

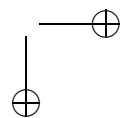
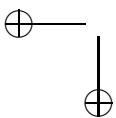
## THE LANGUAGE(S) OF MATHEMATICS

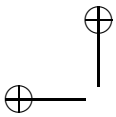
GUEST EDITOR: CATARINA DUTILH NOVAES

In September 2009, a workshop on practice-based philosophy of logic and mathematics took place in Amsterdam; at the talks and during the discussions, participants debated on the possible philosophical insights to be gained from a focus on the actual practices of logicians and mathematicians. One of the aspects of such practices which received quite some attention at the workshop were the languages in which, or through which, logicians and mathematicians conduct their investigations. Two of the speakers who focused on the languages of logic and mathematics were Wilfrid Hodges and Danielle Macbeth, and their thoughts on the topic are now available here. My own presentation at the workshop was on formal languages as used by logicians, but in the meantime I developed an interest in the role of external representational systems for the development of mathematical cognition. Thus, the three papers to follow offer different perspectives on roughly the same issues: what is the role of different ‘languages’ in the practices of mathematics? What are the connections between mathematical thought and public vehicles such as written and spoken languages? What are the connections between mathematics as expressed in vernacular languages and uses of specifically designed mathematical notations? These are all important questions within a practice-based approach to mathematics.

Wilfrid Hodges examines uses of modal vocabulary in mathematics textbooks; modal terms are widely used in such contexts, and yet, in principle at least, mathematics does not deal with modalities or modal phenomena. Why are these terms being so extensively used in mathematical contexts then? In what way, if any, do they impact a student’s learning process? Such observations touch upon the much wider issue of the connections between ‘vernacular mathematics’ and mathematical notation, formalization of mathematics, and several others.

Danielle Macbeth distinguishes three basic positions that can be (and have been) held on the relations between mathematics and external symbolic systems (both special mathematical notations and vernacular languages): they are constitutive of mathematical thought; they merely report independent





processes of mathematical thought; mathematics is done primarily in vernacular languages, and notations are merely convenient short-hands. She argues that the third one in any case is deeply misguided, and that Frege’s idea of a *Begriffsschrift* falls essentially within the first position.

My contribution departs from Macbeth’s taxonomy to explore the impact of external symbols on the development of mathematical cognition, as investigated empirically within cognitive science and related fields. I argue that, in the majority of cases, external symbols are necessary for the development of mathematical knowledge, but that manipulation of external symbols does not exhaust the nature of mathematical insight and mathematical cognition.

Taken together, these contributions illustrate that a focus on aspects of the practices of logicians and mathematicians, in this case the languages used by them, can be a fruitful approach to perennial questions in the philosophy of logic and mathematics.

