Logique & Analyse **228** (2014), 637–661

INQUIRY IN CONVERSATION: TOWARDS A MODELLING IN INQUISITIVE PRAGMATICS

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Abstract

Conversation is one of the main contexts in which we are conducting inquiries. Yet, little attention has been paid so far in pragmatics or epistemology to the process of inquiry in conversation. In this paper, we propose to trigger such an investigation through the development of a formal modelling based on inquisitive pragmatics — a framework offering a semantic representation of questions and answers, along with an analysis of the pragmatic principles that govern questioning and answering moves in conversations geared towards information exchange. Our starting observation is that an interrogative inquiry in a conversation takes the form of a finite *sequence* of questioning and answering steps, and appears thereby as an inherently *temporal process*. The central notion of *interrogative protocol* introduced in this paper precisely aims to capture the temporal dimension of inquiry. Interrogative protocols are defined as branching-time tree structures encoding all the possible sequences of questioning and answering steps — i.e., all the possible inquiry paths — that can subsequently occur in a starting conversational situation in accordance with the principles of inquisitive pragmatics. They provide us with a formal environment to define and investigate the epistemological notions of *interrogative inquiry* and *interrogative consequence* in conversational contexts. One of the main interests of the resulting framework is to enable a formal investigation of central epistemological issues relative to interrogative inquiry under the form of computational problems. We frame three key inquiry problems along this line, and we then propose an algorithmic procedure for solving them in the restricted case where the inquirer has at her disposal a finite number of questions in her inquiry. We conclude by relating our approach to Wiśniewski's Erotetic Search Scenarios and Hintikka's Interrogative Model of Inquiry.

1. Introduction

Conversation is one of the main contexts in which we are conducting inquiries. Our daily life witnesses to this as we often find ourselves engaged in conversation in order to be brought in specific epistemic states, and we do so by requesting information from others through questioning. Such a process of information-seeking by questioning has received particular attention in epistemology through the work of Jaakko Hintikka [12, 13]. In his

doi: 10.2143/LEA.228.0.3078177.

recent book entitled *Socratic Epistemology* [13], Hintikka argues that epistemology should focus on information acquisition, and proposes to conceptualize any information acquisition process as a questioning process, a perspective directly inspired by the Socratic *elenchus* — the method by which Socrates conducts philosophical inquiries by addressing questions to his interlocutors. Yet, even though the Socratic dialogues are probably one of the most characteristic examples of inquiry in conversation and the direct source of inspiration for Socratic Epistemology, little attention has been paid so far to the process of inquiry in the particular context of *conversation*. An investigation of the process of inquiry in conversation at the crossroad of epistemology and pragmatics remains to be developed.

How to model inquiry in a conversational context? In its simplest form, an inquiry in a conversation consists in a sequence of questions successively addressed by an inquirer to designated conversational participants.[1] Thus, modelling inquiry in conversation requires at least the following components: (i) an *epistemic* representation of the informational setting of a conversation, i.e., of the information states of the participants and the common ground of the conversation, (ii) a *semantic* representation of questions and answers, along with an account of how they modify the informational setting of the conversation, (iii) a *pragmatic* representation of the rules governing questioning and answering moves in conversation. A formal framework has recently been developed that precisely offers a modelling of the components (i)-(iii), namely *inquisitive pragmatics* [11]. Inquisitive pragmatics is based on inquisitive semantics which yields a semantic representation of questions and answers, along with an account of how they modify the informational setting of a conversation. On top of this, inquisitive pragmatics provides an analysis of the regulative principles that govern conversations geared towards information exchange, and in particular an account of the rules that govern questioning and answering moves. These features make inquisitive pragmatics an ideal starting point to develop a modelling of inquiry processes in conversational contexts.

Our aim in this paper is to investigate the process of inquiry in conversation through a formal modelling based on inquisitive pragmatics. Even though inquisitive pragmatics provides a formal representation of the components (i)-(iii), modelling inquiry requires an additional formal structure to represent *sequences* of interrogative steps, i.e., to capture the *temporal* dimension of inquiry processes. To this end, our approach will consist in introducing the notion of *interrogative protocol* which aims to encode all the possible sequences of questioning and answering steps, i.e., all the possible inquiry paths, that can subsequently occur in a starting conversational

[1] Notice that this is exactly the form inquiry takes in the Socratic dialogues, namely sequences of questions asked by Socrates and directed towards his interlocutors.

situation in accordance with the rules of inquisitive pragmatics. The notion of protocol is widely used in computer science and refers to a set of rules governing a temporal process. Such protocols are often represented extensively as branching-time tree structures, which encode all the possible paths that a temporal process can take in accordance with some rules. We will follow this practice here, and represent interrogative protocols as branching-time tree structures, where a path in an interrogative protocol will represent a possible sequence of questioning and answering steps starting from an initial conversational situation. Branching in an interrogative protocol can then occur at the level of both questioning and answering steps, respectively when different choices of questions to ask or answers to provide are available. As we shall see, interrogative protocols provide a formal setting in which one can define and investigate inquiry processes in conversations.

This paper is organized as follows. We first provide in section 2 a brief presentation of inquisitive semantics and pragmatics. In section 3, we introduce the notion of *conversational state*, which formalizes the informational setting characterizing a given stage in a conversation, and we provide a description of the way questioning and answering moves modify conversational states in accordance with the principles of inquisitive pragmatics. These elements will be used in section 4 to build the central notion of *interrogative protocol*, which in turn will allow us to define the epistemological notions of *interrogative inquiry* and *interrogative consequence*. Section 5 shows how our formal framework allows to address central epistemological issues relative to the process of inquiry in conversation under the form of computational problems. Section 6 ends this paper with a brief summary, along with some remarks relating our approach to Wiśniewski's Erotetic Search Scenarios [24] and Hintikka's Interrogative Model of Inquiry [12, 13].

2. Inquisitive Pragmatics in a Nutshell

Inquisitive pragmatics [11] is a framework that aims to analyze and capture the rules governing a certain type of conversation geared towards information exchange.[2] The specificity of inquisitive pragmatics is to take in account not only the *informative* content of the sentences uttered in a conversation, i.e., the capacity of such sentences to provide information, but also their *inquisitive* content, i.e., their capacity to raise issues. To enable this, inquisitive pragmatics is based on *inquisitive semantics*, a recently developed semantic theory that provides a notion of meaning which represents

[2] Further developments on inquisitive pragmatics are currently the focus of on-going research, see in particular [22]. In this paper, we will be exclusively concerned with the version of inquisitive pragmatics developed in [11].

both the informative and inquisitive contents of sentences in an integrated way.[3] The inquisitive dimension of inquisitive pragmatics makes it an ideal framework for investigating questioning processes in conversation.

In this paper, we will be exclusively concerned with the questioning and answering moves occurring in this type of conversation, and we will therefore only used part of the whole inquisitive framework. We now present the elements of inquisitive semantics and pragmatics that will be directly useful to our purpose: we first explain how questions and answers are represented in inquisitive semantics, and we then briefly present how inquisitive pragmatics accounts for the regulative principles of conversations concerned with information exchange.

2.1. *Questions and Answers in Inquisitive Semantics*

As a general approach to meaning, inquisitive semantics can serve as a framework for various concrete systems. In this paper, we will make use of the recently developed propositional system called InqD [4]. The particularity of InqD is to define inquisitive semantics for a language that syntactically distinguishes between declaratives and interrogatives.[4] The language $\mathcal{L}$ of InqD is defined as follows:[5]

Definition 2.1 (Language $\mathcal{L}$). *Let $\mathcal{P}$ be a set of atomic sentences. The language $\mathcal{L}$ is defined as the union of the languages $\mathcal{L}_!$ and $\mathcal{L}_?$ where:*

- *$\mathcal{L}_!$ is the declarative fragment of $\mathcal{L}$ and is defined as the smallest set containing $\mathcal{P}$ and $\bot$ and closed under conjunction and implication:*[6]
  - *If $\alpha, \beta \in \mathcal{L}_!$, then $\alpha \wedge \beta \in \mathcal{L}_!$,*
  - *If $\alpha, \beta \in \mathcal{L}_!$, then $\alpha \rightarrow \beta \in \mathcal{L}_!$.*

[3] For an overview of the developments relative to inquisitive semantics, we refer the reader to the lecture notes [3]. For the algebraic foundations of inquisitive semantics see [17], and for inquisitive logic see [5] and [7]. All these references and additional ones are available online at www.illc.uva.nl/inquisitive-semantics.

[4] The basic system InqB of inquisitive semantics [3] does not make such a distinction and provides a semantics for the usual language of propositional logic. Since there is a direct correspondence between the InqB and InqD systems [4], there is no technical reason to favor one particular inquisitive system. Our choice of InqD is guided by the fact that we will only make use of questions and assertions, and therefore distinguishing between these two categories at the syntactic level shall improve the clarity of our discussion.

[5] In this section, we closely follow the presentation of InqD as provided in [4]. In particular, we will use α and β as meta-variables ranging over declaratives, μ and ν as meta-variables ranging over interrogatives, and φ and ψ as meta-variables ranging over arbitrary sentences.

[6] The negation and disjunction connectives are defined as abbreviations: $\neg\alpha$ is an abbreviation for $\alpha \rightarrow \bot$ and $\alpha \vee \beta$ is an abbreviation for $\neg(\neg\alpha \wedge \neg\beta)$.

- *$\mathcal{L}_?$ is is the interrogative fragment of $\mathcal{L}$ and is defined as the smallest set such that:*
  - *If $\alpha_1, \ldots, \alpha_n \in \mathcal{L}_!$ and $\alpha_1 \vee \ldots \vee \alpha_n$ is a classical tautology, then $?\{\alpha_1, \ldots, \alpha_n\} \in \mathcal{L}_?$,*
  - *If $\mu, \nu \in \mathcal{L}_?$, then $\mu \wedge \nu \in \mathcal{L}_?$,*
  - *If $\alpha \in \mathcal{L}_!$ and $\mu \in \mathcal{L}_?$, then $\alpha \rightarrow \mu \in \mathcal{L}_?$.*

The semantics for $\mathcal{L}$ in InqD requires the two notions of *world* and *state*: a $\mathcal{P}$-*world* is a valuation function for $\mathcal{P}$ attributing a truth value to each $p \in \mathcal{P}$, and a $\mathcal{P}$-*state* is a set of $\mathcal{P}$-worlds.[7] The semantics for InqD is defined in terms of *support*, a binary relation between states and formulas:

Definition 2.2 (Support)**.** *Let $s \in \mathcal{S}$.*

$$\begin{array}{rll} s \vDash p & \textit{iff} & \forall w \in s : w(p) = 1 \\ s \vDash \perp & \textit{iff} & s = \emptyset \\ s \vDash ?\{\alpha_1, \ldots, \alpha_n\} & \textit{iff} & s \vDash \alpha_1 \textit{ or} \ldots \textit{ or } s \vDash \alpha_n \\ s \vDash \varphi \wedge \psi & \textit{iff} & s \vDash \varphi \textit{ and } s \vDash \psi \\ s \vDash \alpha \rightarrow \psi & \textit{iff} & \forall t \subseteq s : \textit{if } t \vDash \alpha \textit{ then } t \vDash \psi \end{array}$$

We can now define the central notion of the *proposition* expressed by a formula φ in a state σ, along with the one of *possibility*:

Definition 2.3 (Proposition and possibility)**.** *Let $\varphi \in \mathcal{L}$ and $\sigma \in \mathcal{S}$.*

- *The proposition expressed by φ in σ, denoted by $[\varphi]_\sigma$, is the set of all states $s \subseteq \sigma$ supporting φ.*
- *The maximal states in $[\varphi]_\sigma$ are called the possibilities for φ in σ.*

The proposition expressed by a formula φ in a state σ encodes both its informative and inquisitive contents. The informative content of φ in σ is represented by $\mathsf{info}(\varphi)_\sigma := \bigcup [\varphi]_\sigma$ and is understood as providing the information that the actual world is contained in a state s supporting φ in σ. The inquisitive content of φ in σ is understood as a request of information by the speaker who utters φ in σ to be brought in a state that supports φ. Then, we say that φ is informative in σ whenever its informative content in σ does not coincide with σ itself. We say that φ is inquisitive in σ whenever it is not enough to just accept the informative content of φ in σ in order

[7] In the following, we will drop references to $\mathcal{P}$ whenever this does not lead to any ambiguity. We will denote the set of all worlds by ω and the set of all states by $\mathcal{S}$.

to be brought in a state that supports φ — further information than the one provided by $\mathsf{info}(\varphi)_\sigma$ is required to do so. Whether a formula φ qualifies as informative or inquisitive in a state σ can be defined as follows:

Definition 2.4 (Informativeness and inquisitiveness). *Let $\varphi \in \mathcal{L}$ and $\sigma \in \mathcal{S}$.*

- *φ is informative in σ iff $\mathsf{info}(\varphi)_\sigma \neq \sigma$.*
- *φ is inquisitive in σ iff $\mathsf{info}(\varphi)_\sigma \notin [\varphi]_\sigma$.*

Following [4], the two semantic categories of questions and assertions are defined from the notions of informativeness and inquisitiveness:

Definition 2.5 (Question and assertion). *Let $\varphi \in \mathcal{L}$ and $\sigma \in \mathcal{S}$.*

- *φ is a question in σ if it is non-informative.*
- *φ is an assertion in σ if it is non-inquisitive.*

Notice that, according to this definition, a question is not necessarily inquisitive and an assertion not necessarily informative in a state σ. The following fact states that the syntactic categories of declaratives and interrogatives are semantically categorized respectively as questions and assertions:

Fact 1. *Let $\varphi \in \mathcal{L}$ and $\sigma \in \mathcal{S}$.*

- *If $\varphi \in \mathcal{L}_!$, then φ is an assertion in σ.*
- *If $\varphi \in \mathcal{L}_?$, then φ is a question in σ.*

Finally, the last notion we need to introduce is the one of *answerhood* which specifies when an assertion can be considered as a possible answer to a given question in a state σ:[8]

Definition 2.6 (Answerhood). *Let $\alpha \in \mathcal{L}_!$, $\mu \in \mathcal{L}_?$ and $\sigma \in \mathcal{S}$. We say that α is an answer to μ in σ iff the only possibility for α in σ coincides with the union of a set of possibilities for μ in σ.*

2.2. *Regulative Principles of Information Exchange in Inquisitive Pragmatics*

Inquisitive pragmatics provides an analysis of the rules that govern the behavior of participants engaged in a certain type of conversation geared

[8] The notion of answerhood is a particular case of a more general notion of *compliance* introduced in [11] which aims to characterize when a "given conversation move is related to the foregoing discourse" [11, p. 19].

towards information exchange. The analysis in [11] proposes four regulative principles, which are first stated informally and then formalized using the resources of inquisitive semantics. We now present these four principles informally, omitting subtleties that are not directly relevant to the questioning and answering moves we are primarily concerned with.

Principle 1: Maintain your information state. In order to discuss the regulative principles of any type of conversation, one shall first define the *information state* of the conversational participants involved. In inquisitive pragmatics, these information states are represented as non-empty states — i.e., non-empty sets of worlds — encoding all the information that a conversational participant possesses. The first principle of inquisitive pragmatics states that each participant should *maintain* her information state. This means that each participant should not update her information state with information that would be inconsistent with it, i.e., avoid information updates that would render her information state empty or inconsistent.

Principle 2: Trust and be truthful. The first part of this principle states that each participant should *trust* the information provided by the others. The only limit to this trust is dictated by the first principle, i.e., that a participant should not trust information inconsistent with her own information state. The second part of this principle states that each participant should be *truthful*, which means that each participant should only communicate information that she actually possesses, i.e., information that is supported by her own information state.

Principle 3: Maintain the common ground. Another central notion of pragmatics is the one of the *common ground* of a conversation [18]. The common ground of a conversation is the body of information that has been established in the conversation up to its current state. In inquisitive pragmatics, the common ground is also formalized as a non-empty state. The first part of this principle states that the information supported by the common ground should also be supported by the information states of the conversational participants. Formally, this means that the information states of the participants should all be included in the common ground. The second part of the principle states that this should remain so: if a piece of information is provided by one of the participants, the other participants should either update their information state with this information, or announce their unwillingness to do it. When all participants accept a piece of information uttered in the conversation, the common ground is consequently updated with it.

Principle 4: Enhance the common ground. This principle generally states that each conversational move should be directed towards *enhancing the* *common ground* in order to achieve specific goals, such as specific needs of information from individual participants or for coordinated actions. In our context, we shall specify this principle for the two conversational moves of *asking* and *answering* questions. For asking questions, the only condition is that the question asked be *inquisitive* in the current common ground, i.e., the question should correspond to an actual request of information by proposing two or more alternatives among which the other participants can choose. For answering questions, the condition is threefold: an assertion uttered in response to a question should be (i) an answer to the question according to definition 2.6, which is (ii) supported by the information state of the answerer[9] and (iii) informative in the current common ground whenever this is possible.

3. Modelling Inquiry in Conversation: Setting the Stage

Before we can define the notion of an interrogative protocol, we shall provide a precise description of the components (i)-(iii) necessary to model inquiry in conversation. Component (ii) has already been addressed in the previous section, since the system InqD provides us with a semantic representation of questions and answers. In this section, we address component (i) by giving a formal representation of the informational setting of a conversation through the notion of *conversational state*, and component (iii) by giving a precise description of how an interrogative step — i.e., the succession of a questioning step and an answering step — modifies the current conversational state in accordance with the principles of inquisitive pragmatics.

3.1. *Conversational State*

As we have seen in the previous section, a given stage in a conversation can be characterized by (i) its *common ground* and (ii) the *information states* of its conversational participants. Among the conversational participants, we distinguish one as being the *inquirer*, and the others as being her *interlocutors*. This perspective is directly inspired by the Socratic dialogues, where only one participant is conducting the inquiry, namely Socrates, and where the role of the other conversational participants boils down to answering the questions of the inquirer. The notion of a *conversational state*, which aims to capture the epistemic situation at a given stage of a conversation, can then be defined as follows:

[9] This condition already follows from principle 2.

Definition 3.1 (Conversational state)**.** *A conversational state C is defined as a tuple $C = (\sigma, \tau_I, \tau_1, \ldots, \tau_p) \in \mathcal{S}^{p+2}$, where σ denotes the common ground of the conversation, τ_I denotes the information state of the inquirer, $\tau_1, \ldots, \tau_p$ denote the information states of the other interlocutors, and such that:*

1. $\tau_I, \tau_1, \ldots, \tau_p \subseteq \sigma$,
2. $\left(\bigcap_{1 \leq i \leq p} \tau_i\right) \cap \tau_I \neq \emptyset$.

The set of all conversational states is denoted by $\mathcal{C}$, the set of all conversational states with p interlocutors is denoted by $\mathcal{C}^p$.

In the above definition, clause 1 says that the information states of the participants are contained in the common ground of the conversation. Clause 2 says that the information states of the participants are consistent with each others. To illustrate our definition, we propose a graphical representation of the notion of conversational state in figure 3.1.

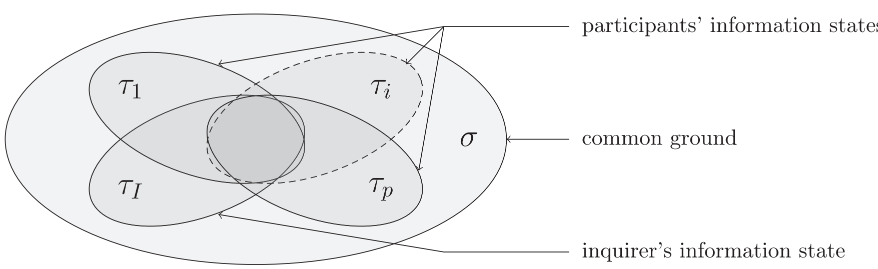

Figure 3.1: A conversational state $C = (\sigma, \tau_I, \tau_1, ..., \tau_p)$.

3.2. *Interrogative Step*

An inquiry in a conversation takes the form of a sequence of *interrogative steps* that move the conversation from one conversational state to another. An interrogative step can be decomposed into a *questioning step*, where the inquirer addresses a question to a designated interlocutor, and an *answering step*, where the designated interlocutor eventually answers the question. Let $C = (\sigma, \tau_I, \tau_1, ..., \tau_p)$ be a conversational state. We now describe how an interrogative step starting in C, i.e., the succession of a questioning step and an answering step, modifies the conversational state C, and we will show that this description is in accordance with the four regulative principles of inquisitive pragmatics.

Questioning step. In the conversational state C, the inquirer can pose different questions to her interlocutors. We introduce as a parameter the notion of an *erotetic awareness set* which is formalized as a set $A_? \subseteq \mathcal{L}_?$ and which represents the interrogatives that the inquirer is aware of in the conversation, i.e., that the inquirer has at her disposal in her inquiry. Now, the interrogatives in $A_?$, even though they are questions as established by fact 1, are not necessarily *inquisitive* in the conversational state C. Following principle 4 of inquisitive pragmatics, the questions that the inquirer can pose in C are thereby the questions in $A_?$ that are inquisitive in σ. In our framework, each question of the inquirer will be directed towards a designated conversational participant. We will then talk of *directed questions* which are pairs (μ, i) composed of an interrogative μ and the index of a participant $i \in \{I, 1, \ldots, p\}$.[10] Thus, in a conversational state C, the conversational moves available to the inquirer are directed questions $(\mu, i) \in A_? \times \{I, 1, \ldots, p\}$ such that μ is inquisitive in σ. This is illustrated in figure 3.2.

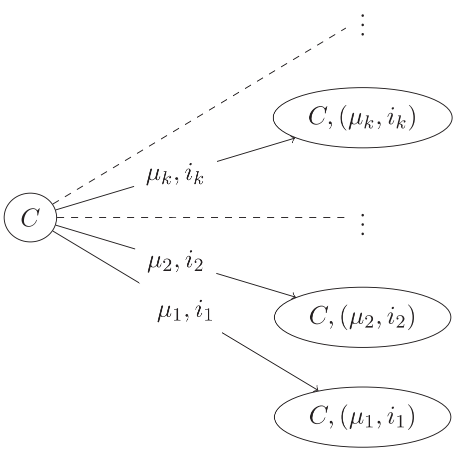

Figure 3.2: A questioning step.

Answering step. After the inquirer has addressed a suitable directed question (μ, i) in C, participant i has to provide an answer to μ. What are then the possible conversational moves available to participant i according to inquisitive pragmatics? We should distinguish between two cases depending on whether or not there exists an informative answer to μ supported by the information state of participant i. If there exists such an answer, then the

[10] We thus allow the inquirer to pose questions to herself. This is a way to allow the inquirer to share information she possesses in the course of the conversation.

available conversational moves for participant *i* are, according to principle 4, the utterance of a declarative $\alpha \in \mathcal{L}_!$ such that (i) α is an answer to μ, (ii) α is supported by τ_i and (iii) α is informative in σ. If participant *i* utters such an assertion, the conversational state C is then updated with the information provided by α. This is represented by erasing in the common ground and in the information states of the different participants all the worlds that are excluded by the informative content of α. Formally, the informational update of a conversational state with a declarative α can be defined as follows:

Definition 3.2 (Informational update)**.** *Let $C \in \mathcal{C}^p$ and $\alpha \in \mathcal{L}_?$. The conversational state $C|\alpha$ resulting from an update of C with the information provided by α is given by:*

$$C|\alpha := (\sigma|\alpha, \tau_I|\alpha, \tau_1|\alpha, \ldots, \tau_p|\alpha)$$

where $s|\alpha := s \cap \mathsf{info}(\alpha)_\sigma$ for $s \subseteq \sigma$.

In the case where no such answer exists, participant *i* should respond to the question by an utterance of the statement $\top := \neg\bot$.[11] The situation resulting from the first case is illustrated in figure 3.3.

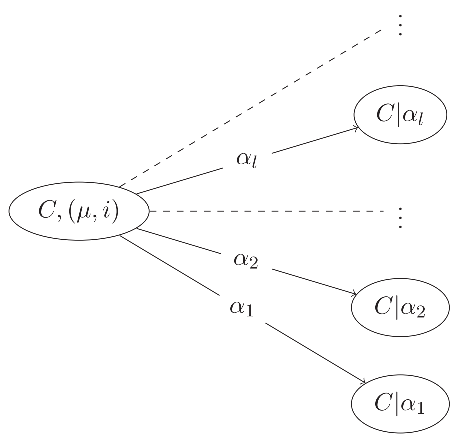

Figure 3.3: An answering step.

Given the definition of conversational states and the way they are modified by interrogative steps, an inquiry in a conversation, i.e., a sequence of interrogative steps, satisfies the principles of inquisitive pragmatics. From the

[11] Notice that this is still a significant conversational move insofar as it informs the inquirer that participant *i* does not have sufficient information for providing an informative answer to question μ.

definition of a conversational state, the information states of the participants are consistent with each other, and since any assertion uttered in an answering step has to be supported by the information state of the answerer, a conversational state is never updated such as to make the information states of some of the participants inconsistent. Principle 1 is thereby satisfied. Principle 2 is satisfied due to the fact that each assertion uttered in an answering step should be truthful and that all the information states of the participants are consequently updated with the provided information. The first part of principle 3 is directly satisfied by the definition of a conversational state, and the second part is satisfied given the way answers update conversational states. Finally, principle 4 has been directly encoded in our description of questioning and answering steps.

4. Interrogative Protocol, Interrogative Inquiry and Interrogative Consequence

In the previous section, we have introduced the notion of a *conversational state* — which aims to characterize the epistemic situation at a given stage in a conversation — and we have described how an *interrogative step* — i.e., the succession of a questioning step and an answering step — moves the ongoing conversation from one conversational state to another. Now, an inquiry in a conversation takes the form of a *sequence* of interrogative steps, and we shall then represent this *temporal dimension* of inquiry processes. To this end, we introduce in this section the notion of *interrogative protocol* which aims to encode all the possible sequences of interrogative steps, i.e., all the possible inquiry paths, that can subsequently occur from a starting conversational state in accordance with the principles of inquisitive pragmatics. This will allow us to define the epistemological notions of *interrogative inquiry* and *interrogative consequence*.

4.1. *Interrogative Protocol*

Following custom practice in computer science, we will define interrogative protocols as *branching-time tree structures*.[12] An interrogative protocol will be represented as a tree, where each finite path in the tree represents a

[12] Protocols are used in computer science to represent and analyze temporal processes governed by specific rules. Protocols are often represented under the form of branching-time tree structures, e.g., in the cases of Epistemic Temporal Logic [16], Interpreted Systems [10] and STIT [2]. In the context of dynamic logic of questions [15, 20], a notion of temporal protocols with questioning operators has been proposed using the methodology developed in [19]. For recent developments on protocols in the context of epistemic dynamics, we refer the reader to the two PhD theses [14] and [21].

possible inquiry path, i.e., a possible sequence of interrogative steps starting from a given conversational state. Besides, an interrogative protocol will be completely determined by two parameters: a starting *conversational state* $C_0 \in \mathcal{C}^p$, representing the starting point of the conversation, and an *erotetic awareness set* $A_? \subseteq \mathcal{L}_?$, representing the interrogatives that the inquirer is aware of in her inquiry. Interrogative protocols are then defined inductively based on these two parameters:

Definition 4.1 (Interrogative protocol)**.** *Let* $C_0 \in \mathcal{C}^p$ *and* $A_? \subseteq \mathcal{L}_?$. *The interrogative protocol* $\mathbf{P}(C_0, A_?)$ *based on* C_0 *and* $A_?$ *is a rooted tree inductively defined as follows:*

Level 0: *The root of the tree is* C_0.

Level 2n: *If* $C = (\sigma, \tau_I, \tau_1, ..., \tau_p)$ *is a node at level* $2n$, *then for each* $(\mu, i) \in A_? \times \{I, 1, \ldots, p\}$ *such that* μ *is inquisitive in* σ, C *has for child the pair* $(C, (\mu, i))$.

Level 2n+1: *If* $(C, (\mu, i))$ *is a node at level* $2n+1$, *then*

- *If there exists an informative answer to* μ *in* σ *supported by* τ_i, *then for each* $\alpha \in \mathcal{L}_i$ *such that* α *is an informative answer to* μ *in* σ *supported by* τ_i, $(C, (\mu, i))$ *has for child the conversational state* $C \mid \alpha$.
- *Otherwise,* $(C, (\mu, i))$ *has for only child the conversational state* C.

The definition of interrogative protocols is built from the description of questioning and answering steps provided in the previous section. Consequently, two types of branching can occur in an interrogative protocol: (i) at the level of questioning steps, where branching represents the possible choices of questions available to the inquirer; (ii) at the level of answering steps, where branching represents the possible choices of answers available to the conversational participant to whom the question has been addressed. These two types of branching are depicted in figure 4.1. It is important to notice that each path in an interrogative protocol represents a possible course of a conversation starting from C_0 which respects the regulative principles of inquisitive pragmatics. This is due to the fact that, as we saw at the end of the previous section, questioning and answering steps have been represented in accordance with the principles of inquisitive pragmatics.

4.2. *Interrogative Inquiry*

In its simplest form, an inquiry in a conversation appears as a *finite sequence* of *directed questions*. Using the resources of InqD and its distinction between declaratives and interrogatives, the notion of *interrogative inquiry* can be defined syntactically as follows:

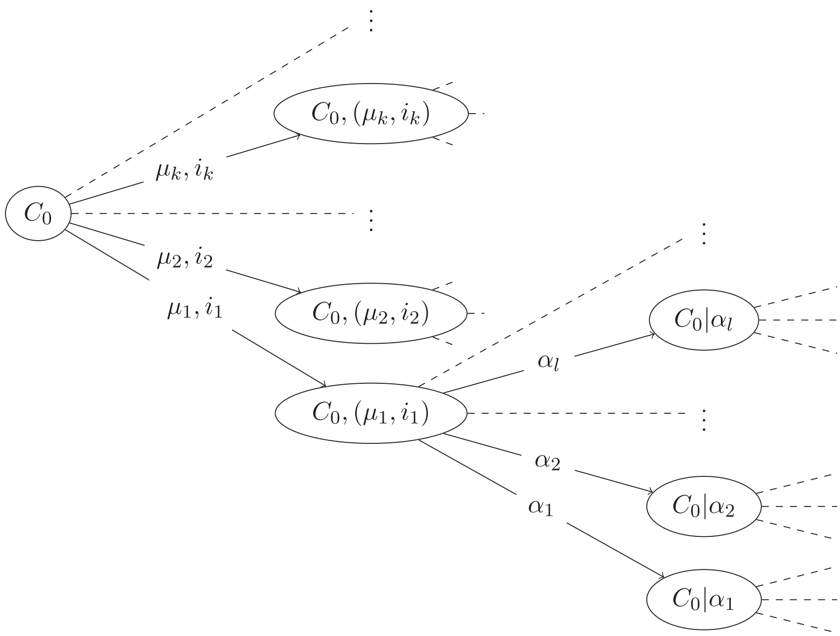

Figure 4.1: A graphical representation of the notion of an interrogative protocol.

Definition 4.2 (Interrogative inquiry)**.** *Let $\{I, 1, \ldots, p\}$ be a set of conversational participant indices. An interrogative inquiry is a finite sequence of directed questions*

$$\langle(\mu_0, i_0), \ldots, (\mu_n, i_n)\rangle$$

where $n \in \mathbb{N}$ and $(\mu_k, i_k) \in \mathcal{L}_? \times \{I, 1, \ldots, p\}$ for all $k \in [\![0, n]\!]$.

With this definition, an inquirer in a conversation will not always be in a position to conduct an interrogative inquiry. This is due to the fact that the questions the inquirer can ask in a conversation are constrained by two things: (i) her erotetic awareness set, and (ii) the principles of inquisitive pragmatics, in particular the fact that a question should be inquisitive in order to be addressed in a given conversational state. Consequently, interrogative inquiries should be investigated *within* the context of interrogative protocols. Of particular interest is the case in which an interrogative inquiry $\Gamma = \langle(\mu_0, i_0), \ldots, (\mu_n, i_n)\rangle$ is *safe* in an interrogative protocol, i.e., when the inquirer is always in a position to pose the question (μ_{k+1}, i_{k+1}) after having addressed the sequence of questions $\langle(\mu_0, i_0), \ldots, (\mu_k, i_k)\rangle$, no matter how the answers to the questions $\langle(\mu_0, i_0), \ldots, (\mu_k, i_k)\rangle$ modified the starting conversational state.

Before we can define the notion of safe interrogative inquiry, we shall introduce some convenient terminology: in an interrogative protocol $\mathbf{P}(C_0, A_?)$, (i) we call the nodes at odd levels the *questioning nodes*; (ii) we call the paths of even lengths the *interrogative paths*; (iii) we say that a questioning node is *interrogatively labelled* with $(\mu,i) \in \mathcal{L}_? \times \{I, 1, \ldots, p\}$ if it is of the form $(C, (\mu, i))$ with $C \in \mathcal{C}$; (iv) we say that an interrogative path in $\mathbf{P}(C_0, A_?)$ is *interrogatively labelled* with interrogative inquiry $\langle(\mu_0, i_0), \ldots, (\mu_n, i_n)\rangle$ if, for every $k \in [\![0, n]\!]$, the questioning node at level $2k+1$ on this path is interrogatively labelled with (μ_k, i_k). The notion of *safe* interrogative inquiry can then be defined as follows:

Definition 4.3 (Safe interrogative inquiry). *Let $\Gamma = \langle(\mu_0, i_0), \ldots, (\mu_n, i_n)\rangle$ be an interrogative inquiry and $\mathbf{P}(C_0, A_?)$ be an interrogative protocol. We say that Γ is a safe interrogative inquiry in $\mathbf{P}(C_0, A_?)$ if (i) C_0 has a child in $\mathbf{P}(C_0, A_?)$ interrogatively labelled with (μ_0, i_0) and (ii) for each $k < n$, all interrogative paths in $\mathbf{P}(C_0, A_?)$ interrogatively labelled with $\langle(\mu_0, i_0), \ldots, (\mu_k, i_k)\rangle$ have for child*[13] *a node interrogatively labelled with (μ_{k+1}, i_{k+1}).*

In the remainder of this paper, we will mostly be concerned with safe interrogative inquiries.

4.3. *Interrogative Consequence*

One of the main interests of modelling inquiry in conversation using interrogative protocols is to frame so-called *reachability problems*. This is of particular interest, as key epistemological issues relative to interrogative inquiry can be framed in this way. A central one consists in asking what *information* the inquirer can obtain through interrogative inquiry, along with what *problems* she can solve, in a given interrogative protocol. To address these issues in our framework, we will define the notions of *interrogative consequence* and *interrogative solvability*. To this end, we first need to state what it means for an interrogative inquiry to establish a given statement, or to solve a given problem, in an interrogative protocol:

Definition 4.4. *Let Γ be an interrogative inquiry, $\mathbf{P}(C_0, A_?)$ be an interrogative protocol, $\alpha \in \mathcal{L}_!$ and $\mu \in \mathcal{L}_?$. We say that Γ establishes α, respectively solves μ, in $\mathbf{P}(C_0, A_?)$ if (i) Γ is a safe interrogative inquiry in $\mathbf{P}(C_0, A_?)$ and (ii) all interrogative paths in $\mathbf{P}(C_0, A_?)$ interrogatively labelled with Γ end up with a conversational state in which α has been established, respectively μ has been settled, in the common ground.*

[13] By a child of an interrogative path, we mean a child of the end-node of the interrogative path.

We can now define the notion of *interrogative consequence*. Intuitively, a declarative α is an interrogative consequence in a given interrogative protocol if the information provided by α can be reached by the process of interrogative inquiry, i.e., there exists a safe interrogative inquiry such that asking successively the directed questions that comprise it will, in all the possible paths that the conversation can take, establish α in the common ground.[14] Formally, this can be defined as follows:

Definition 4.5 (Interrogative consequence)**.** *Let* $\mathbf{P}(C_0, A_?)$ *be an interrogative protocol and* $\alpha \in \mathcal{L}_!$*. We say that* α *is an interrogative consequence in* $\mathbf{P}(C_0, A_?)$ *if there exists a safe interrogative inquiry* Γ *establishing* α *in* $\mathbf{P}(C_0, A_?)$.

In the exact same way, one can define the notion of *interrogative solvability*, which says when the issue raised by an interrogative μ can be solved by the process of interrogative inquiry in a given interrogative protocol:

Definition 4.6 (Interrogative solvability)**.** *Let* $\mathbf{P}(C_0, A_?)$ *be an interrogative protocol and* $\mu \in \mathcal{L}_?$*. We say that* μ *is interrogatively solvable in* $\mathbf{P}(C_0, A_?)$ *if there exists a safe interrogative inquiry* Γ *solving* μ *in* $\mathbf{P}(C_0, A_?)$.

Finally, the notions introduced in this section allow us to define a concept of *optimality* for interrogative inquiries. More specifically, we will say that an interrogative inquiry is *optimal* for establishing a statement or for solving a problem in a given interrogative protocol whenever there is no other interrogative inquiry that can do so using a strictly inferior number of questions:

Definition 4.7 (Optimal interrogative inquiry)**.** *Let* Γ *be an interrogative inquiry,* $\mathbf{P}(C_0, A_?)$ *be an interrogative protocol,* $\alpha \in \mathcal{L}_!$ *and* $\mu \in \mathcal{L}_?$*. We say that* Γ *is an optimal interrogative inquiry establishing* α*, respectively solving* μ*, in* $\mathbf{P}(C_0, A_?)$ *if (i)* Γ *establishes* α*, respectively solves* μ*, in* $\mathbf{P}(C_0, A_?)$ *and (ii) there is no interrogative inquiry establishing* α*, respectively solving* μ*, in* $\mathbf{P}(C_0, A_?)$ *of length strictly smaller than* Γ.[15]

[14] Another alternative is to ask α not to be necessarily established in the common ground, but only to be established in the information state of the inquirer. Such a "private" notion of interrogative consequence is as legitimate as the "public" one defined in this section. Our choice of the public notion is guided by our inspiration from the Socratic dialogues, where the results of Socrates' inquiry are made public to all the participants. Yet, one can straightforwardly modify the present framework for studying the private notion, by simply replacing in the definition of interrogative consequence that α be established in the information state of the inquirer.

[15] By the length of an interrogative inquiry, we mean the number of directed questions comprising it.

5. Computing Interrogative Inquiries

The main reasons we engage in inquiry is to establish *statements* or to solve *problems*. In this respect, three issues of particular epistemological interest can be phrased in our approach: (1) Can a certain statement α be established, or a given problem μ be solved, in an interrogative protocol? (2) How to find an interrogative inquiry that establishes α, or solves μ, when there exists one? (3) How to find an *optimal* interrogative inquiry that establishes α, or solves μ, when there exists one? Within the formal framework proposed in the previous sections, such epistemological issues can be framed in exact terms as *computational problems*. In this section, we first formulate these three issues respectively as decision, search and optimization problems. We then provide an algorithmic procedure for solving them in the restricted case of interrogative protocols with finite erotetic awareness sets.

5.1. *The Decision, Search and Optimization Inquiry Problems*

The epistemological issues (1)-(3) can be framed as computational problems within our framework.[16] Issue (1) can be represented as a *decision* problem defined as follows:

INPUT: An interrogative protocol $\mathbf{P}(C_0, A_?)$ and $\alpha \in \mathcal{L}_!$.

QUESTION: Is α an *interrogative consequence* in $\mathbf{P}(C_0, A_?)$?

Then, issue (2) can be represented as a *search* problem in the following way:

INPUT: An interrogative protocol $\mathbf{P}(C_0, A_?)$ and $\alpha \in \mathcal{L}_!$.

TASK: Find an *interrogative inquiry* Γ in $\mathbf{P}(C_0, A_?)$ establishing α.

Finally, issue (3) can be represented as an *optimization* problem:

INPUT: An interrogative protocol $\mathbf{P}(C_0, A_?)$ and $\alpha \in \mathcal{L}_!$.

TASK: Find an *optimal* interrogative inquiry Γ in $\mathbf{P}(C_0, A_?)$ establishing α.

We will refer to these three computational problems respectively as the decision, search and optimization inquiry problems.

[16] In the following, we frame the different computational problems in terms of interrogative consequence. The case of interrogative solvability can be addressed in the very same way by replacing in the different problems the notion of interrogative consequence with the one of interrogative solvability.

5.2. *An Algorithmic Procedure for Solving the Inquiry Problems*

We now provide an algorithmic procedure for solving the three inquiry problems in the restricted case of interrogative protocols with finite erotetic awareness sets.[17] Before being able to do so, we shall introduce the notions of a *significant* question and an *informatively drained* interrogative inquiry:

Definition 5.1. *Let $\Gamma = \langle(\mu_0, i_0), \ldots, (\mu_n, i_n)\rangle$ be an interrogative inquiry and $\mathbf{P}(C_0, A_?)$ be an interrogative protocol. We say that the directed question (μ_k, i_k) in Γ is significant in $\mathbf{P}(C_0, A_?)$ if there exists an interrogative path in $\mathbf{P}(C_0, A_?)$ interrogatively labelled with Γ such that the response to the question μ_k by participant i_k is informative in the current common ground.*

Definition 5.2. *Let Γ be an interrogative inquiry and $\mathbf{P}(C_0, A_?)$ be an interrogative protocol. We say that Γ is informatively drained in $\mathbf{P}(C_0, A_?)$ if any extension of Γ with a directed question $(\mu, i) \in A_? \times \{I, 1, \ldots, p\}$ is such that (μ, i) in the interrogative inquiry $\Gamma(\mu, i)$ is not significant in $\mathbf{P}(C_0, A_?)$.*[18]

Intuitively, an interrogative inquiry is informatively drained in an interrogative protocol when conducting such an inquiry ends up in a situation where the inquirer cannot obtain further information by pursuing the inquiry with additional interrogative steps. This notion plays a role in the following definition of the *inquiry algorithmic procedure* we consider in this section:

Definition 5.3 (Inquiry algorithmic procedure)**.** *The inquiry algorithmic procedure takes as inputs an interrogative protocol $\mathbf{P}(C_0, A_?)$ and a declarative $\alpha \in \mathcal{L}_!$, and proceeds as follows:*

1. *Starting from $n = 0$, the procedure successively checks for each interrogative inquiry Γ in the list $L(n)$, where $L(n)$ is the list of interrogative inquiries that the procedure considers at step n,*[19] *whether it establishes α:*[20]

[17] Throughout this section, we will make the following assumption on interrogative protocols: for each questioning node, we identify the children of the node that correspond to the same conversational state. An interrogative protocol satisfying this assumption and the one of the finiteness of erotetic awareness sets appears as a *finitely branching tree*: for each node at an even level, there are only finitely many directed questions that the inquirer can pose to the conversational participants; for each node at an odd level, there are only finitely many conversational states that can result from answering a directed question, as the inquisitive proposition expressed by an interrogative μ in $\mathcal{L}_?$ always yields finitely many possibilities for μ in any conversational state.

[18] $\Gamma(\mu, i)$ denotes the interrogative inquiry obtained by extending Γ with (μ, i).

[19] For $n = 0$, $L(0)$ is the list of all interrogative inquiries of length 1 in $\mathbf{P}(C_0, A_?)$, namely $L(0) := \{\langle(\mu_0, i_0)\rangle \mid (\mu_0, i_0) \in A_? \times \{I, 1, \ldots, p\}\}$. For $n > 0$, the list $L(n)$ is defined by the procedure during step $n-1$.

[20] Checking whether a given interrogative inquiry establishes α in $\mathbf{P}(C_0, A_?)$ can be done by exploring all the leafs of a particular subtree of $\mathbf{P}(C_0, A_?)$. When the considered

   - *If it does, then the procedure outputs* Γ.
   - *Otherwise, the procedure checks for each directed question* $(\mu, i) \in A_? \times \{I, 1, \ldots, p\}$ *whether* (μ, i) *in the interrogative inquiry* $\Gamma(\mu, i)$ *is significant in* $\mathbf{P}(C_0, A_?)$*, and if it is* $\Gamma(\mu, i)$ *is added to the list* $L(n+1)$.

2. *If no interrogative inquiry in the list* $L(n)$ *establishes* α*, then the procedure checks whether all the interrogative inquiries in* $L(n)$ *are informatively drained in* $\mathbf{P}(C_0, A_?)$*:*
   - *If they are, then the procedure outputs '*α *is not an interrogative consequence in* $\mathbf{P}(C_0, A_?)$'.
   - *Otherwise, the procedure starts again with the list* $L(n+1)$.

This procedure instantiates a certain type of full search in an extensive list of interrogative inquiries ordered by growing lengths, up to a certain point, to look whether there exists an interrogative inquiry that establishes a certain statement in a given interrogative protocol.[21] Of course, this is far from being an optimized procedure for doing so. However, our objective in this section is not to come up with an optimized procedure, but only to show that the three inquiry problems can be solved by an algorithmic procedure. To this end, our task consists in finding one procedure that solves the three inquiry problems, and we will now show that the inquiry algorithmic procedure just defined is such a procedure.

First of all, we show that the inquiry algorithmic procedure always terminates. We refer to the list of interrogative inquiries $\bigcup_{i\in\mathbb{N}} L(i)$ as the *scan list*[22] for α in $\mathbf{P}(C_0, A_?)$.[23] The following lemma says that the scan list of a declarative in an interrogative protocol is always finite:

Lemma 1. *Let* $\mathbf{P}(C_0, A_?)$ *be an interrogative protocol with* $A_?$ *finite and* $\alpha \in \mathcal{L}_!$*. The scan list for* α *in* $\mathbf{P}(C_0, A_?)$ *is finite.*

interrogative inquiry is safe — if the interrogative inquiry is not safe, then it cannot by definition establish α — this subtree is composed of all the interrogative paths interrogatively labelled with the considered interrogative inquiry. Since we are considering finite interrogative inquiries, and that under the assumptions we made in this section interrogative protocols are finitely branching trees, this subtree has finitely many nodes. Exploring all the leafs of this subtree — to see whether they correspond to conversational states in which α has been established in the common ground — can then be done by a custom procedure for tree traversal such as a breadth-first search (e.g., see section 4.2 of [9]).

[21] Notice that the procedure can straightforwardly be adapted to the case of interrogative solvability, by taking as input an interrogative $\mu \in \mathcal{L}_?$ and by checking at step 1 whether the considered interrogative inquiry Γ solves μ.

[22] We take as a convention that if the procedure terminates at step n, then $L(m) = \varnothing$ for all $m > n$.

[23] When the parameters α and $\mathbf{P}(C_0, A_?)$ are clear from the context, we simply speak of the *scan list*.

Proof. Assume towards a contradiction that the scan list for α in $\mathbf{P}(C_0, A_?)$ is infinite. This means that there exists an infinite interrogative inquiry Γ_∞ such that any initial segment of Γ_∞ is not informatively drained in $\mathbf{P}(C_0, A_?)$ and each directed question in Γ_∞ after the first one is significant in $\mathbf{P}(C_0, A_?)$. Consider the tree composed of all interrogative paths in $\mathbf{P}(C_0, A_?)$ interrogatively labelled with Γ_∞. We perform the following operation on this tree: we 'erase' all questioning nodes $(C, (\mu, i))$ in the tree which have for only child a node labelled with the conversational state C.[24] We then obtain a tree in which all questioning nodes are followed by an informative answer. Since Γ_∞ is composed of infinitely many significant questions in $\mathbf{P}(C_0, A_?)$, this tree has infinitely many nodes due to the fact that infinitely many questioning nodes are followed by an informative answer. Since $\mathbf{P}(C_0, A_?)$ is a finitely branching tree, one can check that the tree resulting from this operation is finitely branching as well. We are thus in a position to apply König's lemma to this tree, which yields that there must exist an infinite path in this tree. Thus, there must exist an interrogative path in $\mathbf{P}(C_0, A_?)$ interrogatively labelled with Γ_∞ such that infinitely many questioning nodes in this path are followed by an informative answer. It follows from this and from the fact that there are finitely many directed questions based on $A_?$, that there must exist a directed question $(\mu, i) \in A_? \times \{I, 1, \ldots, p\}$ and an interrogative path in $\mathbf{P}(C_0, A_?)$ interrogatively labelled with Γ_∞ such that conversational participant i provides an informative answer to μ infinitely many times on this path. However, this is impossible due to the fact that the proposition expressed by μ in any state has always finitely many possibilities and thereby finitely many possible answers, therefore participant i cannot provide infinitely many informative answers to a given question μ on an interrogative path in $\mathbf{P}(C_0, A_?)$. □

It follows directly from this lemma that the inquiry algorithmic procedure always terminates in any interrogative protocol:

Corollary 1. *Let $\mathbf{P}(C_0, A_?)$ be an interrogative protocol with $A_?$ finite and $\alpha \in \mathcal{L}_!$. The inquiry algorithmic procedure with inputs $\mathbf{P}(C_0, A_?)$ and α terminates in a finite amount of time.*

Then, we show that the inquiry algorithmic procedure is doing the job it was designed to do, namely finding an interrogative inquiry for a given $\alpha \in \mathcal{L}_!$ in an interrogative protocol $\mathbf{P}(C_0, A_?)$ when there exists one. The following lemma establishes this:

[24] In other words, we 'erase' the questioning nodes which are not followed by an informative answer.

Lemma 2. *Let* $\mathbf{P}(C_0, A_?)$ *be an interrogative protocol with* $A_?$ *finite and* $\alpha \in \mathcal{L}_!$*. If* α *is an interrogative consequence in* $\mathbf{P}(C_0, A_?)$*, then the inquiry algorithmic procedure with inputs* $\mathbf{P}(C_0, A_?)$ *and* α *will output an interrogative inquiry establishing* α *in* $\mathbf{P}(C_0, A_?)$*.*

Proof. Assume that α is an interrogative consequence in $\mathbf{P}(C_0, A_?)$. By definition, this means that there exists an interrogative inquiry Γ that establishes α in $\mathbf{P}(C_0, A_?)$. Consider the interrogative inquiry Γ^* obtained by 'erasing' in Γ all the directed questions that are not significant in $\mathbf{P}(C_0, A_?)$. Since non-significant questions do not bring any information, and therefore do not contribute to establishing α, we have that if Γ establishes α, then Γ^* establishes α as well. Besides, Γ^* cannot be a proper extension of an informatively drained interrogative inquiry, since such extensions ends with questions that are not significant. Thus, either the inquiry algorithmic procedure will encounter Γ^* and output it, or it will encounter another interrogative inquiry establishing α in $\mathbf{P}(C_0, A_?)$ appearing before Γ^* in the list of interrogative inquiries checked by the procedure. In both cases, the inquiry algorithmic procedure will output an interrogative inquiry establishing α in $\mathbf{P}(C_0, A_?)$. □

We are now in a position to show that the inquiry algorithmic procedure can be used to solve the decision, search and optimization inquiry problems:

Theorem 1. *Let* $\mathbf{P}(C_0, A_?)$ *be an interrogative protocol with* $A_?$ *finite and* $\alpha \in \mathcal{L}_!$*. The decision, search and optimization inquiry problems are solvable by the inquiry algorithmic procedure.*

Proof. By corollary 1 and lemma 2, the inquiry algorithmic procedure solves the search inquiry problem: (1) corollary 1 tells us that the procedure always terminates in a finite amount of time, and (2) lemma 2 tells us that if there exists an interrogative inquiry establishing α in $\mathbf{P}(C_0, A_?)$, then the procedure will encounter one and output it. Moreover, lemma 2 is also telling us that if there is no such interrogative inquiry in the scan list, then α is not an interrogative consequence in $\mathbf{P}(C_0, A_?)$, and in this case the procedure rightly outputs 'α is not an interrogative consequence in $\mathbf{P}(C_0, A_?)$'. From this, it follows immediately that the procedure can also solve the decision inquiry problem, by simply outputting 'α is an interrogative consequence in $\mathbf{P}(C_0, A_?)$' in case it finds one. Finally, by exploring the interrogative inquiries by growing length, if the procedure outputs an interrogative inquiry in $\mathbf{P}(C_0, A_?)$ establishing α, it is necessarily an *optimal* interrogative inquiry according to definition 4.7. Consequently, the procedure solves the optimization inquiry problem as well. □

6. Conclusion

The aim of this paper was to provide a formal modelling of the process of inquiry in conversation based on the framework of inquisitive pragmatics. To this end, we introduced the notion of *interrogative protocols* defined as branching-time tree structures encoding all the possible inquiry paths that an inquirer can pursue in an ongoing conversation, where the conversation is regulated by the principles of inquisitive pragmatics. Interrogative protocols provide us with a formal environment in which the epistemological notions of *interrogative inquiry* and *interrogative consequence* can be formally defined and investigated. In particular, we show how central epistemological issues can be framed as *computational problems*, and we provided an algorithmic procedure for solving them in the restricted case of interrogative protocols with finite erotetic awareness sets. Thus, the formal framework developed in this paper provides a first approach to the modelling of inquiry in conversation within the context of inquisitive pragmatics.

It is important to notice that the notion of an interrogative protocol is susceptible of many variations depending on the features of the inquisitive pragmatics one is starting with. Two main types of variations appear as particularly natural. One concerns the representation of questions and answers for which other inquisitive systems can be adopted. Of particular interest is the InqD_π system [4] which extends InqD with presuppositional interrogatives, and which would bring presuppositions into the picture.[25] Furthermore, the system IDEL [6] would allow to represent inquiry about higher-order information, while first-order inquisitive systems [3, 8] would bring inquiry to the first-order case. At the level of the principles of inquisitive pragmatics, many different choices are available as well. One might want in particular to constrain the answerers in a conversation to behave in a certain way, for instance by being maximally or minimally informative in their choices of answers. One might also require that the questions asked by the inquirer be inquisitive in her own information state. In this paper, we chose to restrict ourselves to a minimal set of assumptions on interrogative protocols. However, the methodology developed here can be used to study different types of inquiry in conversation governed by various sets of pragmatic rules.

In the remainder of this conclusion, we relate our approach to Wiśniewski's *Erotetic Search Scenarios (ESS)* [24] and Hintikka's *Interrogative Model of Inquiry (IMI)* [12, 13].

[25] Working with InqD_π would bring us closer to the perspective of Hintikka, who emphasizes the important role of presuppositions in interrogative inquiry, see for instance chapter 4 of [13].

The notion of ESS has been introduced by Wiśniewski in order to represent "how an initial question can be answered on the basis of a given set of initial premises and by means of asking and answering auxiliary questions." [24, p. 391] An ESS can be conceived as a *plan* that an inquirer can follow in order to solve or answer a "big" or principal question by means of "small" or operative questions. As such, ESS can perfectly be investigated within interrogative protocols, the latter providing an environment to investigate the former in the specific context of conversations governed by the rules of inquisitive pragmatics. To this end, the notion of ESS should be adapted along the two following lines: (i) ESS should be defined in the multi-agent case where questions can be addressed to different sources of information; (ii) the representation of questions in ESS should be shifted from the one of inferential erotetic logic to the one of inquisitive semantics. This appears perfectly feasible since issue (i) can be treated by working with *directed questions*, and issue (ii) has recently been addressed in a paper that compares and connects inferential erotetic logic with inquisitive semantics [23], and which proposes a direct translation between the representations of questions inherent to the two frameworks. Such developments would open the way to an investigation of ESS in conversational contexts.

The IMI developed by Hintikka [12, 13] represents inquiry as a *game* between an Inquirer and Nature. The IMI and the framework developed in this paper aim to model the very same epistemological process. One might even argue that the formal structures identified by the two approaches are essentially the same: an interrogative protocol can be interpreted as the structure of an extensive game where moves consist in asking and answering questions, while an interrogative game, insofar as it describes a questioning process governed by certain rules, can be represented as an interrogative protocol. Thus, interrogative protocols and interrogative games provide two different and complementary perspectives on the same epistemological process: interrogative protocols offer an *external* perspective, adopting a third person perspective and emphasizing reachability problems, while interrogative games offer an *internal* perspective, adopting the first person perspective of the inquirer and emphasizing the strategic aspects of inquiry. Yet, a full formalization of interrogative games as extensive games remains to be developed.[26] Such advances appear as a necessary prerequisite for comparing the respective benefits of using protocols or games to represent and investigate the epistemological process of inquiry in conversation.

[26] A fully formalized framework for representing question-answer games has been proposed in [1], but only for the case of *strategic* games.

Acknowledgments

Earlier versions of this paper have been presented at the Interrogative Model of Inquiry Workshop in Paris (France, January 2012), the DiP Colloquium in Amsterdam (the Netherlands, May 2012), the Workshop on Strategies for Learning, Belief Revision and Preference Change at ESSLLI 2012 in Opole (Poland, August 2012) and the Reasoning Club Conference in Brussels (Belgium, September 2012). I would like to thank the audiences of these events for helpful feedback and comments. I am thankful to Ivano Ciardelli, Jeroen Groenendijk, Floris Roelofsen and Matthijs Westera for many discussions relative to this work. Finally, I am very grateful to Ivano Ciardelli for providing extensive comments and suggestions on a pre-final version of this paper and to Floris Roelofsen for providing feedback during the initial phase of this work. The author of this paper acknowledges support from a doctoral fellowship of the Research Foundation Flanders (FWO).

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