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LOGICAL CONSEQUENCE AND CONDITIONALS FROM A DIALETHEIC PERSPECTIVE

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ABSTRACT

Dialetheism holds the thesis that certain sentences are *dialetheias*, i.e. both true and false, and devises several strategies for avoiding trivialism, the (classical) consequence that *all sentences* are true. Two such strategies are aimed at invalidating one of the most direct arguments for trivialism, viz. Curry's Paradox: a proof that you will win the lottery, a proof that only resorts to naïve truth-principles, Conditional Proof (CP), *modus ponens* (MPP) and the standardly accepted structural rules. The first strategy simply consists in observing that the most well-known dialetheist logic, sometimes referred to as the Logic of Paradox (LP), invalidates MPP. The second strategy consists in rather taking one of the primary senses of 'if' to be captured by an *entailment* connective which does not validate CP. We argue that both strategies are problematic.

1. Introduction

Dialetheism holds that there are dialetheias, i.e. propositions that are both true and false.[1] Among the dialetheists, Priest (for example, in (Priest, 1979), (Priest, 2002a), (Priest, 2002b), (Priest, 2006a), (Priest, 2006b)) claims that dialetheism supplies the best solution to all self-reference paradoxes. The paradigmatic example of a self-reference paradox is the *strengthened liar paradox*, having the form:

(a): (a) is not true,

which is solved, according to Priest, by holding that (a) is both true and not true. In classical logic, the presence of a *dialetheia* entails *trivialism* (the truth of all sentences) and explosion (the derivability of any sentence) according to the classical rule *ex contradictione quodlibet* (ECQ).

Classical logic escapes *trivialism* because of the alleged evidence, rejected by dialetheists, that no contradiction can be true. In standard natural deduction,

[1] Priest uses the terms 'dialetheias' and 'true contradictions' to indicate 'gluts', a term coined by K. Fine in (Fine, 1975). For an introduction to dialetheism, see e.g. (Berto, 2007).

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ECQ can be derived using *reductio ad absurdum* (RAA) and other apparently non-problematic rules. Since a contradiction may be true, RAA is immediately rejected by dialetheists. However, rejecting RAA is insufficient to avoid trivialism: Curry's paradox, from which trivialism follows, can be generated without the help of RAA.

In the *Logic of Paradox* (LP) (Priest, 1979),[2] Priest observes that, in a semantically closed theory, using *modus ponens* (MPP) and *absorption* (ABS), i.e.:ϕ

$$\text{ABS}\ \frac{\phi \rightarrow (\phi \rightarrow \psi)}{\phi \rightarrow \psi}$$

a version of Curry's paradox is derivable.[3]

In LP, $(A \rightarrow B)$ is defined as $(\neg A \vee B)$ (the material conditional), which suffices to establish that MPP can't in general be valid. For, if A is a dialetheia, $(\neg A \vee B)$ is true even if B is not. MPP is labeled in LP as a *quasi-valid* rule, a rule that is valid provided that all truth-values involved are classical (i.e., solely true or solely false).

However, Priest realizes that the material conditional, just because it invalidates MPP, is not a genuine conditional. He emphasizes that "any conditional worth its salt should satisfy the *modus ponens* principle" (Priest, 2006b, p. 83). So, in subsequent works (for example in (Priest, 2006b) and (Priest, 2008)), he introduces a new conditional satisfying MPP, the *entailment connective*, and tries to escape Curry's paradox by rejecting ABS. And since in natural deduction ABS is a rule derived from CP and MPP the rejection of ABS and the acceptance of MPP implies the rejection of CP.

To summarise: Priest's strategy for avoiding Currry's paradox is to refute the general validity of one of the two rules for the use of the conditional (CP and MPP) in natural deduction. The above strategy should be compatible with the following two general Priest's claims:

1. The presence of dialetheias does not entail trivialism;
2. The meaning of logical constants should be dialethically acceptable both in the object language and in the metalanguage.

In the reminder of this paper, we critically assess the foregoing approach to Curry's Paradox. For reasons of space, we will not consider other dialetheic approaches and solutions to Curry's Paradox.[4] Our overall aim is

[2] For general background on LP, see (Asenjo, 1966), (Asenjo & Tamburino, 1975), (Routley, 1979), (Beall, 2009). For an introduction see (Berto, 2007, cap. 8).

[3] Formulations of Curry's paradox that do not rely on ABS typically make an appeal to the *structural* version of the rule, *Structural Contraction*, viz. that if $\Gamma, A, A \vdash B$, then $\Gamma, A \vdash B$. On this, see e.g. Beall and Murzi (forth).

[4] For other dialetheic approaches to the paradox see (Beall, 2007).

to show that there are difficulties both in the strategy of refuting MPP and in that of refuting CP.

On one hand, in a dialetheic framework, Curry's Paradox is blocked by showing that the Curry sentence is a dialetheia, and that, for this reason, MPP is invalidly applied in the Curry derivation. One consequence of this, however, is that, due to the failure of MPP, the material conditional is not in accordance with the use of conditional by working mathematicians. Yet, such a use is essential also for a dialetheist: for instance, it occurs in Priest's metalanguage, e.g. for expressing truth-preservation of the inference rules.

On the other hand, maintaining MPP and avoiding CP, using the entailment connective ($\Rightarrow$) makes it harder for the dialetheist to fulfill claim 2.

2. Curry's paradox and its arithmetical formalization

Curry's paradox belongs to the family of so-called paradoxes of self-reference (or paradoxes of circularity).[5] In short, the paradox is derived in natural language from sentences like the following:

(b): If sentence (b) is true, then Santa Claus exists.

Suppose the antecedent of the conditional in (b) is true, i.e. sentence (b) is true. Then, by MPP, Santa Claus exists. So, we have proved the consequent of (b) under the assumption of its antecedent. By CP, we have then proved (b), i.e. sentence (b) is true. We can now apply MPP once more, and conclude that Santa Claus exists. Of course, we could substitute any arbitrary sentence for 'Santa Claus exists'. As a result, every sentence can be proved and trivialism follows.

We reconstruct Curry's argument in the language of first order arithmetic with a truth predicate.

Let $\mathcal{L}$ be the language of first order arithmetic and $\mathcal{N}$ its standard model. Now extend $\mathcal{L}$ to $\mathcal{L}^*$ by introducing a new predicate T.

Assume a codification of the syntax of $\mathcal{L}^*$ by natural numbers and extend $\mathcal{N}$ to a model $\mathcal{N}^*$ of $\mathcal{L}^*$ by interpreting T as the truth predicate of $\mathcal{L}^*$. So, for all $n \in \mathcal{N}$, $T(\underline{n})$ is true *if and only if* n is the code of a true sentence A of $\mathcal{L}^*$, in symbols $n = \lceil A \rceil$.

To be sure, classically such an interpretation is impossible, since the theory obtained by adding to Peano arithmetic the truth predicate for the extended language $\mathcal{L}^*$ (with Tarski's shema) is inconsistent. This is not so for a dialetheist, however, who merrily accepts inconsistent models.

[5] Curry's original paper in which the paradox was introduced is (Curry, 1942).

We can now show that, if one uses the classical rules of the conditional in natural deduction and Tarski's scheme

$$T(\ulcorner A \urcorner) \leftrightarrow A,$$

the model $\mathcal{N}^*$ turns out to be trivial. In fact, let A be any sentence of $\mathcal{L}^*$. By diagonalization, there is a natural number k such that

$$k = \ulcorner T(\underline{k}) \rightarrow A \urcorner.$$

We can now prove A as follows:

1	(1)	$T(\underline{k}) \leftrightarrow (T(\underline{k}) \rightarrow A)$	Tarski's schema
2	(2)	$T(\underline{k})$	Assumption
1, 2	(3)	$T(\underline{k}) \rightarrow A$	1, 2 MPP
1, 2	(4)	A	2, 3 MPP
1	(5)	$T(\underline{k}) \rightarrow A$	2, 4 CP
1	(6)	$T(\underline{k})$	1, 5 MPP
1	(7)	A	5, 6 MPP

In LP, this derivation is of course invalid. The logic doesn't validate MPP: as we have already observed in §1, if A is a dialetheia, $(\neg A \vee B)$ is true even if B is solely false.

3. A dialetheic criticism to the material conditional

We're not convinced that the material conditional in LP can actually be seen as a *genuine conditional*.

Consider the usual meaning of the conditional in the *metalanguage* of any mathematical theory. Logicians and mathematicians alike use the conditional "if A then B" whenever they wish to say that the truth of A is a sufficient condition for the truth of B *independently of the possible falsity of* A. Indeed, from a proof of A and a proof "if A then B" they can get a proof of B.

This meaning is captured in natural deduction by the introduction rule of the conditional, according to which one gets a proof of "if A then B" by proving B under the assumption A: the possible falsity of A is not at issue.[6] According to this meaning, a genuine conditional should satisfy MPP, quite independently of the presence of a dialetheia; and this is just the sense of truth preservation according to which Priest claims that the material conditional fails to validate MPP.

[6] Sometime Priest observes that a genuine conditional must preserve falsity from the consequent to the antecedent. Observe, however, that this condition is not implicit in the very meaning of the conditional. It is rather a consequence, in classical logic, of the absence of dialetheias, and hence we think should not be endorsed by dialetheism.

However, as already observed, this is not the case for the material conditional. A classicist can maintain that the latter is a genuine conditional because of his rejection of dialetheias: in this case the truth of $(\neg A \vee B)$ guarantees the truth preservation from A to B. Not so for a dialetheist: the possibile presence of dialetheias should lead a dialetheist to reject the classical equivalence between $(A \rightarrow B)$ and $(\neg A \vee B)$. Of course, nothing prevents a dialetheist from *defining* $(A \rightarrow B)$ as $(\neg A \vee B)$, but in this way she cannot transfer the meaning of "if … then …" from the *metalanguage* to the *object language*.

In particular, observe that, when dealing with dialetheias, the material conditional trivializes Tarski's scheme. For if A is a dialetheia, it fails to express truth-preservation from A to $T(\ulcorner A \urcorner)$ and *vice versa*. That is, if

$$T(\ulcorner A \urcorner) \leftrightarrow A$$

is understood as

$$((\neg T(\ulcorner A \urcorner) \vee A) \wedge (\neg A \vee T(\ulcorner A \urcorner))),$$

when A is a dialetheia, then $T(\ulcorner A \urcorner)$ may have any value.

Consider the semantic of LP. Semantically, LP is a three-valued logic whose language is that of classical logic. The set of admissible valuations v_{LP} is composed of all the total maps from the set of well-formed formulae wff to the set $\{1, 0.5, 0\}$ (where '1' means "true and true only" '0' means "false and false only", and '0,5' means "true and false") satisfying:

$$(\vee)\ v(A \vee B) = \max\{v(A), v(B)\};$$
$$(\wedge)\ v(A \wedge B) = \min\{v(A), v(B)\};$$
$$(\neg)\ v(\neg A) = 1 - v(A);$$ [7]

Take, for example, $v(A) = 0{,}5$ and $v(T(\ulcorner A \urcorner)) = 0$. The result of

$$((\neg T(\ulcorner A \urcorner) \vee A) \wedge (\neg A \vee T(\ulcorner A \urcorner)))$$

is = 0.5. According to this evaluation, the biconditional is true, A is true, while $T(\ulcorner A \urcorner)$ fails to be true.

The problem may be avoided by adopting, instead of the Tarskian schema, a Tarskian inference rule according to which A and $T(\ulcorner A \urcorner)$ are interderivable, much like transparent theories of truth based on the paracomplete dual of LP, viz. K3.[8] The difficulty remains, however, that it doesn't look like the dialetheist can stick to the material conditional even in the metalanguage, contrary to what thesis (2) claims.

[7] We make the simplifying assumption that every object on the domain serves as a name of itself.

[8] See e.g. (Kripke, 1975) and (Field, 2008).

For these reasons, we think that the adoption of the material conditional in the *object language* is an *ad hoc* move for avoiding trivialism at the cost of a severe limitation of the expressive power of the logical language.

Priest is aware of this fact, and perhaps for this reason, introduces in subsequent works (for example in (Priest, 2006b, ch.6)) a new conditional satisfying MPP.

4. Recovering (MPP) dialetheically

As already observed, a conditional satisfying CP and MPP allows the derivation of Curry's paradox. If, following Priest, it is thought that MPP is constitutive of the meaning of 'if ', one natural reaction to Curry's Paradox, then, is to define a conditional which validates MPP but not CP. A first attempt by Priest to introduce one such is in (Priest, 2008, 7.4.6). Here the language of LP is extended by means of a conditional $\supset$ defined by the following truth table:

$\supset$	1	i	0
1	1	0	0
i	1	i	0
0	1	1	1

where '1' means "true and true only", '0' means "false and false only", and 'i' means "true and false".

This new conditional is characterized by the condition of preserving truth forward (i.e. from the antecedent to the consequent) and falsity backwards (i.e. from the consequent to the antecedent). It satisfies MPP, but invalidates CP. To see this, consider the following derivation:

1	(1)	A	Assumption
2	(2)	B	Assumption
1, 2	(3)	$(A \wedge B)$	1, 2 $\wedge$ I
2	(4)	$(A \supset (A \wedge B))$	1, 3 $\supset$ Introduction

Now, if A is only true and B is a dialetheia, then $A \wedge B$ is a dialetheia and hence $A \supset (A \wedge B)$ is only false because falsity fails to be preserved backwards.

However, the derivation of Curry's paradox in section 2 still goes through, even if the classical $\rightarrow$ is replaced by $\supset$ (and in Tarski's scheme too).

1	(1)	$(T(\underline{k}) \supset (T(\underline{k}) \supset A)) \wedge ((T(\underline{k}) \supset A) \supset T(\underline{k}))$	Tarski's schema
2	(2)	$T(\underline{k})$	Assumption
1, 2	(3)	$T(\underline{k}) \supset A$	1, 2 MPP
1, 2	(4)	A	2, 3 MPP

1	(5)	$T(\underline{k}) \supset A$	2, 4 $I \supset$
1	(6)	$T(\underline{k})$	1, 5 MPP
1	(7)	A	5, 6 MPP

Here the deduction from step 1 to step 4 preserves truth and falsity in the appropriate directions, so that, at step 5, $(T(\underline{k}) \supset A)$ turns out to be true.

The validity of Curry's paradox can be recognized also semantically by inspecting the truth table of $\supset$. Indeed, if A and $(A \supset B)$ are equivalent (where equivalence is expressed in terms of $\supset$ and $\wedge$), then B is true. Our conditional $\supset$, then, still validates Curry's paradox.

Can the dialetheist do better?

5. Entailment: Logic and semantics

In (Priest, 2006b, ch. 6) Priest proposes a different way to preserve MPP while avoiding Curry's paradox. He introduces a more sophisticated conditional ($\Rightarrow$) which he takes to be an *entailment connective*. Accordingly, Priest suggests that we read $(A \Rightarrow B)$ as "B follows logically from A". As in the case of $\supset$, he imposes to the entailment connective the condition of preserving truth and falsity forward and backwards respectively. But, he observes, due to the fact that logical consequences are such *necessarily*, also preserving truth and falsity is required to hold *necessarily*.

The main feature of the entailment connective, $\Rightarrow$, is that it is a *modal* connective invalidating ABS. The modal force of $\Rightarrow$, however, is quite different from the force of other modal conditionals, such as the strict conditional, or even the counterfactual conditional. Both, in fact, validate ABS.

An interpretation I for a language $\mathcal{L}$ of propositional logic with $\Rightarrow$ is a quadruple $\langle W, R, G, v\rangle$, where W is, as usual, an arbitrary set of objects ("possible worlds"), R is a dyadic relation between members of W ("the accessibility relation"), G is a designated member of W ("the actual world") and v is an evaluation function that assigns to each propositional atom and world w a non-empty subset of $\{0, 1\}$, where 1 is the value "true", 0 is the value "false". Similarly for a first order language.

The semantic clauses for a formula like $\phi \Rightarrow \psi$ are the following:

- $\phi \Rightarrow \psi$ is true in w if, and only if, for every world w' such that $R(w, w')$, if $1 \in v_{w'}(\phi)$, then $1 \in v_{w'}(\psi)$ and if $0 \in v_{w'}(\psi)$, then $0 \in v_{w'}(\phi)$.
- $\phi \Rightarrow \psi$ is false in w if, and only if, for some world w' such that $R(w, w')$, $1 \in v_{w'}(\phi)$ and $0 \in v_{w'}(\psi)$.

In short: $\phi \Rightarrow \psi$ is true in a world w if and only if, for every world w' accessible from w, if ϕ is true in w', so is ψ and if ψ is false in w', so is ϕ.

$\phi \Rightarrow \psi$ is false at a world w if and only if there is at least one accessible world w' where ϕ is true and ψ is false.[9]

The definitions of, respectively, *semantic consequence* and *logical truth* are as follows.

(SC) $\Gamma \vDash \alpha$ if$_{df}$. for all I, if, for every $\beta \in \Gamma$, $1 \in v_G(\beta)$, then $1 \in v_G(\alpha)$, and if $0 \in v_G(\alpha)$ then $0 \in v_G(\beta)$ for some $\beta \in \Gamma$.
(LT) $\vDash \alpha$ if and only if, for every I, $1 \in v_G(\alpha)$.

Note the definition of logical truth as truth in each actual world of every interpretation and logical consequence as consequence in every actual world of every interpretation is in accordance with the standard Kripkean definitions of *semantic consequence* and *logical truth*.

Priest claims that SC validates MPP for $\Rightarrow$. That is obvious if the metalanguage is classical. However, in section 7, we will argue that, in a dialetheic metalanguage, this claim is problematic. Anyway, we will accept, for the moment the validity of MPP for $\Rightarrow$.

Counterexamples to ABS are obtained by means of interpretations with the following two features:

- G is *omniscient*: for every $w \in W$, $R(G, w)$.
- R is *non-reflexive*: there is at least one $w \in W$ such that $\neg R(w, w)$.

Consider now the following interpretation:

- $W = \{G, w\}$
- $R(G, w)$, $\neg R(w, w)$, $R(G, G)$, $R(w, G)$
- $v_G(\phi) = \{0\}$; $v_G(\psi) = \{0\}$; $v_w(\phi) = \{1\}$; $v_w(\psi) = \{0\}$

In such an interpretation, we have that $v_G(\phi \Rightarrow (\phi \Rightarrow \psi)) = \{1\}$, at least in the classical metalanguage. However, $v_G(\phi \Rightarrow \psi) = \{0\}$, since in w, which accessible from G, ϕ is true and ψ is false.

We can then solve Curry's paradox by holding that, if in a semantically closed language ϕ is false only, then the Curry sentence

$$\text{(Curry) } \phi \Leftrightarrow (\phi \Rightarrow \psi)$$

is true, but both ϕ and $\phi \Rightarrow \psi$ are only false and ψ does not follow by MPP.

Observe that the presence of non reflexive worlds is essential for invalidating ABS. For, suppose that all worlds are reflexive and prove ABS. Let $1 \in v_G(\phi \Rightarrow (\phi \Rightarrow \psi))$ and let w be any world. Suppose that $1 \in v_w(\phi)$. Then, $1 \in v_w(\phi \Rightarrow \psi)$ and, by reflexivity, $1 \in v_w(\psi)$; besides, if $0 \in v_w(\psi)$ then $0 \in v_w(\phi)$. Thus, $1 \in v_G(\phi \Rightarrow \psi)$.

[9] On this see also Carrara, Gaio, & Martino (2011), Carrara, Martino, & Morato (2012) and Carrara & Martino (2014).

Moreover, note that no dialetheia is involved in this solution of the paradox, which entails that the foregoing solution to the paradox is not specifically *dialetheist*. Finally, it is worth emphasising that the non-reflexivity of R is *essential* for falsifying ABS.

It is also important to observe that, in other publications, Priest has adopted, again, a different treatment of the conditional. In *Paraconsistent Logic* (Priest, 2002b), for example, Priest specifically concentrates on two ways of defining a many-valued conditional operator. A first definition (Priest, 2002b, p. 320) gives rise to "semi-relevant" logics, i.e. logics avoiding the usual relevance paradoxes. A second one, analysed in detail in (Priest, 2002b, sec. 5.5.), treats $\rightarrow$ intensionally. This treatment is also expanded in the second edition of *In Contradiction*, where Priest gives a technical revision of the conditional adopted — a contraction-free conditional, i.e. a conditional which fails to satisfy ABS — defined by means of a ternary accessibility relation on points-worlds of relevant logics. A modal semantics with non-normal worlds is then proposed for the given conditional.[10]

The main philosophical difficulty arising in this context has traditionally been how to interpret the ternary relation at work in the semantics (in the specification of the truth-conditions of conditionals in non-normal worlds).

We return to this topic in §9. For the time being, we first concentrate on the philosophical analysis of non-reflexivity and omniscience in the given semantics for entailment.

6. The philosophical justification of non-reflexivity and omniscience

In *In contradiction*, Priest aims to give a philosophical justification of omniscience and non-reflexivity for the semantics of the entailment connective. His view is revealed in the following long passage:

> Now, how do we know that all the "possible worlds" in an interpretation are conceivable by people living under those conditions of G? Simply because we are those people (by definition), and we conceive them. It is we who are theorizing, specifying what interpretations are, and we who can spell out any particular [assignment]. If we were to live under a different set of conditions, however, there would be no guarantee that we would be able to think all of this. Indeed, had we not evolved, we might have been maladapted to our environment, and might not even, therefore, have been able to conceive properly of the conditions under which we actually lived. G is omniscient, but there is no reason, therefore, why any other world should be omniscient or even reflexive. (Priest, 2006b, p. 87)

[10] It is worth noticing that this different conditional proposed in *Paraconsistent Logic* is important, since Priest gives there a non-triviality proof for a paraconsistent-dialetheic theory of naive truth including such conditional. We thank a referee for pointing this out to us.

From this quoted passage we can extract the following main points:

Omniscience of G: G is omniscient because the totality of possible worlds accessible from the actual world of an interpretation is the totality of the possible worlds conceivable by the inhabitants of G.

In particular, the reflexivity of G follows from the omniscience of G.

Possible non-reflexivity of some non-actual worlds: We cannot grant the inhabitants of other possible worlds are able to conceive their own situation.

We think, however, that various aspects of the foregoing philosophical picture are problematic.

Let us concede that accessibility is to be understood as conceivability. So we agree with Priest that, since it is we who are theorizing in our world G about possible worlds, G is to be omniscient. But this seems to clash with the non-reflexivity of some non-actual worlds. For, as the above countermodel to ABS shows, the non-reflexivity of w has the effect that the evaluation of $\phi \Rightarrow \psi$ at w be made disregarding the values of ϕ and ψ at w. But since it is we who evaluate $\phi \Rightarrow \psi$ at w and we know the values of ϕ and ψ at w, there is no reason why the latter should not to be taken into account. The reply that these values may be inaccessible to the inhabitants of w seems to be misleading just because it is we who make the evaluations at w. On the other hand, if the evaluation of a sentence at w were to be made by the inhabitants of w, it would be hard to maintain that, while they have access to the values of ϕ and ψ at w for evaluating, for instance, $(\phi \wedge \psi)$ at w, they have no access to them when evaluating $\phi \Rightarrow \psi$. Besides, as we know, $\Rightarrow$ satisfies MPP. This means that ψ is a semantic consequence of ϕ and $\phi \Rightarrow \psi$, i.e. that, in every actual world of any interpretation, if $1 \in v_G(\phi)$, and $1 \in v_G(\phi \Rightarrow \psi)$, then $1 \in v_G(\psi)$. As our countermodel to ABS reveals, however, MPP fails in w, where $1 \in v_w(\phi)$ and $1 \in v_w(\phi \Rightarrow \psi)$ but $1 \notin v_w(\psi)$. Priest seems to accept the failure of MPP in some non-actual world by holding that the logical rules valid in a non-actual world may be deviant with respect to our rules. But that is hardly convincing. When defending the omniscience of G, Priest observes that it is we who are theorizing about possible worlds (see the above long quotation). If so, it is we, the inhabitants of the actual world, who are reasoning about non-actual worlds using our logic. For these reasons, the existence of non reflexive worlds does not seem adequatly justified.

7. Logical consequence and Entailment

We now move to considering Priest's thesis that $\phi \Rightarrow \psi$ is to be interpreted as saying that ψ is a *logical consequence* of ϕ. How to understand this thesis?

To be sure, the truth of $\phi \Rightarrow \psi$ in a specific interpretation cannot mean that ψ is a logical consequence of ϕ. So, it seems to be plausible to understand Priest's thesis in the following way:

$$(+)\ \ \phi \vDash \psi \text{ iff } \phi \Rightarrow \psi \text{ is logically true.}$$

From SC and the truth clauses for $\Rightarrow$ it follows that MPP holds, i.e. that ψ is a logical consequence of $\phi \wedge (\phi \Rightarrow \psi)$, at least in a classical metalanguage (we will discuss later the case of the dialetheic metalanguage). On the other hand, $\phi \wedge (\phi \Rightarrow \psi) \Rightarrow \psi$ is not logically true as the following countermodel shows:

- $W = \{G, w\}$
- $R(G, G)$, $R(G\,w)$, $R(w, G)$, $\neg R(w, w)$
- $v_G(\phi) = v_w(\phi) = v_G(\psi) = \{1\}$, $v_w(\psi) = \{0\}$

Notice that $\phi \wedge (\phi \Rightarrow \psi) \Rightarrow \psi$ is not valid in the above model: since $v_w(\phi \wedge (\phi \Rightarrow \psi)) = \{1\}$ and $v_w(\psi) = \{0\}$, $v_G(\phi \wedge (\phi \Rightarrow \psi) \Rightarrow \psi) = \{0\}$.

Furthermore, it is also worth noting that the model exploits in an essential way the non-reflexivity of *w*.

The failure of the thesis (+) can be ascribed to the inadequacy of SC. Indeed, the introduction of modal semantics for the entailment was motivated by the requirement that logical consequence be such necessarily. Accordingly, $\phi \vDash \psi$ should require truth preservation from ϕ to ψ not only in the actual worlds but also in all worlds accessible from the actual ones. For this reason, the appropriate definition of logical consequence would rather seem to be following:

(SC) $\Gamma \vDash \psi$ *if*$_{df}$. for all worlds w of all models, if, for all $\beta \in \Gamma$, $1 \in v_w(\beta)$, then $1 \in v_w(\psi)$; and if $0 \in v_w(\psi)$ then, for some $\beta \in \Gamma$, $0 \in v_w(\beta)$.

From SC*, the required equivalence immediately follows:

$$(+)\ \ \phi \vDash \psi \text{ iff } \phi \Rightarrow \psi \text{ is logically true.}$$

However, once SC has been replaced by SC*, $\Rightarrow$ fails to satisfy MPP. Indeed, in this case, the above countermodel to (+) becomes a countermodel to MPP.

To sum up: if we assume SC, then $\Rightarrow$ fails to express logical consequence. On the other hand, if we emend SC by replacing it with SC*, $\Rightarrow$ fails to satisfy MPP, and so also fails to be, by Priest's own standards, a genuine conditional.

In any event, we now want to argue that, even adopting SC, the validity of MPP for $\Rightarrow$ — as well as the above counterexample to ABS — are problematic in a dialetheic metalanguage. To see this, remember that, for Priest's dialetheism, the meaning of the logical constants should be dialetheically acceptable both in the object language and in the metalanguage (claim 2). To prove that $\Rightarrow$ satisfies MPP one must show that, given an arbitrary model $\mathcal{M}$, the following holds:

$$(*) \text{ if } 1 \in v_G(\phi \wedge (\phi \Rightarrow \psi)) \text{ then } 1 \in v_G(\psi).$$

Now, suppose that $\mathcal{M}$ has a unique world G and consider the evaluation: $v_G(\phi) = v_G(\psi) = \{0\}$. How can we recognize that (*) holds? Since all we know from the evaluation v is that antecedent of (*) is only false, the only way to recognize the validity of (*) is to invoke the *False antecedent rule*:

(FA) Any conditional with a false antecedent is true.

We now face a problem, however: what kind of conditional is used in the metalanguage when proving that $\Rightarrow$ satisfies MPP?

Remember that, according to Priest, even the metalinguistic logical constants are to be dialetheically understood (claim 2). Since, as Priest maintains, any genuine conditional must validate MPP, it does invalidate, on pain of trivialism, FA. Dialetheism rejects FA by observing that, if ϕ is a dialetheia and ψ is only false, then $\phi \rightarrow \psi$ is only false since it does not preserve truth from ϕ to ψ. So, one could think, at first sight, that, where dialetheias are not involved, FA dialetheically holds. That is not the case, however. Indeed, in the above countermodel to ABS, $\phi \Rightarrow \psi$ is only false even if ϕ is only false. Thus, according to Priest's semantics, FA is rejected independently of the presence of dialetheias. For this reason the metalinguistic conditional cannot be a genuine one.

A typical non-genuine dialetheic conditional satisfying FA is the material conditional. So it would seem plausible to adopt the latter in the metalanguage. However, as seen before, the material conditional invalidates MPP. Moreover, we will now show that, if the material conditional is used in the metalanguage, the entailment connective no longer validates MPP.

Though Priest does not identify falsity with untruth, he holds that certain sentences are both true and untrue. A case in point is in his view, the *strengthened liar*

(a): (a) is not true.

Now consider a model $\mathcal{M}$ with a unique world G, where ϕ is both true and not true and ψ is only false.

Since ϕ is not true at G, the metalinguistic material conditionals

(**) If ϕ is true at G, then so is ψ
If ψ is false at G, then so is ϕ

are true. It follows that $(\phi \Rightarrow \psi)$ is true. So, ϕ and $(\phi \Rightarrow \psi)$ are true but ψ is only false; hence MPP does not hold. It follows, then, that $\Rightarrow$ fails to satisfy MPP.

A dialetheist may perhaps object to our use of FA in establishing the first conditional in (**) as follows. In the semantics at issue, "ϕ is true" is expressed by $1 \in v(\phi)$, and hence $1 \in v(T(\ulcorner\phi\urcorner))$, while "$\phi$ is untrue" is expressed by $1 \in v(\neg T(\ulcorner\phi\urcorner))$ i.e. $0 \in v(T(\ulcorner\phi\urcorner))$. And since, according to Priest, untruth implies falsity, $0 \in v(\phi)$. Summing up, "ϕ is tue and untrue" is expressed by

$$v(\phi) = v(T(\ulcorner\phi\urcorner)) = \{0, 1\}.$$

That is, both ϕ and $T(\ulcorner\phi\urcorner)$ are dialetheias. So the appropriate truth-conditions of $(\phi \Rightarrow \psi)$ are:

(x) If $1 \in v(\phi)$ then $1 \in v(\psi)$; if $0 \in v(\psi)$ then $0 \in v(\phi)$.

With (x) in place, we can no longer resort to FA to establish the truth of the first conditional in (x). In fact, the negation of the antecedent is $1 \notin v(\phi)$, i.e. $1 \notin \{0, 1\}$ which is only false. Hence, $\phi \Rightarrow \psi$ is derivable only by means of FA.

However, this argument shows that the semantics at issue is inadequate to express the metalinguistic notion of untruth and hence to a dialetheic solution of the *strengthened liar*. In fact, if $1 \in v(\phi)$ means that ϕ is true, the untruth of ϕ is properly expressed by $1 \notin v(\phi)$, while the truth of $\neg(T(\ulcorner\phi\urcorner))$ is expressed by $1 \in v(\neg(T(\ulcorner\phi\urcorner))$, i.e. $0 \in v(T(\ulcorner\phi\urcorner))$; and from the latter $1 \notin v(\phi)$ does not follow.

Replying to a criticism by Littman and Simmons (Littman & Simmons, 2004), Priest observes that the treatment of functions in a dialetheic framework is a sensitive matter (Priest, 2006b, p. 288)). As a way to sidestep their use of functions, he suggests to employ relations instead of functions. In particular, in the case of semantic values, instead of an evaluation function, one can take an evaluation relation R from the set of sentences to $\{0,1\}$, such that, for any sentence ϕ, $R(\phi, 0)$ or $R(\phi, 1)$. Priest insists that for a dialetheist even the metalanguage may be inconsistent, so that R may both correlate and not correlate a sentence with a certain truth value. Following this suggestion, the evaluation of an untrue sentence ϕ must satisfy the condition $\neg R(\phi, 1)$; and if T must express metalinguistic truth, $R((T(\ulcorner\phi\urcorner)), 0)$ is to be equivalent to $\neg R(\phi, 1)$.

Priest's suggestion does not help him to circumvent the problem, however. To see this, consider again our model $\mathcal{M}$, this time using R instead of v. Then, the appropriate values of a true and untrue sentence ϕ are

$$R(\phi, 1) \text{ and } \neg R(\phi, 1).$$

Hence, the metalinguistic material conditional

$$\text{if } R(\phi, 1) \text{ then } R(\psi, 1)$$

is true by FA and our conclusion once more follows.

8. Logical consequence and denial

We now move on a different, though, related topic, viz. Priest's suggestion that a dialetheist may express *exclusive denial* by means of a MPP-satisfying conditional and an absurdity constant. Our aim is to argue that Priest's entailment connective is also inadequate for this purpose.

Priest tries to make up for the need of an exclusive negation introducing the notion of *rejection* of a proposition, to be clearly distinguished from the *acceptance* of its negation (on this see (Priest, 1993), (Priest, 1998), (Priest, 2006b), (Restall, 2013); for a general introduction to the topic see (Ripley, 2011)). *Acceptance* and *rejection* are cognitive states expressibile *via* the illocutory linguistic acts of *assertion* and *denial* (Priest, 2006a, p. 104).

According to Priest, while one can accept both a proposition and its negation, one cannot accept and reject the same proposition. By means of this device, a dialetheist can sometimes recover the possibility of expressing that a sentence ϕ is false *only*, precisely when he is in a position to reject it. In addition, Priest argues that the rejection of proposition ϕ is, sometimes, expressible by the assertion of a suitable ψ. Indeed, although the rule RAA, in general, fails dialetheically, Priest describes a limited use of it, which he takes to be dialetheically correct:

> An argument against an opponent who holds α to be true is rationally effective if it can be demonstrated that α entails something that ought, rationally, to be rejected, β. For it then follows that they ought to reject α. (Priest, 2006a, p. 86)

Priest exploits this idea by introducing a logical constant $\perp$ (*falsum*) such that it is a logical truth that, for every ϕ, $(\perp \rightarrow \phi)$ (where $\rightarrow$ is any conditional satisfying MPP). For instance, Priest observes, given the truth predicate T, satisfying the Tarskian schema, $\perp$ can be defined as $\forall x Tx$.

In "normal conditions" (Priest, 2006a, p. 105), the denial of ϕ can be expressed by the assertion of $\phi \rightarrow \perp$. What Priest intends by "normal conditions" and "most contexts" is explained in the following quotation:

> In most contexts, an assertion of […] $\alpha \rightarrow \perp$ would constitute an act of denial. Assuming that the person is normal, they will reject $\perp$, and so, by implication, α. The qualifier "in most contexts" is there because if one were ever to come across a trivialist who accepts $\perp$, this would not be the case. For such a person an assertion of $[\alpha \rightarrow \perp]$ would not constitute a denial: nothing would. (Priest, 2006a, p. 105-106)

It follows that, since the dialetheist is "normal", she is not a trivialist. So she must reject $\perp$ and therefore accept that the assertion of $\phi \rightarrow \perp$ implies the rejection of ϕ. Notice that it does not follow that a dialetheist can always express the denial of a sentence ϕ by asserting $\phi \rightarrow \perp$. In fact, one could have good reasons for denying ϕ without any evidence that ϕ entails trivialism. So, to be in "normal conditions" does not seem a sufficient condition for expressing the denial of ϕ by $\phi \to \bot$. But a dialetheist seems to be entitled to express the denial of ϕ by $\phi \to \bot$ after having recognized that $\bot$ follows from ϕ.[11]

The formal rule the dialetheist should accept is then the following introduction of the conditional ($\to$*Dial* I):

1	(1)	ϕ	Assumption
	$\vdots$	$\vdots$	
1	(n)	$\bot$	
	($n + 1$)	$\phi \to \bot$	(discharging (1))

Now consider again the above derivation of Curry's paradox, taking $\bot$ for ϕ.

1	(1)	$T(\underline{k}) \leftrightarrow T(\underline{k}) \to \bot)$	Tarski's schema
2	(2)	$T(\underline{k})$	Assumption
1, 2	(3)	$T(\underline{k}) \to \bot$	1, 2 MPP
1, 2	(4)	$\bot$	2, 3 MPP
1	(5)	$T(\underline{k}) \to \bot$	2, 4 $\to$ *Dial I*
1	(6)	$T(\underline{k})$	1, 5 MPP
1	(7)	$\bot$	5, 6 MPP

If both Tarski's schema and MPP are in place, one can only escape the paradox by rejecting step (5), i.e. the application of $\to$ *Dial I*. Thus, even the recognition that a sentence ϕ leads to contradiction is insufficient for a dialetheist to express the rejection of ϕ by $\phi \to \bot$.[12]

We conclude that any genuine conditional satisfying Tarski's schema is inadeguate, not only to express logical consequence, but also to express denial.

9. Posssible and impossible worlds

In (Priest, 2008) Priest adopts a different modal semantics based on the distinction between *normal* and *non-normal* worlds (for an introduction to the topic see (Berto, 2007, capp. 6 and 9) and (Berto, 2012)). In this section, we suggest that Priest's possible — and indeed impossible — worlds semantics for his entailment connective lacks philosophical justification.

[11] $\bot$ is a logical constant (*falsum*) such that it is a logical truth that $\bot \to \phi$, (for every ϕ). $\bot$ is basically the symbol for an explosive sentence (i.e., a sentence implying all the others). $\bot$ must be solely false for the dialetheist, because if it were true, trivialism would follow.

[12] We are grateful to an anonymous referee for pointing out to us that Hartry Field makes a similar point, in (Field, 2008, pp. 386-398). We have here expanded Field's original idea.

Historically, the notion of non-normal worlds is due to Saul Kripke, who first introduces it in (Kripke, 1965). In his treatment of possible worlds semantics, Kripke introduces a special kind of worlds, which he calls *non-normal*, in order to provide a semantics for modal logics (also called *non-normal*) weaker than the basic normal modal system **K**, such as C.I. Lewis' systems **S2** and **S3**, i.e. systems not including the *Necessitation Rule*. Kripke's introduction of non-normal worlds is just a techincal device aimed at proving a semantics for Lewis' non-normal modal worlds. But setting aside this technical reason, the philosophical question remains: why introduce non-normal worlds?

In his entry on *impossible worlds* (Berto, 2012), Francesco Berto surveys two main arguments for the introduction of non-normal, or impossible, worlds: the so-called "argument from ways" and an argument from *counter-possible reasoning*.

The first argument is based on David Lewis' proposal on quantifying on ways things could be. Just like our quantification on ways the world could be should be taken at *face value* as evidence for *possible worlds*, so our quantification on ways the world could not be should be taken at *face value* as evidence for *impossible worlds*. On this perspective impossible worlds are ways things could not be: "for any way the world could not be, there is some impossible world which is like that" (Berto, 2012, p. 15).[13]

The second argument comes from the basic idea that we can reason non-trivially from impossible suppositions: we assume that something impossible is the case, and we ask what follows or not from that.

In this second perspective worlds are usually taken as constituents of some paraconsistent logic and they are shaped by some logical structure: they are closed under a paraconsistent consequence relation, normally weaker than the classical one. This position focuses on the definition of *impossible worlds* as worlds where logical laws may fail or be different (see on this Priest, for example, (Priest, 2008, cap. 9)). In these impossible worlds intensional operators as, for example, *strict* or *relevant conditional* behave in a non-standard fashion. By contrast, the truth conditions for standard connectives as conjunction, disjunction, or quantifiers, should remain the same as in all possible worlds. Thus Priest:

> There are no [non-normal] worlds at which $A \wedge B$ is true, but A is not, or at which $\neg\neg A$ is true, but A is not. But it is conditionals that express the laws of logic, not conjunctions or negations. That is why it is their behaviour (and only theirs) that changes at non-normal worlds. (Priest, 2008, p.172)

In what follows, we argue that neither of the above motivations for the introduction of *impossible worlds* justifies Priest's treatment of the *entailment* connective.

[13] For criticism to the proposal see, again, (Berto, 2012).

Let us consider *impossible worlds* as impossible situations. This conception is motivated by the fact that often logicians and mathematicians reason assuming impossible hypotheses. Observe, however, that in these situations one reaons as if they were possible to the purpose of discovering their impossibility: a typical case is an argument by way of contradiction. If usual reasoning starting from impossible hypotheses is correct — i.e. if this way of reasoning holds — in such impossible situations the same logic of normal worlds must hold. Otherwise, since our arguments *by reduction* are carried on within our own logic such arguments should be rejected. It follows that, since — according to Priest — MPP holds in all possible worlds, it should hold in the impossible worlds too. Thus, the conception of *impossible worlds* at issue does not justify the presence of worlds where MPP fails.

However, following Priest, one may consider a conception of *impossible worlds* according to which they are worlds where — by definition — an alternative logic holds (see on this, for example, (Priest, 2008, p. 184)). This is indeed Priest's point of view on *impossible worlds*: in the philosophical *locus classicus* of Priest's conception of *impossible worlds* (Priest, 1992) *non-normal worlds* are described as those where logic is different from ours (see also (Priest, 2008, p. 172) quoted before).

Then, one can justify such commitment to *impossible worlds* on the basis of the folowing argument: we are able to consider, and theorise about worlds where logic is different from our own world, without automatically being forced to apply our own logic. For istance, suppose one is a classical logician, and thinks classical logic is the *One True Logic*; one can still understand fairly well how an intuitionistic world looks like. One would still know intuitionistic logic fairly well, having a precise idea of how a world at which intuitionistic logic holds, and classical logic fails, looks like.[14]

Observe, however, that logical consequence is relative to a well determined logic. And now recall the semantic clauses for $\phi \Rightarrow \psi$:

- $\phi \Rightarrow \psi$ is true in w if, and only if, for every world w' such that $R(w, w')$, if $1 \in v_{w'}(\phi)$, then $1 \in v_{w'}(\psi)$ and if $0 \in v_{w'}(\psi)$, then $0 \in v_{w'}(\phi)$.
- $\phi \Rightarrow \psi$ is false in w if, and only if, for some world w' such that $R(w, w')$, $1 \in v_{w'}(\phi)$ and $0 \in v_{w'}(\psi)$.

The first truth-condition of $\phi \Rightarrow \psi$ at a normal world cannot require the preservation of truth from ϕ to ψ at a non-normal or impossible world. So, for instance, suppose that in a non-normal world w the intuitionistic logic holds. It may happen that $\neg\neg\phi$ is true at u while ψ is not. In this case, if w is accessible to a normal world u, the conditional $\neg\neg\phi \Rightarrow \phi$ – according to the above clauses — would not be true at u. But, according to Priest's logic,

[14] We owe this example to an anonymous referee.

ϕ is a logical consequence of $\neg\neg\phi$; so $\Rightarrow$ would fail to express the dialetheic notion of consequence at a normal world.

Of course, in our actual world we are able to conceive of, and theorizing about, an *impossible world*, where the logic is different from that of our world. But such considerations cannot affect our notion of logical consequence.

From a technical point of view, the objection can be superseded by using a ternary accessibility relation.[15]

As we anticipated earlier, Priest has also adopted a different treatment of the conditional where an impossible worlds semantics with a ternary accessibility relation is specified.[16] The philosophical issue with this kind of framework is how to interpret the ternary relation adopted to characterize such semantics and the truth-conditions of conditionals in non-normal worlds.

The philosophical significance of such ternary accessibility relation, as Priest himself observes, is far from be clear. For this reason, we have restricted our criticism to the first edition of *In contradiction*, where we have found at least a tentative to give a suggestion of how to understand the crucial features of the adopted semantics.

10. Concluding remarks

In this paper we have argued that Priest's solution to the Curry's paradox encounters crucial difficulties both in the strategy of refuting MPP and in that of refuting CP.

On one hand, the material conditional in LP blocks Curry's paradox. However, it is not in accordance with the use of conditional by working mathematicians. We have shown that a dialetheist — unlike a classicist — cannot limit herself, at least in the metalanguage, to the material conditional.

On the other hand, Priest's strategy for recovering a genuine conditional involves a counterintuitive modal semantics — one that allows him to recover MPP only at the cost of a use of the conditional rule FA in the metalanguage.

This use is highly problematic, however, because of Priest's requirement that the inference rules used in the metalanguage should be dialetheically acceptable.

Moreover, the entailment connective is inadequate, against Priest's intentions, to express the notion of *logical consequence*. Furthermore, any conditional satisfying MPP but not CP cannot express *logical consequence* nor can be used for expressing the *denial* of a proposition.[17]

[15] For a survey on the solutions using a ternary accessibility relation see (Mares, 2004) and (Priest, 2008, cap. 10).

[16] For example in the above mentioned (Priest, 2008, cap. 10).

[17] As an anonymous referee suggested, this is an issue for any naive theory of truth based on a non-substructural logic.

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