment of the definition of analyticity in Rosado Haddock (2007).

[6] In my previous papers touching on this issue I have considered only mathematical statements and seem to have tacitly assumed that analytic statements do not have any sort of material content, since that is what is meant when one says that they are true in virtue of their semantic form. An objection to the former version of the definition made by Jairo da

Such a restriction would certainly exclude physical laws of low-level and at most physical laws of higher level – what Husserl called[7] *hypotheses cum fundamento in re*, like the law of gravitation in classical mechanics –, whose relation to experience is somewhat tenuous, though their explanatory power and their role in systematization of our empirical knowledge is fundamental. However, with such an additional clause, the definition would still be too wide, since statements with material content true in any physical world – statements like those Husserl considered synthetic *a priori*[8] – would still be considered analytic. On the other hand, one could try to add a clause excluding the occurrence of all constants in analytic statements. Such an additional clause, however, would not only exclude all statements about any physical world, but will also exclude arithmetical statements from being analytic. In fact, it would make the definition essentially equivalent to Husserl's. Hence, the definition would be too narrow. Thus, one has to find a clause intermediate in strength between those two. The clause should read as follows: (iii) σ, or better $\{\sigma\}$ should not imply or presuppose the existence either of a physical world or of a world of consciousness. Therefore, the definition should now read as follows: A statement σ is analytic if and only, if: (i) $\{\sigma\}$ has a model, (ii) if $\{\sigma\}$ has a model M, then any structure M^* isomorphic to M is also a model of $\{\sigma\}$, and (iii) $\{\sigma\}$ does not imply or presuppose the existence either of a physical world or of a world of consciousness. This new definition, however, forces us to make some additional distinctions.

There are essentially two sorts of analytic statements, namely, those that contain mathematical constants – like the two number-theoretic examples mentioned above – and those that do not contain any constants and could appropriately be called "analytic laws". Let us, following Husserl[9], call a statement an "analytic necessity" if it is obtained from an analytic law by instantiation (or exemplification), that is, by the usual method of replacement of occurrences of a variable by a constant (and the corresponding deletion of the corresponding quantifier – in our case, a universal quantifier). Since such constants do not need to be mathematical constants, it is clear that analytic necessities, in general, do not satisfy the third clause of the definition of analyticity.

Silva, in the sense that the definition was too wide, made me reconsider it and make the assumption explicit.

[7] See, for example Husserl (1900), *Logische Untersuchungen I*, Chapter IV, §23 as well as Chapter XI, §§62-66 for a more thorough discussion of Husserl's views on explanatory versus descriptive sciences.

[8] See *Logische Untersuchungen II*, U. III, §12.

[9] Ibid. It should be pointed out that my use of the expresion "analytic necessity" is just a "façon de parler" and it should be clearly distinguished from my use of the concept of mathematical necessity in the next §. See also Chateaubriand's *Logical Forms II*, Chapter 18, for a distinction parallel to Husserl's but concerning logical truth.

3. Some Conceptual Elucidations

The present definition – even in its now abandoned original version – is certainly immune to the three objections brought against the other three attempts to define analyticity already mentioned. Moreover, the resulting notion of analyticity does not coincide with that of categoricity, since $\{\sigma\}$ can very well be analytic but have models that are not isomorphic. In fact, under this definition, not only all number-theoretic theorems turn out to be analytic, but also statements like the commutative law for groups, true only in all Abelian groups but not in all groups, and Skolem's statement asserting that there is a number larger than any natural number, a statement true only in non-standard models of first-order arithmetic, including their elementary extensions, which can have any infinite cardinality. Hence, the notion of analyticity does not coincide either with that of categoricity or with that of necessity, that is, of truth in any possible world (or in any possible world in which the objects referred to by designators in the statement exist).[10] The commutative law for Abelian groups is certainly not necessary, since it is not true in a world populated by all groups, and Skolem's existence statement for a number larger than all natural numbers is not necessary, since it is not true in the standard model of first-order arithmetic and certainly not true in any model of second-order arithmetic.

A similar example of an analytic but not necessary statement can be obtained from general topology.[11] Hausdorff spaces are topological spaces, but not all topological spaces are Hausdorff spaces. In order for a topological space to be Hausdorff, it has to satisfy the following condition ©: any two distinct points α and β have non-intersecting neighbourhoods, that is, in the topological space there exist open sets A and B such that $\alpha \in A$ and $\beta \in B$ and $A \cap B = \emptyset$, briefly, disjoint points have disjoint neighbourhoods. If a topological space T satisfies condition ©, then all spaces T^* isomorphic to T satisfy ©. Moreover, as any genuine mathematical statement, it does not presuppose or imply the existence of any physical world or world of consciousness. Hence, © is an analytic statement, according to the above definition. Nonetheless, it is not necessary for all topological spaces to satisfy condition ©. Therefore, © is an analytic but not necessary statement.

In order to avoid some misunderstandings, the following should be stressed before continuing. Firstly, since analyticity and necessity have been shown to be different, there is nothing abnormal when an analytic statement

[10] When dealing with abstract mathematical entities Leibniz's characterization of necessity would be sufficient. Nonetheless, the notion of mathematical necessity used below is neither Kripkean nor even Leibnizian in a strict sense, but a sort of analogue of the latter adapted to mathematical structures.

[11] See any good book on general topology, e.g., Thron (1966) or Kelley (1955).

σ is true in a structure *M* and its negation, namely $\neg\sigma$, is true in other structures not isomorphic to the structure *M*. In fact, mathematical statements usually are true only in families of structures, not in all structures, and are either not defined or not true in structures not isomorphic to those in which they are true. I have defined analyticity to capture precisely that "truth in virtue of its semantic form" that presumably distinguishes mathematical from empirical and, in general, synthetic statements, whereas logical truths, which are supposed to be true in any model, are simply a limiting case of analyticity. In fact, logical truths are not only necessary – as is the case of axioms defining general mathematical structures –, but are also true in any possible circumstance, under any interpretation. In fact, we can distinguish here three different concepts corresponding to three different levels of conceptual generality[12], namely, from the more general to the less general: (i) a logically true statement is a statement true in any possible model; (ii) a mathematical necessary statement is a statement true in any model of the axioms of a mathematical theory, for example, true in any topological spaces, in any groups, in any rings, etc.[13]; (iii) an analytic statement is a statement true in at least one model as well as in any model isomorphic to a model of the statement, and such that it does not imply or presuppose the existence of any physical world or world of consciousness, thus, it does not have any content besides mathematical content.

Secondly, it should be stressed that although my definition of analyticity is clothed in model-theoretic vocabulary, that does not mean that we are in any sense bound to classical first-order model theory. In fact, "isomorphism" is not a first-order notion, like its first-order approximation "elementary equivalence", and the notion of model is also not limited to first-order theories. Hence, the definition of analyticity is in no way bound to first-order languages.

However, since the notions of aprioricity and aposterioricity are epistemological, whereas the notions of analyticity and syntheticity are semantic, it still needs to be examined whether there exist statements that are analytic, but are not *a priori*. Since it has already been shown that analyticity and necessity do not coincide, we will examine a collection of statements that are *a posteriori*, but seem to be necessary and even analytic. In fact, it will be shown that though the clause added above prevents analytic laws and other strictly analytic statements from being *a posteriori*, some instantiations of analytic laws, that is, some analytic necessities can be *a posteriori*.

[12] Of course, the extensions of the concepts are in reverse order, being the extension of the concept of analyticity the widest of the three concepts.

[13] I am perfectly conscious that this distinction makes the concept of mathematical necessity have a somewhat fuzzy extension, since what has been called "mathematically necessary" at one moment in history could be 'degraded' to being 'merely' analytic by the consideration of more abstract structures. Nonetheless, the two more important notions, those of logical truth and analyticity, have a fixed meaning and extension once and for all.

4. On Analytic Necessities that are *a posteriori*

It is said that the great Gauß once conceived the possibility of measuring the angle of a triangle formed by three mountains with the hope of definitely establishing whether space was Euclidean or non-Euclidean. After the advent of non-Euclidean geometries and, especially, of general relativity the belief in the empirical nature of physical space has been widely accepted. Thus, on the one hand, there are the geometrical multiplicities, the n-extended magnitudes of which the great Riemann spoke, some of them three-dimensional, some four-dimensional and, in general, for every natural number n, n-dimensional manifolds, some of them Euclidean, others Riemannian[14] and others Lobaschevskian. On the other hand, there is physical space, whose dimensionality and structure are, contrary to our old friends Kant and Frege's views, to be empirically determined. Thus, let us suppose that physicists are able to measure the structure, not of the space between the three mountains near Göttingen, as Gauß hoped, but of a big chunk of intergalactic space. Let us suppose that the result of such measurement is that the sum of the angles of the triangle is less than (or greater than) 180 degrees. Hence, the structure of space is Lobaschevskian (respectively, Riemannian). Therefore, the theorems of three-dimensional Lobaschevskian (respectively, Riemannian) geometry are all true for physical space.[15] Moreover, such theorems of Lobaschevskian (respectively, Riemannian) geometry are true not only for physical space, but for any structure isomorphic to physical space. Thus, if as a result of the measurements we conclude that space (or space-time) is Lobachevskian, the following three statements are true in our physical world and in any world isomorphic to our physical world: (i) rectangles do not exist, and all triangles have angle sum less than 180 degrees; (ii) it is impossible to magnify or shrink a triangle without distortion; (iii) if *l* and *l** are any distinct parallel lines, then any set of points in *l* equidistant from *l** has at most two points in it.[16] According to the first two clauses of my definition of analyticity – that is, to the older now abandoned version of the definition –, those three statements, as all other theorems of Lobaschevskian geometry, would be analytic. On the other hand, our knowledge of the truth of those theorems in physical space was empirically obtained and could not be obtained otherwise. Thus, our knowledge of them is *a posteriori*.

[14] In this paper we only use the term "Riemanian" in the restricted sense of geometrical manifolds with positive curvature.

[15] For simplicity, we speak here of 'space', not of '(four-dimensional) space-time', but nothing in our argument would change if we did.

[16] For those three statements, see, e.g., Greenberg (1973, pp. 150-152).

However, if one takes into account – as one should do – the third clause of my definition of analyticity finer distinctions are required. What was empirically obtained was not a pure statement of Lobaschevskian geometry but one of its instantiations, namely, the statement: 'In our physical universe the sum of the angles of a triangle are less than 180°'. That statement, and the other similar statements, for example, 'In our physical universe if *l* and *l** are two distinct parallel lines, then any set of points in *l* equidistant from *l** has at most two points in it', though not only true in our physical world, but also in any other physical world isomorphic to ours[17], express structural features of our physical world and certainly presuppose the existence of that physical world. They should be clearly distinguished from the corresponding statements of Lobaschevskian geometry, which are pure mathematical statements, do not refer to any world, and are clearly both *a priori* and analytic on the basis of my definition of analyticity. Contrary to the latter, the above quoted statements are not analytic laws of three-dimensional geometric manifolds with negative curvature, since though they satisfy the first two of the three clauses of our definition of analyticity they are not free of all material content and cannot satisfy the third clause. Thus, such statements are really instantiations of analytic laws, that is, they are analytic necessities. Hence, one can conclude that there exist statements that are analytic necessities and are, nonetheless *a posteriori*.[18]

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[17] We are here presupposing that an isomorphism between physical structures does not present any difficulties. Nonetheless, one would need to fix the individuals that are members of the universe, which could very well be either mass points or space-time points, and somehow presuppose that there is a bijection between the individuals of our physical world and the individuals of those other possible physical worlds, as well as that the structure of our physical world is completely given by its physical laws – which could very well not be the currently accepted ones. Thus, there is a lot of idealization when one speaks about isomorphisms between our physical world and other possible physical worlds.

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