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THE KNOWABILITY PARADOX AND TRUTHS DEPENDENT ON THEIR EPISTEMIC STATUS

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ABSTRACT

On the anti-realist notion of truth, truth is not evidence-transcendent. Thus, it seems, the anti-realist should be committed to the claim that all truths are knowable. But Fitch showed that we can derive a contradiction from this knowability claim and two very plausible principles of knowledge. In this paper I argue that Fitch's paradox arises because we wrongly assume the validity of the following inference: one is entitled to assert the existence of unknown truths; therefore, one is entitled to assert for some specific proposition that it is an unknown truth. Based on the invalidity of this inference, I show that we can block the derivation of Fitch's proof without weakening the knowability claim, although I do not endorse the knowability claim itself. I also show that my restriction strategy disarms the paradox in such a way that is free from the criticisms made against Tennant-style restriction strategies.

1. Introduction

According to metaphysical realism, reality exists independently of our means of conceptualizing or knowing it; in other words, things could be just as they actually are, even if there were no minds to know about them. Thus, on the realist view of truth, truth is a property that can transcend our ability to determine whether it obtains or not; in other words, there are statements that can transcend our means of determining their truth values. In this sense, realists take truth to be radically non-epistemic or evidence-transcendent. Anti-realism is the rejection of realism. The rejection of realism may take various possible forms. Thus anti-realism does not denote a specific philosophical doctrine, but rather various rejections of the realist doctrine.[1] Immanuel Kant's Copernican revolution is one such rejection. According to Kant, knowledge is possible only through the synthesis of sensibility and understanding. Thus, on his view, we cannot know a truth independently of our conceptual scheme. For example,

[1] Cf. Michael Dummett [1991] *Logical Basis of Metaphysics*. Cambridge: Harvard University Press, p. 4.

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consider the proposition that the earth is round. This proposition, which is a truth-bearer, is constituted by our concepts *the earth* and *being round*. In his influential paper *On the Very Idea of a Conceptual Scheme*, Donald Davidson takes one more step. He claims that the notion of alternative conceptual scheme does not make sense. If, on the one hand, we can succeed in interpreting an alien conceptual scheme, then it cannot be a conceptual scheme radically different from our own. If, on the other hand, something is not interpretable into our own scheme, there is no intelligible basis on which to judge that it is indeed a conceptual scheme. For this reason, the notion of a non-interpretable conceptual scheme is unintelligible.[2] Along these lines of reasoning, anti-realists take what we call 'world' or 'reality' to be dependent in some significant way on our ways of conceptualizing it. In addition, anti-realists are very concerned with grasp of meanings, or contents. The meaning of a statement must be something people can grasp or understand. Thus, on the anti-realist view, the meaning of a statement is to be understood in terms of what is epistemically accessible to us. In other words, the meaning of a statement must be fixed by the sort of thing that counts as evidence for the statement. Hence, according to anti-realists, notably Michael Dummett, to know the meaning of a statement is to know when one would be warranted in asserting it. Dummett says:

> Realism I characterize as the belief that statements of the disputed class possess an objective truth-value, independently of our means of knowing it: they are true or false in virtue of a reality existing independently of us. The anti-realist opposes to this the view that statements of the disputed class are to be understood only by reference to the sort of thing which we count as evidence for a statement of that class. That is, the realist holds that the meaning of statements of the disputed class are not directly tied to the kind of evidence for them that we can have, but consist in the manner of their determination as true or false by states of affairs whose existence is not dependent on our possession of evidence for them. The anti-realist insists, on the contrary, that the meanings of these statements are tied to directly to what we count as evidence for them, in such a way that a statement of the disputed class, if true at all, can be true only in virtue of something of which we could know and which we should count as evidence for its truth.[3]

Along these lines, anti-realists hold that there is no truth independent of our conceptual scheme, so that a statement cannot possess an objective truth-value independently of our means of conceptualizing or knowing it. In this sense, anti-realists take truth to be epistemically constrained.

[2] Donald Davidson [1984] *Inquiries into Truth and Interpretation*. Oxford: Clarendon Press, pp. 183-198.

[3] Michael Dummett [1978] "Realism", in his *Truth and Other Enigmas*. Cambridge: Harvard University Press, p. 146.

Now, it seems, a good way to express the epistemic character of truth is to say that all truths are at least in principle knowable. If this knowability principle is true, anti-realists can reject the realist, non-epistemic notion of truth. But the knowability principle is facing an important obstacle. Frederic Fitch showed in his 1963 paper that we can easily derive a contradiction from the knowability principle and two other very plausible principles of knowledge.[4] Whether correct or not, the epistemic notion of truth, it seems, is at least coherent. Thus Fitch's proof is usually taken to be a paradox, because it seems to show the incoherence of the epistemic notion of truth. In this paper, however, I argue that Fitch's proof is not a deep paradox concerning the nature of truth but simply a result to be properly understood. I argue, in particular, that the knowability paradox arises because we wrongly assume the validity of the following inference: one is entitled to assert the existence of unknown truths; therefore, one is entitled to assert for some specific proposition that it is an unknown truth.

2. The Knowability Paradox

Let me begin by briefly introducing the knowability paradox, which arises from the following two assumptions:

($\Diamond$K) $(\forall p)(p \rightarrow \Diamond Kp)$ (All truths are knowable)

(NO) $(\exists p)(p \ \& \ {\sim}Kp)$ (Some truths are unknown)

Here 'Kp' means that some being at some time (past, present, or future) knows that p; and '$\Diamond$' is the possibility operator. Fitch showed in his 1963 paper that we can derive a contradiction from the knowability principle ($\Diamond$K) and the non-omniscience claim (NO) if we further assume the following two principles of knowledge, which are hardly controversial.

(i) Knowledge distributes over conjunction: $K(p \ \& \ q) \rightarrow (Kp \ \& \ Kq)$
(ii) Knowledge implies truth: $Kp \rightarrow p$

If a conjunction is known, its conjuncts must also be known; and nothing can be known without being true. Let us now see how a contradiction can be derived.

Proof: First, we can easily prove the following unknowability claim:

(UK) $(\forall p)({\sim}\Diamond K(p \ \& \ {\sim}Kp))$

[4] Frederic Fitch [1963] "A Logical Analysis of Some Value Concepts", *Journal of Symbolic Logic*, 28, pp. 135-142.

Let 'p' be an arbitrary proposition. Suppose for *reductio* that $K(p \& {\sim}Kp)$. Since knowledge distributes over conjunction, we have that $Kp \& K{\sim}Kp$. Since $K{\sim}Kp$ and knowledge implies truth, we also have that ${\sim}Kp$. Thus we can derive a contradiction that $Kp \& {\sim}Kp$. Therefore, by *reductio*, we can have that ${\sim}K(p \& {\sim}Kp)$. Since this result can be derived as a theorem, we can obtain by the rule of necessitation that $\Box{\sim}K(p \& {\sim}Kp)$, which is equivalent to ${\sim}\Diamond K(p \& {\sim}Kp)$.

We can also easily show that $\Diamond K(p \& {\sim}Kp)$, contradicting the above result. By the $\exists$-elimination inference, we have the following instance of (NO): $p \& {\sim}Kp$. By the $\forall$-elimination inference, we also have the following instance of ($\Diamond$K): $(p \& {\sim}Kp) \rightarrow \Diamond K(p \& {\sim}Kp)$. Hence, by applying the $\rightarrow$-elimination rule to these two statements, we can finally derive that $\Diamond K(p \& {\sim}Kp)$.[5]

Therefore, given the aforementioned two principles of knowledge, we can easily derive a contradiction from ($\Diamond$K) and (NO). Here we can hardly deny (NO). We are not omniscient, after all. Besides, there are a lot of unimportant facts which are not worth the trouble to find out. Thus, since a contradiction is generated from ($\Diamond$K) and (NO), it seems, we are required to reject ($\Diamond$K). Hence Fitch's proof is usually presented as a threat to the anti-realist, epistemic notion of truth.

3. The Unassertibility of Unknown truths

In this section, I argue that the knowability paradox arises due to a wrong assumption, which will be explained shortly and also that the anti-realist notion of truth is not threatened by the falsity of the knowability principle ($\Diamond$K).

Let us begin by considering the following existential statement:

> $(\exists p)(p \& {\sim}Bp)$. (There is a proposition 'p' such that p and I don't believe that p.)

Here 'Bp' means that I believe that p. I am certainly entitled to assert that $(\exists p)(p \& {\sim}Bp)$, because I am not omniscient so that there are bound to be a lot of true propositions which I do not believe. Nonetheless, I am not entitled to assert for any specific proposition 'p' that $p \& {\sim}Bp$. The reason is straightforward. The truth of '$p \& {\sim}Bp$' depends on its epistemic status

[5] This proof is essentially due to Fitch 1963. In his paper Fitch mentions the origin of the proof: "This theorem is essentially due to an anonymous referee of an earlier paper, in 1945, that I did not publish" (Fitch 1963, p. 138, footnote 5). According to Joe Salerno, this anonymous referee is Alonzo Church. See Joe Salerno [2009] "Knowability Noir: 1945-1963", in J. Salerno (ed.), *New Essays on the Knowability Paradox*. Oxford: Oxford University Press, pp. 29-48.

of *not being believed by me*. If, however, I sincerely assert the first conjunct '*p*', its epistemic status changes from *not being believed by me* to *being believed by me* to the effect that the second conjunct is no longer true. For this reason, the following inference is invalid:

One is entitled to assert that $(\exists p)(p \ \& \sim Bp)$.
∴ One is entitled to assert for some specific proposition '*p*' that $p \ \& \sim Bp$.

In like manner, although one is entitled to assert that $(\exists p)(p \ \& \sim Kp)$, one is not entitled to assert for any specific proposition '*p*' that $p \ \& \sim Kp$. Remember that 'Kp' means that some being (past, present, or future) knows that *p*. If any proposition '*p*' comes to be known, then its epistemic status must change from *being unknown* to *being known*. Thus it is a conceptual truth that an unknown truth cannot be known unless it loses its epistemic status of being unknown. Now observe that if the first conjunct of '$p \ \& \sim Kp$' comes to be known, it loses its epistemic status of being unknown, and so its second conjunct becomes false. Therefore any proposition '*p*' cannot be known insofar as '$p \ \& \sim Kp$' is true. To put it another way, if any proposition '*p*' comes to be known, it cannot be one of those instances which make the non-omniscience claim '$(\exists p)(p \ \& \sim Kp)$' true. Hence the fact that '$p \ \& \sim Kp$' cannot be known is neither paradoxical nor surprising, because its truth depends on its epistemic status of being unknown.

Along these lines, we can argue that the knowability paradox arises because we wrongly assume the validity of the following inference:

One is entitled to assert that $(\exists p)(p \ \& \sim Kp)$.
∴ One is entitled to assert for some specific proposition '*p*' that $p \ \& \sim Kp$.

Let me elaborate a bit further. As mentioned in section 2, the knowability paradox depends on the following two assumptions:

$(\Diamond K)$ $(\forall p)(p \rightarrow \Diamond Kp)$
(NO) $(\exists p)(p \ \& \sim Kp)$

In Fitch's proof, in order to derive a contradiction, we first need to derive the following instance of (NO) by the ∃-elimination inference:

(i) $p \ \& \sim Kp$

We also need to derive the following instance of $(\Diamond K)$ by the ∀-elimination inference:

(ii) $(p \ \& \sim Kp) \rightarrow \Diamond K(p \ \& \sim Kp)$

And then we need to detach the consequent of (ii) by the →-elimination inference from (i) and (ii). To do so, we must be entitled to assert (i). But, as noted already, we are not entitled to assert for any specific proposition '*p*' that p & ~Kp, although we are entitled to assert that $(\exists p)(p \mathbin{\&} {\sim}Kp)$. Consequently, we are not entitled to assert the antecedent of (ii), and so we cannot detach its consequent. Then we are not entitled to assert that $\Diamond K(p \mathbin{\&} {\sim}Kp)$. Due to this reason, anti-realists can legitimately refuse to accept that $\Diamond K(p \mathbin{\&} {\sim}Kp)$ on the grounds that we are never entitled to assert the antecedent of (ii) in the first place. Hence anti-realists can block the derivation of Fitch's proof without restricting ($\Diamond$K), because they can do so by denying the validity of the aforementioned inference: one is entitled to assert that $(\exists p)(p \mathbin{\&} {\sim}Kp)$; therefore, one is entitled to assert for some specific proposition '*p*' that p & ~Kp.

As argued above, we can block the derivation of Fitch's proof without restricting ($\Diamond$K), but anti-realists don't need to endorse this unrestricted knowability principle. In the remainder of this section I argue that the anti-realist notion of truth is not threatened by the falsity of ($\Diamond$K).

In the first place, the fact that 'p & ~Kp' cannot be known has nothing to do with the debates between realists and anti-realists. The main point of contention between realists and anti-realists is whether our notion of truth has a radically non-epistemic feature relevant to realism. And the unknowability of 'p & ~Kp' has nothing to do with such a realism-relevant feature. Observe that the unknowability of 'p & ~Kp' is not due to our cognitive limitations. Even a super-intelligent being without having any substantial cognitive limitations cannot know that p & ~Kp. This is because coming to know the first conjunct makes the second conjunct false, and even a super-intelligent being cannot know what is not true anymore. Therefore both realists and anti-realists should endorse a conceptual truth that what is not known (by anyone at any time) is not known.

In the second place, (UK) is much more plausible than ($\Diamond$K).

(UK) $(\forall p)({\sim}\Diamond K(p \mathbin{\&} {\sim}Kp))$
($\Diamond$K) $(\forall p)(p \rightarrow \Diamond Kp)$

As shown in the previous section, we can easily prove (UK) just by using two very plausible principles of knowledge and the rule of necessitation; in other words, its proof does not depend on any principle that is really controversial. Thus, if we have to decide between (UK) and ($\Diamond$K) we have to give up ($\Diamond$K). In addition, as noted, it is a conceptual truth that an unknown truth cannot be known unless it loses its epistemic status of being known. Therefore any specific unknown truth of the form 'p & ~Kp' cannot be known. For these reasons, both realists and anti-realists should endorse (UK). Now observe that the non-omniscience claim (NO) and (UK) imply the falsity of ($\Diamond$K). According to (NO), there is a truth of the form 'p & ~Kp'. Thus it follows from (NO) and (UK) that $(p \mathbin{\&} {\sim}Kp) \mathbin{\&} (\forall p)({\sim}\Diamond K(p \mathbin{\&} {\sim}Kp))$. Anti-realism is not threatened by the fact that there is a truth of the form '*p* & ~K*p*'. Nor is it threatened by a conceptual truth such that an unknown truth cannot be known unless it loses its epistemic status of being unknown. Therefore anti-realists can (or should) endorse the falsity of (◊K). To put it another way, the anti-realist notion of truth is not threatened by the falsity of (◊K).

Along these lines, anti-realists can restrict (◊K) in a way that is consistent with the aforementioned conceptual truth, and restricting (◊K) in this way is neither arbitrary nor *ad hoc*. Hence this line of thought naturally leads us to restrict the knowability principle in the following way:

> (◊K*) $(\forall p)(p \rightarrow \Diamond Kp)$ only for propositions '*p*' such that the truth of '*p*' does not depend on its epistemic status of being unknown.

(◊K*) restricts the knowability principle by citing a feature of truth that calls for the restriction in question. For the appropriate knowability principle, to which anti-realists should be committed, must be consistent with the aforementioned conceptual truth, namely, that an unknown truth cannot be known unless it loses its epistemic status of being unknown. Since anti-realism is not threatened by such a conceptual truth, anti-realists have no reason to find the above restriction unpalatable. Hence my restricted principle above can avoid the charge of being *ad hoc*.

4. Comparisons with Tennant's Restriction Strategy

My restricted principle proposed in the previous section is somewhat similar to the restricted principle put forward by Neil Tennant in his book *The Taming of the True*.[6] Thus let me briefly compare his principle with mine. Tennant imposes the following restriction on the knowability principle:

> (◊KC) $(\forall p)(p \;\&\; Cp \vdash \Diamond Kp)$

where '*Cp*' means that '*p*' is Cartesian. What Tennant calls 'a Cartesian proposition' is one such that the assumption that it is known is logically consistent. On this restricted principle, propositions of the form '*p* & ~K*p*' are not Cartesian, because 'K(*p* & ~K*p*)' is not logically consistent. Consequently, unknown truths of the form '*p* & ~K*p*' are compatible with (◊KC).

However, Tennant's proposal above has been criticized for being desperately *ad hoc*. Hand and Kvanvig, for instance, say:

> We should expect [Tennant] to find some feature of truth, antirealistically conceived, that disarms the paradox by allowing some truths to be unknowable.

[6] Neil Tennant [1997] *The Taming of the True*. Oxford: Oxford University Press.

> We find, however, that Tennant cites no such feature of truth and fails to provide a substantive philosophical approach to the paradox.[7]
>
> For example, consider Russell's paradox, which threatens the claim that any grammatically predicative expression defines a set. We can apply Tennant's strategy and avoid the paradox simply by saying that any such expression defines a set except when the assumption that it does so yields a contradiction. Such an approach to Russell's paradox is clearly *ad hoc*. What it does not provide, but what is needed, is some account of the nature of sets that precludes problematic ones. The same is true of the difficulties plaguing the antirealists' conception of knowability. What is needed is *an account of the nature of truth* that prevents problematic ones from making trouble for antirealism. Tennant's proposal simply does not do that.[8]

Against objections of the above sort, Jon Cogburn defends Tennant's restriction strategy.[9] On his view, Tennant distinguishes between logical possibility and the kind of possibility occurring in his restricted principle ($\Diamond$KC), and so we should understand his claim as saying that if the assumption that '*p*' is known is logically consistent, then if '*p*' is true then it is possible *in some weaker modality* that '*p*' is known. To put it another way, what ($\Diamond$KC) is saying differs from the following trivial claim: if the assumption that '*p*' is known is logically consistent, then if '*p*' is true then it is logically possible that '*p*' is known.[10] In addition, Cogburn claims that Hand and Kvanvig's critique of Tennant's proposal is based on a false analogy between Russell's paradox and the knowability paradox. He says:

> In the case of Russell's paradox, solving it involved developing modern set theory. Here we did get an 'account of the nature of sets that precludes the problematic ones,' and so much more. Hand and Kvanvig charge a non *ad hoc* solution to Fitch's proof with similarly illuminating the key concepts in the proof. … Tennant should just respond that epistemic logic is already well understood in a way that the foundations of mathematics weren't at the time of Russell's paradox. … This is not to deny that shedding light upon possibility and knowledge is a goodmaking feature of solutions to Fitch's proof. The problem with Hand and Kvanvig's critique of Tennant is that they assume that it is a necessary condition on solutions that they so deepen our understanding. … Tennant's solution seems *ad hoc* because we already have a rigorous body of knowledge about how knowledge, possibility, and consistency interact. [11]

It is not easy to specify what is exactly the aforementioned weaker modality occurring in Tennant's restricted principle, which is supposed to save his

[7] Michael Hand and Jonathan L. Kvanvig [1999] "Tennant on Knowability", *Australasian Journal of Philosophy*, 77, p. 423.

[8] Ibid., p. 426.

[9] Jon Cogburn [2004] "Paradox Lost", *Canadian Journal of Philosophy*, 34, pp. 195-216.

[10] Ibid., p. 210.

[11] Ibid., pp. 213-214.

principle from being *ad hoc*. In addition, even if it is true that epistemic logic relevant for the knowability paradox is already well developed, whereas modern set theory was not developed at the time of Russell's paradox, it is not clear that this dissimilarity is a relevant respect strong enough to dismiss Hand and Kvanvig's critique based on the analogy between Tennant's restricted principle and the aforementioned approach to avoid Russell's paradox, that is, a way to avoid the paradox simply by restricting set-forming conditions as not being applied to the cases where the assumption that a grammatically predicative expression defines a set yields a contradiction. But I shall not push these points in this paper. We may grant that Tennant's restriction strategy might not be completely trivial, and also that some legitimate solution to a certain paradox might not deepen our understanding of the key concepts involved in the paradox. Nonetheless, as Cogburn himself admits, an illuminating solution is much more desirable if possible. Hand and Kvanvig demand that the appropriate restriction on the knowability principle be motivated by a feature of truth that calls for the restriction in question. Unlike Tennant's proposal, my proposal meets their demand. As argued in the previous section, the knowability principle should be restricted in a way that is consistent with a conceptual truth such that an unknown truth cannot be known unless it loses its epistemic status of being unknown. In this sense, my proposal disarms the paradox by citing some feature of truth that calls for the restriction in question. For any legitimate solution should at least be consistent with such a conceptual truth. More importantly, as argued in the previous section, my proposal can block the derivation of Fitch's proof without restricting the knowability principle. On my view, we can block the derivation of Fitch's proof by denying the validity of the following inference: one is entitled to assert the existence of unknown truths; therefore, one is entitled to assert for some specific proposition that it is an unknown truth. Hence my proposal is free from the criticisms Hand, Kvanvig, and others made against Tennant-style restriction strategies.

5. Kvanvig's Objection against Restriction Strategies In General

Let us consider Kvanvig's another objection, which he raises against restriction strategies in general. In his book *The Knowability Paradox*, Kvanvig insists that the heart of the knowability paradox is a lost distinction between possible knowledge and actual knowledge.[12] His main reason is

[12] See Jonathan L. Kvanvig [2006] *The Knowability Paradox*. Oxford: Clarendon Press. See also Kvanvig [2009] "Restriction Strategies for Knowability: Some Lessons in False Hope", in J. Salerno (ed.), *New Essays on the Knowability Paradox*. Oxford: Oxford University Press, pp. 205-222.

that we can easily derive the omniscience-like claim that every truth is known from the knowability claim that every truth is knowable.

(i)	$(\forall p)(p \to \Diamond Kp)$	$(\Diamond K)$
(ii)	$(p \,\&\, {\sim}Kp) \to \Diamond K(p \,\&\, {\sim}Kp)$	an instance of $(\Diamond K)$
(iii)	${\sim}\Diamond K(p \,\&\, {\sim}Kp)$	an instance of (UK)
(iv)	${\sim}(p \,\&\, {\sim}Kp)$	by denying the consequence from (ii) and (iii)
(v)	$p \to Kp$	logically equivalent to (iv)
(vi)	$(\forall p)(p \to Kp)$	since '*p*' is an arbitrary proposition

The above proof shows that $(\forall p)(p \to \Diamond Kp) \to (\forall p)(p \to Kp)$. In addition, since '$Kp$' implies '$\Diamond Kp$', we can also obtain the other direction, namely, that $(\forall p)(p \to Kp) \to (\forall p)(p \to \Diamond Kp)$. Therefore, on Kvanvig's view, we end up with the following logical equivalence: $(\forall p)(p \to \Diamond Kp) \leftrightarrow (\forall p)(p \to Kp)$. On the basis of this logical result, he claims:

> Such [restriction] approaches maintain … that the claim that all truths are knowable must be restricted in some way in order to express an anti-realist commitment. I argue against all examples of such an approach, and argue further that even if there were a successful restriction strategy, the paradox would remain untouched. For the fundamental paradoxicality we must address is not about whether all truths are knowable. It is, instead, about a lost logical distinction between possible knowledge of all truth and actual knowledge of all truth. The result is that restriction strategies are all red herrings when it comes to the fundamental perplexity engendered by the knowability paradox.[13]

However, the above diagnosis of the paradox is a *non sequitur*, for there is a better way to interpret the above logical result. Either there is a truth of the form '$p \,\&\, {\sim}Kp$' or there is no truth of the form '$p \,\&\, {\sim}Kp$'. The second case is logically equivalent to the claim that $(\forall p)(p \to Kp)$. As mentioned in section 2, we can hardly deny the non-omniscience claim, because we are not omniscient. Thus, granted that the second case is very implausible, no paradoxical result is generated from this case. Now turn to the first case. As also noted in section 3, given the unknowability claim (UK), which is almost uncontroversial, the first case implies that $(\Diamond K)$ is false. Therefore, given that the first case is very likely to be true, we have a good reason to believe that $(\Diamond K)$ is false. Hence the proper conclusion to draw from the equivalence that $(\forall p)(p \to \Diamond Kp) \leftrightarrow (\forall p)(p \to Kp)$ is just that ${\sim}(\forall p)(p \to \Diamond Kp) \,\&\, {\sim}(\forall p)(p \to Kp)$. In short, we have two options. One option, which Kvanvig insists, is to face the fundamental perplexity involved in a lost distinction between possible knowledge and actual knowledge. Another option is to endorse the falsity of $(\Diamond K)$. As noted already, we have a very

[13] Kvanvig 2006, pp. 3-4.

good reason to believe that ($\Diamond$K) is false; and anti-realists can reject ($\Diamond$K) in such a way that they will not find unpalatable. Therefore anti-realists can deny the very problematic claim that $(\forall p)(p \rightarrow \Diamond Kp)$. And this option is much more reasonable than the former option. In addition, this line of reasoning allows us to dismiss Kvanvig's objection to restriction strategies in general, namely, that restriction strategies are all red herrings when it comes to the fundamental perplexity engendered by the knowability paradox. Contrary to Kvanvig's claim, the heart of the paradox is not a lost distinction between possible knowledge and actual knowledge. As argued in section 3, it is rather the invalidity of the following inference: one is entitled to assert that $(\exists p)(p \,\&\, {\sim}Kp)$; therefore, one is entitled to assert for some specific proposition '*p*' that $p \,\&\, {\sim}Kp$.

6. Cogburn on Moorean Inference

Let me finally discuss Jon Cogburn's criticisms of a Moorean strategy. In his recent paper Cogburn argues that what he calls 'Moorean inferences' should be restricted as not being applied within the scope of things assumed hypothetically.[14] Moorean inferences are inferences of the following sort:

p; therefore, '*p*' is believed.
'*p*' is provable; therefore '*p*' is proven.
'*p*' is provable; therefore '*p*' is known.

In each case, if one is entitled to hold the premise, then one is also entitled to hold the conclusion. For example, if one shows that '*p*' is provable, one thereby shows that '*p*' is known.

Cogburn claims that anti-realists should reject unrestricted Moorean validity. For example, consider the claim that there is a provable but unknown proposition. This claim is very plausible because it seems possible for some proposition to be provable even if nobody proves it for some practical or other reasons. If this claim is true, unrestricted Moorean reasoning is incorrect. According to Cogburn, realists can easily explain why such Moorean reasoning is invalid. A realist might say that in some sense a proof of *p* is in Plato's heaven, without ever being accessible to beings like us. [15] But anti-realists cannot offer a similar explanation. Contemporary anti-realists usually utilize the proof-theoretic semantics, according to which one can assert the existential claim such that there is a provable but unknown

[14] Jon Cogburn [2012] "Moore's Paradox as an Argument Against Anti-realism", in Shahid Rahman et. al. (eds.), *The Realism-Antirealism debates in the Age of Alternative Logics, Logic, Epistemology, and the Unity of Science*, 23, pp. 69-84.

[15] Ibid., p, 76.

proposition just in case one possesses a procedure ν for finding a particular proposition 'p' such that ν proves both 'p' and '$\sim Kp$'; that is, ν proves that p and also that it is unknown. But this implication sounds absurd. Thus, on Cogburn's view, anti-realists are in big trouble unless they are able to offer an equally good explanation about Moorean invalidities from their point of view.

Cogburn's proposal is that Moorean inferences should not be applied within the scope of things assumed hypothetically. His main motivation for such restriction is that proofs to contradictions are generated only when Moorean inferences are allowed inside sub-proofs that depend on premises hypothetically entertained. Thus, if Moorean inferences are not used within the scope of things assumed hypothetically, he claims, the derivation of Fitch's proof is blocked. He says:

> Our reaction to Moorean validities suggests a new strategy for Fitch's proof. Perhaps the anti-realist inference [$p \vdash \Diamond Kp$] is itself a Moorean validity, and as such subject to the restriction that it not apply within the scope of things hypothetically assumed. But then Fitch's proof is invalid.[16]

One surprising implication of Cogburn's proposal above is that such important and influential philosophers as Berkeley and Davidson used Moorean inferences in an unacceptable way. He says:

> In *On the Very Idea of a Conceptual Scheme*, Davidson asks us to imagine a language not translatable into ours. But since to imagine a language would be to imagine it as being translatable into ours, we can't do this. Therefore every language is translatable into ours and thus (along with other Davidsonian premises) there are no radically different conceptual schemes. Like Berkeley, he is using a Moorean inference inside the scope of something assumed hypothetically. We are to conclude about a hypothetically presumed language, that since we have a conception of it, it must be translatable into our language. While this may be true of any actual language we can so categorize (for the very Davidsonian reason that we would not be in a position to consider it a language unless we could translate it into ours) to conclude this about a language hypothetically assumed to exist requires improper use of Moorean reasoning.[17]

Cogburn's restriction on Moorean reasoning is not well-motivated, however. The first thing to note is that the reason why Moore's paradox arises is not that Moorean reasoning is used within the scope of things assumed hypothetically. At least some uses of Moorean reasoning within such a scope, it seems, are perfectly alright. For example, consider an object hypothetically assumed to be a dog. Since to conceive something as a dog is to conceive it as an animal, we can say of that object that under the assumption that it

[16] Ibid., p. 78.
[17] Ibid., p. 83.

is a dog one must believe that it is also an animal. Thus the fact that a contradiction can be generated from some uses of Moorean reasoning within the scope of things assumed hypothetically does not show that all Moorean inferences should be banned inside that scope. To put the same point another way, banning all Moorean inferences within the scope of things assumed hypothetically is most likely to be an excessive restriction on our inferential power. More importantly, Moore's paradox arises from the peculiar logical character of such a sentence as 'p & ~Bp'. As noted in section 3, it is because inferences of the following sort are invalid: one is entitled to assert that $(\exists p)(p$ & ~B$p)$; therefore, one is entitled to assert for some specific proposition 'p' that p & ~Bp. Whether an anti-realist or not, one should accept that inferences of this sort are invalid, and also that their invalidities have no direct bearing on whether Moorean inferences are used inside the scope of things assumed hypothetically. Therefore anti-realists do not need to restrict Moorean inferences in such an excessive way Cogburn suggests.

Another thing to note is that Cogburn's proposal has some far-fetched implications. Consider his claim regarding Davidson's famous argument against radically different conceptual schemes. On his view, we can say of any *actual* language that we would not be in a position to consider it as a language unless we could translate it into our own conceptual scheme, whereas for any *hypothetically presumed* language we cannot make the same sort of claim. But Davidson's argument against radically different conceptual schemes is a modal argument intended to establish the impossibility of a radically different conceptual scheme; that is, it is intended to show that if something, *whether it is actual or hypothetical*, is not interpretable into our own scheme, there is no intelligible basis on which to judge that it is indeed a conceptual scheme. Sometimes we need to make such a modal claim. For example, consider again the modal claim that it is impossible for a non-animal to be a dog. Such a modal claim requires considering hypothetical scenarios as well. That is, it requires saying that even for things assumed hypothetically, if they are not animals, they cannot be dogs either. Due to this sort of reason, even about things assumed hypothetically, we can say that under the assumption that it is not an animal it must not be conceived as a dog either. If Cogburn is right, we are not allowed to make such a claim. But this is absurd. At any rate, this sort of consequence is very controversial. A proposal without having such very controversial implications is much more desirable. My proposal is indeed such a one.

7. Conclusion

Fitch's proof is not a paradox but simply a result to be properly understood. The lesson of Fitch's proof is that one is not entitled to assert for any specific proposition '*p*' that p & ~Kp, although one is entitled to assert that $(\exists p)(p$ & ~K$p)$. The main point of contention between realists and anti-realists is whether our notion of truth has a radically non-epistemic feature relevant to realism. But the unknowability of truths of the form 'p & ~Kp' has nothing to do with such a realism-relevant feature. It is mainly due to a conceptual truth that an unknown truth cannot be known unless it loses its epistemic status of being unknown, and this conceptual truth has nothing to do with the debates between realists and anti-realists. Therefore my proposal disarms the knowability paradox by citing some feature of truth that everyone should accept. In addition, my proposal can block the derivation of Fitch's proof without restricting the knowability principle, although I do not endorse the unrestricted knowability principle itself. Consequently, my restriction strategy can successfully disarm the paradox in such a way that is free from the criticisms Kvanvig and others made against Tennant-style restriction strategies.[18]

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