A NOTE ON UNIVERSALLY FREE FIRST ORDER QUANTIFICATION THEORY

A. P. RAO

According to Meyer and Lambert [4], i) existence assumptions with respect to individual constants, and ii) the assumption as to the non-emptiness of the domain of interpretations are "independent". Leonard [3] in his attempt to free classical first order quantification theory (FQT) from i) suggested that instead of accepting

(1) $Fa \rightarrow (\exists x)Fx$

as an axiom, as is done, usually, in formulating (FQT)

(1A) $(Fa \;\&\; E!a) \rightarrow (\exists x)Fx$

be accepted as an axiom; and similarly, instead of

(2) $(\forall x)Fx \rightarrow Fy$,

(2A) $((\forall x)Fx \;\&\; E!y) \rightarrow Fy$

be accepted. As, in his reformulation of (FQT), it is definitionally equivalent to

$$(\exists P)(Px \;\& \Diamond \sim Px),$$

$$E!x$$

is a second order modal predicate. In this note it is intended to show how the same thing can be accomplished without smuggling into (FQT) such a predicate. When (FQT) is reformulated as is suggested below, it will be free from not only i) but also ii). The suggested reformulation of (FQT) — which, as will be obvious, is an incorporation into the formal structure of (FQT) the Quinian pegasizing-strategy — results in what is called in the terminology of [4], universally free first order quantification theory (FFQT). This may be treated as a sufficient reason to believe that i) and ii) are not as independent as they are taken to be in [4].

To have (FFQT), augment the vocabulary of (FQT) by the following:

$A_1, A_2, \ldots, A_n$

(to be called *monadic predicate constants*).

Let (FFQT) have as its Wffs the formulae determined by the following condition:

Φ is a Wff of (FFQT) iff,

1) Φ is a Wff of (FQT) or

2) Φ is $A_i a_i$ ($i = 1, 2, \ldots, n$), or

3) Φ is $A_i x_i$ (,,), or

4) Φ is $(\forall x_i)\ \Phi_i$, where Φ_i is $A_i x_i$,

5) Φ is $(\exists x_i)\ \Phi_i$, where Φ_i is $A_i x_i$.

Leaving the remaining part of the axiomatic framework of (FQT) untouched, instead of either (1) and (2), or (1A) and (2A), accept the following axiom schemata:

(1B) $(\forall x_i)Fx_i \rightarrow ((\exists x_i)A_i x_i \rightarrow Ft_i)$, where t_i is a term free in F,

(2B) $Ft_i \rightarrow ((\exists x_i)A_i x_i \rightarrow (\exists x_i)Fx_i)$.

When (FQT) is reformulated thus,

$$(\forall x_i)Fx_i \rightarrow (\exists x_i)Fx_i$$

will not be theorem of the resultant system, i.e. (FFQT). Nor does it contain 'exists' as a predicate, as it is the case with the formulation of Meyer and Lambert [4].

Quantifiers in (FFQT) are to be interpreted "ontologically". Marrying off the Quinian interpretation [5] to model-theoretic interpretation, we shall take

(a) $(\forall x_i)Fx_i$

as a conjunction, and

(b) $(\exists x_i)Fx_i$

as a disjunction of all

$$Pe_i \ (i = 1, 2, \ldots, n)$$

where 'e_i' is a member of the domain D of interpretation, and 'P' is the property exemplified by (or the relation holding among) the members of D, and is assigned to 'F' in the interpretation I. If D is non-empty truth-value assignment to (a) and (b) is to be carried on in the same fashion as in (FQT); and if D is empty (a) and (b) are to be assigned *T* and *F* respectively, treating, as Hailperin [1] does, (a) as a conjunction and (b) as a disjunction of zero conjuncts and zero disjuncts respectively.

The sentential connectives of (FFQT) are to be allowed to retain the interpretation they got in (FQT). Then (1B) and (2B) turn out to be valid when D is empty. If D is non-empty, let the meta-theory of (FFQT) be the same as that of (FQT), but add wherever they are necessary extra clauses concerning the interpretation of

$A_i a_i$ (for each i).

For example, when 'a_i' is 'Pegasus',

(I) $A_i a_i$, (II) $A_i x_i$, (III) $(\exists x_i) A_i x_i$, and (IV) $(\forall x_i) A_i x_i$ will be

(Ia) Pegasus pegasizes, (IIa) e_i pegasizes (were e_i is $f(x_i)$ when f is the function mapping (FFQT) into D such that $e_i \in D$, under the interpretation I), (IIIa) there exists an x_i such x_i pegasizes, and (IVa) for every x_i, x_i pegasizes, respectively.

This reformulation has at least two additional advantages, firstly, (FQT) can be freed from i) and ii) within the scope of the unextended (FQT), i.e. without invoking the notion of identity, as is not the case with the reformulation suggested by Hintikka [2], and secondly, it tags off existence to quantification.

Indian Institute of Advanced Study A. Pampapathy RAO
Simla-5, India

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