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# STRONG AND WEAK REGRESS ARGUMENTS

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> “Much of the trouble hinges on unclarities about the role of infinite regresses.” (Oliver 1996)

*Abstract*
In the literature, regress arguments often take one of two different forms: either they conclude that a given solution fails to solve *any* problem of a certain kind (the strong conclusion), or they conclude that a given solution fails to solve *all* problems of a certain kind (the weaker conclusion). This gives rise to a logical problem: do regresses entail the strong or the weaker conclusion, or none? In this paper I demonstrate that regress arguments can in fact take both forms, and clearly set out the logical difference between them. Throughout the paper, I confine myself to metaphysical examples from the early Russell. Only now that we know they are valid can we start to discuss whether they are sound.

## 1. *Preamble*

Everyone knows the difference between

- I did not send this paper to *all* of the other journals.
- I did not send this paper to *any* other journal.

Yet the all/any distinction has been overlooked so far in the debate on regress arguments. This oversight is unfortunate, because much follows from it. Or so I will argue in this paper.

## 2. *Four cases*

Let us consider the following four examples of a regress argument invented and/or promoted by the early Russell.

*Case (i): Resemblance.* Suppose items a, b and c have the same property F. According to a certain theory of properties and relations, the fact that these items are all F reduces to the fact that they resemble the standard F-item (call it 'd'). This yields the situation where the pairs {a, d}, {b, d}, {c, d} all stand in the same relation, i.e. of resemblance. By the given theory, this fact reduces to the fact that they all resemble the standard resemblance pair. Regress. Conclusion: the given theory will never reduce all facts in cases where several items have the same property or relation. (1903: §55, 1911–12: 9, 1912: 96–7, cf. 1900: 555–7)

*Case (ii): Asymmetry.* Suppose a is earlier than b. According to a certain theory, the asymmetric relation between a and b is reducible to properties of a and b, say 'existing at $t_1$' of a and 'existing at $t_2$' of b. This yields the situation where $t_1$ must be earlier than $t_2$. By the given theory, this fact is reducible to properties of $t_1$ and $t_2$, say 'existing at $t^*_1$' of $t_1$ and 'existing at $t^*_2$' of $t_2$. Regress. Conclusion: the given theory will never reduce all facts in cases where items stand in an asymmetric relation. (Russell 1899, 1903: §214, 1906–07: 41–2, 1959: 54–5)[1]

*Case (iii): Unity.* Suppose a and b stand in relation R. According to a certain theory, how R forms a unity with a and b can be explained by the fact that there is a relation R* which unifies R with a and b. This yields a situation where a, b and R stand in R*. By the given theory, how R* forms a unity with a, b and R can be explained by the fact that there is a relation R** which unifies R* with a, b and R. Regress. Conclusion: the given theory will never explain how R forms a unity with a and b. (Bradley 1893: chs. 2–3; cf. Russell 1899: 146, 1903: §99, 1904: 210, 1910: 373–4)

*Case (iv): Order.* Suppose a is earlier than b. According to a certain theory, the difference between Rab (i.e. the fact that a is earlier than b) and Rba (i.e. the fact that b is earlier than a) is that the former fact resembles Rdc and the latter resembles Rcd. This yields a situation where the fact that Rab resembles Rcd differs from the fact that Rab resembles Rdc. By the given theory, this difference can be explained by the fact that the pair {Rab, Rcd} resembles another fact Ref, and that the pair {Rab, Rdc} resembles Rfe. Regress. Conclusion: the given theory will never explain the difference between Rab and Rba. (Russell 1913: 84)

[1] Cf. Russell (1903: §214) for why this reasoning does not apply to symmetric relations.

### 3. *All/Any Problem*

I have selected the cases in the previous section for several reasons. First, Russell's regress arguments are interesting in their own right as they motivated his view and defence of relations, which played a crucial role in the history of philosophy. Before Russell, almost everyone assumed that relations are in one way or another reducible to properties of their relata, yet Russell's criticism undermined this assumption (cf. Griffin 1991; Mulligan 1998; Candlish 2007). Furthermore, Russell's cases have received quite some attention in contemporary debates in metaphysics.[2]

I will show in this section, however, that there is a problem with these cases. Let us consider the conclusions of the four regress arguments. In the following I have restated them in terms of 'you will never solve problem X if you carry out solution Y':

(i) You will never reduce *all* properties/relations if you reduce them to resemblances with standard items.
(ii) You will never reduce *all* relations if you reduce them to properties of their relata.
(iii) You will never explain how R (or *any* other relation) forms a unity with its relata if you appeal to further relations.
(iv) You will never explain the difference between Rab and Rba (or *any* other pair of facts where the relation applies differently to the same relata) if you appeal to resemblances with other such facts.

I have emphasized the words 'all' and 'any'. For clearly these words are important: if we substitute 'all' for 'any', or the other way around, we obtain completely different conclusions:

(i*) You will never reduce *any* property/relation if you reduce them to resemblances with standard items.
(ii*) You will never reduce *any* relation if you reduce them to properties of their relata.
(iii*) You will never explain how *all* relations are unified with their relata if you appeal to further relations.

[2] Cf. Armstrong (1974, 1978, 1989); Betti (2014); Cameron (2008); Campbell (1990); Dodd (1999); Gaskin (2008); Hochberg (1980, 1987); Lewis (1983); Loux (1998); MacBride (2007); Maurin (2002, 2010, 2012); Mertz (1996); Nolan (2001, 2008); Oliver (1996); Orilia (2006, 2009a, 2009b); Rodriguez-Pereyra (2001, 2002); Schnieder (2004, 2010); Vallicella (2002, 2004); among many others.

(iv*) You will never explain the difference between *all* facts where the relation applies differently to the same relata if you appeal to resemblances with other such facts.

These are clearly different. For example, to say that one will never reduce all relations given a certain solution does not entail the stronger claim that one will never reduce any relation given that solution. Or to say that one will never explain how all relations are unified with their relata given a certain solution does not entail the stronger claim that one will never explain how any relation is unified with its relata given that solution.[3]

Now the question is: what is the right conclusion in each of these cases? Do regress arguments establish all-conclusions, or the stronger any-conclusions, or perhaps no conclusions at all? Call this the All/Any Problem.[4] This logical problem is clearly important given that Russell's regress arguments (and regress argument generally) are supposed to make a significant difference to the debates in which they are employed (e.g. Armstrong 1974, 1978; Maurin 2002). However, if we do not know whether regress arguments are valid in the first place (and if they are valid, what form they should take), we also do not know whether they are sound, and indeed whether we should care about them in the first place.

In recent years, a few philosophers have tried to clarify the general format of regress arguments (see especially Black 1996; Cling 2009; Gratton 2010). Yet no existing proposal addresses the All/Any Problem, and none provides a framework on the basis of which it might be solved. The main aim of the present paper is to offer just such a framework.[5]

I should say that ideas about regress arguments with all- and any-conclusions have made appearances at other places in the literature. Consider for example the following two texts (again, the crucial terms are emphasized):

> A philosophical explanation of predication must, if it is to be successful, explain *all* instances of predication. [. . . ] The argument purports to show that no matter how often you iterate the explanation in order to include the predication just introduced, you will

[3] Another difference is that the former two are about reduction, while the latter two about explanation. Yet so long as the reduction cases can be stated in terms of explanation, this difference has nothing to do with the form of the arguments.

[4] For a statement of this problem (yet without solution), cf. Wieland (2011).

[5] My solution forms the central part of what I call the Failure Theory (see my 2012, 2013), as distinguished from the Paradox Theory (i.e. my label for the proposals by Black, Cling and Gratton, among others).

> always introduce a new, unexplained predication. (Day 1987: 156–7)

> A regress is said to be vicious if, for example, in order to have something, there is always an additional something one is first required to have. In general, in a vicious regress, one could never be in a position to have *anything* at all, or the requirements for having the first or any additional thing could never be met. (Ruben 1990: 127)

These general reflections on regress arguments have never been compared and made precise. In the following, I will argue that both of these texts get something right, and that regress arguments can establish all-conclusions as well as any-conclusions. In other words, I will show that there are both valid weak regress arguments (i.e. with an all-conclusion) and valid strong regress arguments (i.e. with an any-conclusion). In Sect. 4 I will set out the logical and dialectical details of weak regress arguments, and in Sect. 5 do the same for strong regress arguments. In Sect. 6 I will respond to a worry about such regress arguments voiced in the literature. I will conclude the paper with some remarks on the importance of these results (Sect. 7).

I would like to be clear about the objective of this paper. There are many delicate issues about regresses and regress arguments (e.g. what distinguishes vicious regresses from harmless ones, whether Russell's regress arguments are not only valid but also sound, and thus whether they prove, among other things, that relations are irreducible to properties of their relata, and so on). Nevertheless, no single paper can deal with all these issues at once, and in the following I will focus exclusively on the following, single issue pertaining to regress arguments: *the problem of their logical validity*. What form should regress arguments take if an all-conclusion or any-conclusion is to follow logically from a regress (and the regress in turn from premises and hypotheses)?

## 4. *Weak regress arguments*

The rationale of weak regress arguments can be informally captured as follows: it will never be the case that *all* problems of a certain kind are solved, because the solution under consideration generates a regress of ever more problems of that kind. For example, the rationale of Russell's regress argument (ii) (from Sect. 2) is that it will never be the case that all relations are reduced because the solution of reducing relations to properties of their relata generates a regress of ever more relations that require reduction (namely the relations between those properties).

At this point the question is how things can be made precise. How to phrase this rationale in terms of a valid argument pattern? Or again: what is the valid argument schema that has weak regress arguments as instances?

In the following I will present my solution. To obtain instances of the schema, 'K' is to be replaced by a specific domain, 'S' by a person (or agent that can solve problems), and the Greek letters '$\varphi$' and '$\psi$' by predicates which express actions involving the items in that domain. For example, an easy way to read the schema is by the following key: items in K: propositions; $\varphi$: justify; $\psi$: provide a reason for.

*Weak Regress Schema*

(1) For all x in K, if S has to $\varphi$ x, then S $\psi$-s x.
(2) For all x in K, if S $\psi$-s x, then there is a new item y in K.
(3) For all x in K, S has to $\varphi$ x.
(4) For all x in K, if S has to $\varphi$ x, then S has to $\varphi$ a new item y in K. [from 1–3]
(5) S will never $\varphi$ all items in K. [from 4]
(C) If S $\psi$-s any item in K that S has to $\varphi$, then S will never $\varphi$ *all* items in K. [from 1–5]

This schema has one hypothesis, i.e. line (1); two premises, i.e. lines (2) and (3); and three main inferences, i.e. lines (4), (5) and (C). For a statement of this schema in first-order predicate logic, plus details about the inferences (i.e. the rules of inference and one suppressed premise), I refer to the Appendix.[6] The premise/hypothesis distinction is important for regress arguments. Premises are lines taken to be true; hypotheses are not taken to be true, but merely taken into consideration. In regress arguments, solutions (i.e. instances of line (1)) are considered merely for the purpose of deriving a failure from them (i.e. instances of line (5)), such that we may conclude by Conditional Proof that 'if (1), then (5)' (which is the final line (C)).

Weak regress arguments are *weak* because the failure they conclude to is weak. That is, they conclude that a given solution fails to solve *all* problems of a certain kind (rather than *any* problem of that kind). How does this weak failure arise? According to my proposal, it arises because the solution of $\psi$-ing the items in K that are to be $\varphi$-ed always generates yet another item in K that is to be $\varphi$-ed, such that one never $\varphi$-s all items in K (or, as I put it earlier, the considered solution generates a regress of ever more problems of the same kind). To illustrate this, let us consider a full instance of the Weak

[6] One premise is suppressed because its truth does not usually depend on the content of specific instances, i.e. it is virtually never a point of discussion (at least not in the Russell cases).

Schema, namely Russell's regress argument against Resemblance Nominalism (i.e. case (i) from Sect. 2):

*Resemblance Regress Argument (Weak instance)*

(1) For all universals x, if S has to reduce x instantiated by items a and b, then S reduces x to the fact that a and b resemble the standard x-item c.
(2) For all universals x, if S reduces x to the fact that a and b resemble c, then there is a resemblance universal instantiated by {a, c} and {b, c}.
(3) For all universals x, S has to reduce x.[7]
(4) For all universals x, if S has to reduce x instantiated by a and b, then S has to reduce the resemblance universal instantiated by {a, c} and {b, c}. [from 1–3]
(5) S will never reduce all universals. [from 4]
(C) If S reduces all the universals that S has to reduce to resemblances with standard items, then S will never reduce *all* universals. [from 1–5]

How to read this reconstruction? If we assume that NN1 and NN2 are two persons, then the dialectical setting is as follows:

(1) This is NN1's proposed solution.
(2) NN2 argues that NN1 has to concede this premise.
(3) NN2 argues that NN1 has to concede this premise.
(4) NN2 infers this line from (1)–(3).
(5) NN2 infers this line from (4).
(C) This is NN2's conclusion that NN1's solution fails.

In the case of the Resemblance Regress, the dialectic is that Resemblance Nominalism[8] subscribes to (1) in order to solve the problem of reducing all universals, and that opponents of this theory show that (1), together with further premises (2) and (3), generates a regress which prevents the theory from ever solving that problem.[9]

[7] An alternative interpretation of the problem: For all properties/relations x, S has to explain how it is possible that the distinct items can have/stand in the same x (cf. Armstrong 1978: 11; Wieland 2008). The rest of the argument can be adjusted accordingly.

[8] Rodriguez-Pereyra (2002: 124) labels this theory 'Aristocratic Resemblance Nominalism' to distinguish it from other versions.

[9] Note that (3) and (5) are not inconsistent: it is not inconsistent to say that a problem has to be solved and that a given solution never solves it.

Given the validity of the argument (i.e. that the conclusion (C) follows logically from (1)–(5); see the Appendix), Resemblance Nominalism has only a minimal set of options for resisting the argument. I will say more about this in Sect. 7.

Instances of the Weak Regress Schema are *infinite regress* arguments, because infinite regresses are generated by instances of its lines (1)–(3) (plus an arbitrary hypothesis). Schematically:

(i) S has to $\varphi$ a. [hypothesis]
(ii) S $\psi$-s a. [from i, 1]
(iii) S has to $\varphi$ b. [from ii, 2, 3]
(iv) S $\psi$-s b. [from iii, 1]
(v) S has to $\varphi$ c. [from iv, 2, 3]
etc.

Line (i) is arbitrary in the sense that we could use any arbitrary item from the domain K to generate such a regress. Instances of such lists are series of problem/solution pairs: (i) is a problem, (ii) is introduced as a solution to (i), (ii) entails another problem of the same kind: (iii), (iv) is introduced as a solution to (iii), etc. In terms of this, the Resemblance Regress would run as follows:

(i) S has to reduce F instantiated by a and b. [hypothesis]
(ii) S reduces F to the fact that a and b resemble the standard F-item, i.e. c. [from i, 1]
(iii) S has to reduce the resemblance universal $R_1$ instantiated by {a, c} and {b, c}. [from ii, 2, 3]
(iv) S reduces $R_1$ to the fact that {a, c} and {b, c} resemble the standard $R_1$-pair, i.e. {d, e}. [from iii, 1]
(v) S has to reduce the resemblance universal $R_2$ instantiated by {{a, c}, {d, e}} and {{b, c}, {d, e}}. [from iv, 2, 3]
etc.

This set-up assumes that problems are *tasks*, and that the considered solutions are *actions* meant to accomplish such tasks. Alternatively, regresses could be understood in terms of 'processes'.[10] One worry about the latter proposal is that processes seem to involve the idea of time, while many regresses (such as Russell's cases) do not. The case just mentioned, for example, concludes that Resemblance Nominalism never reduces all universals in

[10] As a referee suggested.

the sense that *at no point in the regress* will it be the case that there is no further universal to be reduced (not in the sense that at no point in time will this be the case). That is, Resemblance Nominalism never reduces all universals, regardless of whether the solution of reducing universals to resemblances with standard items takes time.

Strictly speaking, such regresses play no *logical* role in weak regress arguments (as (4) follows from (1)–(3) without them).[11] Still, it is instructive to spell out a few steps of the regress to see how it is the case that *always* yet another problem of the same kind has to be solved such that at no point will one solve all of them.

## 5. *Strong regress arguments*

The rationale of strong regress arguments, accordingly, is as follows: no *single* problem of a certain kind will ever be solved because the solution under consideration generates a regress of ever more problems that must be solved in order for any initial problem to be solved. For example, the rationale of the regress argument (iii) (from Sect. 2) is that no single problem of explaining how a relation forms a unity with its relata will ever be solved because the solution of appealing to further relations generates a regress of ever more relations for which you must explain how they are unified with their own relata first.

The question, again, is how to make this precise. How can this rationale be stated as a valid argument pattern? What is the valid argument schema that has strong regress arguments as instances? My solution is the following:[12]

*Strong Regress Schema*

(1) For all x in K, if S has to $\varphi$ x, then S $\psi$-s x.
(2) For all x in K, if S $\psi$-s x, then there is a new item y in K *and S first has to $\varphi$ y in order to $\varphi$ x.*
(3) For all x in K, if S has to $\varphi$ x, then there is a new item y in K and S first has to $\varphi$ y in order to $\varphi$ x. [from 1–2]
(4) S will never $\varphi$ any item in K. [from 3]
(C) If S $\psi$-s any item in K that S has to $\varphi$, then S will never $\varphi$ *any* item in K. [from 1–4]

[11] Thanks to Benjamin Schnieder for this point. The same applies to strong regress arguments in the next section.

[12] It could again be read by the following key: items in K: propositions; $\varphi$: justify; $\psi$: provide a reason for. For a first-order rendering, see the Appendix.

Strong regress arguments are *strong* because the failure to which they conclude is strong. That is, they conclude that a given solution fails to solve *any* problem of a certain kind (rather than *all* problems of that kind). How does this strong failure arise? According to my proposal, it arises because the solution of $\psi$-ing the items in K that are to be $\varphi$-ed always generates yet another item in K that is to be $\varphi$-ed *first*, such that one never $\varphi$-s any item in K (or, as I put it earlier, the considered solution generates a regress of ever more problems that must be solved in order for any initial one to be solved). The main differences with the Weak Schema are emphasized in the schema above (see the italics). Before explaining them, let us consider Bradley's instance for illustration:

*Unity Regress Argument (Strong instance)*

(1) For all relations x, if S has to explain how x forms a unity with its relata, then S appeals to a relation y that unifies x with its relata.
(2) For all relations x, if S appeals to a relation y, then S first has to explain how y forms a unity with its relata in order to explain how x forms a unity with its relata.
(3) For all relations x, if S has to explain how x forms a unity with its relata, then S first has to explain how another relation y forms a unity with its relata in order to explain how x forms a unity with its relata. [from 1–2]
(4) S will never explain how any relation forms a unity with its relata. [from 3]
(C) If S appeals to a further relation every time S has to explain how a relation forms a unity with its relata, then S will never explain how *any* relation forms a unity with its relata. [from 1–4]

The dialectical setting here is almost the same as in the weak case: (1) is NN1's solution to explain for at least one relation how it forms a unity with its relata,[13] and NN2 shows on the basis of such a strong argument that NN1's proposal fails. There are *two* main differences between weak and strong regress arguments.

First difference: the infinite regress is generated differently. In the weak case, any problem in the regress is entailed by the solution in the previous step plus premises (2) and (3). In the strong case, any problem in the regress is entailed by the previous solution and, importantly, *only* premise (2) (which

[13] An alternative interpretation of the problem: For at least one relation x, S has to explain the difference between the fact that a stands in x to b in a world w and the mere collection of a, b and x in certain worlds distinct from w (cf. Armstrong 1989: 88; Vallicella 2002: 12). The rest of the argument can be adjusted accordingly.

is substantially longer than the parallel line in the Weak Schema: it also comprises the clause 'and S first has to $\varphi$ y in order to $\varphi$ x'). Schematically:

(i) S has to $\varphi$ a. [hypothesis]
(ii) S $\psi$-s a. [from i, 1]
(iii) S first has to $\varphi$ b. [from ii, 2]
(iv) S $\psi$-s b. [from iii, 1]
(v) S first has to $\varphi$ c. [from iv, 2]
etc.

In terms of this, the Unity Regress would run as follows:

(i) S has to explain how $R_1$ forms a unity with a and b. [hypothesis]
(ii) S appeals to a relation $R_2$. [from i, 1]
(iii) S first has to explain how $R_2$ forms a unity with $R_1$, a and b. [from ii, 2]
(iv) S appeals to a relation $R_3$. [from iii, 1]
(v) S first has to explain how $R_3$ forms a unity with $R_2$, $R_1$, a and b. [from iv, 2]
etc.

Again, such regresses play no logical role (as (3) follows from (1)–(2) without them), yet it is instructive to spell out a few steps of the regress to see how it is the case that *always* yet another problem of the same kind has to be solved *first*, such that at no point will one solve any of them. The term 'first' indicates an *instrumental* order rather than a *temporal* order. It need not be the case that the problem of explaining how $R_{n+1}$ forms a unity with its relata must be solved earlier in time. What matters is the asymmetry between the problems: explaining how $R_{n+1}$ forms a unity with its relata is meant to be a precondition of explaining how $R_n$ forms a unity with its relata, and not the other way around (more on this in the Appendix).

In principle, any case *can* be stated in terms of both schemas, but whether a given case *should* be set out in the strong or the weak way must be decided on the basis of premise (2) of the Strong Schema. For example, the Resemblance Regress Argument is stated in terms of the Weak Schema, because (2) of the Strong Schema seems implausible, i.e. it seems implausible to hold that 'for all universals x, if S reduces x instantiated by a and b to the fact that a and b resemble the standard F-item c, then S first has to reduce the resemblance universal instantiated by {a, c} and {b, c} in order to reduce F instantiated by a and b.'

Second (and expected) difference: the conclusions of weak and strong regress arguments are different. Weak arguments demonstrate that a given solution fails to accomplish *all* problems of a certain kind, while strong ones demonstrate that it fails to accomplish *any single* such problem. Moreover, the Strong Schema's conclusion is stronger (as its name indicates) because it entails the Weak Schema's conclusion, but not vice versa. That is, if you will never $\varphi$ *any* item in K, then (provided there is at least one such item) it cannot be the case that you $\varphi$ *all* items in K (e.g. if you do not send your paper to any journal, then surely you do not send it to all of them). Yet, if you will never $\varphi$ *all* items in K, it may still be the case that you $\varphi$ *some* of them (if you do not send your paper to all journals, it may still be the case that you send it to some of them).

## 6. *Schlesinger's worry*

In this section, I will respond to a worry about regress arguments voiced by Schlesinger (1983: 221–7; cf. Oppy 2006: 289–90). Schlesinger draws the attention to an ambiguity. To explain this, consider his view on the dialectical situation:

S2 NN1's solution to solve an initial problem P1.
P2 NN2 shows that NN1 has to solve this similar problem.
S2 NN1 solves P2 in a similar way.
P3 NN2 shows that NN1 has to solve this similar problem.
etc.

The question is what follows:

> Are we to say that, since essentially the same problem keeps arising no matter how far we progress along the regress, we are faced with an ineradicable problem, or that, since every time we raise a problem we can at once come up with a solution, we are left we no difficulty? (Schlesinger 1983: 221)

Hence, Schlesinger identifies two potential conclusions. First conclusion: Every solution entails the same kind of problem, so in some sense the problem is ineradicable. Call this 'Pessimism'. Second: For every problem there is a solution, so in some sense we are left we no difficulty. Call this conclusion 'Optimism'. Furthermore, as Schlesinger suggests, Pessimism prevails if we stop at one of the problems (for in that case a problem is left unsolved), and Optimism if we stop at one of the solutions (for in that case no problem is left unsolved). Schlesinger does not distinguish between strong and weak cases, so I will assume this worry applies to both varieties.

In my view, however, regress arguments as constructed in this paper do not fall prey to the Optimism/Pessimism ambiguity. This seems clear for four reasons. First, so long as all problems and solutions are entailed, it is not possible just to stop at a problem (and so land in Pessimism) or at a solution (and so end with Optimism). In the schemas presented in Sections 4–5 all problems and solutions are entailed (i.e. if one assumes an arbitrary hypothesis).

Second, the dialectic invoked by Schlesinger is not exactly the one which I myself presented in Sect. 4. Schlesinger's dialectic differs in its being between someone who poses problems and someone who proposes solutions for them. The dialectic I presented, by contrast, is between someone who purports to solve all/any problems of a given kind and someone who shows that the former never succeeds in doing so (as each time yet another, similar problem has to be solved). In the latter case, the Optimism/Pessimism ambiguity does not apply. Either the problem is ever solved, or it is not: it is not solved half of the time.

I consider this second point decisive. However, one might still suspect that Optimism could apply to selected regress arguments. Consider, for example, the Resemblance Regress from Sect. 4. So long as there is an endless number of resemblances with standard items, then it seems *all* resemblance universals can be reduced. In general: so long as the series of solutions is endless, *all* problems can be paired off with a solution. Does this constitute a worry for regress arguments? It would do so only if it were to conflict with something stated in a regress argument. Yet this may not be the case. Consider the following two claims:

(i) There is a solution for all problems.
(ii) There is always yet another problem to be solved.

The first claim is Optimism, and applies to a selected number of regress arguments. The second claim is what is demonstrated by a regress, in *all* cases. Here there is no conflict, for neither claim excludes the other. Still, the implications of these claims do seem to conflict:

(iii) If (i), then all problems are solved.
(iv) If (ii), then it is never the case that all problems are solved.

Indeed, it cannot be both that all problems are solved and that it is never the case that all problems are solved. Nonetheless, I do not think that regress arguments are afflicted by this problem. What regress arguments show is that it is never the case that all/any problems of a given kind are solved *in the sense that* there is always a further problem to be solved, whether or not there is a solution for all of them. For example, even if all resemblance universals can be reduced to resemblances with standard items, still it would hold that there are always further resemblance universals to deal with and that one will never deal with all of them in this sense.

My fourth and final point is that similar queries have been raised in the discussion of *supertasks*. Namely: various supertasks seem possible, even though they consist of an infinity of tasks. To recall a classic example, Achilles is able to catch the Tortoise within a finite period of time even though he has to traverse an infinite number of distances. Do my reconstructions have room for such exceptions? They do: if a certain regress does not lead to a failure, then this does not mean that the reconstruction (based on one of my schemas from Sections 4–5) is logically invalid, but that the suppressed premise which licences the step to the failure (i.e. the step from (4) to (5) in the Weak Schema, and from (3) to (4) in the Strong Schema) is false. I further explain this in the Appendix.

## 7. *Relevance*

In this paper I have presented two formats that regress arguments can take: the Weak Regress Schema and the Strong Regress Schema. In this final section I would like to explain why these schemas are relevant. *What is their use?* I will identify four possible uses.

First of all: *logical validity*. The schemas show that regress arguments are valid arguments, i.e. if the latter are set out as instances of the former. For example, they show that Russell's regress arguments are valid and that the substantive conclusions he associates with them in the metaphysical debates on properties and relations (e.g. that the complete reduction of relations to properties of their relata or to standard items fails) can indeed be drawn (i.e. if the relevant premises are in place). Moreover, my investigation shows not only that weak regress arguments with all-conclusions are valid, but also that strong regress arguments with any-conclusions are valid. I regard this as the most significant result of this study: only now that we know they are valid can we start to discuss whether they are sound.

Second: *resistance*. If you know what kind of premises are part of an argument, you know what has to be attacked if you would like to resist its conclusion (or know what has to be defended if you would like to endorse its conclusion). Given that the conclusion follows logically from the premises (2) and (3) and the hypothesis (1), there is only a minimal and fixed set of options to resist regress arguments.[14] Specifically, NN1 (i.e. the person who

[14] In some selected cases, an additional, suppressed premise could also be questioned (see the Appendix). This will be ignored here, as it does not apply to Russell's cases.

wants to resist the regress argument by NN2) has the following three options: she may deny that

- premise (2) holds;
- premise (3) holds;[15]
- she is committed to (1) if fully universally quantified.

Consider for example Russell's regress argument against Resemblance Nominalism. The latter has the following three logical options to resist the argument. First, it may deny that there is a resemblance universal instantiated by {a, the standard F-item} and {b, the standard F-item} if the items a and b resemble the standard F-item (e.g. by denying that anything with many instances is a universal). Second, it may deny the problem that *all* universals are to be reduced (perhaps only a group of them must be reduced, cf. Lewis 1983: 353–4). Third, it may deny that its solution is meant to apply across the board (perhaps it is meant to apply to all universals except the resemblance universal, cf. Price 1953: 26).

Hence, the schemas not only clarify what and how anything can be established by a regress argument, they also define what exactly can be disagreed about when there is a dispute about a certain regress argument (such as Russell's arguments that purport to refute many theories of properties and relations).

Third: *further conclusions*. The schemas are relevant not only for those who wish to resist regress arguments, but also for those who believe they are sound and want to draw further conclusions from them. There are at least two options:

(*) If the solution will never solve the given problem, and if it is also shown that there is an alternative regress-free solution which does solve the problem, then this favours the alternative solution.

(**) If the solution will never solve the given problem, and if it is also shown that no alternative solution is possible, then it can be concluded that problem cannot be solved.

Several texts hint at such expanded arguments. Consider for example the following two texts (the expanded steps are just after 'and' in the last cited sentences):

[15] This second option is only available in the weak case: there is no second premise in the strong case. Still, there are two different ways in which the premise of the latter (i.e. line (2)) could be denied: one may deny that 'there is a new item y in K', or accept this yet deny that 'S first has to $\varphi$ y in order to $\varphi$ x'.

> We explain the likeness of two terms as consisting in the likeness which their likeness bears to the likeness of two other terms, and such a regress is plainly vicious. Likeness at least, therefore, must be admitted as a universal, and, having admitted one universal, we have no longer any reason to reject others. (Russell 1911–12: 9)

> The links are united by a link, and this bond of union is a link which also has two ends; and these require each a fresh link to connect them with the old. The problem is to find how the relation can stand to its qualities; and this problem is insoluble. (Bradley 1893: 28)

The first text is an instance of expansion (*). Namely: Russell concludes in favour of an alternative regress-free solution: Realism about Universals. Bradley's text is an instance of expansion (**), i.e. he draws the sceptical conclusion that the given problem, i.e. the problem of how relations form a unity with their relata, cannot be solved.

Fourth: *generality*. The applicability of the schemas is not at all restricted to Russell's metaphysical cases. For example, the regresses generated by solutions to the Liar Paradox (e.g. Beall 2008) are arguably to be spelled out in terms of the Weak Regress Schema, and the widely discussed Regress of Reasons (e.g. Sextus Empiricus, *Outlines* 1.166–7) is arguably to be spelled out in terms of the Strong Regress Schema. If so, the relevant conclusions would be the following two:

- S will never resolve *all* Liar Paradoxes if S introduces an extra truth value every time S has to resolve a Liar Paradox.
- S will never justify *any* proposition if S appeals to another proposition every time S has to justify a proposition.

The choice of schema proves crucial, as the weaker or stronger counterpart conclusions are quite different (compare: 'S will never resolve *any* Liar Paradox', 'S will never justify *all* propositions').

This concludes my defence of the all/any distinction in the debate on regress arguments.

*Appendix*

In this Appendix, I show that the argument schemas presented in this paper are valid according to classical first-order predicate logic. I use the propositional calculus by Nolt et al. (1988: ch. 4), and the first-order extension by Gamut (1982: 142–7). So I employ standard natural deduction abbreviations of the inference rules and a strict distinction between premises (PREM) and hypotheses (HYP). All portions of hypothetical reasoning are clearly marked by vertical lines. Some of the predicates and premises need some explanation. These explanations are provided right after the formalisation. Please note that the numbering of the lines does *not* correspond to the numbering used in Sections 4–5.

Key:
Kx: x is in domain K
Tx: S has to carry out task T regarding x
Rxy: S first has to carry out T regarding y in order to carry out T regarding x
Ax: S performs action A regarding x
Cx: S carries out T regarding x

Example:
Kx: x is a dispute
Tx: S has to settle x
Rxy: S first has to settle y in order to settle x
Ax: S invokes a proposition to settle x
Cx: S settles x

*Weak Regress Schema*

| | | |
|---|---|---|
| (1) | $\forall x(Ax \rightarrow (\exists yKy \wedge x \neq y))$ | PREM |
| (2) | $\forall x(Kx \rightarrow Tx)$ | PREM |
| (3) | $\forall x((Kx \wedge Tx) \rightarrow \exists y(Ky \wedge Ty \wedge x \neq y)) \rightarrow \neg\forall x(Kx \wedge Cx)$ | PREM |
| (4) | $\mid \forall x((Kx \wedge Tx) \rightarrow Ax)$ | HYP $\rightarrow$ I |
| (5) | $\mid \mid Ka \wedge Ta$ | HYP $\rightarrow$ I |
| (6) | $\mid \mid (Ka \wedge Ta) \rightarrow Aa$ | 4; $\forall$E |
| (7) | $\mid \mid Aa$ | 5, 6; $\rightarrow$ E |
| (8) | $\mid \mid Aa \rightarrow (\exists yKy \wedge a \neq y)$ | 1; $\forall$E |
| (9) | $\mid \mid \exists yKy \wedge a \neq y$ | 7, 8; $\rightarrow$ E |
| (10) | $\mid \mid \mid Kb \wedge a \neq b$ | HYP $\rightarrow$ I |
| (11) | $\mid \mid \mid Kb$ | 10; $\wedge$ E |
| (12) | $\mid \mid \mid Kb \rightarrow Tb$ | 2; $\forall$E |
| (13) | $\mid \mid \mid Tb$ | 11, 12; $\rightarrow$ E |
| (14) | $\mid \mid \mid Kb \wedge Tb \wedge a \neq b$ | 10, 13; $\wedge$ I |
| (15) | $\mid \mid \mid \exists y(Ky \wedge Ty \wedge a \neq y)$ | 14; $\exists$I |
| (16) | $\mid \mid (Kb \wedge a \neq b) \rightarrow \exists y(Ky \wedge Ty \wedge a \neq y)$ | 10–15; $\rightarrow$ I |
| (17) | $\mid \mid \exists y(Ky \wedge Ty \wedge a \neq y)$ | 9, 16; $\exists$E |
| (18) | $\mid (Ka \wedge Ta) \rightarrow \exists y(Ky \wedge Ty \wedge a \neq y)$ | 5–17; $\rightarrow$ I |
| (19) | $\mid \forall x((Kx \wedge Tx) \rightarrow \exists y(Ky \wedge Ty \wedge x \neq y))$ | 18; $\forall$I |
| (20) | $\mid \neg\forall x(Kx \wedge Cx)$ | 19, 3; $\rightarrow$ E |
| (21) | $\forall x((Kx \wedge Tx) \rightarrow Ax) \rightarrow \neg\forall x(Kx \wedge Cx)$ | 4–20; $\rightarrow$ I |

*Strong Regress Schema*

| | | |
|---|---|---|
| (1) | $\forall x(Ax \rightarrow \exists y(Ky \wedge Ty \wedge Rxy))$ | PREM |
| (2) | $\forall x((Kx \wedge Tx) \rightarrow \exists y(Ky \wedge Ty \wedge Rxy)) \rightarrow \neg\exists x(Kx \wedge Cx)$ | PREM |
| (3) | $\mid \forall x((Kx \wedge Tx) \rightarrow Ax)$ | HYP $\rightarrow$ I |
| (4) | $\mid \mid Ka \wedge Ta$ | HYP $\rightarrow$ I |
| (5) | $\mid \mid (Ka \wedge Ta) \rightarrow Aa$ | 3; $\forall$E |
| (6) | $\mid \mid Aa$ | 4, 5; $\rightarrow$ E |
| (7) | $\mid \mid Aa \rightarrow \exists y(Ky \wedge Ty \wedge Ray)$ | 1; $\forall$E |
| (8) | $\mid \mid \exists y(Ky \wedge Ty \wedge Ray)$ | 6, 7; $\rightarrow$ E |
| (9) | $\mid (Ka \wedge Ta) \rightarrow \exists y(Ky \wedge Ty \wedge Ray)$ | 4–8; $\rightarrow$ I |
| (10) | $\mid \forall x((Kx \wedge Tx) \rightarrow \exists y(Ky \wedge Ty \wedge Rxy))$ | 9; $\forall$I |
| (11) | $\mid \neg\exists x(Kx \wedge Cx)$ | 2, 10; $\rightarrow$ E |
| (12) | $\forall x((Kx \wedge Tx) \rightarrow Ax) \rightarrow \neg\exists x(Kx \wedge Cx)$ | 3–11; $\rightarrow$ I |

Premise (3) of the Weak Schema and (2) of the Strong Schema were suppressed in the semi-first-order schemas and thus require some further explanation. In terms of the latter schemas, they read:

- If S has to $\varphi$ a new item in K for any item in K that S has to $\varphi$, then S will never $\varphi$ all items in K.
- If S first has to $\varphi$ a new item in K for any item in K that S has to $\varphi$, then S will never $\varphi$ any item in K.

For example:

- If S has to settle a new dispute for any dispute that S has to settle, then S will never settle all disputes.
- If S first has to settle a new dispute for any dispute that S has to settle, then S will never settle any dispute.

According to this construction, the predicates 'T' and 'C' do not depend on each other. That is, if S has to carry out T, then she may or may not in fact carry out T. Relatedly, 'T' does not carry modal or deontic connotations, or at least none of the inferences rely on such connotations. For example, they do not make use of the consideration that ought-implies-can (i.e. that if S has to carry out T, then S should be able to carry out T). According to my reconstructions, 'S fails to carry out T regarding any/all K(s)' does not mean 'S lacks a certain ability', but rather 'S always has to accomplish a further task of the same sort in order to carry out T regarding any/all K(s), and so S will never carry out T regarding any/all K(s) in this sense'.[16]

The main difference between the schemas lies in the predicate 'R'. 'R' cannot be expressed purely in terms of the predicate 'T', given that 'R' imposes an order on tasks (i.e. something that the tasks themselves do not have). Again, the term 'first' in 'S first has to settle dispute y in order to settle dispute x' (i.e. 'Rxy') indicates an instrumental order rather than a temporal order. It need not be the case that the problem of settling y must be solved earlier in time. What matters is the asymmetry between the problems: settling y is meant to be a precondition of settling x, and not the other way around. Thus: $\forall x \forall y (Rxy \rightarrow \neg Ryx)$.

Finally, premises (1) and (3) of the Weak Schema explicitly assume that x and y are distinct items. First, (3)'s antecedent would automatically be satisfied without this assumption (which is undesirable, because in that case the failure would follow at once). Second, in this schema we have no asymmetric relation between the tasks that can ensure that $x \neq y$. Yet, there remains

[16] Further research should show whether there are alternatives to this 'T'/'C' construction.

a problem about (3), as its antecedent does not say what it should say. It should say that there is always a *new* task of the same kind to be carried out, while in fact it merely says that for each task, there is a *distinct* task of the same kind to be carried out.

To solve this, we could introduce an additional relation ‘<’, distinct from R, whose only job is to order the Ks, and make sure that all items introduced in the regress are new items (such that they form an infinite, non-circular series). To do this, ‘x<y’ can be read as ‘x occurs earlier in the regress than y’ and has to satisfy the following conditions:[17]

- $\forall x \neg x < x$
- $\forall x \forall y \forall z((x < y \wedge y < z) \rightarrow x < z)$
- $\forall x \forall y((x \neq y \wedge Kx \wedge Ky) \rightarrow (x < y \vee y < x))$
- $\forall x \forall y(x < y \rightarrow (Kx \wedge Ky))$

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[17] These ensure that ‘<’ is irreflexive and transitive, and that all and only Ks stand in ‘<’. Thanks to Christian Straßer for suggesting this solution.

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