TRUTH-FUNCTIONAL PERTURBATIONS

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The horseshoe, dot, wedge, and three-bar operators are all introduced and defined differently in truth-functional logic. Yet the following are tautologies:

$$(p \supset p) \equiv (p \equiv p),$$
$$(p \cdot p) \equiv (p \vee p).$$

One way out is to say that there is no contradiction between the definitions and the logical equivalences because the definitions involve nonreiterative forms, while the equivalences are reiterations. "$p \vee q$" is not said to be equivalent to "$p \cdot q$", nor "$p \supset q$" to "$p \equiv q$". That would be real trouble.

Nevertheless there is still an air of unsatisfactoriness, since it still looks as though the equivalences contradict the definitions. They seem to say that differently defined and introduced connectives are identical, that "$\supset$" and "$\vee$" are the same as "$\equiv$" and "." respectively.

It might be replied that reiterations form exceptions, that when involved, connectives which are differently introduced may form identities. But why should reiterations make such a big difference ? Why should they identify discernibles ? After all, when we define connectives we define *them,* not the variables flanking, or associate with, them.

And there is this too: while "$p \supset q$" and "$p \equiv q$" are contingencies, their corresponding reiterations ("$p \supset p$", "$p \equiv p$") are tautologies.

All these perturbations or seeming perturbations may be due to the truth-value orientation (definitions).

It is interesting to note that in antiquity some defined the conditional (and or implication ?) so that reiterative conditionals (and or implicatons ?) were excluded or counted as false. According to these logicians, Sextus Empiricus says, the statement " 'If it is day, it is day' and every duplicated conditional (*synèm-*

menon) will perhaps be false; for it is impossible for something to be contained in itself" (*Outlines of Pyrrhonism* 2.112). In other words, these unidentified logicians defined the relation between the antecedent and consequent of a conditional as one of containment: the consequent is *implicit* (*dynamei*) in the antecedent. An exemple might be: "If this is a good pencil, it writes". The notion involved here seems to be that of implication in the ordinary sense, when it connotes a necessary relation *and* when "implies" contrasts with "states" (i.e. "states explicitly").

But though this move may avoid the above seeming paradoxes within truth-functional logic (the post-Aristotelian logicians gravitated toward extensionalism), it creates another difficulty, since it in effect does away with the propositional and perhaps class laws of identity. To eliminate the form "$p \supset p$" is certainly at least as drastic as the elimination of "$p \vee \sim p$". The latter of course, as my assistant Cary Debessonet among others noticed, has its own difficulties insofar as it is interpreted to mean that the same proposition is (can be) true or false or *both*. Its replacement by strong disjunction won't save the boat, since the weak form can be validly obtained from the strong, as well as from "$p \supset p$" and "$(p \cdot \sim p)$", assuming Simp., M.I., deM, and D.N.

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