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TENSE AND TEMPORAL REFERENCE HYBRID TEMPORAL LOGIC

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Abstract
Prior's approach to time has been neglected by semanticists for several reasons. The main one, we believe, is the inability of Priorean tense logic to refer to times. The second one, is the inability to account for some important features of natural language such as temporal anaphora and the role of temporal constructions in discourse. Priorean tense logic has, however, one important advantage over other accounts: the internal perspective of time (due to its modal nature). This paper examines extensions of Priorean tense logic in which reference to times is possible, focusing on the so-called hybrid temporal logic. We will outline some of its main features and analyze some of its philosophical implications.

1. *Introduction*

Prior's approach to time has been neglected by semanticists for several reasons. The main one, we believe, is the inability of Priorean tense logic to refer to times. The second one, is the inability to account for some important features of natural language such as temporal anaphora and the role of temporal constructions in discourse. Priorean tense logic has, however, one important advantage over other accounts: the internal perspective on time. By using a modal approach, rather than a first-order one, Prior emphasized the internal perspective on time and this emphasis seems to cohere with our daily uses of time.

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Actually, this is the main semantic idea behind Prior's tense logic, the claim that our everyday discourse presupposes an internal, local, perspective on time. Past, present and future thus are not to be taken as absolute notions: they make sense only relative to a context. By uttering a sentence we implicitly fix a time, an *utterance* time, and we typically specify temporal information relative to this utterance time (also called the *deictic centre*). Priorean tense logic, because of its modal character and especially through the use of Kripke semantics, neatly mirrors this semantic intuition.

To put it succinctly, tense logic keeps this internal perspective by evaluating formulas inside models, at some particular point or moment in time. In contrast, first order logic adopts an external perspective; a sentence in first-order logic does not depend on the contextual information contained in assignments of values to variables: sentences take a God's-eye-view of structure, and, irrespective of the variable assignment we use, are simply true or false of a given model.[1]

But, as we said, the price of taking this internal perspective is rather high, for we lose expressivity. Take a sentence like *Sara had spoken*. Allegedly, from an internalist stance, it is impossible to establish the truth or falsity of it. We are not capable of doing so because we can't make reference to the particular moment t before which it was true that Sara had spoken. On the other hand it seems the problem evaporates once we read this and similar sentences from an external point of view, according to which we should not evaluate the sentence within the system (within the time flow) but from an external point of view. Thus, from this external point of view, we can say that if it was indeed true that at a moment t_0 in time Sara spoke, it must also be true that there was a later moment t_1 in which it was true that she had spoken and hence, at an even later time t_2, it is true that at t_1 Sara had spoken.

The internalists, with their modal approach, cannot do this. They value locality over expressivity. They cannot (without the aid of hybridization) check whether the sentence was true or false at t_1, for they cannot even make reference to that point. In other, more colloquial, words, the subject cannot go to t_1 to check the validity of the sentence there.[2]

[1] A nice way of expressing the internal character of modal logic is saying that a modal formula is like a creature (or an automaton) placed inside a structure at some point *t*, and forced to explore by making transitions to accessible points, as expressed, for example, in [6].

[2] Note that Prior would represent this sentence by PP(*Sara speak*). This representation, which might be correct when considering isolated sentences, does not capture the referentiality of the tense and thus it is insufficient when considering sentences within a discourse, because it does not allow for anaphoric temporal reference. Thanks to one anonymous referee for comments on this point.

Take another example, even more relevant, of the lack of expressiveness of tense logic. One cannot say *now*, *then* and *when* in the classical temporal language. And this can be particularly troublesome because some very basic natural features of the temporal frame cannot be expressed syntactically. For instance, "*now* will not occur again", or, formally, the flow of time is irreflexive, is beyond the expressive powers of classical temporal logic. In other words, some important (essential) properties of frames, such as irreflexivity, asymmetry and antisymmetry, are not syntactically definable in tense logic.

There are at least three possible ways to enrich tense logic in order to overcome this limitation (We are following here Goranko [14]):

1. We can extend the language with new particular operators intended to express and formalize specific temporal phenomena. See for instance Kamp's binary temporal modalities *Until* and *Since* (see e.g. [9]).
2. We can add new sorts of syntactic objects having specific interpretations in the temporal framework and thus be able to increase generally the expressive power of the language. This is the route we will follow in this paper. The first attempts to extend temporal language this way can be found in Prior's so-called *clock variables* (nominals) that are true at exactly one instant in the flow of time (see e.g. [27] [6] [8]).
3. We can add more rules of inference intended to depict semantic features not expressible by means of formulae. See for instance Gabbay's *irreflexivity rule* (see e.g. [13]).

In considering possible extensions of Priorean tense logic, two things must be kept in mind. First, the purpose of the extension, what we want to achieve by extending the language. In this case, as we said, we want to obtain a logic that allows us to refer to points in time and thus, in doing so, gives a better account of our uses of temporal expressions in natural language. Second, it is important that in doing this, we do not lose the properties that made Prior's logic attractive in the first place. That is, it is important not to lose the internal perspective on time and Prior's logical simplicity. The strategy followed by the defenders of hybrid logic (which will be the one we'll follow in this paper) consists in

> *[S]orting* the atomic symbols of the language. Different sorts of propositional symbols are introduced, and their interpretation is constrained in various ways to achieve the desired forms of temporal reference (in [4]).

The aim of this paper is twofold. First, we will try to present in a brief but comprehensive way the main features of hybrid logic (assuming its minimal form and avoiding most technical debates and difficulties) and we will analyze its applications to the problem of temporal reference and temporal indexicality. The first thing to do is to introduce a new type of atomic symbol to our language, nominals; here we'll briefly see some of its main properties (syntactic, but mainly semantic). For the most part, we will focus only on this aspect of hybridization, that is, on nominal tense logic; but we will mention two other main features of hybrid logic, the introduction of a new operator @, called the satisfaction operator, and the introduction of quantification into the logic, focusing on the so-called binder $\downarrow$. With these tools settled, we'll turn on their applications for the semantic analysis of tense in natural language.

Temporal logic in general, and hybrid logic in particular, is a very lively field, and new developments are being presented constantly. However, logicians working on hybrid logic are mostly concerned with the computational aspects of it. The philosophical implications, or the linguistic ones, have been somewhat neglected. Only very recently a group of philosophers and logicians have started to work on them and so it is still unclear to what extent hybrid temporal logic can be a useful tool to analyze topics such as temporal reference, temporal indexicals or temporal anaphora. We believe it is indeed a useful tool and that it can give us some interesting insights on these topics. Lots of work remains to be done of course, the logic needs to be refined and developed further. We will outline here some of its achievements, its main applications (when it comes to reference), some drawbacks and some possible lines of work.

One basic question, both from a philosophical and a linguistic perspective, is whether treating tenses as modal operators is a compelling view at all or, in other words, what are the advantages we get by adopting it. That is, even if we can prove that the proposal, with the aid of hybridization, is indeed sound and logically compelling, the question remains of whether it captures the way we use tenses in natural language. Jefrey C. King [17] for one, has presented strong arguments against the idea of treating tenses as operators. He admits that the question of expressivity cannot be considered as a reason for rejecting the view of tenses as operators, for there are several options, as we mentioned above, to overcome these limitations. What he claims is that there is

> [D]ata that shows that tenses and temporal expressions do not work like the standard operators of tense logic

and that

> virtually all recent attempts to handle complex data involving tenses [...] have rejected the view that tenses are operators (in [17])

We will go through the *data* mentioned by King, that is, three objections raised against the modal treatment of tenses, and we will show how they can be overcome with the aid of hybridization. Of course this, relevant as it is, will not be enough to convince either philosophers or linguists that the modal approach is to be preferred over, say, the quantificational one. By overcoming the objections, and by doing so in a simple and elegant way, the hybrid approach postulates itself as an attractive and promising account of tense, but further arguments are required to explain why this approach is to be preferred; specially since, as King affirms, the view that tenses are operators is not the predominant one among semanticists. In the last section of the paper, we will present and discuss some philosophical arguments which, we will claim, go indeed in favor of the modal approach. Roughly, we shall claim that the modal approach to tense captures both the way we perceive time and the way we express it. That is, it captures the way we use tense in natural language, its indexical nature and, with it, the cognitive role it plays.

2. *Basic Modal Language*

Let's outline a basic propositional modal logic.[3] Given propositional symbols PROP = {p, q, r, ... }, and modality symbols MOD = $\{m, m', m'', \ldots\}$ we define the *basic modal language* as follows:

$$\text{WFF} := \text{p} \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid \varphi \rightarrow \psi \mid < m > \varphi \mid [m]\varphi$$

The $< m >$ and $[m]$ symbols are called the diamond and box symbols respectively. If there is just one modal symbol in the language we write $\Diamond$ and $\Box$. But as we'll see, we often use other symbols as for example in temporal logic, where we write F and P for the diamond forms and G and H for the box forms.

Let's see now briefly the semantics for modal logic with just one modal symbol in the language (thus, with $\Diamond$ and $\Box$). We define a modal model (or a *Kripke model*) $\mathcal{M}$ as an structure (M, R, v), from a frame (M, R), where

[3] We are following here what is commonly called the Amsterdam Perspective on Modal logic. Very roughly, this perspective attributes to modal logic the following characteristics: modal languages are simple ways for talking about relational structures; modal languages provide an internal perspective on relational structures; modal languages are not isolated formal systems. For a much more detailed exposition see [7].

1. $M \neq \emptyset$ is a set of worlds,[4] $M = \{m_1, m_2, m_3, \ldots\}$.
2. $R \subseteq M^2$ is the accessibility relation between worlds and has different properties depending on the system,[5]
   (a) Reflexivity: $\forall m_i \in M(m_i R m_i)$
   (b) Transitivity: $\forall m_i, m_j, m_k \in M((m_i R m_j \wedge m_j R m_k) \rightarrow (m_i R m_k))$
   (c) Symmetry: $\forall m_i, m_j \in M(m_i R m_j \rightarrow m_j R m_i)$
3. v is an assignment, that gives the value true or false to each wff in a world (being F the set of all wff, $v : F \times M \rightarrow \{1, 0\}$).[6] From here, we can get the subsets of $v(p) \subseteq M$ in which an atomic formula is true. The assignment satisfies the following conditions, for any $m_i \in M$, $A, B \in F$ and propositional variable p:
   (i) $v(p, m_i) = 1$ or $v(p, m_i) = 0$.
   (ii) $v(\neg A, m_i) = 1$ if and only if $v(A, m_i) = 0$; otherwise $v(\neg A, m_i) = 0$.
   (iii) $v(A \wedge B, m_i) = 1$ if and only if both $v(A, m_i) = 1$ and $v(B, m_i) = 1$; otherwise $v(A \wedge B, m_i) = 0$.
   (iv) $v(A \vee B, m_i) = 1$ if and only if either $v(A, m_i) = 1$ or $v(B, m_i) = 1$; otherwise $v(A \vee B, m_i) = 0$.
   (v) $v(A \rightarrow B, m_i) = 1$ if and only if either $v(A, m_i) = 0$ or $v(B, m_i) = 1$; otherwise $v(A \rightarrow B, m_i) = 0$.
   (vi) $v(\Diamond A, m_i) = 1$ if and only if $\exists m_j \in M(m_i R m_j \wedge v(A, m_j) = 1)$; otherwise $v(\Diamond A, m_i) = 0$.

   We assume the classical propositional logic definitions and the definition of $\Box A$ by means of $\neg\Diamond\neg A$.

A wff A is valid on a frame (M, R) if and only if, for every model $\mathcal{M}$ based on that frame and for every world $m_i \in M$, $v(A, m_i) = 1$. We define T-validity by saying that a wff is T-valid if it is valid on every reflexive frame; S4 validity by saying that a wff is S4-valid if it is valid on every reflexive and

[4] Note that we talk of a set of worlds. This is the traditional interpretation, for one-dimensional modal languages. In the next section, when talking about modal logic with temporal symbols P and F we will talk of a set of times.

[5] For T, reflexivity; for S4, reflexivity and transitivity; and for S5, reflexivity, transitivity and symmetry.

[6] 1 means that the formula is true and 0 that is false.

transitive frame; and S5 validity by saying that a wff is S5-valid if it is valid on every frame whose accessibility relation is an equivalent relation (that is, reflexive, transitive and symmetric).

3. *Temporal Logic*

Tense logic[7] was introduced by Prior in the sixties. Very briefly, it is a simple form of modal logic used for reasoning about time. Prior introduced new modalities: F and P (meaning "at some future time" and "at some past time" respectively) and their respective duals G and H (meaning "it is always going to be the case" and "it has always been the case" respectively). Tense logic has been further developed; new operators have been added (like *Until* and *Since*) as well as new and different axiomatic systems.

We won't get here into the details of the different temporal logics. We will first only present the semantics for a one-dimensional temporal system. We define a flow of time, a temporal model T as a structure $(T, <, v)$, from a frame $(T, <)$, where

1. $T \neq \emptyset$ is a set of moments, $T = \{\ldots, t_{-2}, t_{-1}, t_0, t_1, t_2, t_3, ..\}$. t_0 is usually taken to be the present moment or some special moment of time.
2. $<\subseteq T^2$ is the earlier/later relation on moments and has different properties depending on the flow of time[8], being always irreflexive $(\forall t_i \in T \neg(t_i R t_i))$. For the simple model we are analyzing, we shall place some constraints on the relation. We shall assume that $<$ is always a *strict total order*; that is, a relation that is not only irreflexive but also transitive and trichotomous. In short, we are assuming a linear flow of time (later, we will need more demanding conditions).
3. v is an assignment, that gives the value true or false to each well formed formula (wff) in a moment of time (being F the set of all wff, $v : F \times T \to \{1, 0\}$). From here, we can get the subsets of $v(p) \subseteq$ T in which an atomic formula is true. The assignment satisfies the following conditions, for any $t_i \in T$, $A, B \in F$ and propositional variable p:

[7] Tense logic is a specific kind of temporal logic, the one developed by A. Prior. Sometimes these two terms are used interchangeably. In this paper, whenever we use "tense" logic we mean Prior's logic.

[8] Depending on the flow of time, the frame can be dense, transitive, past-infinite, future-infinite, past-linear, and so on.

(i) $v(p, t_i) = 1$ or $v(p, t_i) = 0$.

(ii) $v(\neg A, t_i) = 1$ if and only if $v(A, t_i) = 0$; otherwise $v(\neg A, t_i) = 0$.

(iii) $v(A \rightarrow B, t_i) = 1$ if and only if either $v(A, t_i) = 0$ or $v(B, t_i) = 1$; otherwise $v(A \rightarrow B, t_i) = 0$.

(iv) $v(FA, t_i) = 1$ if and only if $\exists t_j \in T(ti < tj \wedge v(A, t_j) = 1)$; otherwise $v(FA, t_i) = 0$.

(v) $v(PA, t_i) = 1$ if and only if $\exists t_j \in T(t_i > t_j \wedge v(A, t_j) = 1)$; otherwise $v(PA, t_i) = 0$.

We assume here also the classical propositional logic definitions and the definition of HA and GA by means of $\neg P \neg A$ and $\neg F \neg A$.

4. *Hybrid Temporal Logic*

Hybrid logics are modal logics that allow referring to individual states in models. In the case of temporal logic, they allow referring to a particular point of time, an instant. The principal ideas behind hybrid logics were already introduced by Prior in 1967,[9] although they were not fully developed. After him, it was developed by Bull and reinvented by a group of logicians from the Sofia School. In the 1990s, the research papers about this topic increased due to the work of authors like Blackburn, Areces, Goranko, Marx and other researchers linked to the University of Amsterdam.[10] Doubtless, the main innovation of hybrid logic, already present (albeit somehow hidden) in Arthur Prior's work, is the introduction of a new set of atomic symbols, called "nominals" as a tool for naming or reasoning about points or instants in the models (in the set of semantic possible worlds, in the flow of time, etc.). These nominals appear in the syntax, and we can build well-formed formulas with them. The set of nominals, usually represented as $\{i, j, k, \ldots\}$ is disjoint from the set of standard propositional variables, extending the expressive power of the logic ($NOM \cap VAR = \emptyset$). Nominals are true at one point, and only at one point, in any model. This way, they *name* this unique point; by being true there and nowhere else. This point is usually called the "denotation" of the nominal. Formally, the set of times used to interpret a nominal must be a *singleton set*.

By introducing nominals we get a more expressive logic. In order to see that this is so, let's consider a standard example, as presented by Areces and Blackburn.

[9] [27].

[10] See [8], [5], [14] and [1].

Consider the following (orthodox) tense logical formula:

$$F(r \wedge p) \wedge F(r \wedge q) \rightarrow F(p \wedge q)$$

> This can be falsified. The first conjunct in the antecedent says that in the future there is time where both r and p are true together, and the second asserts that in the future there is a time where p and q are true together. But this is obviously unjustified: the future times that witness p and q may be distinct. Now consider the formula of nominal tense logic:
>
> $$F(i \wedge p) \wedge F(i \wedge q) \rightarrow F(p \wedge q)$$
>
> [...] the resulting formula is impossible to falsify. We now have some extra information: the p-witnessing and q-witnessing future times both make i true, and there is only one time which does, for i is a nominal. Hence the future times must be identical, and the conclusion follows.(in [2])

With the addition of nominals we get what has been called "Nominal Tense Logic", and their introduction is the key step towards basic hybrid (temporal) language, but most hybrid logics involve more elements than just nominals. There are a number of options for enlarging the logic, the most common of them being the introduction of the *satisfaction operators* @. These operators are of the form $@_i$ (i being a nominal) and allow us to *jump* to the point named by i. The formula $@_i\varphi$ (read 'at i, φ') asserts that φ is satisfied at the (unique) point named by the nominal i or, in other words, moves the point of evaluation to the state named by i and evaluates φ there. These operators satisfy many nice logical properties; syntactically they are simply modalities and semantically they also turn out to be well-behaved modalities:

- they satisfy the distributivity axiom: $@_i(\varphi \rightarrow \psi) \rightarrow (@_i\varphi \rightarrow @_i\psi)$
- they satisfy the necessitation rule (admit modal generalization): if φ is valid, then $@_i\varphi$ is also valid
- they are self-dual: $@_i\varphi \leftrightarrow \neg@_i\neg\varphi$

In an intuitive sense, the $@_i$ operators provide a bridge between semantics and syntax by internalizing the satisfaction relation $\models$ into the logical language:

$$\mathcal{M}, w \models \varphi \text{ iff } \mathcal{M} \models @_i\varphi, \text{ where } i \text{ is a nominal naming } w$$

To further enrich the language, one obvious step would be to have not only names for individual states, but also variables ranging over states, with the corresponding quantifiers. Adding the $\forall$ quantifier we would be able to write:

$$\forall y.\Diamond y.$$

Which is translated into first-order language as $\forall y.R(x, y)$, forcing the current state to be related to all states in the domain. The $\forall$ quantifier is very expressive. Together with @ it gives us already full first-order expressive power. But it is too "classical". Modally, other quantifiers, besides the $\forall$ are possible. The $\downarrow$ binder binds variables to the *current* point of evaluation. Or, in other words, it links a nominal to a concrete point, naming only this point, as the denotation of the nominal. More precisely:

> It enables us to create a name for the here-and-now, and refer to it later in the formula. For example, the formula
>
> $$\downarrow y.\Diamond y$$
>
> is true at a state m iff m is related to itself. The intuitive reading is quite straightforward: the formula says: "call the current state y and check that y is reachable". [...] Note that the satisfaction operators works in perfect coordination with $\downarrow$. Whereas $\downarrow$ "stores" the current point of evaluation (by binding a variable to it), the satisfaction operators enable us to "retrieve" stored information by shifting the point of evaluation in the model. (in [3])

So, in a sense, $\downarrow$ gives us a sort of *generalized present tense*; it enables us to "store" an evaluation point, thereby making it possible to insist later that certain events happened at *that* time, or that certain other events must be viewed from that particular perspective. We won't get into the details of the syntax nor the semantics of hybrid logic, but notice that with the aid of both the satisfaction operators and the binder $\downarrow$ it is possible to define properties of frames that are not definable in ordinary modal and temporal logic, such as

(i) Irreflexivity: $(\downarrow x.\Box\neg x)$

(ii) Asymmetry: $(\downarrow x.\Box\Box\neg x)$

(iii) Antisymmetry: $(\downarrow x.\Box(\Diamond x \rightarrow x)$

5. *Problems with Orthodox Temporal Logic*

In the introduction, we said that orthodox modal — and hence temporal — logic has an important weakness: the inability to name points. This limitation is actually just one aspect of a more general problem: the fact that tenses in natural language do not always seem to behave like the operators of standard tense logic. All this is not new of course; actually one of the most pervasive issues in the research into the semantics of tense has been whether tense ought to be treated as an operator or rather as *something* that refers to moments of times. And part of the attractiveness of hybridization is precisely the possibility of "having it all"; that is of keeping the idea of tenses as operators and also as making reference to precise moments of time. But before getting into the details, let us briefly summarize the state of the debate in contemporary semantics regarding this issue.

Two of the most important semanticists to argue in favor of the application of modern logic to the analysis of natural language were Montague [22] and Reichenbach [28], and they have been the source of inspiration for two different trends within the semantics of tense.[11] Very roughly, those defending that tenses should be viewed as operators and thus analyzed in terms of Prior's temporal logic follow mostly Montague's approach while those who take the semantics of tense to involve reference to moments of time have mostly been inspired by Reichenbach's ideas. As we have said, this second perspective is the most common in semantics, led mostly by defenders of the Discourse Representation Theory (Kamp, and others). Now, the relations between these two approaches are multifarious, and so are their possible interpretations. We will not get into this debate. But we want to claim that hybridization offers a compelling way to *improve* tense logic so that it becomes capable to overcome some of its limitations. In other words, it offers a neat way to link both approaches.

So, we do not intend to offer here a complete account of the semantics of tense, our goal is much more modest. We aim to show how hybrid temporal logic can deal with three objections commonly raised against the modal approach:

1. Temporal reference
2. Temporal anaphora (temporal discourse)
3. Scope problems

These objections are quite persistent and common in the literature and the three of them are present in Jeffrey King's paper *Tense, Modality and*

[11] See [20].

Semantic Values. Similarly, the solutions we offer here are already present in the literature though not in a systematic way, and they are not always used to solve the objections we intend to overcome here. So, all in all, the conclusions to be extracted from them will be our main contribution to the discussion.

5.1. *Modality and Temporal Reference*

The inability to account for temporal reference is the oldest and most basic objection of the three and also the one with a straight answer, not only through hybrid logic but also through some other possible enrichments of temporal logic. Let's analyze this shortcoming a bit further. In modal logic we cannot say that some *particular* individual has some property, or that something happened *then*. Notice that first-order logic can do all this. We just need to use constants to name the individuals or the moments in question. As a result, we would have to conclude that temporal logic is not expressive enough to handle the temporal semantics of natural language properly. Temporal logic cannot account for *temporal reference*. As we said, this is the main reason why Prior's tense logic has been largely overlooked by semanticists.

In the standard (Kripke) semantics for modal logic, truth is relative to points in a set. These points, as we have seen, can be taken to represent possible worlds, times, knowledge, states in a computer, etc. Hence propositional symbols can have different truth-values relative to different points, that is, relative to different possible worlds, times, etc. This allows us to formalize natural language statements like *it is raining* which has different truth-values at different times.

But in natural language we often explicitly introduce the time against which the statement has to be evaluated. This can be done in several ways, we can for instance, introduce a temporal indexicals, such as "yesterday" or "then", or we can introduce some specific information about the date or the time, like in *it is raining at two o'clock on 15 April 2009*, which is true at just one time (two o'clock 15 April 2009). Once made true (or false) it remains eternally true, for, if it was indeed the case that it was raining at two o'clock on 15 April 2009, it will always be the case that it was so. This second kind of natural language statements cannot be formalized in orthodox modal logic. Orthodox modal logic is not capable of representing the reference to a specific moment of time.

But it is easy to see how this limitation can be overcame by hybridization, since nominals are precisely used as a mechanisms for referring to moments of time. To see how this works and how it helps with temporal reference let's begin with a simple example. Take the example *Marta bought some shoes*. In order to formalize this statement, it is important to have a referential point

(only one) before the current one where it is true that Marta buys some shoes. In (orthodox) temporal logic, we would write:

$$\mathrm{P}(\textit{Marta buys some shoes})$$

But this clearly does not specify any particular moment of time. In hybrid temporal logic however, we write:

$$\mathrm{P}(i \wedge \textit{Marta buys some shoes})$$

where P is the monadic operator for sometime in the past, and i is the nominal. We thus know, because i is a nominal, that i is true in only one point of the model and in that point it is true that Marta buys some shoes.

Consider again the example we used in the introduction: *Sara had spoken*. We claimed that it was impossible to capture its truth-conditions in (orthodox) temporal logic, for we cannot make reference to the particular moment t in which it was true that Sara spoke. Formally, in orthodox temporal logic, this statement is translated as P (Sara speaks) and this is obviously insufficient, for what we are claiming is that in the past it is true that Sara speaks. What we need is to specify a time t_1 where it is true that at some previous time t_0 it is true that at an even earlier time t_{-1} it is true that Sara speaks. We can do this adding nominals:

$$P(i \wedge P\varphi)$$

("there is some time in the past labelled i and that the event φ happened *before that*").

5.2. *Nominals and Temporal Anaphora*

Hybrid temporal logic can also be used to represent temporal intersentential anaphora. The basic idea here is to use nominals as discourse markers. In order to see how it works, we shall use the examples provided by Blackburn [4].[12] Let's begin with temporal anaphora in state sentences. Consider the discourse: *The shutters were closed. It was dark*. The second sentence (a state sentence) anaphorically picks out the time referred to by the tense of the first sentence. We can represent this discourse with the aid of nominals as follows:

$$P(i \wedge \textit{The shutters be closed}) \wedge P(i \wedge \textit{It be dark})$$

[12] Blackburn, in turn, approaches the subject via Partee's discussion. See [23].

Since nominals are linked to a unique time, we can reuse the i nominal provided by the first sentence in the representation of the second. This gives us what we wanted, reference to a unique past point that is both a shutters-being-closed-time and it-being-dark-time. In other words, nominals are always available for reuse. It doesn't matter if the two occurrences of the nominal are in different conjuncts and embedded within the scopes of different past tense operators for it follows from the semantics of hybrid temporal logic that the above representation is logically equivalent to

$$P(i \wedge \textit{the shutters be closed} \wedge \textit{it be dark})$$

Now, things are not so simple when it comes to two or more *event* sentences. Let's consider again an example given by Blackburn: *Mary woke up sometime during the night. She turned on the light*. This can be represented as:

$$P(i \wedge \textit{Mary wake up sometime during the night} \\ \wedge P(j \wedge Pi \wedge \textit{she turn on the light}))$$

Notice that we need here two distinct nominals, for we are talking about two different times. The turning of the light clearly occurs *after* Mary wakes up, so we need to introduce a second nominal j to represent a time t' later than the moment t (represented by i). In order to state that the second nominal names a more recent time than the old, we introduce the conjunction $j \wedge Pi$. This is a typical fact of event sentences, they move the story along.

Lastly, let's consider a more complex example (again, Blackburn's) where we have both state and event sentences: *John got up, went to the window, and raised the blind. It was light out. He pulled the blind down and went back to bed. He wasn't ready to face the day. He was too depressed*. This is the representation that Blackburn[4] proposes:

$$\begin{array}{ll} & P(i \wedge \textit{John get up}) \\ \wedge & P(j \wedge Pi \wedge \textit{go to the window}) \\ \wedge & P(k \wedge Pj \wedge \textit{raise the blinds}) \\ \wedge & P(k \wedge \textit{It be light out}) \\ \wedge & P(i_1 \wedge Pk \wedge \textit{He pull blind down}) \\ \wedge & P(j_1 \wedge Pi_1 \wedge \textit{go back to bed}) \\ \wedge & P(j_1 \wedge \textit{He not ready to face the day}) \\ \wedge & P(j_1 \wedge \textit{He be too depressed}) \end{array}$$

5.3. *Scope Problems*

The last of the three objections we shall consider in this paper was first raised by Dowty [10] but it is also mentioned in King [17] as one further reason to abandon the modal approach on tense. Shortly put, the objection claims that treating temporal indexicals, such as "yesterday" or "tomorrow", as operators yields the wrong predictions. To see why this is so, consider the following definition of "yesterday" as an operator

(Yesterday) Y (φ) is true at t iff φ is true at some t' such that t' is within the day preceding the day that includes t

Now, when we apply this definition to an example of natural language, such as (to follow with the examples above): *Yesterday, John turned off the stove* (uttered at a certain time t on a day d), we get two possible and incompatible interpretations,

1 Y(P(John turns off the stove))
   - Would be true in a situation in which at ANY time prior to a time included in the day before d, John turned off the stove

2 P(Y(John turns off the stove))
   - Would be true in a situation in which there is some past time t' (any t') such that John turned off the stove on the day d' that precedes the day that includes t'

And obviously, none of these interpretations captures the meaning of the utterance mentioned before. Indeed, the mere fact that there are two possible interpretations for such a clear and non-ambiguous sentence is a reason to reject the approach.

But, once again, this objection does not affect the modal perspective when enlarged by hybridization. The trick is treating "yesterday" not as an operator but as a nominal. Nominals are the perfect tools to capture the role of terms such as "yesterday" in natural language, their indexical and referential nature. Actually, through hybridization the apparent problem of scope dissolves.The so-called scope problem comes from assuming that both the tenses and temporal adverbs are nested and thus must take scope over one

another. But hybridization questions this assumption, and it does so in a simple way.[13] Basically, once again, what hybridization does is to combine two forms of information in a uniform way.

Blackburn [4] develops an account of temporal adverbs from the perspective of hybrid temporal logic (more precisely, nominal temporal logic). The basic idea is that our language deals with arbitrary information (via propositional symbols) and labeling information (via state symbols) and, in hybrid language, both types of information are regarded as propositions. In particular, it has been shown that we can take labels to be just a special sort of proposition that is true at exactly one state in any model.

Notice however that treating temporal adverbs like "yesterday" as nominals, entails working with intervals of time, instead of moments of time and that brings some complications when it comes to the semantic interpretation. But, roughly the application is quite straightforward. Assuming the general idea of treating both types of information as propositions, it is only natural to ask whether this can also work for other sorts of information, like intervals of time.

Therefore, the idea is sorting to model the indexicals "yesterday", "today" and "tomorrow". At the syntactic level, Blackburn's proposal is simple enough. He adds three new symbols, that will be atomic wff, *yesterday, today* and *tomorrow* to our nominal tense logic.The set of atomic symbols of this language is defined as follows:

$$\text{ATOM} = \text{VAR} \cup \text{NOM} \cup \{\textit{yesterday, today, tomorrow}\}$$

We make wffs from the stock of symbols exactly as before. These new symbols allow us to represent English sentences containing indexicals, such as *Marta went to the cinema yesterday* as

$$\text{P}(\textit{Yesterday} \wedge \textit{Marta go to the cinema})$$

Now, semantics is a bit more complicated, Blackburn presents a proposal based on the so-called "California theory of reference" (see for instance, Montague [22], Kamp [15], Kaplan [16]), with the novelty of not treating temporal adverbs as modal operators, but rather as nominals. In order to do so, some new terminology has to be introduced to take into account the notion of "dayhood". That is, by taking "yesterday" as a nominal, equating it thus with other temporal indexicals, temporal ontology needs to be somehow modified. Instead of working with a traditional notion of a linear flow of time, we need to work with close intervals that hence require to

[13] Dowty himself proposed a way out of this claiming that tense morphemes and temporal adverbs should be taken as "sister nodes". See Dowty [11].

be defined beforehand. Blackburn proposes a simple account, based on the sequence of rational numbers (see Blackburn [4]), the technical details are not relevant here, but it is however important to see, if only briefly, how he accommodates the account of a temporal adverb like "yesterday" into the framework of a theory of reference for natural language. In particular, how the hybrid account fits naturally into a theory about reference of indexicals like the "California theory". It is important because one of our claims is that, even though there might be other feasible explanations of tense and temporal reference, the account presented here has as one of its assets the fact that, contrary to the traditional objections raised against the modal approach, it captures the way we use tense in natural language and thus it can be integrated naturally into some classic semantic explanations of tense and reference in natural language.

In order to apply the ideas of the "California theory of reference" to temporal indexicals, the first element to take into account is the notion and role of context when evaluating the truth of utterances. The idea here is to introduce contexts into Kripke models so that when wffs are evaluated they are not only related to times but also to contexts. Again Blackburn's proposal is,

> Fix some non-empty set $\mathcal{C}$, the set of *contexts*, or *contexts of utterance*, and specify a function $g : C \to Q$. The function g is to be thought of as specifying the *utterance time* of each context of utterance (in [4])

We get then a very weak (but sufficient for our purposes) notion of context, *Contextualization* of Q $(Q, <, C, g)$ and with this in place he proposes an interpretation of the language, starting with a definition of a valuation,

> A valuation is a function $V : ATOMxC \to Por(Q)$ that respects the following constraints. First, for each nominal i and each context c, $V(i, c)$ must be a singleton. Second, for each context c:
>
> 1. $V(today, c) = day(g(c))$
> 2. $V(tomorrow, c) = next(day(g(c)))$
> 3. $V(yesterday, c) = prev(day(g(c)))$
>
> The intuition behind should be clear. In any context c the $today$ atom is to be true at all points in the day containing the utterance time and at no others. $yesterday$ and $tomorrow$ are to be true at, respectively, precisely those points in the day preceding, and the day succeeding, 'the day of utterance'. (in [4])

Let's see how these enrichments work. Take again the sentence *Marta went to the cinema yesterday*, as we said we can represent this as

$$P(Yesterday \wedge \textit{Marta go to the cinema})$$

Suppose we are in some context of utterance c, and suppose we evaluate the sentence at the pair [g(c),c] (we are asking ourselves what happens if we utter this wff in the context c at the utterance time for c). It follows from the above definition that this wff is true iff there is a time t' in the day preceding the utterance day such that *Marta goes to the cinema* is true at [t',c] (this wff is true iff Marta did go to the cinema during the day classified as yesterday in the context of utterance).

One of the main advantages of this approach is that the indexicals here don't quantify, so there is no scope problem. They rather act as range restrictors of the quantification performed by the familiar temporal operators. Our indexicals function in a similar way as nominals, they restrict the range of the operators. So for instance the role of $yesterday$ in the wff above is to ensure that only those times classified as being "yesterday" are relevant to the truth or falsity of the wff.

6. *Temporal Innocence or the Speaker's point of view*

We would like to conclude with some philosophical reflections on the role that hybridization can play for the semantic analysis of tense in natural language. So far, we've focused on showing how hybridization could help temporal logic in overcoming some of the main problems raised against it as applied to the study of tense in natural language. But even assuming that these solutions are convincing and thus that we can develop an appropriate semantics for tense in natural language, a question would remain: why the effort? What are we to gain by doing so? Or rather, what are the reasons to claim that tenses should be treated as operators?

Well, as we said at the beginning, we would gain the internal, speaker-oriented, perspective on time. Notice that most detractors, such as King, are aware of the fact that the modal approach can overcome the objections, either through hybridization or by other means. What they claim is something "deeper", that is, they claim that treating tenses as operators just doesn't capture the way tenses work in natural language. We want to claim exactly the opposite. The main advantage of the modal approach is precisely that it captures the way speakers perceive and talk about time and tense. This in

principle attractive idea, brings a problem on its own though: it appears to require the adoption of some sort of relativism (usually called "temporalism"), but we do believe this can be avoided.

The opposition between the modal and first-order approaches to tense, their differences in formalizing the logic of time, reflects a philosophical difference in their attitudes toward time. The modal approach is linked to what has been traditionally called the "A-theory" of time, according to which time is to be taken as a flow, characterizing the events as past, present or future. The first-order approach is linked with the so called "B-theory" according to which events are to be characterized through the relations of "earlier" and "later".[14] Regardless of what metaphysical picture of time we want to defend, it is a fact that we live within the flow of time, we perceive events as happening in the present, we remember them as part of the past and we anticipate them as future events, yet to happen. In other words, we live immersed in the flow of time. Because of the internal perspective, the modal approach captures this neatly and with this, some further characteristics of tense in natural language, i.e. its indexical character and its cognitive role.

These features, and their relevance, were brought about by work in the semantics of indexical expressions. Arguments by Prior [27] and Perry [24], among others, have shown that certain thoughts are essentially tensed, and thus they cannot be adequately characterized in tenseless terms. Therefore, all attempts to reduce tensed talk into tenseless one (by reducing expressions such as "now" or "tomorrow" to, for instance, dates or "tokens" such as "the time t of the utterance", as Reichenbach [28] did) seem to be doomed to failure. Of course that does not entail that there must be (irreducible) tensed facts, but rather, that some kind of explanation of the role of tensed talk is in order.

Prior was well aware of this, of the non-reducibility of tense. This point was famously made by him in his paper "Thank Goodness that's over":

> One says, e.g. "Thank goodness that's over!", and says something which it is impossible that any use of a tenseless copula with a date should convey. It certainly doesn't mean the same as, e.g. "Thank goodness the date of the conclusion of that thing is Friday, June 15, 1954", even if it be said then. (Nor, for that matter, does it mean "Thank goodness the conclusion of that thing is contemporaneous with this utterance". Why would anyone thank goodness for that?) (in [26])

[14] This distinction was introduced by McTaggart in his now classic paper "The unreality of time" [21].

Notice that Prior introduces here a criticism to the reduction of tenses to reflexive tokens, that is, to Reichenbach's proposal. Prior's aim was to defend the non-reducibility of tense. With the aid of hybridization, the modal approach can keep the non-reducibility of tense. Indeed, notice that what hybridization does is introducing some "B-elements" into an "A-semantic theory". Combining thus ideas from Reichenbach with a general Priorean approach.

However, this "have it all" strategy brings about some philosophical problems. As Kaplan [16] claimed and Recanati [29] has recently put it, treating tenses as operator requires "time-neutral" propositions for such operators to operate on. Temporal operators shift the point of evaluation, specify a time at which the proposition must be evaluated. If the proposition already contains the time at which it is to be evaluated (it is "time-specific"), then the operator would be vacuous. But if tense operators actually operate over propositions this leads to a problem, for it goes against the traditional view concerning them. According to it, propositions are eternal, that is, any proposition has in any possible world a fixed and thus eternal truth value. So the idea of "temporal propositions", that is, propositions that change their truth values depending on the time of utterance is at odds with the standard view. It is our contention that we can safely combine these two elements: keep the idea that propositions are eternal while defending that there can be indeed non-eternal (non temporally specific) contents for temporal operators to operate on. And the best way to do so, we believe, is by adopting a Korta and Perry-inspired pluripropositional view.[15] Explaining the details of how the hybrid approach can be combined with Korta and Perry's proposal would have to be left for a future paper, but it is worth mentioning here some important aspects.

Let's begin by explaining a bit further what we mean by a proposition having to be "eternal". Notice that it is a central element of the very nature of what a proposition is supposed to be. According to Frege for instance, the idea of a proposition that is true at some times and not others, that cannot be eternally evaluated unless we are given an extra element — a time —, is incoherent. Temporal propositions are semantically incomplete under this view,

> Now is a thought changeable or is it timeless? The thought he express by the Pythagorean Theorem is surely timeless, eternal, unvarying, "But are there not thoughts which are true today but false in six months' time? The thought, for example, that the tree is covered with green leaves, will surely be false in six months' time". No, for it is not the same thought at all. The words 'this tree is covered with green leaves' are not sufficient by themselves to constitute the

[15] See [25] [19] [18].

> expression of thought, for the time of utterance is involved as well. Without the time-specification thus given we have not a complete thought, i.e., we have no thought at all. Only a sentence with the time-specification filled out, a sentence complete in every respect, expresses a thought. But this thought, if it is true, is true not only today or tomorrow but timelessly. ("Thoughts", in [12])

Now, it is our contention that we can affirm both that propositions are eternal (à la Frege) and that temporal sentential operators operate on non-eternal semantic contents. And we do this by allowing each utterance to express a variety of contents. Tense operators operate upon one of such contents, which is indeed a temporal content and thus — in accordance with Frege's view — cannot be the full content of what is said. Rather, tense operators operate upon minimal contents: true at some points (times) and false at others.

This is where Korta and Perry's ideas come handy. The idea behind their pluripropositionalism is rather simple, it stems from Perry's ideas on unarticulated constituents (see [25]). According to Perry, there is a clear difference between, say, "it is raining here" and "it is raining". In the first case, the place is explicitly included in the content expressed but not in the second. The first is thus location-specific and the second is not. So, following with the terminology introduced above, the first expresses the full content of what is said, a proposition that once true stays so eternally. The second, on the other hand, expresses a minimal content, that can be true at some locations and not at others.

Notice that this proposal fits perfectly with the modal approach defended here. On the one hand, according to Korta and Perry's view, the relevant time (on his example, the relevant location) is not part of the content of the utterance, but of the *circumstance* in which the content is evaluated. That allows us to keep the time neutrality of the content (minimal content), needed for the operators to operate on. This is an important fact for it highlights an essential element of our everyday use of tense. On many occasions, even though time is needed to evaluate the sentences expressed by our utterances, we do not *talk about* time when using them. Even though they might *concern* time, our utterances are not *about* time in any significant sense.[16] To put it briefly, using Recanati's words, the modal approach preserves the temporal innocence of the speaker.[17]

[16] See Perry [25] for the distinction about/concerns.

[17] So far, we have emphasized on the referential role of tenses but this is not their only possible role; there is also a quantificational role for instance; and that can only be captured by treating tenses as operators.

There are of course occasions in which the speaker wants the relevant time to be part of the "content expressed", part of what is said by the utterance (i.e. "it is raining now" or "it is raining at 3 p.m."). But there are other occasions in which even though the content expressed by the utterance is relative to a particular moment of time, it is not *about* that particular moment of time (i.e. "it is raining").

However, this difference does not necessarily entail that we have to renounce to the idea of propositions having eternal truth values. Granted, a sentence like "it is raining" changes its truth value depending on the time (and place) of evaluation; but once this time (and place) is settled, then its truth value remains stable. This difference can be taken into account by considering multiple contents. Following Korta and Perry, and very roughly, there are different levels of content for each utterance. The "minimal" level will give us the "utterance-bound truth conditions" and those are basically what a hearer of the utterance would grasp if she only had access to the meaning of the words and the syntactic structure of the sentence. This minimal content does not include, in a case like "it is raining", any information about time or location, for these are given by the context of utterance and are not explicitly included in the sentence expressed. Clearly though, this is not "what is said" by the utterance "it is raining" and thus a hearer that only grasps the utterance-bound truth conditions, cannot be considered as having "understood" what the speaker said. In order to grasp the full meaning of the utterance, the hearer needs to have access to some elements of the context, in this case, a location and a time. Once she knows them, she can have access to the full propositional truth conditions, which, once settled remain so eternally.[18]

Needless to say, further discussion is needed, but it is our contention that by differentiating different levels of content for each utterance, with different truth conditions, it would be possible to defend a modal approach to tense, without falling into relativism (temporalism). A whole account on how to apply the modal operators on the minimal semantic content, and how to combine this with Korta and Perry's proposal is required, but the chances of success look very promising. Much has been said on the debate between temporalists and eternalists and the different possible ways to accommodate

[18] In [29] Recanati argues along these lines, and he uses these arguments to defend not only that tenses can and should be treated as operators but also that this entails some sort of relativism concerning tense that he labels (moderate) temporalism. Fortunately for the modal approach, we do believe there are other alternatives, like the one we are outlining here. Regardless of whether temporalism is correct or not (and we are not convinced it is) it certainly would be a limitation of the modal approach if it were a direct and necessary consequence.

our intuitions about time with an adequate semantic analysis.[19] Our aim in this paper was simply to defend the view of tenses as operators as a viable semantics for tense in natural language; that is, how, with the aid some basic tools from hybrid temporal logic, hybrid temporal logic can easily overcome some of the objections traditionally raised against it and present some philosophical reasons as to why it is worth developing such an account.

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[19] See [29] for a nice review of some of these debates and a defense of a moderate temporalist view. See also Richard's paper [30] for some objections to temporalism and King's [17] for some criticisms on the view of tenses as operators and a defense of an externalist view.

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