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THE TRUTH-TELLERS PARADOX

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Abstract
Ttler='Ttler is true' *says of itself that it is true*. It is a truth-teller. I argue that we have equally telling arguments (i) to the effect that all truth-tellers must have the same truth-value (ii) and the effect that truth-tellers differ in truth-value. This is what I call the Truth-Tellers paradox. This paradox stems from the fact that the truth-value of a truth-teller like Ttler should be determined by the fact that it says of itself that it is true (which entails (i)) but that it cannot be determined by that fact (as witnessed by (ii)). The Truth-Tellers paradox resembles the classical semantic paradoxes like the Liar. In both cases, a form of self-reference allows us to derive a contradiction from otherwise plausible semantic and logical principles. Furthermore the Truth-Tellers paradox can be formulated without using sentences which are in an intuitive sense ungrounded, it thus severs the link, almost universally taken for granted, between the semantic paradoxes and ungroundedness. Finally, some classical solutions to the Liar do not generalize to the Truth-Tellers paradox.

Ttler Ttler is true.

Ttler is a truth-teller: it says of itself that it is true. It could be wrong however, and it is hard to see any reason why we should prefer an option to the other. It seems that it could admit arbitrary truth-value assignments and it is in that sense pathological. Some philosophers have argued that it is nevertheless false (Priest (2006, 64–66) Yablo (1993a, 387)). Others have argued that it is true (Smith, 1984). Yet others suggest that it is neither true nor false.[1] Priest and Mortensen (1981) have argued, finally, that although this suggestion is well motivated for there is no proof that Ttler is true, and no proof that it is false, there is a proof that it is either true or false. Ttler would thus be paradoxical.

[1] Goldstein (2000) claimed that Ttler does not make a statement. Read (2008b) concurred but he has retracted since then (Read, 2008a, 213).

They are, I believe, not quite right. Ttler is indeed paradoxical, but not for the reason Priest and Mortensen (1981) put forward. I will indeed argue that we have equally telling proofs of the two following inconsistent claims

1. The Semantic Sameness of Truth-Tellers (Semantic Sameness). All the sentences, which say of themselves, like Ttler, that they are true must have the same truth-value.
2. The Semantic Diversity of Truth-Tellers (Semantic Diversity).
   (a) some truth-tellers must be true
   (b) and others must be false.

(Notice that one could argue from 1 and 2a that Ttler is true, and from 1 and 2b that it is false). This antinomy stems from the fact that the truth-value of a truth-teller Ttler should be determined by the fact that it says of itself that it is true (which implies *Semantic Sameness*) but that it cannot be determined by that fact (as witnessed by *Semantic Diversity*). It is paradoxical. As this paradox involves various truth-teller*s*, and in order to distinguish it from Priest and Mortensen (1981)’s Truth-Teller paradox, we can call it the Truth-Teller*s* paradox.

We shall see that this paradox displays some interesting differences with the Liar paradox. In particular, it can be formulated without using sentences which are in an intuitive sense ungrounded. Some influential solutions to the Liar are furthermore ineffective on it. Here is how I will proceed. After arguing for *Semantic Diversity* (§ 1) and for *Semantic Sameness* (§ 2), I will articulate the Truth-Tellers paradox (§ 3). I will then show that the paradox is resistant to some influential treatments to the Liar paradox and that even if it relies on a particularly strong version of the equivalence schema, this strong version is plausible and there is no reason to reject it while retaining the weaker version that is involved in the Liar paradox (§ 4).

1. *Semantic Diversity*

In order to show that truth-tellers should have different truth-values we will need a precise characterization of truth-tellers, and for that, a precise characterization of sentential meaning.

Let us say that a sentence analytically implies another one if a rational subject cannot assent to the first one without assenting to the second one, and that two sentences are analytically equivalent if they analytically imply each other. By ‘sentence’, I will mean sentence *type* and for simplicity I will consider, by default, that the sentences under scrutiny are not context-sensitive (I will relax this assumption in due course). I will assume that the content of a sentence, what it expresses or what it says, is individuated by equivalences that are both strict and analytic. This is quite a fine grained notion but any coarser grained notion should make the class of truth-tellers larger and the semantic diversity of truth-tellers easier to establish. I will also assume that all sentences are interpreted sentences (the language is fixed) and, except otherwise noted (in section 4.3), that the logic is classical.

Truth-Tellers are sentences which say that they are true. This includes not only those sentences which say *that* they are themselves true (what we might call *de dicto* truth-tellers) but also those sentences which merely say *of* themselves that they are true (*de re* truth-tellers). It is not trivial to give a useful semantic characterization of this whole category. Luckily, it is easy to characterize *de dicto* truth-tellers, and being a *de dicto* truth-teller is a sufficient condition for being a truth-teller *tout court*. Let us say that an expression '$X(s)$', which refers to s, is *transparent* if one cannot understand it without understanding that it refers to s. A *de dicto* truth-teller s says exactly that $X(s)$ is true, where '$X(s)$' refers rigidly and transparently to s. We have accordingly the following characterization:

De dicto truth-tellers. A sentence s is a *de dicto* truth-teller iff it is strictly and analytically equivalent to a sentence of the form 'X(s) is true', where 'X(s)' refers rigidly and transparently to s. *De dicto* truth-tellers are truth-tellers.

In order to establish that various truth-tellers must have different truth values I will construct various sentences and show that although (i) they are *de dicto* truth-tellers, (ii) they must nevertheless differ in truth-value.

Very roughly, the sentences in question will be of the form 'the grass is red and this very sentence is true' and 'the grass is green or this very sentence is true', and the argument to the effect that such sentences are truth-tellers will rely on the claim that instances of the equivalence schema (ES) involving those sentences express strict and analytic equivalences. Let us proceed slowly now.

In its unrestricted form, the equivalence schema says that that every instance of the following pair of conditionals is true:

$$\begin{cases} Y(q) \;\; \text{is true} \;\; \rightarrow \;\; q \qquad \text{(T-out)} \\ Y(q) \;\; \text{is true} \;\; \leftarrow \;\; q \qquad \text{(T-in)} \end{cases}$$

where '$\rightarrow$' stands for material implication and instances of 'q' are replaced with a sentence, and instances of 'Y(q)' are replaced by an expression type that refers rigidly and transparently to q.

If, as some redundancy theorists have claimed, every instance of the equivalence schema expresses a strict and analytic equivalence, then both hands of the instances of this schema say the same thing and any sentence to which the equivalence schema applies is, by definition, a *de dicto* truth-teller: it says exactly that it is true. This would make every sentence a truth-teller. Fortunately, it can easily be argued that even if all instances of (ES) express strict equivalences and are *analytically true*, they do not all express *analytic equivalences*. For example

- 'the grass is green' is true $\leftrightarrow$ the grass is green

is analytically true as anyone who understands it should assent to it. But the left hand side of the biconditional, and not its right hand side, involves the concept of truth and the concept of sentence. One can accordingly claim that both hand sides do not express the same content and that the biconditional does not express an analytic equivalence.[2] Similarly, if

- ' 'the grass is green' is true' is true $\leftrightarrow$ 'the grass is green' is true

expressed a strict and analytic equivalence, ' 'the grass is green' is true' would be a truth-teller. It can however be claimed, again, that understanding the left hand side of the biconditional, but not understanding the right hand side, requires the mastery of the concept of second-order truth (truth applied to a sentence asserting truth). That would prevent the biconditional from expressing an analytic equivalence.

Consider however sentences of the form:

$\mathsf{Ttler}_{\wedge,\mathsf{red},X}$ the grass is red and X($\text{Ttler}_{\wedge,\text{red},X}$) is true

where 'X($\text{Ttler}_{\wedge,\text{red},X}$)' stands for an expression type that refers rigidly and transparently to $\text{Ttler}_{\wedge,\text{red},X}$. We could for example choose X as a subscripted demonstrative which refers to the first sentence in which it is tokened (X=D):

[2] Developmental psychology has provided reliable evidence that children under the age of three can believe that the grass is green and yet fail to master the semantic concepts in general and the concepts of truth and truth-bearers in particular (Mascaro and Sperber, 2009). If one grants that they are nevertheless rational, this implies that (ES) does not in general express an analytic equivalence.

$\mathsf{Ttler}_{\wedge,\mathsf{red},D}$ (the grass is red) and this sentence$_{\wedge,\text{red},D}$ is true[3]

We can show that thanks to self-reference, the preceding objection cannot apply to the following instance of the equivalence schema:

- X(Ttler$_{\wedge,\text{red},X}$) is true $\leftrightarrow$ the grass is red and X(Ttler$_{\wedge,\text{red},X}$) is true

As ‘X(Ttler$_{\wedge,\text{red},X}$)’ refers transparently to the right hand side of the biconditional, one cannot understand the left hand side without understanding the right hand side. As the right hand side involves all the concepts involved in the left hand side, one cannot understand the former without understanding the latter. So the biconditional is not only analytically true here, it actually expresses an analytic equivalence. As it also expresses a strict equivalence, this means that Ttler$_{\wedge,\text{red},X}$ says the same thing as the left hand side of the biconditional, namely that it is itself true. In other words, Ttler$_{\wedge,\text{red},X}$ will be a *de dicto* truth-teller. It will be a *de dicto* truth-teller which analytically implies that the grass is red and which is false.

In the same way, a sentence of the following form will be a *de dicto* truth-teller:

$\mathsf{Ttler}_{\vee,\mathsf{green},X}$ the grass is green or X(Ttler$_{\vee,\text{green},X}$) is true

(where ‘X(Ttler$_{\vee,\text{green},X}$)’ stands for an expression type that refers transparently to Ttler$_{\vee,\text{green},X}$). It will however be analytically implied by ‘the grass is green’ and it will be true.

Ttler$_{\vee,\text{green},X}$ and Ttler$_{\wedge,\text{red},X}$ will thus be *de dicto* truth-tellers with different truth-values, which establishes *Semantic Diversity*.

This result generalizes to sentences of the form:

$\mathsf{Ttler}_{*,\text{‘p’},X}$ p * X(Ttler$_{*,\text{‘p’},X}$) is true

[3] If you doubt that there are demonstratives which behave that way, you can use, instead, other diagonalization devices. Following Smullyan (1984), we can for example call *translation* of a sentence p the result of substituting the name ‘p’ of p for every occurrence of ‘this sentence’ in p. The translation of ‘this sentence is true’ is, for example, ‘ ‘this sentence is true’ is true’ (notice that it is not an indexical sentence for the expression ‘this’ is only mentioned here). Instead of taking X=D, you can take X=S, where

- S(Ttler$_{\wedge,\text{red},S}$)= ‘the translation of ‘the grass is red and the translation of this sentence is true’ ’

because whoever understands S(Ttler$_{\wedge,\text{red},S}$) should know that it refers to ‘the grass is red and S(Ttler$_{\wedge,\text{red},S}$) is true’=Ttler$_{\wedge,\text{red},S}$.

where instances of 'p' are substituted by a sentence, instances '*' are substituted by either 'and' or 'or' and where 'X(Ttler$_{*,\text{'p'},X}$)' is substituted by an expression type that refers transparently and rigidly to Ttler$_{*,\text{'p'},X}$. Those sentences will be *de dicto* truth-tellers because the following instances (ES$_{*,\text{'p'},X}$) of (ES) express strict and analytic equivalences:

$$\begin{cases} \text{X(Ttler}_{*,\text{'p'},X}) \text{ is true} \rightarrow \text{p * X(Ttler}_{*,\text{'p'},X}) \text{ is true} & \text{(T-out)} \\ \text{X(Ttler}_{*,\text{'p'},X}) \text{ is true} \leftarrow \text{p * X(Ttler}_{*,\text{'p'},X}) \text{is true} & \text{(T-in)} \end{cases}$$

Interestingly, we can also produce true and false truth-tellers using infinite conjunctions instead of explicit self-reference. Consider

Ttler$^{\omega}_{\wedge,\text{'p'}}$ p and 'p' is true and ' 'p' is true' is true and ' 'p' is true' is true' is true, and...

where infinite conjunctions of the form 'q_1 and q_2 and q_3...' are naturally interpreted as being true iff all their conjuncts q_i are true. Thanks to infinity (rather than self-reference), both hands of the following instances of (ES) involve the same concepts, and those instances should accordingly express analytic equivalences:

$$\begin{cases} \text{'p and 'p' is true and...' is true} \rightarrow \text{p and 'p' is true and...} & \text{(T-out)} \\ \text{'p and 'p' is true and...' is true} \leftarrow \text{p and 'p' is true and...} & \text{(T-in)} \end{cases}$$

Because of this analytic equivalence Ttler$^{\omega}_{\wedge,\text{'p'}}$ says that 'p and 'p' is true and...' is true. But 'p and 'p' is true and...' refers transparently to Ttler$^{\omega}_{\wedge,\text{'p'}}$, so Ttler$^{\omega}_{\wedge,\text{'p'}}$ says that it is itself true. It is a (*de dicto*) truth-teller. Because it is a conjunction of sentences which have the same truth value as p, Ttler$^{\omega}_{\wedge,\text{'p'}}$ will however have the same truth-value as p. Interestingly, such infinite truth-tellers are, unlike their finite cousins, in an intuitive sense *grounded* (they are grounded on 'p'). They shall prove quite useful in the next sections.[4]

[4] The fact that we can use infinity instead of explicit self-reference should not come as a surprise. There is a form of parenthood between self-reference and infinity which comes from the connection between being the value of an infinite number of iterations of a function and being a fixed point of that function.

Let f be a function which distributes over $+$. And let g be a function such that for all y on which it is defined $g(y) = x + f(y)$. Let g^n be the nth iteration of g. By distributivity $g^n(x) = x + \sum_{i=1}^{n} f^i(x)$. Furthermore, as $g^{n+1}(x) = g(g^n(x))$, in many cases, if we can make sense of $g^{\omega}(x)$, we will have $g^{\omega}(x) = g(g^{\omega}(x))$ so $g^{\omega}(x)$ will be a fixed point of g and $x + f(g^{\omega}(x)) = g^{\omega}(x)$.

2. *Semantic Sameness*

Even though different sentences which say of themselves that they are true have different truth-values, there is a good argument to the effect that they should all have the same truth-value. This argument relies on the fact that the property which those sentences self-ascribe, namely truth, is a semantic property, and it actually shows that all sentences saying that they have a given semantic property must have the same truth-value.

Let s and t be sentences, P a property of sentences, and let us suppose that t says of s that it is P.

If P is a morphological property like containing more than five words, or being written in black, the truth value of t will depend on the morphological features of s. If however P is a semantic property, the truth value of t will only depend on the content s:

Semantic Property. A property P is a semantic property iff the following equivalent conditions hold:

- if u and v are content-bearers with the same content, then sentences saying of u that it is P and sentences saying of v is P will have the same truth-value.
- for any content α, the truth-value of sentences saying of a content-bearer u, whose content is α, that it is P depends functionally on α: there is a function I_P which associates to any content α, the truth-value of those sentences which say of a content-bearer u, whose content is α, that it is P.

Truth and falsity are semantic properties in that sense: two sentences having the same content will have the same truth-conditions and they will accordingly have the same truth-value. Notice however that 'meaning that the earth is flat' or 'meaning that the grass is green' are also semantic properties in that sense.

This is the case for example if we can make sense of $g^\omega(x)$ by using the notion of convergence and if f is continuous (take for example $x = \frac{1}{2}$ and $f(y) = \frac{1}{2}y$, then we can define $g^\omega(x)$ as the limit of a series: $g^\omega(x) = \sum_{i=1}^{\infty}(\frac{1}{2})^i = g(g^\omega(x)) = \frac{1}{2} + \frac{1}{2}g^\omega(x) = 1$). This will also be the case if we resort to actual infinity and define $g^\omega(x)$ as an actually infinite sum. For example, if as in the main text, $x = p$, $f(y) = Tr(y)$, and we interpret '$+$' as a conjunction sign, we can make sense of $g^\omega(x)$ as an actually infinite conjunction (with natural notations: $\bigwedge_{0 \leq i < \omega 1} Tr^i(\text{`..`}p\text{'} \ldots ')$) in the simple Hilbert type extension of propositional logic $L_{\omega 1, \omega}$ (Hinman, 2005, 293–308).

Notice also that the claim that truth is a semantic property is strictly weaker than (ES): if p and q have the same content, $p \leftrightarrow q$; as, by (ES), $p \leftrightarrow Tr(p)$ and $q \leftrightarrow Tr(q)$ we will have $Tr(p) \leftrightarrow Tr(q)$.

Let us suppose, now, that P is indeed a semantic property. Let us suppose, moreover that s is not context-sensitive so that it has a constant, determinate, content $[s]$. Then t also has a constant truth-value $[[t]]$ (I use simple square brackets for content, double square brackets for truth-value) and its truth value is, by *Semantic Property*, determined by the fact that

$$[[t]] = I_P([s])$$

The same holds if t is self-referential so that $s = t$. In that case the truth-value $[[t]]$ of t will be *determined* by the fact that

$$[[t]] = I_P([t])$$

It will be, in other words, implicitly defined by this condition. The fact that this condition is circular is not problematic: it should no more worry us than the fact that a real number x is determined by the circular condition $x = 2x + 1$. It means however that the equation $[[t]] = I_P([t])$ should have a unique solution in the following sense: every sentence s whose meaning $[s]$ satisfies $[[s]] = I_P([s])$ should have the same truth-value as t.

Yet, if t' is another (use-insensitive) sentence saying of itself that it is P, t' will also satisfy $[[t']] = I_P([t'])$ (the same reasoning that we applied to t applies to t'). This means that all the sentences saying of themselves that they are P, where P, is a semantic property, must have the same truth-value. With P=truth, we get that all truth-tellers must have the same truth-value.

3. *The Truth-Tellers paradox*

Although all truth-tellers say of themselves that they are true, and should accordingly have the same truth-value, they have different truth-values. This is the Truth-Tellers paradox.

This paradox actually conceals two different problems. The first one is the most obvious: both *Semantic Sameness* and *Semantic Diversity* of truth-tellers are, as we saw, quite plausible but they entail that any truth-teller is both true and false and they are inconsistent. Call that the *extensional problem*.

One might suspect that this conundrum is rooted in a more fundamental problem which already arises with *individual de dicto* truth-tellers like $\text{Ttler}_{\wedge,\text{red},D}$. As $\text{Ttler}_{\wedge,\text{red},D}$ is a *de dicto* truth-teller, we said, it says exactly that it is itself true. But it seems that it also says something else, namely that the grass is red. It might be argued, however, that a sentence cannot say exactly one thing and also say something else. Call this the *problem of uniformity*.

The Truth-Tellers paradox resembles the classical semantic paradoxes like the Liar. In both cases, a form of self-reference allows us to derive contradictions from otherwise plausible semantic and logical principles. The main difference is that in the case of the Truth-Tellers, in order to derive a contradiction, we need to accept a particularly strong form of the equivalence schema (ES), whose application to the self-referential or infinite sentences under consideration yields a strict and analytic, rather than a merely material, equivalence. Given this additional assumption, one might quibble over whether the Truth-Tellers paradox really deserves to be called a paradox. Someone who considers, for example, that this additional assumptions is not as plausible as the semantic principles on which the Liar paradox rely might want to reserve the pedigree "paradox" for the latter and say that truth-tellers rather constitute a "puzzle". The important point, however, is that this assumption is plausible enough. We shall see further that it is not to blame for the Truth-Tellers paradox.

The Truth-Tellers paradox also differs from the Liar paradox in that it can be formulated without using sentences which are intuitively ungrounded sentences. $\text{Ttler}^{\omega}_{\wedge,\text{green}}$ and $\text{Ttler}^{\omega}_{\wedge,\text{red}}$ are indeed intuitively grounded, yet (i) they self-ascribe a semantic property, (ii) they differ in truth value (iii) and they must have the same truth-value (by *Semantic Sameness*). This is important, for ungroundedness is almost universally considered as our best characterization of what goes wrong with the sentences involved in the semantic pathologies. $\text{Ttler}^{\omega}_{\wedge,\text{green}}$ and $\text{Ttler}^{\omega}_{\wedge,\text{red}}$ show that self-ascription of a semantic property and ungroundedness are not equivalent and that our diagnosis of the pathologies should actually focus on the former feature rather than on the latter (see also the "Grounded Liar" in fn. 10).

Finally, let me stress that even though it is responsible for the Truth-Tellers paradox, the tension between *Semantic Sameness* and *Semantic Diversity* has something intuitively appealing, something which should invite us not to reject any of those two conflicting assumptions too quickly. It captures well, in particular, what we might call the 'semantic phenomenology' of truth-tellers. What is intuitively troubling with truth-tellers is not so much that we don't know, when we first encounter them, whether they are true or false. It is that their saying that they are true does not seem to determine what they say and whether they are true, even though nothing else but that should determine whether they are true. Another way to make the same point. Ttler is a sentence whose meaning and truth-value is defined in a circular way, by something like an equation. We seem to understand the sentence, as it is defined: it says of itself that it is true. This seems to imply that this equation constitutes a genuine implicit definition of the content of Ttler. Assuming that the content of Ttler should determine its truth-value, it follows that Ttler should have a definite truth-value. Yet the truth-value of Ttler seems totally unconstrained.

4. *How not to solve this paradox?*

I do not intend to solve the Truth-Tellers paradox. I intend, however, to exclude some diagnoses, as well as their corresponding treatments. I am thinking, in particular, of those selective diagnoses and treatments that would target the specific assumption on which the Truth-Tellers paradox, but not the Liar paradox, relies on.

4.1. *A weaker equivalence schema*

The Truth-Tellers paradox depends on the assumption that if the instances ($ES_{*,\text{‘p’},X}$) of (ES) are true at all, then they express strict and analytic equivalences.

There is a classical objection against the assumption that true instances of the equivalence schema have such a strong force. This objection claims that the equivalence schema cannot express a strict or analytic equivalence because it is not necessarily true nor analytically true (biconditionals which express analytic equivalences are analytically true, even if, as we saw, the converse does not hold). If ‘Y(the grass is red)’ is substituted by an expression which refers to ‘the grass is red’, the objection goes, it is true, but *not* analytically true that

- Y(the grass is red) is true iff the grass is red

for a rational subject could believe that Y(the grass is red) is true but that the grass is green, not red. This could happen, for example, if he does not know that Y(the grass is red) refers to ‘the grass is red’ (say he just knows that it refers to the last sentence his guru said before dying) or if he doesn’t speak English (substitute ‘Y(the grass is red)’ with ‘the grass is red’ and suppose that he is a monolingual Dutch speaker who heard his English guru say ‘the grass is red’). A parallel objection threatens the claim that the equivalence is strict.

This objection, or couple of objections, has a long history (Halbach (2001) traces it back to Lewy and Church). The clearest answer I know comes from Raatikainen (2003). This objection depends, he notices, on one of the two following assumptions:

- *Uninterpreted sentences.* The equivalence schema deals with uninterpreted sentences, or equivalently, it is *not* relativized to a given language.
- *Opaque / non rigid reference.* The expression 'Y(q)' used to refer to the sentence q to which truth is ascribed in the equivalence-schema is not transparent or not rigid (Raatikainen (2003) concentrates on the particular case of descriptive-structural names but his point extends to all transparent and rigid names).

It is true that the relation between the symbols we use and their meaning is not analytic nor necessary, but once this relation is fixed by a language, and provided that the expression 'Y(q)' is rigid and transparent, true instances of the equivalence schema should be analytically and necessarily true. As our version (ES) of the equivalence schema was formulated so as to explicitly reject both problematic assumptions, it is not threatened by this objection.

4.2. *Semantic pluralism*

We saw that $\text{Ttler}_{\wedge,\text{red},X}$ is already problematic because it says exactly one thing (that it is itself true) and it also says something else (that the grass is red). This "problem of uniformity" might suggest that the Truth-Tellers paradox compels us to withdraw the assumption that every sentence, or more precisely every sentence-use, says exactly one thing. Assuming that meaning is individuated by strict and analytic equivalences, this assumption, which has been called the *principle of uniformity*[5] can be given the following formulation:

> Principle of Uniformity. All things said by a sentence are strictly and analytically equivalent.

Interestingly, it has been argued that the equivalence schema, or more precisely T-in, tacitly presupposes this principle (Williamson and Anjelovic, 2000; Dutilh Novaes, 2008): T-in implies that it just needs to be the case that p in order for a use of 'p' to be true, which suggests that such a use of 'p' says only p, rather than p and a complement c. Read (2008b) has further claimed that one could solve the Liar paradox by rejecting T-in in the name of semantic pluralism. One might be tempted to conclude that the uniformity assumption is responsible not only for the uniformity problem but for the extensional problem as well and for the Truth-Tellers paradox in general.

[5] I take the name from Williamson and Anjelovic (2000) but they seem to have in mind a weaker, extensional principle, to the effect that the things said by a sentence use all have the same *truth-value*.

Assuming that all truth-tellers say the same thing we could conclude that they all say both true and false things, and that they are accordingly all false.

Such a pluralistic solution would be misguided. First, the truth-tellers would *still* be paradoxical if we rejected the principle of uniformity and adopted Read’s semantic pluralism instead. Second, and appearances notwithstanding, the claim to the effect that $\text{Ttler}_{\wedge,f,X}$ says exactly that it is itself true is consistent with the claim to the effect that it (also) says that the grass is red. So the problem of uniformity is only shallow and does not compel us to revise the principle of uniformity. I tackle those two points in turn.

We can use ∧-out instead of T-in to establish that $(\text{ES}_{\wedge,p,X})$ expresses a strict and analytic equivalence. We indeed have:

$$\begin{cases} \text{X}_{\text{‘p’},\wedge} \text{ is true} \rightarrow \text{p} \wedge \text{X}_{\text{‘p’},\wedge} \text{ is true} & \text{(T-out)} \\ \text{X}_{\text{‘p’},\wedge} \text{ is true} \leftarrow \text{p} \wedge \text{X}_{\text{‘p’},\wedge} \text{is true} & (\wedge\text{-out}) \end{cases}$$

So even if we renounced T-in and only kept T-out, as Read’s pluralism urged we should, we could still show that all substitution instances of $\text{Ttler}_{\wedge,\text{‘p’},X}$ (if not those of $\text{Ttler}_{\vee,\text{‘p’},X}$) are *de dicto* truth-tellers. This would allow us to construct a *false* truth-teller (take $\text{Ttler}_{\wedge,\text{red},X}$=‘the grass is red and X($\text{Ttler}_{\wedge,\text{red},X}$) is true’, for any acceptable X). We could however still show that there must be *true* truth-tellers as well. Consider the infinite conjunction:

$\mathsf{Ttler}^{\omega}_{\wedge,\mathsf{green}}$ The grass is green and ‘the grass is green’ is true and ‘ ‘the grass is green’ is true’ is true and. . .

Consider also the following sentence which attributes truth to $\mathsf{Ttler}^{\omega}_{\wedge,\mathsf{green}}$:

$\mathsf{Tr}(\mathsf{Ttler})^{\omega}_{\wedge,\mathsf{green}})$ ‘The grass is green and ‘the grass is green’ is true and ‘ ‘the grass is green’ is true’ is true and. . . ’ is true

By T-out, Tr($\text{Ttler}^{\omega}_{\wedge,\text{green}}$) strictly and analytically implies $\text{Ttler}^{\omega}_{\wedge,\text{green}}$. But we can show, *without using T-in* that the converse strict and analytic entailment holds as well, and accordingly that $\text{Ttler}^{\omega}_{\wedge,\text{green}}$ is a truth-teller. By ∧-out, $\text{Ttler}^{\omega}_{\wedge,\text{green}}$ indeed strictly and analytically entails

- ‘The grass is green’ is true and ‘ ‘the grass is green’ is true’ is true and ‘ ‘ ‘the grass is green’ is true’ is true’ is true. . .

which, as truth distributes over conjunction, strictly and analytically entails Tr($\text{Ttler}^{\omega}_{\wedge,\text{green}}$). So we do not need T-in to show that the following biconditional expresses a strict and analytic equivalence

- $\text{Ttler}^{\omega}_{\wedge,\text{green}} \leftrightarrow \text{Tr}(\text{Ttler}^{\omega}_{\wedge,\text{green}})$

and that $\text{Ttler}^{\omega}_{\wedge,\text{green}}$ is accordingly a *de dicto* truth-teller.

Now assuming that ‘the grass is green’ only says true things, and that if a sentence only says true things, the sentences that assert its truth only says true things, all conjuncts of $\text{Ttler}^{\omega}_{\wedge,\text{green}}$ will be true. Accordingly, the conjunction $\text{Ttler}^{\omega}_{\wedge,\text{green}}$ will be true. It will be a true truth-teller.[6] This shows that even if we accept Read’s pluralism, we can still prove that there are both true and false truth-tellers, which reinstates the original paradox.[7]

Finally, the problem of uniformity can be explained away without rejecting the principle of uniformity. Notice first that even an anti-pluralist can accommodate a notion of *partial* meaning or saying. The sentence ‘the snow is white and the grass is green’ for example partially says that the snow is white, partially says that the grass is green, but it exactly says that the snow is white and the grass is green. The classical equivalence schema can remain in place, provided that the following is acknowledged: inasmuch as it implicitly concerns meaning, it concerns exact, not partial, meaning: a sentence is true iff what it *exactly* says is true. Second, by saying exactly something about an object, a sentence can partially say something *about another object*. This happens with sentences using the truth predicate: if you say that the grass is red, by saying that what you just said is true I can (partially) say the same thing as you. It is certainly a strange feature of truth, which can turn talk about a sentence into talk about the grass, but once T-out is taken to express a strict and analytic implication, it is a feature that must be granted.[8] Now I take it that in the very same way, $\text{Ttler}_{\wedge,\text{red},X}$ exactly says that it is true, but by saying this, it also *partially* says something else, something about the grass.

[6] Foes of infinite sentences like $\text{Ttler}^{\omega}_{\wedge,\text{green}}$ can obtain the very same conclusion using the Yabloesque $\text{Ttler}^{\omega'}_{\wedge,\text{green}}$ defined in section 4.3, p. 385.

[7] A slight loophole. I have just shown that Read’s semantic pluralism does not thwart *Semantic Diversity*, it might be wondered whether it does not threaten the other claim we need to derive the paradox, namely *Semantic Sameness*. It is true that one of the arguments I provided in favor of the claim that truth is a semantic property, on which *Semantic Sameness* hinges, relied on (ES) and thus on Tr-in. I do not see, however, why semantic pluralism would be inconsistant with the claim that if two sentences have the same content (say the same things) then sentences saying that they are true have the same truth-value. I thank an anonymous referee of this journal for pressing me on that point.

[8] It is one of the features that have partisans of the redundancy theory say that truth is not a predicate.

4.3. *Truth-Value Gaps*

At that point one might be tempted to acknowledge that truth-tellers really are paradoxical, and that the Truth-Tellers paradox does not hinge on the assumptions that distinguish it from the Liar paradox. One might accordingly be tempted to apply some influential solutions of the latter to the former.

It might be suggested, for example, that truth-tellers are gappy, that is, neither true nor false. There are two ways to interpret that claim. It could mean either that some truth-tellers have a third, non classical truth-value, which is what partisans of paraconsistant solutions contend. Alternatively, it could mean that they have no truth-value at all because they do not make a statement. This is the so-called *cassationist* strategy (Goldstein, 2008).

This ‘gappist’ approach does not threaten the argument for *Semantic Sameness*.[9] Yet, under both of its interpretations, it undermines the argument for *Semantic Diversity*. If X(Ttler$_{*,\text{‘p’},X}$) is gappy, ‘p* X(Ttler$'_{*,\text{‘p’},X}$) is true’ might indeed be gappy as well, instead of having the same truth-value as p.

Like the gappist approaches to the Liar, the gappist approach to the Truth-Tellers paradox is exposed to a form of revenge problem. One can indeed argue that sentences of the form

$\mathsf{Ttler}'_{*,\text{‘p’},X}$ p * X(Ttler$'_{*,\text{‘p’},X}$) is not false

cannot be gappy if ‘p’ is not gappy. I have provided a detailed justification of that claim elsewhere (Billon, 2012).

I believe that there are contextualist ways to circumvent such revenge problems (one could for example generalize the strategies developed by Burge (1979) or Goldstein (2000) to deal with revenge problems affecting gappist solutions to the Liar). There is however an additional, and I believe decisive, objection against gappist approach to the Truth-Tellers paradox. It is that even in a gappist, contextualist setting, we can easily vindicate the argument for *Semantic Diversity* if we appeal to truth-tellers which involve infinite conjunctions instead of explicit self-reference. We saw indeed that Ttler$^{\omega}_{\wedge,\text{green}}$ (=‘the grass is green and ‘the grass is green’ is true and ‘ ‘the grass is green’ is true’ is true, etc.’) and Ttler$^{\omega}_{\wedge,\text{red}}$ are truth-tellers (section 1, p. 376). Unlike the ones which are explicitly self-referential, however, those truth-tellers cannot be gappy. As all its conjuncts are true, Ttler$^{\omega}_{\wedge,\text{green}}$ is true. As all its conjuncts are false, Ttler$^{\omega}_{\wedge,\text{red}}$ is false. They cannot hence have a

[9] If some truth-tellers were gappy because they do not make a statement, however, we would have to modify slightly the formulation of this argument and say that sentences which *purport* to say (rather than say) of themselves that they are true must all have the same *semantic status* (rather than truth-value), where that includes making no statement and having no truth-value.

third, gappy, truth-value.[10] Furthermore, the fact that they have a classical truth-value is a good reason to claim that they *do* make a stement.

It has been noted by Stephen Yablo that we can use infinity to mimic some semantic phenomena that long seemed the province of explicit self-reference (Yablo, 1993b). The infinite sentences introduced here are an illustration of that claim. They differ importantly, however, from the device used by Yablo. A Yablo style truth-teller would for example take the following form:

$\mathsf{Ttler}^{\omega'}_{\wedge,\text{`p'}}$	p and (2) is true
(2)	p and (3) is true
(3)	p and (4) is true
(...)	...

Goldstein (2008) who argues that problematic self-referential sentences like the Liar or Ttler do not make a statement notes that his case generalizes to this kind of Yabloesque infinite sentence sequences. I concur. The argument to the effect that $\text{Ttler}^{\omega'}_{\wedge,\text{`p'}}$ is true if 'p' is true would indeed fail in a non bivalent setting, as one could argue that for some i, (i) is gappy, which would make $\text{Ttler}^{\omega'}_{\wedge,\text{`p'}}$ gappy as well. His case does not generalize to the infinite sentences we considered here, though. This difference stems from the fact that $\text{Ttler}^{\omega}_{\wedge,\text{`p'}}$, unlike the Yabloesque $\text{Ttler}^{\omega'}_{\wedge,\text{`p'}}$, is, in an intuitive sense, grounded. It is grounded on 'p'.

[10] Notice that it is similarly possible to use infinite sentences instead of explicit self-reference to construct paradoxical liar-like sentences. Consider for example (for readability's sake I will use the truth predicate as a prefix and write 'not true 'p' ' for ' 'p ' is not true')

$\mathsf{Liar}^{\omega}_{\wedge,\text{`p'}}$	p and not true 'p and not true 'p and not true 'p and ... ' '... '

$\text{Liar}^{\omega}_{\wedge,\text{`p'}}$ is a conjunction whose second term negates $\text{Liar}^{\omega}_{\wedge,\text{`p'}}$ itself, thanks to infinity. So if it is true, it is not true. Supposing that 'p' is true we have conversely that if $\text{Liar}^{\omega}_{\wedge,\text{`p'}}$ is not true, it is true. So $\text{Liar}^{\omega}_{\wedge,\text{`p'}}$ is paradoxical. (Interestingly, as we saw, it is also, at least in an intuitive sense, grounded.)

However such a construction is not as easy to exploit against the gappist approaches to the Liar as is $\text{Ttler}^{\omega}_{\wedge,\text{`p'}}$ with respect to the Truth-Tellers paradox. First, extensions of propositional logic which can accomodate such sentences as $\text{Ttler}^{\omega}_{\wedge,\text{`p'}}$, which are just formed by conjunction over an infinity of formulas, are much less complex than the ones which could accomodate $\text{Liar}^{\omega}_{\wedge,\text{`p'}}$. Second, the truth-value of the partial sums of the form $\text{Liar}^{1}_{\wedge,\text{`p'}}$='p', $\text{Liar}^{2}_{\wedge,\text{`p'}}$='p and 'p' is not true, $\text{Liar}^{3}_{\wedge,\text{`p'}}$='p and 'p and 'p' is not true' is not true', $\text{Liar}^{4}_{\wedge,\text{`p'}}$, etc. oscillates between truth and falsity so it is not intuitively obvious, to say the least, that the 'limit sentence' $\text{Liar}^{\omega}_{\wedge,\text{`p'}}$ should be either true or false.

4.4. *Use sensitivity*

Another popular strategy put forward to deal with the Liar paradox relies on the claim that different sentence tokens of the same type or, more broadly, different uses of the same sentence type (a sentence token can be used in different contexts) might differ in truth-value, and that the Liar, despite appearances, displays such a use-sensitivity (Burge, 1979; Gaifman, 1992).

This strategy, it seems, even more straightforwardly applies to the Truth-Tellers paradox than to the Liar paradox. The argument for *Semantic Sameness* indeed relied on the assumption that sentences which say of themselves that they have a given semantic property, and in particular sentences which say of themselves that they are true, are not context sensitive. We needed this assumption to assert that if s is such a sentence then its (morphological) definition determines its truth-value. Against our argument it might be claimed, that if s says of itself that it is true, then s is use-sensitive: its truth-value depends on the context in which it is used.

I would like to suggest, however, that the use-sensitivity strategy is ineffective to solve the Truth-Tellers paradox. We can indeed rephrase the argument for *Semantic Sameness* and *Semantic Diversity* in a way that is acceptable even to someone who suspects that truth-tellers are use sensitive. Even he or she should indeed accept the following: If a given sentence has a truth value *in a given context of use*, this truth-value should be determined by the definition of this sentence *and this context of use*. Accordingly, if C is the following context of use, and if we define the sentence s_c as

(s_c) s_c, as used in C, is P C

then, if the self-referential use of s_c in C has a truth-value, the latter is determined by C and by the definition of s_c.

This is enough to argue that all self-referential sentence uses which self-ascribe a given semantic property must have the same truth-value. The argument for this sentence-use version of *Semantic Sameness* exactly parallels the one for the original *Semantic Sameness*: we just need to substitute sentences with sentence uses in the former. However, just like we argued that different sentences which say that they are true must have different truth-values (*Semantic Diversity*), we can argue that different sentence *uses* which say that they are true must have different truth-values.

The partisan of the use-sensitivity objection is thus confronted with what we might call a *revenge problem* that exactly mimics the original one.[11] He has displaced the paradox but he has not solved it.[12]

5. *Conclusion*

Truth-Tellers are not usually considered as genuinely paradoxical. Like the liars, they would exhibit some kind of semantic deficiency, but whereas the liars are paradoxical because they only admit inconsistent truth-value assignments, truth-tellers would admit arbitrary truth-value assignments and they would be merely pathological. The present argument blurs a little this distinction between paradoxes and mere pathologies.

Given that the Truth-Tellers paradox can be formulated without relying on ungrounded sentences, it also severs the link, usually assumed, between the paradoxes or pathologies of self-reference and ungroundedness.

Furthermore, if we grant that the Truth-Tellers paradox is of the same family as the Liar and that similar paradoxes should receive similar solutions, then solutions to the Liar should generalize to the Truth-Tellers paradox. As we saw, this winnows the acceptable approaches to the Liar.[13]

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[11] This revenge problem parallels the one threatening use-sensitivity solutions to the Liar (Gauker, 2006).

[12] This is not to say that one cannot solve the Truth-Tellers paradox by appealing to *context* sensitivity. There might indeed be sentences which are context sensitive not because they are *use*-sensitive (because their truth value depends on the context in which they are used) but because they are *assessment*-sensitive (because their truth-value depends on the context in which they are assessed) (MacFarlane, 2005). I have outlined such an assessment sensitivity solution to related paradoxes of self-reference elsewhere Billon (2012).

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