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# VERBAL DISPUTES IN LOGIC: AGAINST MINIMALISM FOR LOGICAL CONNECTIVES*

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ABSTRACT

Quine's famous meaning-variance thesis has it that when a classical and a nonclassical logician argue about a logical law, say, the law of excluded middle, the apparent disagreement is a 'mere verbal dispute'. Here we explore a popular response to the meaning-variance thesis, minimalism for logical connectives, as developed for example by Hilary Putnam, Susan Haack, and more recently Francesco Paoli. We use a new variant of Quine's argument — a meta-Quinean argument — to show that the minimalist's position is ultimately untenable. We then outline an alternative response to the meaning-variance thesis along structuralist lines.

## Introduction

What do we mean when we say that we disagree about logic? Philosophers occasionally disagree about whether or not something is a logical law.[1] For example, paracomplete and intuitionist logicians reject the law of excluded middle (*LEM*), $A \vee \neg A$, while classical and supervaluationist logicians accept it. There are, in fact, almost no logical laws that remain uncontested in the philosophy of logic, and the debates are often connected with broader philosophical issues: vagueness, truth, information theory, presuppositions, reference failure, and indeterminacy, to name just a few.

There can be no doubt, therefore, that there are *disputes* about logic. But is there also *genuine disagreement?*[2] And, if so, what is the disagreement about? Is the disagreement about a fact, about whether a proposition is

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[1] I take 'logical law' to refer both to (purported) logical truths such as *LEM,* and to (purported) valid inferences such as conditional proof.

[2] I will be assuming that there is something like a distinction between verbal disputes and genuine disagreement. There is, however, an ongoing debate about the distinction (cf. Chalmers [10], Jenkins [24]).

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(logically) true or whether an argument necessarily preserves truth? If so, what sort of facts — normative or descriptive? Is there even a fact of the matter at all? Non-cognitivists about logic say ‘no’ (cf. Resnik [43], Field [17]), while realists say ‘yes’ (cf. Priest [34], Williamson [55]). Nevertheless, both parties of the metaphysical debate agree that there is genuine disagreement about logic. And so they face a well-known argument to the contrary. According to Quine’s ([39]) *meaning-variance argument* disputes about logic are *merely verbal disputes*. Under the provocative vignette ‘change of logic, change of subject’, Quine claims that the classical and non-classical logician are ‘talking past each other’.

There are a number of recent attempts at blocking the meaning-variance argument. The type of theory that will be discussed below is what has become known as *minimalism for logical connectives* (cf. [45], [30], [31]). I have previously argued that the minimalist theories have some crucial shortcomings [20], and in what follows I reply to a number of counters by Paoli [32]. After briefly revisiting the details of the meaning-variance argument (§1), I discuss the precise formulation of the minimalist theory and how it is supposed to block the argument (§§2-4). I then address the particular replies made by Paoli (§§5-6) before I offer what I think are improved versions of my original objections (§§7-8). In conclusion I offer an outline of what I think is a more promising theory of shared meaning for logical connectives: a form of structuralism (§9).

## 1. The Meaning-variance argument

Contrary to Quine’s original formulation, later versions of the meaning-variance argument are given in terms of the *meaning* of logical connectives. The details of the argument, therefore, will depend on the theory of meaning for the connectives. Haack [19], for example, states the argument in terms of *inferential semantics* or *conceptual role semantics*, a reformulation that is at least in the spirit of Quine’s original argument. According to inferential semantics the meaning of a connective, say, negation, is fixed exhaustively by its inference rules (e.g. introduction and elimination rules). Suppose then that the classical translator encounters a non-classical native. The two use the same expression for negation, but ascribe non-equivalent inference rules to it, and hence assent and dissent to different laws. The meaning-variance argument then concludes that the two negation expressions in play have different meanings, and that it is therefore a mistake for the classicist to translate the non-classicist *homophonically*, i.e. to map the negation expression in the nonclassical vernacular to its classical counterpart. There is a verbal dispute waiting to happen: The classicist accepts $A \vee \neg_C A$, while the nonclassicist rejects $A \vee \neg_{NC} A$, and a homophonic translation will lead the parties to talk past each other.

The meaning-variance argument also has an equally problematic truth-conditional version. One formulation is given by Priest [34]. The classical theory and non-classical theory typically ascribe distinct truth-conditions to the negation. If a change of truth-condition is sufficient for a change of meaning, the two theories ascribe different meanings to the negation expression. The result is meaning-variance, and again there is no genuine disagreement when the classicist and nonclassicist accept and reject what appears to be the same logical law.

Yet it would be uncharitable to conclude that philosophers of logic fail to genuinely disagree about their subject matter. The challenge is to formulate the content of the disagreement in a way that does justice to the parties of the debate. There are several options.

THE SEMANTIC OPTION: One would be to bite the bullet and concede that disagreements about logic are always disagreements about meaning. That is not, of course, to say that they are merely verbal disputes, but rather that they are substantial disagreements about semantics. This is arguably the view defended by Dummett [14], and perhaps also Carnap [9]. But the semantic option is not faithful to what many philosophers of logic claim to be disagreeing about. There are examples both in the classical camp (e.g. Williamson [55]) and in the nonclassical camp (e.g. [16]).

THE DESCRIPTIVIST OPTION: A second approach is suggested by Priest [34]: There is genuine disagreement, but about which logical theory correctly captures reasoning in natural language. If we take that account of the disagreement seriously, however, it threatens to make disagreement about logic a descriptive rather than a normative matter. Maybe that is accurate for some logical disagreements (e.g. about paraconsistency), but many of the most entrenched debates (say, between intuitionists and classicists) are about how we ought to reason, not about how we actually reason. Dummett's argument for intuitionism is a case in point.

THE MINIMALIST OPTION: The type of response I will discuss tries to block the meaning-variance argument by finessing the connection between the semantics and the logic of a connective. I will follow Restall [45] and call this position *minimalism for logical connectives* (minimalism for short). In short it will involve a distinction between *meaning-constitutive* and *non-meaning-constitutive* logical laws. The former are the laws which semantically characterize the connective, while other laws are part of the logical theory, but do not hold any semantic significance.

## 2. Minimalism for Logical Connectives

Minimalism for logical connectives has received substantial attention in the literature. Both Putnam [35], [37] and Haack [19] formulated early versions of the idea, and it has been defended more recently by Restall [45], Read

[42], and Paoli [30], [31]. Let us start with Putnam as an example of proto-minimalism. Putnam [37, 189-90] argues that a change of theory from classical logic to quantum logic does not involve a change of meaning. Although quantum logic rejects the law of distributivity, $A \wedge (B \vee C) \rightarrow (A \wedge B) \vee (A \wedge C)$ and classical logic accepts it, he maintains that the quantum logician is offering a different logical theory for the *same* disjunction as the classical logician. Change of theory, same old subject.

Putnam's reason is that distributivity is not a meaning-constitutive law for disjunction. He observes that many important laws of classical disjunction are preserved in quantum logic, e.g. disjunction introduction, *LEM,* and disjunctive syllogism. In fact, the failure of distributivity is entirely due to a minor restriction on the rule of disjunction elimination:

$$\dfrac{A \vee B \quad \begin{matrix}[A]^u \\ \vdots \\ C\end{matrix} \quad \begin{matrix}[B^u] \\ \vdots \\ C\end{matrix}}{A \vee B}\,(Q\vee E) \qquad \dfrac{A \vee B \quad \begin{matrix}\Gamma, [A]^u \\ \vdots \\ C\end{matrix} \quad \begin{matrix}\Gamma', [B^u] \\ \vdots \\ C\end{matrix}}{A \vee B}\,(\vee E)$$

The right-most disjunction elimination rule is standard in classical and intuitionistic logic. The left-most rule is different only with respect to the auxiliary formulae disallowed in the subderivations of the minor premises. With the restriction in place, distributivity becomes underivable. Hence, Putnam is right that even disjunction elimination is preserved, or at least its characteristic features.

But Putnam does not take these facts to be conclusive, simply because he thinks the distinction between meaning-constitutive and non-meaning-constitutive laws is too unclear. "[W]e do not posses a notion of 'change in the meaning' refined enough to handle this issue" (ibid., 190). A similar worry about the very idea of meaning-change is expressed by Field:

> The question [of meaning change] is clear only to the extent that we know how to divide up such firmly held principles into those that are "meaning constitutive" or "analytic" and those which aren't, and this is notoriously difficult. ([16], 17)

What the minimalist needs is some principled identity condition for the meaning-constitutive laws of a logical connective.[3] Putnam did not offer such a criterion, nor did he think it was plausible. His proposal did, however, contain the kernel of a later development of the minimalist position.

[3] Can we assume that the sets of meaning-constitutive laws of connectives are disjoint? That is, can it be the case that *LEM* is a (partial) meaning-constitutive law for both disjunction and negation? I will bracket this and related worries for now. For a related discussion, see Dummett [14].

## 3. Structural Minimalism

Actual proposals for how to draw a line between core and peripheral principles for a logical connective are few and far between. But there is one concrete proposal I think is worth taking seriously. I will call it *structural minimalism*. To motivate this position I want to start off by drawing a distinction between the theoretical contribution made by a deducibility relation and by a logical connective. The latter is a privileged expression in the formal language, together with a set of laws. The laws are broadly speaking an axiomatization, and can consist of axioms and inference rules. The inference rules are typically (but not exclusively) a relation between a set of formulae of the language and single formula, together with some properties, say, reflexivity and transitivity. The manner in which the notion of a deducibility relation can be generalized will play a significant role later, but for now we can focus on the orthodoxy.

The extension of a deducibility relation depends, in part, on which logical connectives are present and what their rules are. What is more interesting for present purposes, however, is the nature of deducibility relations prior to the introduction of any logical connectives. Following Belnap [5], let us call this the *antecedent context of deducibility:*

> Even on the synthetic view, we are not defining our connectives *ab initio*, but rather in terms of an antecedently given context of deducibility, concerning which we have some definite notions. By that I mean that before arriving at the problem of characterizing connectives, we have already made some assumptions about the nature of deducibility (ibid., 131).

Which assumptions are commonly made about the antecedent context of deducibility? There are a number, but the most standard ones are summed up by what is known as a *Tarski relation*. A Tarski relation is of the form $\Gamma \vdash A$ where $\Gamma$ is a set of formulae, $A$ a single formula, and the relation itself has the following structural properties:

Reflexivity: $A \vdash A$;

Transitivity: if $\Gamma \vdash A$ and $\Pi, A \vdash B$, then $\Gamma, \Pi \vdash B$;

Monotonicity: if $\Gamma \vdash A$, then $\Gamma, B \vdash A$.

The antecedent context of deducibility for classical logic is a Tarski relation, but also for intuitionistic logic and a range of other logics. These properties are, in other words, satisfied in these logics when the language is restricted to propositional variables. They are independent of the particular choice of language.

The deducibility relation can be supplemented by extending the language and adding inference rules governing the logical connectives. For the minimalist it is natural to think of the inference rules for a connective as its meaning-constitutive laws. Hence, in a natural deduction calculus, the meaning-constitutive laws would be the connective's introduction and elimination rules; in a sequent calculus it would be the corresponding left- and right-rules. Restall [45] describes a divide between meaning-constitutive laws, and laws that form part of the theory of the deducibility relation without affecting the semantics of the language:

> If any set of rules is sufficient to pick out a single meaning for the connective, take that set of rules and accept those as meaning determining. The other rules are important when it comes to giving an account of a kind of logical consequence, but they are not used to determine meaning. ([45], 11)

Restall's example is the sequent rules for negation:

$$\frac{\Gamma \Rightarrow A, \Delta}{\Gamma, \neg A \Rightarrow \Delta}\,(L\neg) \qquad \frac{\Gamma, A \Rightarrow \Delta}{\Gamma \Rightarrow \neg A, \Delta}\,(R\neg)$$

According to Restall these two negation rules can be taken as 'encoding' the basic properties of negation. "[T]he inference patterns determine the meaning of the connective" (ibid). Yet, these negation rules encode not only the basic properties of classical negation, but also of the negation of relevant logics, linear logics, etc. Restall's point is that the variation in the deducibility relation between these logics is exclusively due to the antecedent context of deducibility, or in particular, the *structural rules* of the systems. There is therefore no reason to conclude that the different logical theories ascribe a different meaning to the negation expression.

On this picture, rules corresponding to properties of the antecedent deducibility relation are non-meaning-determining. Think for example of the identity axiom, the weakening rule or the cut rule of a sequent calculus system:

$$\frac{}{A \Rightarrow A}\,(Id) \qquad \frac{\Gamma \Rightarrow A}{\Gamma, B \Rightarrow A}\,(K) \qquad \frac{\Gamma \Rightarrow A \quad \Pi, A \Rightarrow B}{\Gamma, \Pi \Rightarrow B}\,(Cut)$$

These structural inference rules correspond roughly to reflexivity, monotonicity, and transitivity respectively. Crucially, the structural rules are formulated schematically without any occurrence of logical connectives. In sequent calculus, unlike natural deduction, they are an explicit part of the system. Structural minimalism for connectives would then have it that these rules form part of the antecedent deducibility relation, and are therefore not meaning-constitutive.

There are many well known examples of how the presence or absence of structural rules make a difference to logical theory. Consider the standard sequent calculus rules for a conditional $\rightarrow$:

$$\frac{\Gamma \Rightarrow A \quad \Pi, B \Rightarrow C}{\Gamma, \Pi, A \rightarrow B \Rightarrow C}\,(L\rightarrow) \qquad \frac{\Gamma, A \Rightarrow B}{\Gamma \Rightarrow A \rightarrow B}\,(R\rightarrow)$$

These rules are the sequent calculus counterparts of modus ponens and conditional proof in natural deduction respectively. The rules govern the intuitionistic conditional, but only if we assume an antecedent context of deducibility where the identity axiom and the weakening rule are present (the cut rule is typically admissible). If we remove the weakening rule, however, the resulting logic for the conditional changes in important ways. Most significantly, the following Hilbert axiom no longer holds:

$$\Rightarrow A \rightarrow (B \rightarrow A)$$

For the structural minimalist the above theorem is therefore not a meaning-constitutive law of the intuitionistic conditional. Rather, it is one which follows partly as a result of the assumptions made about the antecedent context of deducibility. In contrast, the minimalist would think of the $R \rightarrow$ and $L \rightarrow$ rules themselves as meaning-constitutive inference rules for the conditional.

Sometimes properties of the antecedent context of deducibility is not explicitly formulated as a structural rule, but rather occurs as a *structural property* of the calculus. As will become clear, there is no obvious definition of what counts as a structural property (or a structural rule, for that matter), but it is easy to find plausible examples. Haack [19] suggests another feature of the inference rules that can be considered 'structural'. The classical conditional rules in sequent calculus differ from the intuitionistic rules only in being *multiple conclusion* (or multiple-succedent). Thus the following are inference rules for the classical conditional:

$$\frac{\Gamma \vdash A, \Delta \quad \Pi, B \vdash \Sigma}{\Gamma, \Pi, A \rightarrow B \vdash \Delta, \Sigma}\,(L\rightarrow) \qquad \frac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \rightarrow B, \Delta}\,(R\rightarrow)$$

With the latter set of rules we can for example derive Peirce's Law, $((A \rightarrow B) \rightarrow A) \rightarrow A$, which is classically but not intuitionistically valid.[4] Haack thinks that the classical and intuitionistic conditional have the same meaning. The reason is precisely that of the structural minimalist. She says about the restriction on the succedent set that "[s]ince this restriction involves no essential reference to any connectives, it is hard to see how it could be explicable as arising from divergence of meaning of connectives" (ibid., 10).[5] Again,

[4] There is a reason why the structural minimalist prefers to make this comparison in sequent calculus. In standard natural deduction axiomatizations the classical and intuitionistic conditional rules are the same. However, the classical negation is nonconservative over the conditional, and therefore allows a derivation of, say, Peirce's Law. This goes to show how presentation sensitive the structural properties of a logical system are.

[5] Let me add that dual intuitionistic logic also shares the same conditional rules, but now with the antecedent of sequents restricted to singleton or empty sets.

the structural minimalist has it that any change in the logic is merely due to a change in the antecedent context of deducibility — in this case from single to multiple conclusion.

Yet another example is the structural rule of *contraction*:

$$\frac{\Gamma, A, A \Rightarrow \Delta}{\Gamma, A \Rightarrow \Delta}(LW) \qquad \frac{\Gamma \Rightarrow A, A, \Delta}{\Gamma \Rightarrow A, \Delta}(RW)$$

When we introduced the notion of a Tarski deducibility relation we talked about a relation between a set of formulae and a formula. We have then considered a generalization to a deducibility relation between two sets of formulae. But notice that if the relata are sets we are absorbing the above structural rule of contraction into the calculus. If we instead consider deducibility relations from *multi-sets to multi-sets* we distinguish between the sequents $A, A \Rightarrow B$ and $A \Rightarrow B$. Again, the structural property of multi-sets plays the role of structural contraction in the antecedent context of deducibility. The result is that classical logic — in both its set or multi-set formulation — has the law of absorption as a theorem: $\vdash (A \to (A \to B)) \to (A \to B)$.

## 4. Operational and Global Meaning

We have seen that the structural minimalist has a rough guideline for a divide between meaning-constitutive and non-meaning-constitutive inference rules. But it is still short of a precise formulation. A more sophisticated theory of structural minimalism is advanced by Paoli [30], [31]. He introduces a distinction between *operational* and *global* meaning. The former is constituted by the specific inference rules for a connective $\lambda$ (e.g. $R\to$ and $L\to$). The latter is defined as the class of all $\lambda$-theorems (for a connective $\lambda$) derivable in a logical theory $S$ (e.g. all classically derivable sequents that include a conditional).

What does it mean that a theorem is a $\lambda$-theorem? It could just mean that the connective $\lambda$ occurs in the theorem, but any instance of a theorem expressed with meta-variables can have any formula occurring in it, and therefore any connective. For example, $A \to (B \to A)$ can have occurrences of $\vee$ or $\wedge$ or both, depending on which formulae instantiate $A$ and $B$. Perhaps, then, it is better to say that a theorem is a $\lambda$-theorem if the connective $\lambda$ figures in the schematic formulation of the theorem, e.g. as $\to$ occurs in $A \to (B \to A)$. Moreover, we might want to restrict $\lambda$-theorems to theorems where $\lambda$ is the *only* connective that figures (in the schema). Thus, the classical theorem $(A \to B) \vee (B \to A)$ is neither a $\to$-theorem or $\vee$-theorem. If we instead insisted that $\lambda$ had to be the principal connective figuring (in the schema), then $(A \to B) \vee (B \to A)$ would be a $\vee$-theorem but not a $\to$-theorem. Paoli recognizes that there are many ways one could state the precise definition, but the details do not matter much for the subsequent discussion.

For the minimalist, operational meaning is the meaning *simpliciter* of a connective. ‘Global meaning’, on the other hand, is not part of the semantics of a connective at all.[6] A change in a meaning-constitutive operational rule can lead to a change in the class of theorems (and therefore in the global meaning). But the class of theorems can also change without any change to the operational rules, in particular, because of a change in the antecedent context of deducibility. One of Paoli’s examples is a comparison between the sequent calculi of classical and (sub-exponential) linear logic. In classical logic there are two formulations of the inference rules for conjunction: *additive* rules and *multiplicative* rules. The additive conjunction, ∧, is characterized by being *context-sharing*:

$$\frac{\Gamma, A_i \Rightarrow \Delta}{\Gamma, A_0 \wedge A_1 \Rightarrow \Delta} \qquad \frac{\Gamma \Rightarrow A, \Delta \quad \Gamma \Rightarrow B, \Delta}{\Gamma \Rightarrow A \wedge B, \Delta}$$

where $i \in \{0, 1\}$. The multiplicative conjunction, ⊗, is *context-independent*.

$$\frac{\Gamma, A, B \Rightarrow \Delta}{\Gamma, A \otimes B \Rightarrow \Delta} \qquad \frac{\Gamma \Rightarrow A, \Delta \quad \Pi \Rightarrow B, \Sigma}{\Gamma, \Pi \Rightarrow A \otimes B, \Delta, \Sigma}$$

In the presence of the classical deducibility relation the additive and multiplicative rules are equivalent. More precisely, if we add weakening and contraction, the two rule pairs are interderivable. On the other hand, linear logic has both additive and multiplicative conjunctions, but without either weakening or contraction. The result is two independent logical connectives, where classical logic had only one.

According to Paoli’s minimalism, classical logic (with additive conjunction) and linear logic (with additive conjunction) determine the same meaning for the conjunction. Nevertheless, the two logics give two different classes of ∧-theorems. Different logical theories — same meaning. Thus, the structural minimalist can maintain that the dispute between the classical logician and the linear logician is non-verbal. The theories are, to use Paoli’s term, ‘genuine rivals’. Similarly, if we are willing to chalk up the difference between single- and multiple-conclusion as structural, the structural minimalist can claim that classical and intuitionistic connectives all have the same (operational) meaning, even if the logical theories are different. The classicist and the intuitionist are having a genuine disagreement.

[6] For a semantic holist, however, the notion of ‘global meaning’ would perhaps be more appropriate.

## 5. What Do We Count As Structural?

So far so good for the structural minimalist. Paoli has given an improved articulation of the distinction between meaning-determining and non-meaning-determining properties of inference rules. As long as there exists a formalization of two logics in which their operational rules are the same, they can be considered genuine rivals. And a number of logics do have such formalizations. We have already seen a comparison of classical logic with linear logic and with intuitionistic logic, but also quantum logic ([47]), dual-intuitionistic logic ([52]), and relevant logics ([44], [29]) can arguably be axiomatized in ways that isolate their disagreement with classical logic as 'merely structural'. The operational rules, and therefore the meaning of the connectives, are constant, but the antecedent context of deducibility varies. In fact, Paoli suggests that the minimalist can do even better. If we consider generalizations of sequent calculus such as hyper-sequents ([2]) or $n$-sided sequents ([3], [21]) as structural, then further logics can be considered genuine rivals, e.g. various modalities can be given the same operational meaning, and inference rules for many-valued connectives can be give as variants of the classical operational rules.[7]

But how clear is the distinction between operational and structural rules (and properties)? And how semantically well-motivated is the distinction? I have previously expressed doubts about the semantic significance of the structural divide ([20], §5.3), and Paoli [32] has subsequently replied to my objections. He has labelled the first of them the 'scope objection'. Recall that the ultimate aim of minimalism is to support the claim that logical disagreements in important philosophical debates are substantive disagreements, rather than mere verbal disputes. With this I agree, but I am not convinced that the structural divide helps the minimalist cover the cases of disagreement we would consider substantive. If structural minimalism turns out to be too revisionist with respect to what counts as a genuine disagreement in logic, so much the worse for structural minimalism.

### 5.1. *The Scope Objection*

So how faithful is structural minimalism to ongoing debates in the philosophy of logic? For some disagreements in the philosophy of logic there are no obvious formalizations which render the dispute 'structural'. Supervaluational

[7] Read [42] has defended a view on which a set of labelled natural deduction rules give the same meaning to all normal modal logic operators, e.g. **S4** and **S5**. Structural inference rules for labels are said to change the logical theory, but without change in the semantics of the operators. Given the wide span of interpretations and applications for modal operators, however, perhaps it is too ambitious to attribute the same semantic content to them.

and subvaluational logics, for example, are reasonably popular theories of correct reasoning with theoretical terms or vague expressions (cf. [53], [18], [23]). For the structural minimalist the question is whether there are formalizations of super- and subvaluational logics in which the difference with classical logic is 'merely structural'. As far as I know there is no obvious way to make the difference a matter of structural properties, but even if there were it appears a bit artificial that the existence of one type of formalization is decisive in the question of genuine disagreement.

Worse, it is not the case that disputes in the philosophy of logic are always neutral with respect to formalizations. A change in the formalization of the logics might prove unfaithful to the content of the debate. An example is the revisionist argument in favour of intuitionistic logic, due to Prawitz and Dummett (e.g. [33], [14]). Their argument relies on a formalization of classical and intuitionistic logic in standard natural deduction. The intuitionists consider the lack of normal form derivations a critical problem for the coherence of classical negation. Classicists have in turn replied by developing non-standard formalizations of classical negation (e.g. [28], [41]). The problem for the structural minimalist is that the original debate requires a formalization in which the logical difference is not structural. In fact, Dummett has expressed independent worries about classical sequent calculus, which for him are reasons to rule out these formalizations as contenders.[8] Whatever the quality of Dummett's arguments, however, it goes to show that the mere existence of a formalization which suits the structural minimalist might not be enough to capture the content of the disagreement. At least in some cases, the formalization might be philosophically objectionable to one party in the debate.

Paoli has a reasonable counter. Structural minimalism only claims to be giving a *sufficient* condition for genuine rivalry between logics — not a necessary condition. In other words, there could be non-verbal disputes even for logical theories without shared operational rules. But sufficiency is still a strong claim, and it runs into another objection. Does structural minimalism count some disputes as genuine that ought to be considered merely verbal? Let us consider Quine's example of a merely verbal dispute. Two theories have simply swapped the inference rules for '$\wedge$' and '$\vee$'. Theory $S_1$ is the $\wedge, \vee$-fragment of classical natural deduction. Theory $S_2$ has the same logical connectives, but with the conjunctive inference rules $\vee$ and vice versa. With this set-up there is no difference in the context of deducibility. Nevertheless, the most natural proposal is that $\wedge$ in $S_1$ and $\vee$ in $S_2$ have the same meaning. This makes sense for the structural minimalist. The two connectives have the same operational rules (i.e. intro- and elim-rules).

[8] See in particular [14, 187]. For the record I do not agree with Dummett's argument, and I have criticized it elsewhere [22].

But most examples are not this straightforward. Even for connectives that appear to have distinct meanings, we cannot rule out that there exists formalizations in which they share the same operational rules, and yet yield different logics only due to structural properties. When that *is* possible, the two connectives have the same meaning by minimalist standard. If the requirement was that the two connectives must have *exactly* the same operational rules, this would hardly make sense. But that is not the requirement. Instead, the structural minimalist will accept inference rules as 'the same' when they differ only with respect to structural properties, e.g. single- vs multiple-succedent (-antecedent), hypersequents, or labelled sequents. For the minimalist, the single-conclusion intuitionistic conditional has 'the same' operational rules as its multi-conclusion classical counterpart. There can in other words be a difference between two operational rules that count as 'the same'. But how big a difference? It better not be big enough to allow disjunction and conjunction to share its operational rules while the corresponding provable sequents vary because of structural properties.

Let me give an example, albeit an artificial one. Let us suppose we have system of *pre-directional sequents* $\Gamma \Rightarrow\Leftarrow \Delta$, read as informally as either $\Gamma \Rightarrow \Delta$ or $\Delta \Rightarrow \Gamma$. The system consists of three inference rules for the connective $\triangledown$, together with the identity axiom $A \Rightarrow\Leftarrow A$:

$$\frac{A \Rightarrow\Leftarrow C}{A \triangledown B \Rightarrow\Leftarrow C} \qquad \frac{B \Rightarrow\Leftarrow C}{A \triangledown B \Rightarrow\Leftarrow C} \qquad \frac{C \Rightarrow\Leftarrow A \quad C \Rightarrow\Leftarrow B}{C \Rightarrow\Leftarrow A \triangledown B}$$

The idea is that in the pre-directional sequent system, the sequents do not yet have a direction. Read from left-to-right these inference rules are those of conjunction, whereas from right-to-left they are disjunction rules. Now consider two systems $S_1$ and $S_2$ extending this system with the structural rules *LR* and *RL* respectively:

$$\frac{\Gamma \Rightarrow\Leftarrow \Delta}{\Gamma \Rightarrow \Delta}\,(LR) \qquad \frac{\Gamma \Rightarrow\Leftarrow \Delta}{\Gamma \Leftarrow \Delta}\,(RL)$$

In the system $S_1$ the resulting class of derivable standard sequents are those of conjunction, while in $S_2$ it is the class of disjunction sequents. The example is rigged. The upshot is that the same operational rules give rise to disjunction and conjunction merely by varying the structural rules of direction.

### 5.2. *The Cut-Off Objection*

Can the structural minimalist simply reject the example? Even though it is not among the standard generalizations of sequents, it is not clear why the example is in principle different from known generalizations of sequent calculus.[9] In fact, it puts the pressure elsewhere. This is essentially the second objection, labelled the 'cut-off objection'. We only have an open-ended grasp of what counts as a structural property, and an equally open-ended grasp of what counts as an operational rule. As a result there is no sharp delineation of the logics that can be formalized in such a way as to qualify as 'genuine rivals' by the minimalist standard. Going on the example of classical vs linear logic alone, such a worry appears exaggerated. However, other examples will help to underwrite how many logical distinctions can be formalized as 'structural'.

In the debate between the relevant logicians and the classicist, the former do not subscribe to a single logic, but have in common that they advocate logics which broadly speaking belong to the same family of systems ([40], [49], [15]). Some of these systems can be axiomatized as standard weakening-free sequent calculi, but other — and in particular those with distributivity — require more elaborate formal frameworks, such as display logic ([6]) or bunch-theoretic calculi ([49]). Both formalizations rely on introducing an inflated stock of 'structural connectives'. These operators are like the comma in sequent calculus in that they do not belong to the object language. Yet, they correspond to object language connectives in the way that the antecedent comma corresponds to conjunction and the the succedent comma to disjunction in classical sequents. For example, while the standard lattice conjunction, '$\wedge$', corresponds to the sequent comma on the left, the relevant logic fusion, '$\circ$', corresponds to the semicolon to the left.[10] Put differently, these calculi allow more than one premise-combinator, and different combinators permit different structural rules. For example, the comma typically permits weakening, whereas the semicolon does not. The upshot is that sets of connectives can be associated with different structural rules in the same calculus.

But although this appears to offer more flexibility for the structural minimalist, it erodes the distinction between what counts as 'structural' and what counts as 'operational'. And this is a problem. The bunch-theoretic and

[9] In particular, why are display calculi legitimate generalizations whereas the pre-directional sequents are not (see below for details)? An anonymous referee has suggested that the difference between standard sequents and pre-directional sequents is that the latter have no informal interpretation in terms of assertions and commitments. I take that point, but if the requirement is that the generalizations must have an assertional reading, we have similar problems with the structural connectives of display or bunch-theoretic calculi.

[10] Below are the inference rules for conjunction and fusion in a bunch-theoretic systems:

$$\frac{X \vdash A \wedge B}{X \vdash A} \qquad \frac{X \vdash A \wedge B}{X \vdash B} \qquad \frac{X \vdash A \quad Y \vdash B}{X, Y \vdash A \wedge B}$$

$$\frac{X \vdash A \quad Y \vdash B}{X; Y \vdash A \circ B} \qquad \frac{X \vdash A \circ B \quad Y(A;B) \vdash C}{Y(X) \vdash C}$$

display calculi can in principle assign a distinct set of structural properties to each logical connective. We are left with a situation where structural properties do not belong to the context of deducibility, but rather to a particular connective. But what is then the reason to insist that these properties do not affect the meaning of the connective? For structural minimalism to be plausible, a more principled demarcation between the structural and the non-structural has to be given. One which not only has a suitable range of applications, but which is also accounts for why certain properties of inference rules are meaning-constitutive and others are not.

## 6. The Meta-Quinean Objection

Let us for the time being grant that the structural minimalist can reply to the cut-off objection in a satisfactory manner. Another objection still remains — the 'meta-Quinean objection'. Recall that the ordinary meaning-variance argument questions the homophonic translation of the same connective, say ∨, across logical theories. The structural minimalist replies by pointing to shared operational rules and variation in the antecedent context of deducibility. But now the Quinean has a revenge argument. Take the sequent rules for the classical conjunction and the linear logic multiplicative conjunction:

$$\frac{\Gamma, A, B \Rightarrow \Delta}{\Gamma, A \wedge B \Rightarrow \Delta} \qquad \frac{\Gamma \Rightarrow A, \Delta \quad \Pi \Rightarrow B, \Sigma}{\Gamma, \Pi \Rightarrow A \wedge B, \Delta, \Sigma}$$

$$\frac{\Gamma, A, B \Rightarrow \Delta}{\Gamma, A \otimes B \Rightarrow \Delta} \qquad \frac{\Gamma \Rightarrow A, \Delta \quad \Pi \Rightarrow B, \Sigma}{\Gamma, \Pi \Rightarrow A \otimes B, \Delta, \Sigma}$$

The operational rules are identical (notwithstanding the symbol for the conjunctions). But because of the linear logic restrictions on structural rules some classical sequents cannot be derived, e.g. $A \Rightarrow A \otimes A$. The reason is the absence of the structural rule of contraction $LW$:

$$\frac{\Gamma, A, A \Rightarrow \Delta}{\Gamma, A \Rightarrow \Delta}\ (LW)$$

The meta-Quinean turns the attention from the object language connectives to the structural connectives of the proof theoretic framework, e.g. the comma ',' and the sequent arrow '⇒' in the above inference rules. The structural minimalist has shifted the logical disagreement from the operational rules for the (object language) connectives to the structural properties. These properties, and in particular the structural rules, govern the structural connectives. The comma of the classical calculus is contractive, the linear comma is not. Why is it that these expressions, although not in the object language, are immune to a meaning-variance argument? The question gains importance as the structural minimalist becomes more reliant on the introduction of non-standard structural connectives, such as those in display logic.

Here is one type of answer to the meta-Quinean: Structural connectives are not part of the logical vocabulary in question, and thus not in dispute. We could of course make the dispute about them, but that wouldn't be a dispute about semantics since these objects are not *bearers of meaning*. But there is at least informally a logical affinity between structural connectives and their object language counterparts, e.g. the sequent arrow and the conditional. In fact, a number of authors have suggested that logical connectives directly *internalize* structural connectives. For example, it appears natural to say that the standard conjunction $\wedge$ is an objection language expression of the structural comma on the left:

$$\frac{\Gamma, A, B \Rightarrow \Delta}{\Gamma, A \wedge B \Rightarrow \Delta}$$

The bi-directional rule simply says that the two sequents are equivalent, and therefore ensures that the conjunction will inherit the structural properties of the comma. To this Paoli has a sophisticated answer, one based on a suggestion by Dosen [12], and developed further by Avron [1]. First, note that the rule for the conjunction is the same, regardless of which structural properties the comma has (albeit the ensuing logic might be different). Second, there is a 'solution' of this equivalence which produces the right-side sequent rules for $\wedge$. First, we assume that $A \wedge B \Rightarrow A \wedge B$. The bottom-up inference from the above rule gives us $A, B \Rightarrow A \wedge B$, from where we can apply cut twice with $\Gamma \Rightarrow A, \Delta$ and $\Pi \Rightarrow B, \Sigma$ respectively. The result is the following 'derivation' of the inference rules $R\wedge$:

$$\dfrac{\dfrac{}{\Pi \Rightarrow B, \Sigma} \qquad \dfrac{\dfrac{}{\Gamma \Rightarrow A, \Delta} \quad \dfrac{A \wedge B \Rightarrow A \wedge B}{A, B \Rightarrow A \wedge B}}{\Gamma, B \Rightarrow A \wedge B, \Delta}}{\Gamma, \Pi \Rightarrow A \wedge B, \Delta, \Sigma}$$

The advantage of the equational solution is that the right-side rule was produced from the left-side rule without any applications of the structural rules of weakening and contraction.[11] The structural minimalist can claim this as further evidence that the operational rules are sufficiently independent

[11] Although note that the structural properties of the sequent arrow must include reflexivity and transitivity in order for the equational solutions to work. This is relevant for what follows in section 7.

of the context of deducibility. Corresponding solutions exist also for other logical connectives, e.g. the additive conjunctions, disjunction, the conditional, etc.

Nevertheless, matters are not as straightforward as the structural minimalist would hope. First, some structural assumptions about the sequents are required for the solutions to work. Bonnay and Simmenauer [8] point out that there is no solution for the conditional unless one assumes that the rule allows auxiliary formulae. Assume that the rule is as follows:

$$\frac{A \Rightarrow B}{\Rightarrow A \to B}$$

Then we cannot move from $A \to B \Rightarrow A \to B$ to $A \to B, A \Rightarrow B$ since the rule only applies to sequents with empty antecedent. One can object that the presence of a set of auxiliary formulae $\Gamma$ is innocuous, but that is not obvious. In the formalizations developed by Sambin [47], for instance, the *visibility* of rules plays a crucial role, i.e., that there are no formula present other than the principal formula.

## 7. Internalization of Validity

The meta-Quinean objection is at its gravest for the sequent arrow. The meta-Quinean might insist that even if two theories can align the meanings assigned to logical connectives, they will assign different meanings to the sequent arrow $\Rightarrow$ insofar as they yield different logics. As before it threatens to make the logical dispute a merely verbal dispute. This, I think, is the heart of the meta-Quinean challenge. The meta-Quinean's point is that the original meaning-variance was not directed against logical connectives in particular. It merely tried to establish that disputes about logic are merely verbal. Now the meaning-variance argument is attaching to more than just the connectives. It targets the notions of 'validity' or 'following from'. Even if the classicist and intuitionist mean the same thing by 'or' and by 'not', it does not follow that they mean the same thing by '*A* or not-*A* is valid'.

Elsewhere I have called the attention to a distinction between meaning-variance for logical connectives and meaning-variance for 'validity' ([21]). I there suggested two different meaning-variation theses — type A and type B:

(A) The meaning of 'valid' varies across logical theories;

(B) the meaning of some logical connective $\lambda$ varies across logical theories.

The meta-Quinean is suggesting that even if the structural minimalist can deflect the meaning-variance argument for *B*, there is a related argument for *A*. And it is equally damning. If the concept of validity it not stable across the rival theories, there is no reason to think that the same subject matter is under dispute. Whether or not the theories then happen to attach the same meanings to the logical connectives is not relevant. Presumably, what we want is an account of logical disagreement that avoids making the dispute a verbal one, even for expressions such as ‘valid’.

It is no good to reply that the sequent arrow is not a logical connective and therefore not a bearer of meaning. The meta-Quinean can demand that the predicate ‘$x$ is valid’ has to be part of the rival logical theories. After all, validity is logic’s chief subject matter. Let us denote this new predicate $Val(\ulcorner A \urcorner, \ulcorner B \urcorner)$, where $\ulcorner\,\urcorner$ is used for the names of sentences.[12] In the object language, we can then express that $B$ follows from $A$, and indeed quantify into sentence position. Alternatively, we could introduce a new connective which has the status of an ‘entailment connective’ along the lines explored in relevant logic.

The natural extension of the minimalist position is that whatever plays the internalized role of validity in the objection language has a set of meaning-constitutive laws. But now the problem is that any structural property is a potential meaning-constitutive property. Is the structural rule of weakening meaning-constitutive? Presumably not, as it would follow that classical and linear logic assign different meanings to the validity predicate. What about transitivity? The cut rule is a strong candidate for a core property of validity. Formally, it is present — albeit admissible — in almost any logical theory, and it has an intuitive connection to the chaining of argument steps.

Paoli’s response to the meta-Quinean objection is, in part, to take reflexivity and transitivity as meaning-constitutive properties of validity. In fact, he seems to suggest that these properties are constitutive of a deducibility relation. That is not an uncommon view. Both properties are part of the traditional Tarskian analysis. Preserving these two properties also fits the strategy of ‘equational solutions’ for sequent rules discussed above. But as a strategy for the structural minimalist it is also a concession. The meta-Quinean argument has now forced the minimalist to admit that any logical theory with a non-transitive consequence relation cannot be a genuine rival to, say, classical logic. How bad is this concession? Although non-transitive systems are not popular, they have advocates in a number of philosophical debates. Tennant [51], [50] has developed a non-transitive relevant system for so-called ‘epistemic gain’ in inference; Weir [54] is a proponent of a non-transitive logic for unrestricted set comprehension; Zardini [57] and Cobreros et al. [11] have defended non-transitive logical theories for vague expressions; and, finally, Ripley [46] gives a non-transitive theory for an

[12] Validity predicates are susceptible to semantic paradox, much like the related truth predicate. But the resolution of that problem — itself controversial — is irrelevant for the strengthened meaning-variance argument. For some recent discussions, see [16], [48], and [4].

unrestricted truth predicate. In Zardini [58] there are also more comprehensive arguments for why transitivity should not hold unrestrictedly of logical consequence.

Although most philosophers of logic consider non-transitivity too high a cost, it strikes me as premature rule out such consequence relations apriori. Yet this essentially what the structural minimalist proposes to do. If consequence relations are analytically transitive, then it is simply a change of subject to offer a non-transitive logical theory. A similar case can be made for reflexivity, but I will bracket that for now.[13] The overarching point is this: The meaning-variance thesis applies as much to expressions such as '*x* is valid or '*x* is inconsistent with *y*' as it does to conjunctions, disjunctions, and negations. A satisfactory response to the meaning-variance argument is only forthcoming when disputes about validity as well as logical laws are non-verbal disputes.

## 8. Validity in Sequent Calculus

If we want to internalize validity in the object language, however, we are left with some interesting choices. Paoli points out that there are at least two candidate notions of validity associated with a sequent calculus. The first he calls *internal validity* (not to be confused with my use of 'internalized' validity). Internal validity for the system **S** is defined as follows: The argument from $\Gamma$ to $A$ is internally valid ($\Gamma \vdash_S^I A$) just in case the sequent $\Gamma \Rightarrow A$ is derivable in **S**. So far I have assumed that this is the notion of validity that we were interested in internalizing. It is evident that internal validity inherits some structural properties straight from structural rules of **S**, e.g. weakening. But there is an alternative notion of validity that Paoli cleverly uses to parry the meta-Quinean objection. Let us say that an argument is *externally valid*, $\Gamma \vdash_S^E A$, just in case $\Rightarrow A$ is derivable in the system obtained by adding $\Rightarrow B$ as initial sequent for each $B \in \Gamma$ to **S**. As it happens, these two notions of validity are equivalent in classical sequent calculus, but come apart elsewhere.

External validity has structural properties which are unaffected by the structural rules of the system in question. Paoli observes that even if **S** does not have weakening it will be the case that $A, B \vdash_S^E A$. We also have that if $A \vdash_S^E B$ and $B \vdash_S^E C$, then $A \vdash_S^E C$. If $\Rightarrow B$ is derivable in **S** plus $\Rightarrow A$, and $\Rightarrow C$ is derivable in **S** plus $\Rightarrow B$, then $\Rightarrow C$ is derivable in **S** plus $\Rightarrow A$. Put informally, the transitivity of external validity follows from the transitivity of the fraction line in the sequent system, not from the sequent arrow. In

[13] Furthermore, structural exchange of premises (and conclusions) is admissible in systems where the antecedent and succedent are multi-sets. But a number of logical theories do not allow unrestricted exchange, e.g. distributive relevant logics weaker than **T**.

other words, the cut rule does not figure in the argument. Paoli further motivates this distinction between 'horizontal inference' (internal validity) and 'vertical inference' (internal validity) in terms of an informational view and a warrant-preservation view of validity respectively ([27]). I will leave the particular interpretation out of my discussion, however, since the main advantage of the distinction between internal validity and external validity lies elsewhere.

For Paoli's structural minimalist there is an elegant fact available to argue that classical and linear logic have the same antecedent context of deducibility. Where every $q \in \Gamma$ and $p$ are propositional variables, $\Gamma \vdash_{CL} p$ if and only if $\Gamma \vdash^{E}_{LL} p$. That is, in the pre-logical language, $p$ is a classical consequence of $\Gamma$ just in case $p$ is an *external* linear consequence of $\Gamma$. The idea is that there is now structural alignment between classical and linear logic at the level of external validity prior to the introduction of logical connectives. The minimalist can then reply to the meta-Quinean that there is a genuine rivalry even at the level of validity. As Paoli puts it, '[a]lthough the sequent separator $\Rightarrow$ may be a false friend, the fraction line separating the sequent premisses from the conclusion in an operational rule is not.'

But how robust is this victory? It is true that sequent calculi have a shared sequent-derivation relation — the fraction line — which has some structural properties merely by definition. As far as these are the same for two logics, the meta-Quinean cannot argue that there is change of meaning. The meta-Quinean is limited to the level of internal validity and the sequent arrow, just as the regular Quinean was limited to the object language. But the target of the meaning-variance argument was not what the logical rivals *agree* on, but what they disagree on. The meta-Quinean approach should be to claim that a change of internal validity is a change of subject. The minimalist can reply that the sequent arrow is not a bearer of meaning, but that is besides the point. As long as it gives rise to a validity predicate, it can be the target of a meaning-variance argument. So in order to succeed, the structural minimalist would have to insist that logical theories are really only about external validity, not internal validity. That is simply not reasonable, however. The susbtructuralist literature is a case in point. The logical attention is on the internal validity relation, where the structural revisions are made.

## 9. Structuralism for Logical Connectives

Even if I disagree with the details, I still have a lot of sympathy for the minimalist cause. There should be genuine disagreement between logical theories. But minimalism is not the only strategy to establish sameness of meaning. I will quickly outline what I think is a promising alternative, one with some interesting precedents in a related literature. The backdrop of the proposal is, broadly speaking, an *anti-exceptionalism* for logic. Logical theory is not fundamentally different from other scientific theories. The logical theories are, as a rule, subject to the same desiderata and conditions as theories in natural science. Moreover, logical theories are in principle *revisable* in much the same ways as other theories.[14] In contrast, the *exceptionalist* position is typified by Dummett [13], and the idea that logic is epistemologically privileged. Logic might still be revisable, but under conditions essentially different from those of scientific theories.

Details aside, anti-exceptionalism has been defended by a variety of philosophers, including Field [16], Priest [34], Putnam [36], and Williamson [55]. Although anti-exceptionalism in and of itself is no reply to the meaning-variance argument, it does point to an important analogy. The Quinean meaning-variance argument about logic sorts under a larger debate about meaning-variance in scientific theories.[15] Kuhn [25] argued that when one scientific theory overtakes another there might be a change in the meaning of theoretical terms. In a word, there is *semantic incommensurability*. Kuhn used the example of the term ‘mass’ in physical theories. This theoretical term, he claimed, picked out a different concept in Einsteinian theory than it did in Newtonian theory. It is no surprise that Kuhn developed his idea of incommensurability with explicit reference to Quine’s indeterminacy of translation.

The post-Kuhnian literature has seen an industrious literature on anti- incommensurability arguments. As with the case of Quine’s meaning-variance thesis for logic, many commentators think it is uncharitable to scientists to maintain that their theoretical disagreements are merely verbal disputes. The challenge is the same. How can we establish preservation of meaning from a scientific theory to its successor? There are plenty of enterprising responses to the incommensurability thesis, but here I want to draw on some ideas associated with *structural realism*. Worral [56] formulates a realist position for theoretical terms by drawing a distinction between the *structure* and the *content* of scientific theories. He points to examples where new theories have preserved the mathematical structure of the preceding theory, either by simply reinterpreting a set of equations, or, more frequently, by preserving the old equations only as limit cases of the new equations. If such a case of structural preservation can be made to work between Newtonian mechanics and Special Relativity, for instance, then the term ‘mass’ might be said to have shared meaning in both theories through its role in a set of equations.

[14] In fact, this was Quine’s position in *Two Dogmas* [38], prior to the formulation to the meaning-variance argument.

[15] This has not gone unnoticed. Boghossian [7], for example, gives a joint theory of theoretical terms and logical connectives. Priest [34] also points to the connection between incommensurability and the meaning-variance argument.

The structural realists have produced a number of examples of alleged structural preservation in scientific theories (cf. [26]). Whether the history of science actually bears out such a distinction between structure and content is of course controversial, and at any rate well beyond the scope of this paper. Instead I want to consider whether there is an analogy to logical theories. The key structuralist insight is to require not that one theory be a cumulative extension of another, but that structure is preserved. In particular, the examples of the structural realist are typical ones where the more recent theory *generalizes* the equations of the old theory. The equations — or more generally, the mathematical structure — of the preceding theory survives as a special, limiting case of the new theory. In logic the mathematical structure is not equations, but there are a number of other candidates. For the proof theoretically minded, it might be derivational structure, but it could equally well be algebraic structure, model theoretic structure, etc. Indeed, logical theory is rife with examples of structure preservation.

(1) MANY-VALUED LOGICS are generalizations of classical logic in a straightforward sense. The set of truth values $\mathcal{V}$ is extended with non-Boolean values, and the subset $\mathcal{D} \subset \mathcal{V}$ are the designated values. We can define consequence as $\Gamma \models A$ just in case, for every many-valued valuation $v$, whenever $v(B) \in \mathcal{D}$, for each $B \in \Gamma$, then $v(A) \in \mathcal{D}$. Classical consequence is the special case where $\mathcal{V} = \{1, 0\}$ and $\mathcal{V} = \{1\}$. Furthermore, the many-valued truth-functions for the logical connectives remain classical in the special case where the argument formulae have classical values (e.g. Kleene 3-valued logic, Logic of Paradox **LP**).

(2) As we have already seen above, INTUITIONISTIC LOGIC can be construed as a special case of classical logic by moving from multiple-succedent to single- or-empty-succedent sequent calculus. But that is a perspective limited to sequent calculus. Semantically, the Boolean algebras are a special case of Heyting algebras where $a = \neg\neg a$. The point is also underwritten by Glivenko's theorem which tells us that $A$ follows classically from $\Gamma$ if and only if $\neg\neg A$ follows intuitionistically from $\Gamma$.

(3) SUPERVALUATIONAL LOGIC is another example of generalization on the classical valuations. A supervaluation $v^+$ for a set $V$ of classical valuations is defined as a three value valuation such that $v^+(A) = i$ if $v(A) = i$ for every $v \in V$ (where $i \in \{1, 0\}$); otherwise $v^+(A) = \frac{1}{2}$. Again, classical logic is preserved as a limit case where the set of valuations $V$ is a singleton $\{v\}$.

(4) In a more proof-theoretic vein, HYPER-SEQUENTS and $n$-SIDED SEQUENTS are both generalizations of standard classical sequents. The former is calculus on sequents with sets of sequents as additional context; the latter is a sequent where we generalize from the two-sided to the $n$-sided. In both cases, the classical sequent calculus is preserved as a limit case.

On the structuralist account the identification of logical connectives across theories is a matter of structure preservation. I think that has some plausibility in the above cases. The reason why the Strong Kleene ¬-expression is considered the negation counterpart of classical logic is precisely because it preserves the classical semantics in special cases. The reason why the intuitionist conditional is identified as the counterpart of classical material implication is because it is, essentially, the classical truth condition generalized to a set of valuations. An advantage of the structuralist account is that it offers the same story about consequence and consistency as it does for the logical connectives, and it is a theory that applies to theory-change regardless of whether it is proof theoretic or model theoretic. That is not yet a full theory of meaning-variance, but it is an outline that promises to treat logical expressions on par with other theoretical terms.

## Conclusion

Putnam's challenge still haunts the minimalist. What, if anything, separates the meaning-constitutive from the non-meaning-constitutive laws of a connective? The structural minimalist has what to my mind is the most promising answer, but nonetheless one that fails to do justice to the subtleties of our disagreements about logic. Disagreements about logic are not exclusively about principles about negation, or principles about disjunction. They are disagreements about the nature of validity and consistency. If we take the meaning-variance argument seriously we have to accept that it attaches as much to these latter concepts as to negation and disjunction.

## References

[1] Arnon Avron (1991). Simple consequence relations. *Information and Computation*, 92:105-139.
[2] Arnon Avron (1996). The method of hypersequents in the proof theory of propositional nonclassical logics. In *Logic: from Foundations to Applications, European Logic Colloquium*, pages 1-32.
[3] M. Baaz, C.G. Fermuller & R. Zach (1993). Systematic construction of natural deduction systems for many-valued logics. In *Proc. 23rd International Symposium on Multiple-valued Logic*, pages 208-213. IEEE Press.
[4] J.C. Beall & Julien Murzi (2013). Two flavors of curry paradox. *Journal of Philosophy*, CX(3):143-165.
[5] Nuel Belnap (1962). Tonk, plonk and plink. *Analysis*, 22:30-34.
[6] Nuel Belnap (1982). Display logic. *Journal of Philosophical Logic*, 11(4): 375-417.
[7] Paul Boghossian (2003). Blind reasoning. *Proceedings of the Aristotelian Society Supplementary Volume*, 77:225-248.

[8] Denis BONNAY & Benjamin SIMMENAUER (2005). Tonk strikes back. *Australasian Journal of Logic*, 3:33-44.
[9] Rudolf CARNAP (1937). *The logical syntax of language*. Routledge & Kegan Paul Ltd, New York.
[10] David J. CHALMERS (2011). Verbal disputes. *Philosophical Review*, 120(4): 515-566.
[11] P. COBREROS, P. EGRÉ, D. RIPLEY & R. VAN ROOIJ (2011). Tolerant, classical, strict. *Journal of Philosophical Logic*, pages 1-39.
[12] Kosta DOSEN (1989). Logical constants as punctuation marks. *Notre Dame Journal of Formal Logic*, 30:362-381.
[13] Michael DUMMETT (1978). Is logic empirical? *Truth and Other Enigmas*, pages 269-289.
[14] Michael DUMMETT (1991). *The logical basis of metaphysics*. Harvard University Press, Cambridge, MA.
[15] J. Michael DUNN & Greg RESTALL (2002). Relevance logic. *Handbook of philosophical logic*, 6:1-128.
[16] Hartry FIELD (2008). *Saving Truth from Paradox*. OUP, Oxford.
[17] Hartry FIELD (2009). Pluralism in logic. *Review of Symbolic Logic*, 2(2):342-359.
[18] Kit FINE (1975). Vagueness, truth, and logic. *Synthese*, 30:265-300.
[19] Susan HAACK (1974). *Deviant logic*. Cambridge University Press, London.
[20] Ole Thomassen HJORTLAND (2010). The structure of logical consequence: proof-theoretic conceptions.
[21] Ole Thomassen HJORTLAND (2012). Logical pluralism, meaning-variance, and verbal disputes. *Australasian Journal of Philosophy*, Online first.
[22] Ole Thomassen HJORTLAND (forthcoming). Speech acts, categoricity, and the meanings of logical connectives. *Notre Dame Journal of Formal Logic*.
[23] Dominic HYDE (1997). From heaps and gaps to heaps of gluts. *Mind*, 106(424): 641-660.
[24] C.S.I. JENKINS (2013). Merely verbal disputes. *Erkenntnis*, pages 1-20.
[25] Thomas KUHN (1970). *The structure of scientific revolutions*. University of Chicago Press, Chicago.
[26] James LADYMAN (2009). Structural realism. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Summer 2009 edition.
[27] Ed MARES & Francesco PAOLI (2013). Logical consequence and the paradoxes. ms.
[28] Peter MILNE (1994). Classical harmony: Rules of inference and the meaning of the logical constants. *Synthese*, 100:49-94.
[29] Francesco PAOLI (2002). *Substructural Logics: A Primer*. Kluwer Academic Publishers.
[30] Francesco PAOLI (2003). Quine and Slater on paraconsistency and deviance. *Journal of Philosophical Logic*, 32:531-548.
[31] Francesco PAOLI (2007). Implicational paradoxes and the meaning of logical constants. *Australasian Journal of Philosophy*, 85(4):553-579, 2007.
[32] Francesco PAOLI (2013). Semantic minimalism for logical constants. *Logique et Analyse*.
[33] Dag PRAWITZ (1977). Meaning and proofs: on the conflict between classical and intuitionistic logic. *Theoria*, 43:2-40.
[34] Graham PRIEST (2006). *Doubt Truth to be a Liar*. OUP, Oxford.

[35] Hilary PUTNAM (1957). Three-valued logic. *Philosophical Studies*, 8(5):73-80.
[36] Hilary PUTNAM (1969). Is logic empirical? *Boston Studies in the Philosophy of Science*, page 216.
[37] Hilary PUTNAM (1976). The logic of quantum mechanics. In *Mathematics, Matter and Method*, chapter 10, pages 174-197. Cambridge University Press, Cambridge.
[38] W.V. QUINE (1951). Two dogmas of empiricism. *Philosophical Review*, 60:20-43.
[39] W.V. QUINE (1970). *Philosophy of logic*. Oxford University Press, Oxford.
[40] Stephen READ (1988). *Relevant Logic*. Blackwell, Oxford.
[41] Stephen READ (2000). Harmony and autonomy in classical logic. *Journal of Philosophical Logic*, 29:123-154.
[42] Stephen READ (2008). Harmony and modality. In L. Keiff C. Dégremont and H. Rückert, editors, *On Dialogues, Logics and other Strange Things*. Kings College Publications.
[43] M.D. RESNIK (1999). Against Logical Realism. *History and Philosophy of Logic*, 20(3):181-194.
[44] Greg RESTALL (2000). *An introduction to substructural logics*. Routledge.
[45] Greg RESTALL (2002). Carnap's tolerance, language change, and logical pluralism. *Journal of Philosophy*, 99:426-443.
[46] David RIPLEY (2011). Conservatively extending classical logic with transparent truth. *Review of Symbolic Logic*.
[47] G. SAMBIN, G. BATTILOTTI & C. FAGGIAN (2000). Basic logic: reflection, symmetry, visibility. *Journal of Symbolic Logic*, 65(3):979-1013.
[48] Lionel SHAPIRO (2010). Deflating logical consequence. *The Philosophical Quarterly*.
[49] John SLANEY (1990). A general logic. *Australasian Journal of Philosophy*, 68(1):74-88.
[50] Neil TENNANT (1994). The transmission of truth and the transitivity of deduction. In D.M. Gabbay, editor, *What is a Logical System*, pages 161-178. OUP, Oxford.
[51] Neil TENNANT (1997). *The Taming of the True*. Oxford University Press, Oxford.
[52] I. URBAS (1996). Dual-intuitionistic logic. *Notre Dame Journal of Formal Logic*, 37(3):440-451.
[53] B.C. VAN FRAASSEN (1966). Singular terms, truth-value gaps, and free logic. *The Journal of Philosophy*, 63(17):481-495.
[54] Alan WEIR (1998). Naive set theory is innocent! *Mind*, 107:763-798.
[55] Timothy WILLIAMSON (2013). Logic, metalogic and neutrality. *Erkenntnis*, pages 1-21.
[56] John WORRALL (1989). Structural realism: The best of both worlds?*. *Dialectica*, 43(1-2):99-124.
[57] Elia ZARDINI (2008). A model of tolerance. *Studia Logica*, 90(3):337-368.
[58] Elia ZARDINI (forthcoming). Breaking the chains: Following-from and transitivity. In C. Caret & O. Hjortland, editor, *Foundations of Logical Consequence*. OUP.



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