NOTE ON THE INTERPRETATION OF SO.5

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Richard Routley in [5] claims that Lemmon's SO.5 (v. [3], [4] and [2, pp. 256 f, 286-288]) is incomplete under the intended interpretation of L ($\Box$) as 'it is tautologous (by truth table) that'. It is the purpose of this short note to suggest that it is not so far-fetched as Routley seems to think to suppose that SO.5 does capture this interpretation adequately and that the formal semantics of [1] reflects it. I should first emphasize though, that nothing I say need detract from the very valuable work Routley has done in [5] and [6] in elucidating the various senses L might have, and I am indebted to his articles for forcing a clearer statement of what I, perhaps presumptuously, might call the 'orthodox' interpretation of SO.5.

Routley makes several objections to SO.5. One of these rests on a rather elementary confusion and can be got quickly out of the way. On p. 419 of [5] he says that $\sim Lp$ ought, on the orthodox interpretation, to be a theorem of SO.5 on the ground that 'it is not the case that p is tautologous' is true. This objection is a simple confusion between language and metalanguage. What is true is the metalinguistic statement that 'p' is not a tautology. The formula $\sim Lp$ is an open formula which means that p is not a tautology. This open formula will be false when the variable has as its value a proposition which is a tautology and true when it has one which is not. And since some propositions are tautologies then the schema $\sim Lp$ cannot be valid under the intended interpretation. What has misled Routley is the fact that 'is a tautology' *can* function as a metalinguistic property of formulae of some propositional or predicate logic. By analogy if we were to confuse 'is valid' with 'is necessarily true' we could move from ''p' is not valid' to $\sim Lp$ *in* any standard system of modal logic. In SO.5 Lp must mean 'it is tautologous that p' where this in turn means something like 'p (a proposition) has the form of a valid PC-schema'.

However this objection is not Routley's main one and in fact the account of validity which he offers as the 'correct' interpretation on pp. 419 f. is one which does not verify $\sim Lp$. An example of a non-theorem of SO.5 which does come out as valid by his semantics is $\sim LLp$ (p. 421) and intuitively expressed the reasons for this are as follows: Whatever formula we substitute for p in Lp we shall always have a formula of the form $L\alpha$ and no formula of the form $L\alpha$ is a truth-functional tautology and so $\sim LL\alpha$ is true no matter what formula α may be; i.e. $\sim LLp$ is valid in the sense that it remains true whatever proposition is substituted for p. Routley's semantics is in fact a formal setting out of this position.

$\sim LLp$ cannot be dealt with quite in the way we did with $\sim Lp$ for the claim in the present case is that Lp can never be a tautology no matter what proposition p may be. On the face of it this has a certain plausibility but again it is not clear that we may not be being hoodwinked by a language/metalanguage confusion; this time a rather more subtle one. It is clear that no *formula* of the form $L\alpha$ is a PC-tautology. What is not so clear is that no proposition of the form Lp has the form of a PC-tautology. A formula after all can have only one form (except in the rather trivial sense in which $\sim \alpha \vee \sim \beta$ can be said to have the form $\gamma \vee \sim \beta$ and the form $\sim \alpha \vee \gamma$, or $\gamma \vee \delta$ or even γ) but how many forms can a proposition have? Can it have more than one? And if so can the same proposition have the form Lp and also, say, the form $q \supset q$? I don't know the answer to this question. But even if I did I think I should still prefer a logic which did not presuppose the answer.

If we adopt Routley's semantics we seem obliged to deny that a proposition of the form Lp can have any other form; we seem that is obliged to assume that each proposition has its own logical form and that it has only one. Now it may be that this metaphysical assumption is a plausible one, though in the absence of an adequate account of the nature of propositions or criteria for propositional identity it's a little difficult to see what precisely it means, let alone how it could be defended. And certainly if one does accept it then Routley has given a logic for 'it is tautologous that' which fits it. ([1]) But in choosing such a logic we may not wish to commit ourselves to a specific view about propositions. We may want a logic which gives us those and only those laws which 'it is tautologous that' must satisfy whatever decision we come to about these very difficult metaphysical issues. And the SO.5-models of [1], in treating formulae of the form $L\alpha$ in non-normal worlds as if they were propositional variables, do precisely this.

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REFERENCES

[1] M. J. CRESSWELL, The completeness of SO.5, *Logique et Analyse* no. 34 (1966) pp. 263-266.

[2] G. E. HUGHES and M. J. CRESSWELL, *An Introduction to Modal Logic*, Methuen, London 1968.

[3] E. J. LEMMON, New foundations for Lewis modal systems. *The Journal of Symbolic Logic*, vol 22 (1957) pp. 176-186.

[4] E. J. LEMMON, Is there only one correct system of modal logic ? *Aristotelian Society Supplementary volume* 33 (1959) pp. 23-40.

[5] F. R. ROUTLEY, The decidability and semantical incompleteness of Lemmon's system SO.5, *Logique et Analyse* no 43 (1968) pp. 413-421.

[6] F. R. ROUTLEY, Decision procedures and semantics for **C1 E1** and SO.5°, *ibid* no. 44 (1968) pp. 468-471.

[1] It should be unnecessary to point out that SO.5 is perfectly compatible with the Routley view of proposition. All the theorems remain true under his interpretation and so there is no question of the orthodox view putting up a rival account. The orthodox view (quite properly in my opinion) refuses to put up any account at all.