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TOLERATING DEONTIC CONFLICTS BY ADAPTIVELY RESTRICTING INHERITANCE*

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Abstract
In order to deal with the possibility of deontic conflicts Lou Goble developed a group of logics (DPM) that are characterized by a restriction of the inheritance principle. While they approximate the deductive power of standard deontic logic, they do so only if the user adds certain statements to the premises. By adaptively strengthening the DPM logics, this paper presents logics that overcome this shortcoming. Furthermore, they are capable of modeling the dynamic and defeasible aspect of our normative reasoning by their dynamic proof theory. This way they enable us to have a better insight in the relations between obligations and thus to localize deontic conflicts.

1. *Introduction*

Recent work in deontic logics has shown a growing interest in systems that are able to deal with deontic conflicts (e.g., [9, 11, 12, 14, 15, 18, 20, 22, 27, 32]). A deontic conflict between obligations occurs when the obligations cannot be jointly realized. Note that deontic conflicts are not just an abstruse philosophical notion, but that they occur quite commonly in our every-day moral lives (see e.g. [16, 29]). This has for instance to do with the fact that different obligations and behavioral codices may stem from different moral systems and institutions. Sartre famously reports on one of his students who found himself in an unfortunate situation. On the one hand, he felt obliged to support the French army in their resistance against Nazi Germany. On the other hand, however, there was the obligation to stay at home in order to support his ill mother. Obviously, it was not possible for him to fulfill both obligations simultaneously.

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In deontic logics a modal operator O is used where $\mathsf{O}A$ expresses the obligation to bring about A. In order to accommodate deontic conflicts systems that tolerate them need to be developed, i.e., systems that do not lead to triviality when applied to conflicting obligations. Formally, conflict-tolerant deontic logics do not validate the following principle of deontic explosion:

$$\vdash (\mathsf{O}A \wedge \mathsf{O}\neg A) \supset \mathsf{O}B \tag{DEX}$$

Note that standard deontic logic (SDL)[1] is not conflict-tolerant. One reason is that it validates the principle (D), $\vdash \mathsf{O}A \supset \neg\mathsf{O}\neg A$. Thus, $\neg(\mathsf{O}A \wedge \mathsf{O}\neg A)$ is a theorem of SDL and all conflicts of the form $\mathsf{O}A \wedge \mathsf{O}\neg A$ lead to explosion.

There are various proposals for conflict-tolerant deontic logics. First, one could restrict or reject the *ex contradictione quodlibet* principle ($(A \wedge \neg A) \vdash B$, for any B), i.e., go paraconsistent (see e.g. [9, 11, 22]). Another approach is to restrict the aggregation principle (if $\mathsf{O}A$ and $\mathsf{O}B$, then $\mathsf{O}(A \wedge B)$) or to abandon it (see [12, 13, 20, 27]).

Yet another approach is given by Goble's logics DPM (see [14, 15]). They prevent deontic explosion by restricting the inheritance principle

$$\text{If } \vdash A \supset B, \text{ then } \vdash \mathsf{O}A \supset \mathsf{O}B \tag{RM}$$

Note that any system that validates full aggregation, full inheritance as well as *ex contradictione quodlibet* leads to explosion when applied to conflicts of the form $\mathsf{O}A \wedge \mathsf{O}\neg A$. By aggregation $\mathsf{O}(A \wedge \neg A)$ is derivable from $\mathsf{O}A \wedge \mathsf{O}\neg A$ and, in view of *ex contradictione quodlibet*, $\mathsf{O}B$ follows from $\mathsf{O}(A \wedge \neg A)$ by (RM).

We will argue in this paper that, although Goble's DPM logics are conflict-tolerant with respect to conflicting obligations, they are suboptimal in other respects. In order to overcome this, we will present adaptive strenghtenings of the DPM logics. The idea behind adaptive logics (see [3, 4]) is to interpret a given premise set "as normally as possible". In our case obligations are interpreted as non-conflicting as possible. It will be demonstrated that the adaptive systems are significantly stronger than the DPM logics and approximate SDL. For instance, for premise sets that are conflict-free, the adaptive versions of the DPM systems lead to exactly the same consequence set as SDL.

Let us outline the structure of this paper. In Section 2, we introduce Goble's DPM systems and explain their semantics in Section 3. We show that the DPM systems have some shortcomings in Section 4. Motivated by

[1] SDL is obtained by adding the principle (D), $\vdash \mathsf{O}A \supset \neg\mathsf{O}\neg A$ to the normal modal logic K. There are various alternative axiomatizations of SDL, cfr. footnote 5.

the limitations of the DPM systems, we introduce the reader into adaptive logics in Section 5. In Sections 6 and 7 we present the adaptive strengthenings ADPM.1 and ADPM.2′. We list some meta-theoretical properties of the adaptive logics in Section 8. In Section 9 we discuss some shortcomings of our logics and relate them to other systems. Finally, in Section 10 we offer a conclusion. The appendix features the proofs of our results.

2. *Dealing With Deontic Conflicts by Restricting Inheritance*

In the remainder we work with a propositional language enriched by a monadic obligation operator O. Where $\mathcal{S}$ is the set of sentential letters, our set of well-formed formulas $\mathcal{W}$ is given by the $\langle \neg, \wedge, \vee, \supset, \mathsf{O}\rangle$-closure of $\mathcal{S}$, $\bot$ and $\top$ with the usual rules for brackets. We define $A \equiv B$ by $(A \supset B) \wedge (B \supset A)$ and the permission operator $\mathsf{P}A$ by $\neg\mathsf{O}\neg A$.

The idea behind Goble's DPM systems is to restrict the inheritance principle via permission statements. The full inheritance principle (RM) is replaced by the following 'rule of permitted inheritance'

$$\text{If } \vdash A \supset B, \text{ then } \vdash \mathsf{P}A \supset (\mathsf{O}A \supset \mathsf{O}B) \tag{RPM}$$

What the rule (RPM) comes to is this: if A is obligatory and A entails B, then B is also obligatory *provided that* it is explicitly stated that A is permitted, or what comes to the same, that the obligation to bring about A is unconflicted.[2] Thus, $\mathsf{O}B$ follows neither from $\Gamma_1 = \{\mathsf{O}(A \wedge B), \mathsf{O}\neg(A \wedge B)\}$ nor from $\Gamma_2 = \{\mathsf{O}(A \wedge B)\}$, but it does follow from $\Gamma_3 = \{\mathsf{O}(A \wedge B), \mathsf{P}(A \wedge B)\}$.

Classical propositional logic enriched with the rules (RPM), and

$$\text{If } \vdash A \equiv B, \text{ then } \vdash \mathsf{O}A \equiv \mathsf{O}B \tag{RE}$$

and the axioms

$$\vdash \mathsf{O}\top \tag{N}$$

$$\vdash (\mathsf{O}A \wedge \mathsf{O}B) \supset \mathsf{O}(A \wedge B) \tag{AND}$$

defines the system DPM.1. More precisely, DPM.1 is the least set of formulas containing all classical tautologies of formulas of $\mathcal{W}$, plus all instances of (N) and (AND), that is closed under Modus Ponens, (RE), and (RPM) with '$\vdash$' indicating membership in DPM.1. We define in a canonical way, $\vdash_{\mathsf{DPM.1}}$

[2] In view of the definition of $\mathsf{P}A$, $\mathsf{O}A \wedge \mathsf{P}A$ expresses that the obligation OA is unconflicted.

A iff A is a member of $\mathsf{DPM.1}$. Furthermore, where $\Gamma \subseteq \mathcal{W}$, $\Gamma \vdash_{\mathsf{DPM.1}} A$ iff for some $B_1, \ldots, B_n \in \Gamma$ we have $\vdash_{\mathsf{DPM.1}} (B_1 \wedge \cdots \wedge B_n) \supset A$.[3]

Besides $\mathsf{DPM.1}$ Goble presented another system, $\mathsf{DPM.2}$, that also employs the restricted inheritance principle (RPM), but that moreover restricts aggregation. We have motivated the restriction of the inheritance principle and of the aggregation principle as a way to gain conflict-tolerant deontic logics. As will be stated in Theorem 1, $\mathsf{DPM.1}$ does not validate (DEX). Hence, since $\mathsf{DPM.1}$ is already a conflict-tolerant deontic logic, the question arises concerning the use of this further restriction. Let us give some reasons. First, it is not clear that aggregation should hold unrestrictedly. For instance, should aggregation be applied to conflicting obligations? Example: do we want to derive $\mathsf{O}(A \wedge B)$ from $\{\mathsf{O}A,\ \mathsf{O}\neg A,\ \mathsf{O}B\}$? Analogously, should aggregation be applied in cases where it leads to (additional) deontic conflicts? For instance, should one allow that $\mathsf{O}(A \wedge B)$ is derivable from $\{\mathsf{O}A,\ \mathsf{O}B,\ \mathsf{O}\neg(A \wedge B)\}$, thus creating an additional conflict? A negative answer to these questions motivates the restriction of aggregation. Secondly, principle (P), $\neg\mathsf{O}\bot$, has quite some intuitive appeal. Obviously it is impossible to bring about $\bot$. The Kantian principle 'ought implies can' says that we are not obliged to bring about things that are impossible to realize. However, allowing for unrestricted aggregation in the presence of a conflict $\mathsf{O}A \wedge \mathsf{O}\neg A$ leads to $\mathsf{O}(A \wedge \neg A)$ and hence to $\mathsf{O}\bot$. Thus, adding (P) as an axiom to $\mathsf{DPM.1}$ leads to explosion when applied to deontic conflicts. This can be avoided by restricting aggregation.

Due to the fact that there are various conflict-tolerant deontic logics that *only* restrict (or abandon) aggregation, the reader may still wonder why in $\mathsf{DPM.2}$ both principles are restricted. One reason is, as Goble pointed out in his [15], that many systems that restrict (but do not abandon) aggregation are not conflict-tolerant *enough*. In his critical analysis he elaborated various refined explosion principles. Besides the very strict notion of deontic explosion that underlies (DEX), namely situations in which all obligations are derivable, there are weaker notions. Take for instance the following explosion principle:[4]

$$\text{If } \nvdash \neg B \text{ then } \mathsf{O}A, \mathsf{O}\neg A \vdash \mathsf{O}B \qquad \text{(DEX-1)}$$

[3] See also [28] where the authors define consequence relations for rank-1 modal logics in this way and prove strong completeness.

[4] We slightly adjusted the criteria (DEX-1)–(DEX-3) (the latter two will be introduced in a moment) offered by Goble since his criteria were formulated in terms of theoremhood while we focus on the consequences of premise sets.

Another notion of deontic explosion is given if, for every B, $\mathsf{O}B \vee \mathsf{O}\neg B$ is derivable. Semantically speaking this corresponds to the case where all models are such that for every B there is either the obligation to bring about B or there is the obligation to bring about not-B. Although weaker than (DEX) it is equally counter-intuitive that $\mathsf{O}B \vee \mathsf{O}\neg B$ is derivable from $\{\mathsf{O}A \wedge \mathsf{O}\neg A\}$. Hence, we expect from conflict-tolerant deontic logics that they do not validate the following explosion principle:

$$\mathsf{O}A, \mathsf{O}\neg A \vdash \mathsf{O}B \vee \mathsf{O}\neg B \qquad \text{(DEX-2)}$$

This may be weakened further. Facing a deontic conflict, $\mathsf{O}A \wedge \mathsf{O}\neg A$ as well as an unconflicted obligation $\mathsf{O}C \wedge \neg\mathsf{O}\neg C$, it would be undesired that, for every B, the formula $\mathsf{O}B \vee \mathsf{O}\neg B$ would be derivable. This is expressed as follows:

$$\mathsf{O}A, \mathsf{O}\neg A, \mathsf{O}C, \neg\mathsf{O}\neg C \vdash \mathsf{O}B \vee \mathsf{O}\neg B \qquad \text{(DEX-3)}$$

Validating (DEX-3) is counter-intuitive, since for some arbitrary B the conflict $\mathsf{O}A \wedge \mathsf{O}\neg A$ together with the other, otherwise unrelated and unproblematic obligation $\mathsf{O}C$ does not entail that we are either obliged to bring about B or to bring about $\neg B$: $\mathsf{O}B \vee \mathsf{O}\neg B$.

By restricting aggregation along with inheritance the various advantages can be combined. In this way we gain systems that follow the Kantian intuition 'ought implies can', that hence validate (P), and that are strongly conflict-tolerant such that they do not validate any of the explosion principles (DEX), (DEX-1)–(DEX-3).

In order to achieve such a conflict-tolerant logic, Goble uses the following permitted aggregation principle:

$$\vdash \big(\mathsf{O}A \wedge \mathsf{O}B \wedge \mathsf{P}(A \wedge B)\big) \supset \mathsf{O}(A \wedge B) \qquad \text{(PAND)}$$

The idea is to apply aggregation to $\mathsf{O}A$ and $\mathsf{O}B$, *provided that* $A \wedge B$ is explicitly permitted. Goble's logic DPM.2 is defined by (RPM), (RE), (N), (P), and (PAND). The consequence relation $\vdash_{\mathsf{DPM.2}}$ is defined analogous to $\vdash_{\mathsf{DPM.1}}$.

There is an alternative way of restricting aggregation that offers several advantages over (PAND), namely:

$$\vdash \big(\mathsf{O}A \wedge \mathsf{O}B \wedge \mathsf{P}A \wedge \mathsf{P}B\big) \supset \mathsf{O}(A \wedge B) \qquad \text{(PAND}')$$

Here, the idea is to apply aggregation to $\mathsf{O}A$ and $\mathsf{O}B$ provided that *both* A and B are explicitly permitted.

The logic $\mathsf{DPM.2'}$ is defined by (RPM), (RE), (N), (P), and (PAND′). The consequence relation $\vdash_{\mathsf{DPM.2'}}$ is defined analogous to $\vdash_{\mathsf{DPM.1}}$.

Henceforth we will use DPM as a generic term for $\mathsf{DPM.1}$, $\mathsf{DPM.2}$ and $\mathsf{DPM.2'}$.

As announced already, DPM is sufficiently conflict-tolerant not to validate any of the introduced explosion principles.

Theorem 1: *Where* $\mathsf{L} \in \{\mathsf{DPM.1}, \mathsf{DPM.2}, \mathsf{DPM.2'}\}$, L *does not validate any of the explosion principles* (DEX), (DEX-1)–(DEX-3).

Lou Goble argued in [14] in favor of the following criterion of adequacy for conflict-tolerant deontic logics:

> $(\star)$: A conflict-tolerant deontic logic should be such that the result of adding (D), namely $\vdash \mathsf{O}A \supset \neg\mathsf{O}\neg A$, as an axiom leads to the same consequence relation as SDL.[5]

Theorem 2: *Where* $\alpha \in \{1, 2'\}$, $\mathsf{DPM}.\alpha$ *satisfies* $(\star)$.

The logic $\mathsf{DPM.2'}$ has several advantages as compared to Goble's $\mathsf{DPM.2}$. First, the restricted aggregation principle of $\mathsf{DPM.2'}$, i.e. (PAND′), coheres better with the idea underlying (RPM) than (PAND). Note that the idea underlying (RPM) was to restrict inheritance to those obligations that are explicitly permitted, or what comes to the same, are explicitly unconflicted. This idea is applied to the aggregation principle by (PAND′) — aggregation can only be applied if both obligations are explicitly unconflicted. In contrast, the idea underlying (PAND) is to apply aggregation to $\mathsf{O}A$ and $\mathsf{O}B$ provided that the *outcome* of the aggregation, $A \wedge B$, is explicitly permitted. Thus in the case of (PAND), but not in the case of (PAND′), $\mathsf{O}(A \wedge B)$ is derivable from $\{\mathsf{O}A,\ \mathsf{O}\neg A,\ \mathsf{O}B, \mathsf{P}(A \wedge B)\}$.[6] Second, $\mathsf{DPM.2'}$ satisfies Goble's criterion $(\star)$ while $\mathsf{DPM.2}$ does not. Third, by choosing $\mathsf{DPM.2'}$ instead of $\mathsf{DPM.2}$ as a basis for the adaptive strengthenings that are introduced in Section 5 we will avoid some technical problems.

[5] Goble axiomatizes SDL by adding (D), (N), (RE), (RM), and (AND) to full propositional logic. A consequence relation $\vdash_{\mathsf{SDL}}$ can be defined analogous to $\vdash_{\mathsf{DPM.1}}$. We slightly adjusted Goble's $(\star)$ since he is mainly interested in theoremhood, while we focus on consequence relations.

[6] A restricted inheritance principle following the intuition of (PAND) would be: If $\vdash A \supset B$, then $\vdash \mathsf{P}B \supset (\mathsf{O}A \supset \mathsf{O}B)$. Inheritance is applied to $\mathsf{O}A$ in order to derive $\mathsf{O}B$ if it does not result in a deontic conflict $\mathsf{O}B \wedge \mathsf{O}\neg B$.

Before we take a look at some of the shortcomings of the DPM logics, let us introduce the semantics.

3. *The Semantics of* DPM

The semantics that we introduce in this section are very similar to Goble's neighborhood semantics for his DPM logics in [14]. The only difference is that we employ an actual world. This makes the semantics philosophically more intuitive for our application, since we are not only interested in modeling theoremhood but also in defining a semantic consequence relation.

One of the basic ideas for the neighborhood semantics is that propositions are interpreted in terms of sets of worlds. Moreover, each world has associated with it propositions, i.e., sets of worlds. The idea is that an obligation $\mathsf{O}A$ is true at a world w, in case A is one of its associated propositions.

Let $\wp(X)$ be the power-set of some set X. A neighborhood frame F is a tuple $\langle W, \mathcal{O}\rangle$ where W is a set of points and $\mathcal{O} : W \to \wp(\wp(W))$. We call elements of W worlds. Thus, $\mathcal{O}$ assigns to each world $w \in W$ a set of propositions, i.e., $\mathcal{O}(w) \subseteq \wp(W)$. We write from now on $\mathcal{O}_w$ instead of $\mathcal{O}(w)$. An F-model M on a frame F is a triple $\langle F, v, @\rangle$ where $@ \in W$ is called the actual world and $v : \mathcal{S} \to \wp(W)$. A propositional atom is mapped by v into the set of worlds in which it is supposed to hold. Where $w \in W$ and $|A|_M =_{\text{df}} \{w \in W \mid M, w \models A\}$, we define:

(M-$\mathcal{P}$) $M, w \models A$ iff $w \in v(A)$, where $A \in \mathcal{S}$
(M-$\mathcal{O}$) $M, w \models \mathsf{O}A$ iff $|A|_M \in \mathcal{O}_w$
(M-$\neg$) $M, w \models \neg A$ iff $M, w \not\models A$
(M-$\vee$) $M, w \models A \vee B$ iff $M, w \models A$ or $M, w \models B$
(M-$\wedge$) $M, w \models A \wedge B$ iff $M, w \models A$ and $M, w \models B$
(M-$\supset$) $M, w \models A \supset B$ iff $M, w \models \neg A \vee B$
(M-$\top$) $M, w \models \top$
(M-$\bot$) $M, w \not\models \bot$

Furthermore, $M \models A$ iff $M, @ \models A$. Where $\Gamma \subseteq \mathcal{W}$, we say that M is an F-model of Γ iff M is an F-model and $M \models A$ for all $A \in \Gamma$.

We define the following requirements on frames $F = \langle W, \mathcal{O}\rangle$. For all $w \in W$:

a) $W \in \mathcal{O}_w$
b) If $X \in \mathcal{O}_w$ and $Y \in \mathcal{O}_w$, then $X \cap Y \in \mathcal{O}_w$
b′) If $X \in \mathcal{O}_w$; $Y \in \mathcal{O}_w$; $W \setminus X \notin \mathcal{O}_w$; and $W \setminus Y \notin \mathcal{O}_w$, then $X \cap Y \in \mathcal{O}_w$
b″) If $X \in \mathcal{O}_w$; $Y \in \mathcal{O}_w$; $W \setminus (X \cap Y) \notin \mathcal{O}_w$, then $X \cap Y \in \mathcal{O}_w$

c) If $X \subseteq Y$; $X \in \mathcal{O}_w$ and $W \setminus X \notin \mathcal{O}_w$, then $Y \in \mathcal{O}_w$
d) $\emptyset \notin \mathcal{O}_w$

Condition a) corresponds to (N), b) corresponds to (AND), b′) corresponds to (PAND′), b″) corresponds to (PAND), c) corresponds to (RPM), and d) corresponds to (P). We call the class of all frames that satisfy a), b) and c) the DPM.1-frames, the ones that satisfy a), b′), c) and d) the DPM.2′-frames, and the ones that satisfy a), b″), c) and d) the DPM.2-frames.

Let $\Gamma \subseteq \mathcal{W}$. A semantic consequence relation can be defined as follows. Where F is a frame, $\Gamma \Vdash_F A$ iff for all F-models M of Γ, $M \models A$. Moreover, where $\alpha \in \{1, 2, 2'\}$, $\Gamma \Vdash_{\mathsf{DPM}.\alpha} A$ iff $\Gamma \Vdash_F A$ for all DPM.α-frames F.

Theorem 3: *Where* $\alpha \in \{1, 2, 2'\}$ *and* $\Gamma \subseteq \mathcal{W}$, $\Gamma \vdash_{\mathsf{DPM}.\alpha} A$ *iff* $\Gamma \Vdash_{\mathsf{DPM}.\alpha} A$.

4. *Some Shortcomings of* DPM

In order to apply the weakened inheritance principle (resp. also the weakened aggregation principle in the case of DPM.2 and DPM.2′) the user has to "manually" add permission statements. For instance, in order to apply the restricted inheritance principle (RPM) to $\mathsf{O}A$ we also need $\mathsf{P}A$. In cases in which $\mathsf{P}A$ is not derivable from the premises by means of DPM, the user has to add manually $\mathsf{P}A$ to the premises. This is suboptimal for various reasons.[7]

(1) For all interesting cases, determining which permission statements can safely be added to a set of premises (that is, in such a way that no explosion follows) requires *reasoning*. This kind of reasoning falls entirely outside the scope of the DPM systems and is therefore left to the user of the DPM systems. So, the DPM systems are inadequate to fully explicate the reasoning processes that are needed to apply the DPM systems in a sensible way (that is, in a way that modal inheritance is applied "as much as possible").

(2) The fact that permissions have to be added manually is especially problematic in cases where the relationship between the premises is interwoven. For instance in complicated setups it might not be obvious at all that $\mathsf{O}A \wedge \mathsf{O}\neg A$ is derivable. However, suppose that in this case the user naïvely

[7] In [32], van der Torre and Tan presented a sequential system which, in a first phase, disables the application of (RM) and allows for the application of a restricted aggregation rule. In a second phase, it disables this aggregation rule and allows for the application of (RM). Although this system overcomes this problem, it can do so only by introducing two different O-operators and by requiring that (RM) is never applied before the restricted aggregation rule. As the authors themselves admit, this is rather strange from an intuitive point of view (see also [15], pp. 470–471).

added $\mathsf{P}A$ to the premises in order to apply (RPM) to $\mathsf{O}A$. Since $\mathsf{P}A$ is equivalent to $\neg\mathsf{O}\neg A$ the user caused in this way an explosion.

(3) For most premise sets the DPM systems are rather weak. Recall that in order to achieve the deductive strength of SDL we had to add (D) to the axiomatization of DPM.1 (resp. DPM.2′). Suppose we accept SDL as the normative standard for the modeling of non-conflicting obligations. It would be desirable then that conflict-tolerant logics apply all rules of SDL to non-conflicting obligations. For instance, given a premise set $\Gamma \subseteq \mathcal{W}$ that is conflict-free, we expect a deontic logic to lead to a consequence set that is the same as that of SDL without the need of strengthening the premise set manually by adding instances of (D), or by adding premises. Thus, if the premises are conflict-free, the logic should apply all the rules of SDL unrestrictedly.

The discussion above motivates the following strengthening of Goble's requirement ($\star$).[8]

> ($\star\star$): For SDL-consistent premise sets[9] $\Gamma \subseteq \mathcal{W}$ a conflict-tolerant deontic logic should lead to the same consequence set as SDL.

Note that neither of the introduced DPM logics satisfies ($\star\star$). We will in the next section adaptively strengthen both logics so that they satisfy criterion ($\star\star$).

As should have become evident from the discussion above, there is a certain trade-off for monotonic deontic logics such as DPM. In order to offer conflict-tolerance certain SDL-principles such as (D) and inheritance have to be restricted or abandoned. In return, this weakens the logics even in cases in which it would be unproblematic to apply the principles in question.

The adaptive strengthenings introduced in the next sections will overcome this trade-off. On the one hand, applying the logic does not involve any user interference. On the other hand, by interpreting a premise set as non-conflicting as possible, principles such as inheritance, (D) and aggregation in the case of the adaptive strengthening of DPM.2′ will be applied as much as possible.

[8] To stay in line with Goble's ($\star$), we formulate the strengthened requirement in terms of SDL. If one's preferred logic is different from SDL, the requirement may easily be adapted. The basic idea is that, where one's preferred deontic logic (for conflict-free premise sets) is L, one expects from a conflict-tolerant deontic logic on the basis of L that it leads to the same consequence set as L for all L-consistent premise sets. This is exactly what adaptive logics allow for.

[9] Γ is SDL-consistent iff $\Gamma \nvdash_{\mathsf{SDL}} \bot$.

5. *Adaptive Logics*

The main feature of adaptive logics is that they interpret a given premise set "as normally as possible". The standard of normality depends on the application. For instance, there are inconsistency-adaptive logics that, while allowing for classical inconsistencies, interpret a given premise set as consistently as possible (see e.g. [1, 4]). In our case the idea is to interpret premise sets "as non-conflicting as possible". We will in the following give a precise meaning to this vague notion. Adaptive logics in standard format are defined as a triple:

1. the lower limit logic, i.e., a reflexive, transitive, monotonic, and compact logic[10] that has a characteristic semantics and contains classical logic,
2. the set of abnormalities Ω, characterized by a (possibly restricted) logical form,
3. the adaptive strategy, in this paper the minimal abnormality strategy.

The adaptive logic strengthens its lower limit logic. Since we are interested in interpreting premise sets as non-conflicting as possible, a minimal requirement for our lower limit is that it is conflict-tolerant. We will use $\mathsf{DPM.1}$ or $\mathsf{DPM.2'}$ for this purpose.[11] Let henceforth $\alpha \in \{1, 2'\}$. In order to avoid an unnecessary level of abstraction we introduce the reader into adaptive logics immediately by means of the systems that we are going to introduce in this paper.[12]

The adaptive strategy together with the abnormalities determine what it means to interpret the premises as normally as possible. The standard format currently is defined for two strategies: the reliability strategy and the minimal abnormality strategy. Choosing the latter results in a stronger consequence relation. However, the proof theory is more involving and it is computationally more complex compared to the one on the basis of the reliability strategy [34]. The reason why we prefer the minimal abnormality strategy over the reliability strategy for our application is that it allows for the derivation of $\mathsf{O}(A \vee B)$ from two incompatible obligations $\mathsf{O}A$ and $\mathsf{O}B$

[10] Where $Cn_{\mathsf{L}}(\Gamma)$ denotes the consequence set of some premise set Γ for the logic L, L is reflexive iff $\Gamma \subseteq Cn_{\mathsf{L}}(\Gamma)$, it is monotonic iff $Cn_{\mathsf{L}}(\Gamma) \subseteq Cn_{\mathsf{L}}(\Gamma \cup \Gamma')$, it is transitive iff if $\Gamma' \subseteq Cn_{\mathsf{L}}(\Gamma)$ then $Cn_{\mathsf{L}}(\Gamma') \subseteq Cn_{\mathsf{L}}(\Gamma)$ and it is compact iff if $A \in Cn_{\mathsf{L}}(\Gamma)$ then there is a finite $\Gamma' \subseteq \Gamma$ such that $A \in Cn_{\mathsf{L}}(\Gamma')$.

[11] We will discuss the case of $\mathsf{DPM.2}$ being a lower limit logic shortly in Section 7.

[12] For a more generic introduction see [3, 4].

while reliability does not. We will give an example of this later on (see Example 7).

Since our aim is to interpret a given premise set Γ as non-conflicting as possible, we define our abnormalities to be deontic conflicts, $\mathsf{O}A \wedge \mathsf{O}\neg A$. We denote the set of all abnormalities by Ω.

Now we have the three elements needed to define the adaptive logics that are explicated in this paper: (1) the lower limit logic is either $\mathsf{DPM.1}$ or $\mathsf{DPM.2'}$, (2) the set of abnormalities is $\Omega = \{\mathsf{O}A \wedge \mathsf{O}\neg A \mid A \in \mathcal{W}\}$, and (3) the strategy is minimal abnormality. We dub these systems $\mathsf{ADPM.1}$ and $\mathsf{ADPM.2'}$.

Semantically the minimal abnormality strategy is realized by selecting a certain well-defined set of $\mathsf{DPM}.\alpha$-models of a given premise set Γ, namely the ones that are "minimally abnormal". The models are selected with respect to their abnormal part, i.e. the abnormalities they verify: where M is a $\mathsf{DPM}.\alpha$-model, $\text{Ab}(M) = \{A \in \Omega \mid M \models A\}$. For a given logic L, we write $\mathcal{M}_{\mathsf{L}}(\Gamma)$ for the set of L-models verifying all members of Γ.

Definition 1: A $\mathsf{DPM}.\alpha$-model $M \in \mathcal{M}_{\mathsf{DPM}.\alpha}(\Gamma)$ is *minimally abnormal* iff there is no $\mathsf{DPM}.\alpha$-model $M' \in \mathcal{M}_{\mathsf{DPM}.\alpha}(\Gamma)$ such that $\text{Ab}(M') \subset \text{Ab}(M)$.[13] We write $\mathcal{M}_{\mathsf{ADPM}.\alpha}(\Gamma)$ for all the minimal abnormal $\mathsf{DPM}.\alpha$-models of Γ.

$\Gamma \Vdash_{\mathsf{ADPM}.\alpha} A$ (A is an $\mathsf{ADPM}.\alpha$-*semantic consequence of* Γ) iff A is verified by all $M \in \mathcal{M}_{\mathsf{ADPM}.\alpha}(\Gamma)$.

Hence all the selected models are such that they validate a minimal amount of conflicts (in the set-theoretical sense). This justifies our claim that the adaptive logics interpret the premises as non-conflicting as possible.

Let us proceed with the syntactic counter-part to the semantic selection. It is realized by dynamic adaptive proofs. While all the rules of $\mathsf{DPM}.\alpha$ are valid, a key feature of adaptive proofs is that they allow for certain additional rules to be applied *conditionally*. In our case the idea is to apply the inheritance principle or principle (D) conditionally. Recall that $\mathsf{DPM}.\alpha$, in order to avoid deontic explosions, only validates a restricted version of (RM). This led to the problem (discussed in Section 4) that in many cases the user needs to add manually permission statements. Note that the following is valid in $\mathsf{DPM}.\alpha$:

$$\text{If } \vdash_{\mathsf{DPM}.\alpha} A \supset B, \text{ then } \mathsf{O}A \vdash_{\mathsf{DPM}.\alpha} \mathsf{O}B \vee (\mathsf{O}A \wedge \mathsf{O}\neg A).$$

[13] As has been shown in [3, 4], the lower limit logic models of a premise set Γ are smooth with respect to their abnormal part and set inclusion. That is to say, for every lower limit model M of Γ there is a minimally abnormal model M' of Γ such that $\text{Ab}(M') \subseteq \text{Ab}(M)$. This property is called strong reassurance by adaptive logicians.

The underlying idea of the restriction is that inheritance is applicable to $\mathsf{O}A$ if there is no deontic conflict concerning $\mathsf{O}A$. In the adaptive logic we make use of this: the inheritance principle is applied to $\mathsf{O}A$ on the condition $\{\mathsf{O}A \wedge \mathsf{O}\neg A\}$. That is to say, on the condition that there is no reason to suppose that there is a deontic conflict concerning $\mathsf{O}A$. This is still very vague, but we will make it more precise in a moment. Suppose $\mathsf{O}A$ is one of the premises. A fragment of an adaptive proof may look as follows:

1	$\mathsf{O}A$	PREM	$\emptyset$
2	$\mathsf{O}(A \vee B)$	1; RC	$\{\mathsf{O}A \wedge \mathsf{O}\neg A\}$

Line 2 contains a conditional application of the inheritance rule to $\mathsf{O}A$. This is indicated by the generic rule RC (that is discussed in more detail later) and by a fourth column in which the conditions of the respective lines are contained. Conditions are finite sets of abnormalities. The condition of line 1 is empty since it is the result of a premise introduction. The condition of line 2 is $\{\mathsf{O}A \wedge \mathsf{O}\neg A\}$. Now suppose we are able to derive the following disjunction of abnormalities from the given premises at a line l:

l	$(\mathsf{O}A \wedge \mathsf{O}\neg A) \vee (\mathsf{O}C \wedge \mathsf{O}\neg C)$	$\ldots$; RU	$\emptyset$

By the generic rule RU we indicate the (unconditional) applications of the $\mathsf{DPM}.\alpha$ rules. Note that the disjunction of abnormalities that has been derived at line l also features the condition of line 2. This gives us reasons to suspect that $\mathsf{O}A$ may after all be part of a deontic conflict. In this case line 2 is marked according to the marking definition. We will define the marking for the minimal abnormality strategy in a moment. Formulas that are the second element of marked lines are not considered to be derived at that stage. Note however that markings may come and go. Assume for instance that $\mathsf{O}C \wedge \mathsf{O}\neg C$ has been derived at a later stage of the proof. In this case there is no reason anymore to suspect that $\mathsf{O}A$ is part of a deontic conflict and the conditional application of inheritance at line 2 can again be considered as valid. The marking will thus be defined on basis of the minimal disjunctions of abnormalities that have been derived at a given stage of the proof.

We give now a precise account of the generic rules RU and RC and the marking definition.

We have already indicated that the rules of the lower limit logic $\mathsf{DPM}.\alpha$ are unrestrictedly valid in the adaptive logic. This is characterized by the generic rule RU:

$$\text{RU} \quad \text{If } A_1, \ldots, A_n \vdash_{\mathsf{DPM}.\alpha} B: \quad \begin{array}{ll} A_1 & \Delta_1 \\ \vdots & \vdots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \end{array}$$

Note that the conditions of the used lines featuring the A_i's are carried forward, resulting in B being derived on the condition $\Delta_1 \cup \ldots \cup \Delta_n$. Moreover, premises are introduced by PREM on the empty condition.

The conditional rule allows for conditional applications of certain rules that are invalid in DPM. As is usually done by adaptive logic scholars we use the notation $\mathsf{Dab}(\Delta) =_{\text{df}} \bigvee_{A \in \Delta} A$ where Δ is a finite set of abnormalities ($\Delta \subset \Omega$). Conditional applications of rules are handled by the generic rule RC:

$$\text{RC} \quad \text{If } A_1, \ldots, A_n \vdash_{\mathsf{DPM}.\alpha} B \vee \mathsf{Dab}(\Theta): \quad \begin{array}{ll} A_1 & \Delta_1 \\ \vdots & \vdots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \cup \Theta \end{array}$$

In addition to the conditional rule, we also need a *marking definition*. This determines when a condition of a line is violated and hence has to be marked. The marking indicates that the formula that occurs on that line is no longer considered as derived in the proof. In order to formulate the marking definition, we need a few more notions.

A formula $\mathsf{Dab}(\Delta)$ is a *minimal Dab-formula* at a stage s of the proof iff it is the formula of a line with condition $\emptyset$ and no $\mathsf{Dab}(\Delta')$ with $\Delta' \subset \Delta$ is the formula of a line with condition $\emptyset$. A *choice set* of $\Sigma = \{\Delta_1, \Delta_2, \ldots\}$ is a set that contains an element out of each member of Σ. A *minimal choice set* of Σ is a choice set of Σ of which no proper subset is a choice set of Σ. Where $\mathsf{Dab}(\Delta_1), \mathsf{Dab}(\Delta_2), \ldots$ are the minimal Dab-formulas that are derived at stage s, $\Phi_s(\Gamma)$ is the set of minimal choice sets of $\{\Delta_1, \Delta_2, \ldots\}$ for a premise set Γ.

Definition 2: (Marking for minimal abnormality) Line i is marked at stage s iff, where A is the second element and Δ is the condition of line i,

(i) there is no $\Delta' \in \Phi_s(\Gamma)$ such that $\Delta' \cap \Delta = \emptyset$, or
(ii) for some $\Delta' \in \Phi_s(\Gamma)$, there is no line at which A is derived on a condition Θ for which $\Delta' \cap \Theta = \emptyset$.

Given a set of abnormalities Ω, the marking definition determines whether lines are "in" or "out" of the proof at a certain stage, i.e., it governs the dynamics of the proof procedure.

In our introductory example it was illustrated that markings in an adaptive proof may come and go. While line 2 is marked as long as $(\mathsf{O}A \wedge \mathsf{O}\neg A) \vee (\mathsf{O}C \wedge \mathsf{O}\neg C)$ is a minimal Dab-formula, it is unmarked as soon as $\mathsf{O}C \wedge \mathsf{O}\neg C$ is derived.

In order to define the consequence set of an adaptive logic we are interested in a stable criterion for derivability.

Definition 3: A is *finally derived* from Γ on line i of a proof at a finite stage s iff (i) A is the second element of line i, (ii) line i is not marked at stage s and (iii) every extension of the proof in which line i is marked may be further extended in such a way that line i is unmarked.

$\Gamma \vdash_{\mathsf{ADPM}.\alpha} A$ (A is *finally* $\mathsf{ADPM}.\alpha$*-derivable* from Γ) iff A is finally derived on a line of a proof from Γ.

Let us state a central representational result for adaptive logics in standard format proven in [3]:

Theorem 4: *Where* $\Gamma \subseteq \mathcal{W}$*,* $\Gamma \vdash_{\mathsf{ADPM}.\alpha} A$ *iff* $\Gamma \Vdash_{\mathsf{ADPM}.\alpha} A$*.*

Besides the lower limit logic there is also an upper limit logic for each adaptive logic. It is the strengthening $\mathsf{UDPM}.\alpha$ of $\mathsf{DPM}.\alpha$ that trivializes abnormalities. It is defined by, $\Gamma \vdash_{\mathsf{UDPM}.\alpha} A$ iff $\Gamma \cup \{\neg B \mid B \in \Omega\} \vdash_{\mathsf{DPM}.\alpha} A$.

We will show that for the adaptive systems introduced in this paper the upper limit $\mathsf{UDPM}.\alpha$ is SDL. For a proof of the following result see Theorem 12 in [3] (note that there premise sets that are consistent with respect to the upper limit logic are called "normal").

Theorem 5: *For* $\mathsf{UDPM}.\alpha$*-consistent premise sets* Γ*,* $\mathsf{ADPM}.\alpha$ *leads to the same consequence set as* $\mathsf{UDPM}.\alpha$*.*

In the remainder we will sometimes speak about adding an axiom to $\mathsf{ADPM}.\alpha$. What we mean is the adaptive logic that is the result of adding the axiom to the lower limit logic, i.e., $\langle \mathsf{DPM}.\alpha*, \Omega,$ minimal abnormality$\rangle$ where $\mathsf{DPM}.\alpha*$ is $\mathsf{DPM}.\alpha$ strengthened by the axiom.

6. *The Adaptive Logic* ADPM.1

In this section we introduce a concrete adaptive system on the basis of Lou Goble's DPM.1. As discussed above, ADPM.1 is defined by the triple

$$\langle \mathsf{DPM.1}, \Omega, \text{minimal abnormality} \rangle.$$

We have already mentioned the basic idea. In order to interpret a given premise set as non-conflicting as possible we define the abnormalities to be deontic conflicts, $\Omega = \{\mathsf{O}A \wedge \mathsf{O}\neg A \mid A \in \mathcal{W}\}$. This makes it possible to apply the inheritance principle to $\mathsf{O}A$ on the condition $\{\mathsf{O}A \wedge \mathsf{O}\neg A\}$ by the generic rule RC. Moreover, also (D) may be applied conditionally. Note that the following is valid:

$$\mathsf{O}A \vdash_{\mathsf{DPM}} \mathsf{P}A \vee (\mathsf{O}A \wedge \mathsf{O}\neg A)$$

This allows to derive $\mathsf{P}A$ from $\mathsf{O}A$ on the condition $\{\mathsf{O}A \wedge \mathsf{O}\neg A\}$ by the generic rule RC.

Example 1: Let us take a look at a first concrete example of a proof in ADPM.1:

1	$\mathsf{O}A$	PREM	$\emptyset$
2	$\mathsf{O}\neg A$	PREM	$\emptyset$
3	$\mathsf{O}(B \wedge C)$	PREM	$\emptyset$
4	$\mathsf{O}B$	3; RC	$\{\mathsf{O}(B \wedge C) \wedge \mathsf{O}\neg(B \wedge C)\}$
5	$\mathsf{P}B$	4; RC	$\{\mathsf{O}(B \wedge C) \wedge \mathsf{O}\neg(B \wedge C), \mathsf{O}B \wedge \mathsf{O}\neg B\}$

Note that $\mathsf{O}A, \mathsf{O}\neg A, \mathsf{O}(B \wedge C) \nvdash_{\mathsf{DPM.1}} \mathsf{O}B$ and $\mathsf{O}A, \mathsf{O}\neg A, \mathsf{O}(B \wedge C) \nvdash_{\mathsf{DPM.1}} \mathsf{P}B$. In Section 4 we have pointed out that this is suboptimal. A deontic logic should apply the rules of SDL to non-conflicting parts of the premise set and there should be no need for the user to add permission statements. In order to derive $\mathsf{O}B$ from $\mathsf{O}(B \wedge C)$ the user of DPM.1 would have to manually add $\mathsf{P}(B \wedge C)$ to the premises. Moreover, there is no way of deriving $\mathsf{P}B$ from the given premises in DPM.1.

In contrast, the adaptive logic ADPM.1 applies inheritance conditionally to $\mathsf{O}(B \wedge C)$ in order to derive $\mathsf{O}B$ at line 4. Moreover, (D) is applied conditionally to $\mathsf{O}B$ in order to derive $\mathsf{P}B$ at line 5. Note that $\mathsf{O}(B \wedge C)$ and $\mathsf{O}B$ are unrelated to the deontic conflict $\mathsf{O}A \wedge \mathsf{O}\neg A$, and hence, as discussed in Section 4, inheritance and (D) should be applicable to them. It can easily be seen that lines 4 and 5 are finally derived.

In the remainder we will frequently make use of the following fact and of the abbreviation $!A$ for $\mathsf{O}A \wedge \mathsf{O}\neg A$.

Fact 1: *Where* $\alpha \in \{1, 2'\}$*, the following holds in* $\mathsf{DPM}.\alpha$*:*[14]

$$\textit{If } \vdash_{\mathsf{CL}} \neg(A \wedge B), \textit{ then } \mathsf{O}A, \mathsf{O}B \vdash_{\mathsf{DPM}.\alpha} !A \vee !B.$$

The fact shows that a pair of conflicting obligations gives rise to a disjunction of abnormalities in the DPM logics, even in case they are not directly conflicting (such as $\mathsf{O}A$ and $\mathsf{O}\neg A$).

Example 2: In the example above it was demonstrated that ADPM.1 delivers all the consequences of a premise ($\mathsf{O}(B\wedge C)$) that is consistent with the other two directly conflicting premises ($\mathsf{O}A, \mathsf{O}\neg A$). In the following example we have two conflicting premises: $\mathsf{O}\neg A$ and $\mathsf{O}(A \wedge B)$. Note that $\mathsf{O}B$ is — via unrestricted inheritance — a consequence of $\mathsf{O}(A \wedge B)$ and that it is not conflicting with the premise $\mathsf{O}\neg A$. This poses the question whether $\mathsf{O}B$ is a desired consequence of these premises. We now demonstrate that ADPM.1 takes a skeptical stance and does not lead to the consequence $\mathsf{O}B$.

By Fact 1 the premises lead to the Dab-formula $!A\vee!(A\wedge B)$. Semantically speaking we have thus two types of minimally abnormal models: models with abnormal part $\{!A\}$ on the one hand and models with abnormal part $\{!(A \wedge B)\}$ on the other hand. In the latter models inheritance is blocked from $\mathsf{O}(A\wedge B)$ and whence it is easy to see that there are minimal abnormal models of this type that do not verify $\mathsf{O}B$. Thus, $\mathsf{O}B$ is not valid in all minimal abnormal models. Let us take a look at an adaptive proof:

1	$\mathsf{O}\neg A$	PREM	$\emptyset$
2	$\mathsf{O}(A \wedge B)$	PREM	$\emptyset$
^{4}3	$\mathsf{O}B$	2; RC	$\{!(A \wedge B)\}$
4	$!A \vee !(A \wedge B)$	1,2; RU	$\emptyset$

Although $\mathsf{O}B$ is conditionally derivable at line 3, it is marked since the condition of line 3 is part of the minimal Dab-consequence derived at line 4.

What motivates the skeptical stance according to which $\mathsf{O}B$ is not a consequence of the given premises is that there may be certain causal connections between A and B. This is for instance the case where A stands for taking a train to London and B for buying the ticket for it. If there is also the obligation not to take the train, then despite the fact that there is no direct conflict between $\neg A$ and B in terms of being jointly realizable, there is nevertheless clearly a tension. As long as we don't have reasons to prefer the obligation to take the train there is no reason to derive the obligation to buy the ticket.

[14] This can easily be seen: from $\mathsf{O}A$ and restricted inheritance we get $\mathsf{O}\neg B \vee !A$. The latter together with $\mathsf{O}B$ results in $!A \vee !B$.

Example 3: Let us alter the example slightly by adding $\mathsf{O}A$ to the premises. This way we make the conflict between $\mathsf{O}A$ and $\mathsf{O}\neg A$ explicit in the premises: now $\mathsf{O}A \wedge \mathsf{O}\neg A$ is DPM.1-derivable.

1	$\mathsf{O}A$	PREM	$\emptyset$
2	$\mathsf{O}\neg A$	PREM	$\emptyset$
3	$\mathsf{O}(A \wedge B)$	PREM	$\emptyset$
^{8}4	$\mathsf{O}B$	3; RC	$\{!(A \wedge B)\}$
5	$\mathsf{O}B \vee !(A \wedge B)$	3; RU	$\emptyset$
6	$\mathsf{O}(\neg A \wedge B) \vee !(A \wedge B)$	2,5; RU	$\emptyset$
7	$\mathsf{O}(\neg A \vee \neg B) \vee !(\neg A \wedge B) \vee !(A \wedge B)$	6; RU	$\emptyset$
8	$!(\neg A \wedge B) \vee !(A \wedge B)$	3,7; RU	$\emptyset$

Note that the adaptive strengthening of DPM.1 interprets the premises as non-conflicting as possible. One may suspect that in this case it is possible to derive $\mathsf{O}B$ by applying inheritance to $\mathsf{O}(A \wedge B)$ conditionally since — opposite to the example above — $!A \vee !(A \wedge B)$ is not anymore a minimal Dab-consequence. Indeed, would $!(A \wedge B)$ not be part of any minimal Dab-consequence line 4 would be unmarked. However, due to the fact that $!(\neg A \wedge B) \vee !(A \wedge B)$ is a minimal Dab-consequence, line 4 is marked and $\mathsf{O}B$ is not a ADPM.1-consequence. Hence, ADPM.1 is also in this example coherent with its skeptical stance that was illustrated already in the example above.

Example 4: Examples 2 and 3 have demonstrated that in some cases we cannot derive non-conflicting conjuncts of conflicting premises by means of ADPM.1. This opens the question whether $\mathsf{O}B$ is derivable if it is part of each conflicting premise.

1	$\mathsf{O}(A \wedge B)$	PREM	$\emptyset$
2	$\mathsf{O}(\neg A \wedge B)$	PREM	$\emptyset$
^{7}3	$\mathsf{O}A$	1; RC	$\{!(A \wedge B)\}$
^{7}4	$\mathsf{O}\neg A$	2; RC	$\{!(\neg A \wedge B)\}$
5	$\mathsf{O}B$	1; RC	$\{!(A \wedge B)\}$
6	$\mathsf{O}B$	2; RC	$\{!(\neg A \wedge B)\}$
7	$!(A \wedge B) \vee !(\neg A \wedge B)$	1,2; RU	$\emptyset$

As the proof indicates, $\mathsf{O}B$ is derivable from $\mathsf{O}(A \wedge B)$ and $\mathsf{O}(\neg A \wedge B)$. The reason why lines 5 and 6 are unmarked is that for each minimal choice set $\{!(A \wedge B)\}$ and $\{!(\neg A \wedge B)\}$ at this stage, $\mathsf{O}B$ is derivable on a condition which has an empty intersection with it. As can easily be checked, $\mathsf{O}B$ is finally derived at line 6.

Example 5: Let us generalize the example above. We take a look at the premises $\mathsf{O}(A \wedge B)$ and $\mathsf{O}(\neg A \wedge C)$. Although $B \vee C$ is not a conjunct of any of the two premises, it is a consequence of both $A \wedge B$ and $\neg A \wedge C$. In coherence with the example above we expect thus $\mathsf{O}(B \vee C)$ to be an ADPM.1-consequence. On the other hand we expect $\mathsf{O}B$ and $\mathsf{O}C$ not to be a consequence.

1	$\mathsf{O}(A \wedge B)$	PREM	$\emptyset$
2	$\mathsf{O}(\neg A \wedge C)$	PREM	$\emptyset$
^{7}3	$\mathsf{O}B$	1; RC	$\{!(A \wedge B)\}$
4	$\mathsf{O}(B \vee C)$	1; RU	$\{!(A \wedge B)\}$
^{7}5	$\mathsf{O}C$	2; RC	$\{!(\neg A \wedge C)\}$
6	$\mathsf{O}(B \vee C)$	2; RU	$\{!(\neg A \wedge C)\}$
7	$!(A \wedge B) \vee !(\neg A \wedge C)$	1,2; RU	$\emptyset$

Indeed, the conditional derivation of $\mathsf{O}B$ and $\mathsf{O}C$ is marked, while $\mathsf{O}(B \vee C)$ is derivable since it is derived on both conditions, $\{!(A \wedge B)\}$ and $\{!(\neg A \wedge C)\}$. Since the minimal choice sets in this example are $\{!(A \wedge B)\}$ and $\{!(\neg A \wedge C)\}$ this ensures that line 6 with formula $\mathsf{O}(B \vee C)$ is unmarked. It can easily be seen that $\mathsf{O}(B \vee C)$ is finally derived at line 6.

Example 6: The following example of an ADPM.1-proof features a more complex setup.

1	$\mathsf{O}A$	PREM	$\emptyset$
2	$\mathsf{O}B$	PREM	$\emptyset$
3	$\mathsf{O}(C \vee D)$	PREM	$\emptyset$
4	$\mathsf{O}\neg(A \wedge C)$	PREM	$\emptyset$
5	$\mathsf{O}\neg(B \wedge D)$	PREM	$\emptyset$
6	$\mathsf{O}(A \wedge \neg C)$	1,4; RU	$\emptyset$
7	$\mathsf{O}(B \wedge \neg D)$	2,5; RU	$\emptyset$
^{16}8	$\mathsf{P}A \wedge \mathsf{P}B \wedge \mathsf{P}(C \vee D)$	1,2,3; RC	$\{!A, !B, !(C \vee D)\}$
9	$\mathsf{O}(A \vee \neg C) \vee !A$	1; RU	$\emptyset$
10	$\mathsf{O}(B \vee \neg D) \vee !B$	2; RU	$\emptyset$
11	$\mathsf{O}\neg C \vee !A$	4,9;RU	$\emptyset$
^{16}12	$\mathsf{O}\neg C$	11; RC	$\{!A\}$
13	$\mathsf{O}\neg D \vee !B$	5,10; RU	$\emptyset$
^{16}14	$\mathsf{O}\neg D$	13; RC	$\{!B\}$
15	$\mathsf{O}\neg(C \vee D) \vee !A \vee !B$	11,13; RU	$\emptyset$
16	$!A \vee !B \vee !(C \vee D)$	3,15; RU	$\emptyset$
17	$\neg\mathsf{P}A \vee \neg\mathsf{P}B \vee \neg\mathsf{P}(C \vee D)$	16; RU	$\emptyset$

Without engaging in more advanced reasoning processes it is for a user of DPM.1 in no way clear which permission statements may be added without causing deontic conflicts or explosion. We have pointed out this problem in Section 4 (point 2). Would (s)he, for instance, add $\mathsf{P}A$, $\mathsf{P}B$, and $\mathsf{P}(C \vee D)$ it would cause explosion, since via DPM.1, $\neg\mathsf{P}A \vee \neg\mathsf{P}B \vee \neg\mathsf{P}(C \vee D)$ is derivable at line 17. Note that in the adaptive proof line 8 is marked in view of line 16 and does therefore not cause any harm. Thus, the adaptive logic identifies given deontic conflicts and blocks undesired consequences from them. For instance, the counterintuitive derivations of $\mathsf{O}\neg C$ and $\mathsf{O}\neg D$ at lines 12 and 14 are marked.

7. *The Adaptive Logic* ADPM.2′

We have already pointed out various advantages of our DPM.2′ logic over Goble's DPM.2. Besides these points DPM.2′ is also more apt as a lower limit logic.

One idea to define an adaptive logic on the basis of DPM.2 would be in terms of the triple ⟨DPM.2, Ω, minimal abnormality⟩. However, this has a severe shortcoming. Suppose our premises are $\Gamma = \{\mathsf{O}A, \mathsf{O}B\}$. Since these premises do not give rise to any deontic conflicts we expect from the logic to apply aggregation to them. However, $\Gamma \nvdash_{\mathsf{ADPM.2}} \mathsf{O}(A \wedge B)$. Since DPM.2 also restricts aggregation beside inheritance it is desirable that the logic is able to apply aggregation conditionally in a similar way as ADPM.1 applies inheritance and (D) conditionally. The way aggregation is restricted in DPM.2, namely by (PAND), does not allow for utilizing the same set of abnormalities as for ADPM.1. A way around this problem is to define the abnormalities in a different way, for instance by $\Omega' = \{\mathsf{O}A \wedge \mathsf{O}B \wedge \mathsf{O}\neg(A \wedge B) \mid A, B \in \mathcal{W}\}$. This would allow to apply the aggregation principle conditionally. However, this logic is very weak. For instance, given two incompatible obligations $\mathsf{O}A, \mathsf{O}B$, where $\mathsf{O}\neg(A \wedge B)$ expresses their incompatibility, we are not able to derive $\mathsf{O}(A \vee B)$. The upshot is that, with DPM.2 as the lower limit logic, we were not able to define a set of abnormalities in such a way that the resulting adaptive logic is sufficiently strong and aggregation is conditionally applicable.

The situation is different if we employ DPM.2′ as lower limit logic. In that case, we can use the set of abnormalities Ω of ADPM.1. This contributes to a more unifying adaptive framework. Let ADPM.2′ be defined by the triple $\langle \mathsf{DPM.2}', \Omega, \text{minimal abnormality}\rangle$.

Note that in DPM.2′ the following is a consequence of (PAND′):

$$\vdash_{\mathsf{DPM.2}'} \big(\mathsf{O}A \wedge \mathsf{O}B\big) \supset \big(\mathsf{O}(A \wedge B) \vee \big((\mathsf{O}A \wedge \mathsf{O}\neg A) \vee (\mathsf{O}B \wedge \mathsf{O}\neg B)\big)\big)$$

This makes it possible to apply aggregation to $\mathsf{O}A \wedge \mathsf{O}B$ on the condition $\{\mathsf{O}A \wedge \mathsf{O}\neg A, \mathsf{O}B \wedge \mathsf{O}\neg B\}$ by the rule RC. Again, inheritance and (D) are applied conditionally as demonstrated already for $\mathsf{ADPM.1}$.

In order to demonstrate the modus operandi of $\mathsf{ADPM.2'}$ we take a look at a concrete example.

Example 7: Let us come back to Sartre's unfortunate student. On the one hand, he has the obligation to stay with his sick mother ($\mathsf{O}M$). However, on the other hand he has the obligation to fight at the front against Nazi Germany ($\mathsf{O}F$). Due to the lack of alethic modalities we have to find a way to model the fact that both obligations are not simultaneously realizable. This can be done by $\mathsf{O}\neg(M \wedge F)$. We add another premise which is independent of the first two: as a good citizen he is obliged to pay taxes and to vote, $\mathsf{O}(T \wedge V)$.[15]

1	$\mathsf{O}M$	PREM	$\emptyset$
2	$\mathsf{O}F$	PREM	$\emptyset$
3	$\mathsf{O}\neg(M \wedge F)$	PREM	$\emptyset$
4	$\mathsf{O}(T \wedge V)$	PREM	$\emptyset$
5	$\mathsf{O}T$	4; RC	$\{!(T \wedge V)\}$
^{7}6	$\mathsf{O}(M \wedge F)$	1,2; RC	$\{!M, !F\}$
7	$!(M \wedge F) \vee !M \vee !F$	1,2,3; RU	$\emptyset$
8	$\mathsf{O}(M \vee F)$	1; RC	$\{!M\}$
9	$\mathsf{O}(M \vee F)$	2; RC	$\{!F\}$

As discussed in Section 4, we expect that all rules of SDL can be applied to non-conflicting obligations such as $\mathsf{O}(T \wedge V)$. Indeed, after having introduced the premises at lines 1–4, inheritance is applied to $\mathsf{O}(T \wedge V)$ in order to derive $\mathsf{O}T$, the student's duty to pay taxes. Furthermore, the application of aggregation to $\mathsf{O}M$ and $\mathsf{O}F$ at line 6 gets marked. This accords with the fact that (PAND′) is a rule that isolates deontic conflicts, i.e. it is not applicable to conflicting obligations.

$\mathsf{ADPM.2'}$ realizes our design requirements. On the one hand, it blocks rules from being applied to conflicting obligations (such as aggregation at line 6). On the other hand it allows for applications of rules to non-conflicting premises (such as inheritance at line 5 to $\mathsf{O}(T \wedge V)$) without requiring the manual addition of permission statements.

Moreover, note that it is desired to derive $\mathsf{O}(M \vee F)$, the student's obligation to either stay with his mother or to fight the Nazis. Indeed, since $\mathsf{O}(M \vee F)$ is derivable on both conditions, $\{\mathsf{O}M \wedge \mathsf{O}\neg M\}$ and $\{\mathsf{O}F \wedge \mathsf{O}\neg F\}$,

[15] The proofs of $\mathsf{O}(M \vee F)$ and $\mathsf{O}T$ for $\mathsf{ADPM.1}$ are left to the reader.

lines 8 and 9 are not marked. Similarly for instance $\mathsf{O}(T \wedge (M \vee F))$ and $\mathsf{O}(V \wedge (M \vee F))$ are derivable.

In contrast, using one of the DPM logics, the user would have to add manually $\mathsf{P}(T \wedge V)$ to the premises in order to derive $\mathsf{O}T$. Furthermore, it is unclear how to derive $\mathsf{O}(M \vee F)$ in DPM. Only by adding either $\mathsf{P}M$ or $\mathsf{P}F$ permitted inheritance is applicable to $\mathsf{O}M$ or resp. $\mathsf{O}F$. However, there is no reason to prefer $\mathsf{P}M$ over $\mathsf{P}F$ or vice versa. Moreover, would the user add both of them, it may lead to explosion. Consider the case where also $\mathsf{P}\neg(M \wedge F)$ is a premise. It can be argued that, since M and F cannot be mutually realized an agent is allowed either to bring about not-M or to bring about not-F. However, in view of the final derivability of the formula at line 7, this leads to the derivability of $(\mathsf{O}M \wedge \mathsf{O}\neg M) \vee (\mathsf{O}F \wedge \mathsf{O}\neg F)$. In case the user would have added both, $\mathsf{P}M$ and $\mathsf{P}F$, explosion would result.

Example 8: ADPM.2′ leads to similar results in examples 1, 2, 4 and 5 as ADPM.1:

$$\mathsf{O}A, \mathsf{O}\neg A, \mathsf{O}(B \wedge C) \vdash_{\mathsf{ADPM.2'}} \mathsf{O}B, \mathsf{P}B$$
$$\mathsf{O}\neg A, \mathsf{O}(A \wedge B) \nvdash_{\mathsf{ADPM.2'}} \mathsf{O}B$$
$$\mathsf{O}(A \wedge B), \mathsf{O}(\neg A \wedge B) \vdash_{\mathsf{ADPM.2'}} \mathsf{O}B$$
$$\mathsf{O}(A \wedge B), \mathsf{O}(\neg A \wedge C) \nvdash_{\mathsf{ADPM.2'}} \mathsf{O}B, \mathsf{O}C$$
$$\mathsf{O}(A \wedge B), \mathsf{O}(\neg A \wedge C) \vdash_{\mathsf{ADPM.2'}} \mathsf{O}(B \vee C)$$

Example 9: A difference between ADPM.2′ and ADPM.1 is in the handling of the premise set consisting of $\mathsf{O}A, \mathsf{O}\neg A$ and $\mathsf{O}(A \wedge B)$:

1	$\mathsf{O}A$	PREM	$\emptyset$
2	$\mathsf{O}\neg A$	PREM	$\emptyset$
3	$\mathsf{O}(A \wedge B)$	PREM	$\emptyset$
4	$\mathsf{O}A \wedge \mathsf{O}\neg A$	1,2; RU	$\emptyset$
5	$\mathsf{O}B$	3; RC	$\{!(A \wedge B)\}$

As explicated in Example 3, due to the unrestricted aggregation principle that is valid in DPM.1, $!(A \wedge B) \vee !(\neg A \wedge B)$ is a minimal Dab-consequence in DPM.1. Note that we cannot derive this Dab-formula in ADPM.2′. With the weaker aggregation principle of ADPM.2′ the argument explicated in the proof of Example 3 is not valid in ADPM.2′. The reason is that at line 6 aggregation is applied to $\mathsf{O}B$ and $\mathsf{O}\neg A$. However, $\mathsf{O}\neg A$ conflicts with $\mathsf{O}A$ and thus (PAND′) is not applicable to it.

8. *Some Meta-Theoretical Properties*

This section features some meta-theoretic properties of the introduced logics. The meta-theory of adaptive logics in standard format equips our systems immediately with soundness and completeness (see Theorem 4). As desired, the adaptive strengthenings are conflict-tolerant.

Theorem 6: *Where* $\alpha \in \{1, 2'\}$, *none of Goble's explosion principles* (DEX), (DEX-1)–(DEX-3) *is valid in* $\mathsf{ADPM}.\alpha$.

Theorem 7: *Where* $\alpha \in \{1, 2'\}$, *the upper limit logic of* $\mathsf{ADPM}.\alpha$ *is* SDL.

Corollary 1: *Where* $\alpha \in \{1, 2'\}$, $\mathsf{ADPM}.\alpha$ *satisfies* $(\star)$.

We introduced another, in a sense more demanding criterion, $(\star\star)$. For SDL-consistent premise sets the given logics should have the same derivative power as SDL. This criterion is not fulfilled by the DPM logics. However, as the following Corollary shows, it is fulfilled by our adaptive strengthenings. The Corollary is a direct consequence of Theorem 5 and Theorem 7.

Corollary 2: *Where* $\alpha \in \{1, 2'\}$, $\mathsf{ADPM}.\alpha$ *satisfies* $(\star\star)$.

9. *Shortcomings and Related Work*

The starting point and main motivation of this paper was to present a technically straightforward way to equip Goble's DPM systems with a defeasible mechanism that strengthens the logics in a non-monotonic fashion. In this section, we point out some parallels and differences between the $\mathsf{ADPM}.\alpha$-systems resulting from our enterprise on the one hand, and existing approaches for dealing with deontic conflicts on the other hand.

The behavior of $\mathsf{ADPM.1}$ explicated in examples 1–5 may suggest that the consequence set of $\mathsf{ADPM.1}$ can be characterized in the following way: $\mathsf{O}B$ is a consequence of a set of obligations $\{\mathsf{O}A \mid A \in \Gamma\}$ (where Γ is a set of propositions) iff A is (classically) derivable from each maximal consistent subset[16] of Γ, or A is in $\Gamma^\wedge$, where $\Gamma^\wedge$ is the closure of Γ under the aggregation rule. In this formulation we need to consider $\Gamma^\wedge$ instead of Γ since the unrestricted aggregation rule of $\mathsf{ADPM.1}$ allows to derive $\mathsf{O}B$ for each $B \in \Gamma^\wedge$. This characterization is similar to approaches making use of

[16] A subset Γ' of Γ is maximally consistent iff Γ' is consistent and for every consistent $\Gamma'' \subseteq \Gamma$, if $\Gamma' \subseteq \Gamma''$ then $\Gamma'' = \Gamma'$.

maximally consistent subsets with a 'skeptical' operator for obligation, e.g. [17, 18] or the "full meet constraint output" of Input/Output logics in [19].[17]

However, there are more complex settings which show that ADPM.1 is too weak.[18] It is a question for future research to find ways to strengthen this logic so that the rationale above is realized and a defeasible proof theory is obtained for the well-known conflict-tolerant approach in terms of maximal consistent subsets.

We have mentioned that for examples 1, 2, 4, and 5 ADPM.2′ has a similar behavior as ADPM.1. This may suggest that the consequences of ADPM.2′ can be characterized as follows: $\mathsf{O}B$ is a consequence of a set of obligations $\{\mathsf{O}A \mid A \in \Gamma\}$ iff A is in Γ or it is (classically) derivable from each maximal consistent subset of Γ. However, again there are examples in which the consequence set of ADPM.2′ is too weak (see Footnote 18).

Moreover, we have already mentioned the rather peculiar asymmetry of ADPM.2′ in the handling of the premise sets $\Gamma_1 = \{\mathsf{O}A, \mathsf{O}\neg A, \mathsf{O}(A \wedge B)\}$ and $\Gamma_2 = \{\mathsf{O}\neg A, \mathsf{O}(A \wedge B)\}$ (see Example 9). While in the first case $\mathsf{O}B$ is derivable, in the second case it isn't. One way to make the behavior of ADPM.2′ more coherent is to restrict inheritance in the lower limit logic more severely by also blocking it from obligations that display "subconflicts", i.e. conflicts concerning subformulas of two obligations. In our example inheritance should be blocked from $\mathsf{O}(A \wedge B)$ since this conflicts with $\mathsf{O}\neg A$. In the adaptive system this would be mirrored by a type of abnormality that is not only restricted to direct conflicts but also considers conflicts such as the one between $\mathsf{O}(A \wedge B)$ and $\mathsf{O}\neg A$. A deontic adaptive logic that takes into account such conflicts can be found in [21].

The treatment of Γ_1 points also to another direction. Namely to design an adaptive logic for which $\mathsf{O}B$ is derivable from a set of obligations $\{\mathsf{O}A \mid A \in \Gamma\}$ iff A is (classically) derivable from some maximal consistent subset of Γ and A is consistent with all maximal consistent subsets of Γ.[19] One way to realize this is to adjust the restricted inheritance principle in the following

[17] In the deontic logic literature on maximally consistent subsets, skeptical operators are contrasted with more 'credulous' operators. The latter allow one to derive the obligation $\mathsf{O}A$ as soon as A is derivable from some maximal consistent subset of Γ (see e.g. [33, 17] or the "full join constraint output" of Input/Output logics in [19]) while the former allow to derive $\mathsf{O}A$ iff A is derivable in all maximal consistent subsets of Γ.

[18] One such counterexample is given by the premise set $\{\mathsf{O}A, \mathsf{O}B, \mathsf{O}\neg(A \wedge B), \mathsf{O}(\neg(A \wedge B) \wedge D), \mathsf{O}(C_1 \wedge C_2)\}$. Here it is not possible to derive $\mathsf{O}C_1$ by means of ADPM.1 or ADPM.2′.

[19] In a non-deontic setting, this approach was taken up by Rescher & Manor in their definition of an 'argued' consequence relation [26]. Adaptive characterizations of Rescher & Manor's consequence relations are given in [2, 5].

way. Instead of applying inheritance if the antecedent is consistent the idea is to apply inheritance if the consequent is consistent:

$$\text{If } \vdash A \supset B, \text{ then } \mathsf{O}A \supset (\mathsf{P}B \supset \mathsf{O}B).$$

We will investigate in these systems in the future.

Recently various other adaptive deontic systems have been presented. In [20] and the forthcoming [21] strictly non-aggregative deontic logics were adaptively strengthened. These systems are based on the lower limit logic $\mathsf{P_eSDL_a}$ from [13]. Two modalities are used: O_e and O_a where the former e.g. expresses obligations that stem from some authority while the latter expresses obligations upon which all authorities agree. These systems validate full inheritance but have no aggregation for the O_e-operator. In [24] these systems are formulated in the prioritized format for adaptive logics from [23].[20] This way they are able to deal with preferences among obligations. The abnormalities used in all of the systems mentioned in this paragraph are of a more complex nature since they also take into account conflicts between subformulas of obligations. One advantage of the systems presented in the present paper is that by means of ADPM.1 and ADPM.2′ we are able to derive $\mathsf{O}(A \vee B)$ from $\mathsf{O}A, \mathsf{O}B$ and $\mathsf{O}\neg(A \wedge B)$. This 'disjunctive solution' has been defended for instance in [10, 18]. Moreover, deontic adaptive logics have been devised that are based on paraconsistent modal logics [9, 8]. This approach has also been applied to a multi-agent setting [6, 7].

In the meantime some other systems have been devised that offer generalizations of the logics presented in the present paper.

Lou Goble presented in [14] conditional versions of his DPM systems. Also there he operates with a restricted version of the inheritance principle. The adaptive handling of inheritance and aggregation introduced in this paper can be applied in the conditional context straightforwardly. Similarly, an adaptive approach can be used in order to apply strengthening the antecedent (if $\mathsf{O}(A \mid B)$, then $\mathsf{O}(A \mid B \wedge C)$) "as much as possible", i.e., to apply it whenever the factual premises do no describe an exceptional situation. This has been presented in [30].

One problem for conditional deontic logics is related to the detaching of conditional obligations. Given a conditional obligation to bring about A if B is the case, written $\mathsf{O}(A \mid B)$, and given that the condition B is fulfilled, one may want to derive the 'actual obligation' to bring about A. However, detachment is not universally valid, there are exceptional circumstances in which doing what normally would be forbidden is permitted or even obligatory. Again, applying adaptively detachment to $\mathsf{O}(A \mid B)$ and B "as much

[20] See also [25] for relations of this format to other ways of prioritizing adaptive logics.

as possible" leads to solutions to this problem. Furthermore, the semantics with an actual world introduced in this paper can easily be generalized for the conditional case. This way factual premises can be handled semantically. This application of adaptive logics has been presented in [31].

10. *Conclusion*

In this paper the adaptive strengthenings ADPM.1 and ADPM.2′ of Goble's conflict-tolerant logic DPM.1 and our variant DPM.2′ of Goble's DPM.2 were introduced. We have demonstrated various advantages of the adaptive systems:

– ADPM.α (where henceforth $\alpha \in \{1, 2'\}$) is significantly stronger than DPM.α. It is not just the case that adding (D) to the axiomatization leads to equivalent systems to SDL. Moreover, for any SDL-consistent premise set, ADPM.α leads to the same consequence set as SDL. Furthermore, ADPM.α applies restricted inheritance "as much as possible". In contrast, in Goble's system many permission statements have to be added by the user in order to apply the inheritance principle. The needed permission statements are generated automatically in the adaptive systems. This brings us to another point:

– The adaptive systems ADPM.α have the design virtue that the logics model all the reasoning for the user. In contrast, in DPM.α the user not just has to interfere in order to enable as much consequences as possible (by adding permission statements). Moreover, finding out what permission statements are harmless and may be added to a given premise set involves advanced reasoning by the user. This is especially the case for complicated setups.

– The meta-theory of adaptive logics in standard format is well-established and actively researched. Many key-features do not have to be proven (any-more) for the ADPM.α logics since they have been shown to be valid for all adaptive logics in standard format. For instance the completeness and soundness of ADPM.α follows immediately from the completeness and soundness of DPM.α.

– The dynamic adaptive proofs mirror actual reasoning processes. While the insight in a given premise set grows, some lines of the proof may get marked, others unmarked due to the fact that their conditions turn out to be (not) trustworthy. Furthermore, the adaptive proofs are able to deal with new information in the form of new premises and thus to handle the defeasibility that comes with it.

The disadvantages of ADPM.1 and ADPM.2′ were discussed in Section 9. The main drawback of ADPM.1 is that its consequence relation is rather weak compared to some existing systems from the literature. Moreover, both

ADPM.1 and ADPM.2′ are not always consequential in their treatment of some (complex) examples. We hope to overcome these problems in the near future by investigating in detail the suggestions for improvement presented in Section 9.

Appendix

In order to prove soundness and completeness with respect to our semantics for DPM, we will show that it is equivalent to Goble's original DPM semantics. Since the authors in [28] have proven soundness and strong completeness for Goble's semantics this is sufficient.

Goble's original neighborhood semantics is very similar to the one presented here: the key difference is that we employ an actual world. Where frames are defined as before, an F-G-model M is a pair $\langle F, v\rangle$ where F is a frame and $v : \mathcal{S} \to \wp(W)$ as before. The essential difference concerns the definition of model-validity. While in our semantics it is defined in terms of validity with respect to the actual world, in Goble's semantics it is defined in terms of validity with respect to all given worlds: $M \models^G A$ iff $M, w \models A$ for all $w \in W$. All other definitions concerning validity are analogous. For a given frame $F = \langle W, \mathcal{O}\rangle$ and $\Gamma \subseteq \mathcal{W}$, $\Gamma \Vdash^G_F A$ iff for all F-G-models M and for all $w \in W$, if $M, w \models B$ for all $B \in \Gamma$, then $M, w \models A$. Moreover, where $\alpha \in \{1, 2'\}$, $\Gamma \Vdash^G_{\mathsf{DPM}.\alpha} A$ iff $\Gamma \Vdash^G_F A$ for all DPM.α-frames F. Schröder and Pattinson have shown the following strong completeness and soundness result in [28]:

Theorem 8: *Where* $\alpha \in \{1, 2'\}$ *and* $\Gamma \subseteq \mathcal{W}$, $\Gamma \Vdash^G_{\mathsf{DPM}.\alpha} A$ *iff* $\Gamma \vdash_{\mathsf{DPM}.\alpha} A$.

Theorem 9: *Where* $\alpha \in \{1, 2'\}$ *and* $\Gamma \subseteq \mathcal{W}$, $\Gamma \Vdash^G_{\mathsf{DPM}.\alpha} A$ *iff* $\Gamma \Vdash_{\mathsf{DPM}.\alpha} A$.

Proof. Let $\mathcal{F}$ be the class of DPM.α-frames. "⇐": Let $\Gamma \Vdash_{\mathsf{DPM}.\alpha} A$ and $F = \langle W, \mathcal{O}\rangle \in \mathcal{F}$. Suppose there is an F-G-model $M = \langle F, v\rangle$ and a world $w \in W$ for which $M, w \not\models A$ and $M, w \models B$ for all $B \in \Gamma$. Note that $M' = \langle F, v, w\rangle$ is a DPM.α-model of Γ for which $M' \not\models A$, — a contradiction.

"⇒": Let $\Gamma \Vdash^G_{\mathsf{DPM}.\alpha} A$. Suppose for some frame $F = \langle W, \mathcal{O}\rangle \in \mathcal{F}$ there is an F-model $M_w = \langle F, v, w\rangle$ of Γ for which $M_w \not\models A$. Let $M = \langle F, v\rangle$. Note that $M, w \models B$ for all $B \in \Gamma$ and $M, w \not\models A$, — a contradiction. □

Proof of Theorem 3 (Soundness and Completeness). Follows immediately by Theorem 8 and Theorem 9. □

Proof of Theorem 2. For $\mathsf{DPM.1}$ this has already been shown by Goble in [14]. Note that (D) together with (PAND) results in (AND). Since $\mathsf{DPM.1}$ strengthened by (D) has the same corresponding consequence relation as SDL, it also validates (N). Thus, $\mathsf{DPM.1}$ strengthened by (D) and $\mathsf{DPM.2}'$ strengthened by (D) have the same corresponding consequence relation. Thus, $\mathsf{DPM.2}'$ strengthened by (D) has the same corresponding consequence relation as SDL. □

Proof of Theorem 6. Let us first consider the case for $\mathsf{ADPM.2}'$. Let $W = \wp(\mathcal{S})$ and p_1 and p_2 are sentential letters. We define

$$\begin{aligned} W_a &= \{w \in W \mid p_1 \notin w, p_2 \notin w\}, \\ W_b &= \{w \in W \mid p_1 \notin w, p_2 \in w\}, \\ W_c &= \{w \in W \mid p_1 \in w, p_2 \notin w\}, \\ W_d &= \{w \in W \mid p_1 \in w, p_2 \in w\}. \end{aligned}$$

We define a frame $F = \langle W, \mathcal{O} \rangle$ where $\mathcal{O}_w = \{W_a \cup W_b, W_c \cup W_d, W\}$ for all $w \in W$. Note that F is a $\mathsf{DPM.2}'$-frame. Let $M = \langle F, v, @ \rangle$ where $v(p_i) = \{w \in W \mid p_i \in w\}$ and @ is any world in W. Note first that $M \models \mathsf{O}p_1, \mathsf{O}\neg p_1, \mathsf{O}\top, \mathsf{P}\top, \mathsf{P}(p_1 \wedge p_2)$ and $M \not\models \mathsf{O}(p_1 \wedge p_2)$. Thus, M models a counter-instance to (DEX), (DEX-1)–(DEX-3). Note furthermore that M is a minimally abnormal model of $\{\mathsf{O}p_1, \mathsf{O}\neg p_1\}$ and also of $\{\mathsf{O}p_1, \mathsf{O}\neg p_1, \mathsf{O}\top, \mathsf{P}\top\}$ since $\mathrm{Ab}(M) = \{\mathsf{O}A \wedge \mathsf{O}\neg A \mid \vdash A \equiv p_1\}$ and $\{\mathsf{O}p_1, \mathsf{O}\neg p_1\} \vdash_{\mathsf{DPM.2}'} A$ for all $A \in \mathrm{Ab}(M)$.

The proof for $\mathsf{ADPM.1}$ is similar. Where $W = \wp(\mathcal{S})$ and $v : \mathcal{S} \to \wp(W)$, $p_i \mapsto \{w \in W \mid p_i \in w\}$, we define a frame $F = \langle W, \mathcal{O} \rangle$ where for all $w \in W$,

$$\mathcal{O}_w = \{W' \mid W' \supseteq v(p_1)\} \cup \{\emptyset\}$$

Note that F is a $\mathsf{DPM.1}$-frame. Let $M = \langle F, v, @ \rangle$ where @ is any world in W. Evidently, $M \models \mathsf{O}p_1, \mathsf{O}\top, \mathsf{O}\bot, \mathsf{P}p_1, \mathsf{P}p_2$ and $M \not\models \mathsf{O}p_2$. Thus, M models a counter-instance to (DEX), (DEX-1)–(DEX-3). Also, M is a minimally abnormal model of $\{\mathsf{O}\top, \mathsf{O}\bot\}$ and of $\{\mathsf{O}\top, \mathsf{O}\bot, \mathsf{O}p_1, \mathsf{P}p_1\}$ since $\mathrm{Ab}(M) = \{\mathsf{O}A \wedge \mathsf{O}\neg A \mid \vdash A \equiv \top\}$ and $\{\mathsf{O}\top, \mathsf{O}\bot\} \vdash_{\mathsf{DPM.1}} A$ for all $A \in \mathrm{Ab}(M)$. □

Proof of Theorem 1. Due to the fact that all adaptively selected models are models of the lower limit logic, the counter-models to (DEX) and (DEX-1)–(DEX-3) constructed for $\mathsf{ADPM.1}$ and $\mathsf{ADPM.2}'$ in the proof of Theorem 6 are also counter-models for $\mathsf{DPM.1}$ and $\mathsf{DPM.2}'$. Theorem 1 was proven for $\mathsf{DPM.2}$ by Goble in [14]. □

Proof of Theorem 7. Given a $\Gamma \subseteq \mathcal{W}$ we have to show that $\Gamma \cup \{\neg(\mathsf{O}A \wedge \mathsf{O}\neg A) \mid A \in \mathcal{W}\} \vdash_{\mathsf{DPM}.\alpha} B$ iff $\Gamma \vdash_{\mathsf{SDL}} B$. Note that $\vdash_{\mathsf{DPM}.\alpha} \neg(\mathsf{O}A \wedge \mathsf{O}\neg A) \equiv (\mathsf{O}A \supset \mathsf{P}A)$. Thus, $\Gamma \cup \{\neg(\mathsf{O}A \wedge \mathsf{O}\neg A)\} \vdash_{\mathsf{DPM}.\alpha} B$ iff $\Gamma \vdash_{\mathsf{DDPM}.\alpha} B$ where $\mathsf{DDPM}.\alpha$ is $\mathsf{DPM}.\alpha$ enriched by (D). However, since by Theorem 2 $\mathsf{DDPM}.\alpha$ has the same corresponding consequence relation as SDL, we are finished. □

Proof of Corollary 1. For SDL-consistent premise sets Γ this is an immediate consequence of Corollary 2. Let $\Gamma \subseteq \mathcal{W}$ be SDL-inconsistent. Where $\mathsf{DADPM}.\alpha$ (resp. $\mathsf{DDPM}.\alpha$) is $\mathsf{ADPM}.\alpha$ (resp. $\mathsf{DPM}.\alpha$) enriched by (D) and $\Gamma' = \Gamma \cup \{\mathsf{O}A \supset \mathsf{P}A \mid A \in \mathcal{W}\}$, note that $\mathcal{M}_{\mathsf{SDL}}(\Gamma) = \emptyset = \mathcal{M}_{\mathsf{DDPM}.\alpha}(\Gamma) = \mathcal{M}_{\mathsf{DPM}.\alpha}(\Gamma') \supseteq \mathcal{M}_{\mathsf{ADPM}.\alpha}(\Gamma') = \mathcal{M}_{\mathsf{DADPM}.\alpha}(\Gamma)$. □

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