



ON INTERNAL RELATIONS IN LEIBNIZ, BRITISH NEO-REALISM
AND WHITEHEAD

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Abstract

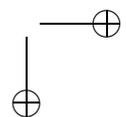
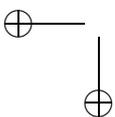
The aim of this paper consists in asking whether Whitehead’s philosophy of organism would be a reenactment of Leibniz’s doctrine of internal relations. As this terminology depends on Russell’s reading of Leibniz, it is first necessary to restore the so-called thesis of reducibility of relations through the opposition of the main commentaries. Secondly, we distinguish Russell’s refusal of the axiom of internal relations from Moore’s criticism, which is rather related to the exclusiveness of this axiom. Thirdly, we follow Whitehead on relations from 1905 to 1925, agreeing with Russell about the untenable supremacy of the judgement of predication, but getting closer to Moore about the axiom, for the very reason that if there were any internal relation, there could be no process in the nature.

1. *Introduction*

An obvious familial resemblance seems to relate Leibniz’s universe and Whitehead’s cosmology, allowing that both of them shared a same holistic view on the “general interconnectedness of things”.¹ The inter-expressing monads have such a connection that they are together related by internal relations, i.e. relations that constitute the very nature of the terms which they connect. According to this, Whitehead wrote in *Modes of Thought*:

“The notion of a mere fact is the triumph of the abstractive intellect [...]. A single fact in isolation is the primary myth required for

¹ Alfred North Whitehead, *Adventures of Ideas*, New York, Macmillan Publishing Co., 1933, II, ix, §4, p. 192.



finite thought [...]. Connectedness is of the essence of all things of all types [...]. No fact is merely itself".²

Even before this, a leitmotiv in *Process and Reality* was already that every actual entity is present in every other actual entity.

Bertrand Russell had believed that he had definitively got rid of internal relations. Dealing with this kind of relations, any form of pluralism becomes inconceivable and leads to an idealist monism, as it was the case of Bradley's neo-Hegelianism. In fact we owe to Whitehead, especially in the chapters X and XI of *Science and the Modern World*, a subtle rehabilitation of the internal relations, however not exclusive of the external relations. Given their so-called "relational essence", the eternal objects hold together through internal relations. Whitehead's main conceptual innovation puts forward that some relations could be internal in one direction but external from the converse side. Indeed, whilst all relations, from the eternal object to the actual occasion, are *external*, Whitehead qualified as *internal* the "ingression" of the eternal objects, gradually determined by each actual occasion.

This distinction between two classes of relations surely involves some old metaphysical oppositions, particularly the one of essence and accidents. While essential attributes determine the identity of a thing, which could not remain the same if its internal relations would be changed, external relations turn out to be an accidental predication, since a thing remains identical to itself through some superficial modifications.

As a result of the special relativity theory, Whitehead conceived the essence of an event to be constituted by the whole of its relations to others events. Such was the great lesson of physical modern theory: it would be impossible from now on to maintain that the spatio-temporal relations are external. Therefore the relations among the events must be characterized as internal, so that internal relatedness constitutes the essence of all events:

"This internal relatedness is the reason why an event can be found only just where it is and how it is, — that is to say, in just one definite set of relationships. For each relationship enters into the essence of the event; so that, apart from that relationship, the event would not be itself. This is what is meant by the very notion of internal relations. It has been usual, indeed, universal, to hold that

² Alfred North Whitehead, *Modes of Thought*, Cambridge, Cambridge University Press, 1938, pp. 12–13.

spatio-temporal relationships are external. This doctrine is what is here denied".³

For the present, my purpose is to establish why Whitehead's views on relations should be brought closer to George Edward Moore's analysis in his famous paper *External and Internal Relations* (1919) and why Whitehead could no longer hold Russell's criticism of internal relations about Leibniz as insuperable. In a first part of this text, I will come back to the topic of internal relations, concerning Leibniz's theory of relations. In a second part, I will discuss Moore's moderate position by contrast with Russell's radical criticism of internal relations. And finally, I will show how, before writing *Science and the Modern World*, Whitehead has freed himself from what we could name Russell's new dogma about the exclusiveness of external relations.

2. *Internal relations and the problem of the reducibility of relations in Leibniz*

In his book on Leibniz, which Whitehead knew very well, Russell identified his own external relations with the Leibnizian extrinsic denominations. Leibniz's leading axiom, according to which "Every extrinsic denomination — i.e. every relation — has an intrinsic foundation, i.e. a corresponding predicate", amounts to the denial of any purely extrinsic relation.⁴ Here it appears that we meet a tension, between, on one side, the pluralism of substances which implies relations between them, and on the other side, the analytic conception of truth, i.e. a definition of the true proposition as one whose subject expressly or virtually contains its predicates. Russell claimed that Leibniz had intended to reduce all relational propositions to predicative judgements.

Following Russell, this problem became a mandatory topic for the scholars: did Leibniz actually intend to reduce the relations? Indeed Leibniz used

³ Alfred North Whitehead, *Science and the Modern World* [SMW], Lowell Lectures, 1925, New York, A Mentor Book, 10th printing, 1960, chap. vii, p. 155.

⁴ G.W. Leibniz, *Die philosophischen Schriften* [GP] published by C.I. Gerhardt, Berlin, 1875–1885, reprint Hildesheim, Georg Olms Verlag, 1968, vol. II, p. 240, quoted by Bertrand Russell, *A Critical Exposition of the Philosophy of Leibniz. With an Appendix of Leading Passages*, Cambridge, 1900, reprint London, Routledge, 2002, §24, p. 68.

the Latin word "reductio" and the verb "reducere" about relations.⁵ By "reduction", Leibniz was supposed to mean a systematic project of rewriting all polyadic sentences under the form of monadic sentences. According to the conceptions of the *Principia Mathematica*, the opposition between these two kinds of sentences does not seem insuperable, at least because both of them take part in the same scheme of a propositional function $F(x)$, $F(x, y)$... etc. But later on, Church's demonstration of the undecidability of the calculus of relations will bring to the fore a metalogical reason to distinguish the calculus of predicates which is itself decidable from the one dealing with relations, which is not.⁶

Other scholars also claimed that Leibniz had upheld the reduction of relations: like Russell, G.H.R. Parkinson came back to the conclusion that Leibniz failed to reduce all the asymmetric relations.⁷ Then came N. Rescher, for whom, at the opposite, Leibniz had succeeded even in this last case, followed by B. Mates, who understood the endeavours of reduction inside a nominalistic reenactment of Leibniz's philosophy.⁸

In 1900, Russell did not mention different issues depending on whether kind of relation was retained, as he would notoriously do later in his *Principles of Mathematics* (§§213–215) by distinguishing "monadistic" and "monistic" reduction of a relation. Russell picked up exactly the same example of an asymmetric relation as Leibniz in a famous passage of his correspondence with Clarke.⁹ In the case of a judgment of comparison between two

⁵ Louis Couturat, *Opuscules et fragments inédits de Leibniz. Extraits des manuscrits de la Bibliothèque royale de Hanovre* [Op. & Fr.], Paris, 1903, reprint Hildesheim, Georg Olms Verlag, 1961, p. 244.

⁶ Alonzo Church, "A Note on the *Entscheidungsproblem*", in *The Journal of Symbolic Logic*, vol. 1, n° 1, 1936, pp. 40–41.

⁷ G.H.R. Parkinson, *Logic and Reality in Leibniz's Metaphysics*, Oxford, Clarendon Press, 1965, pp. 39–55.

⁸ Nicolas Rescher, *Leibniz's Metaphysics of Nature. A Group of Essays*, Dordrecht, Reidel, 1981, pp. 56–83; Benson Mates, *The Philosophy of Leibniz. Metaphysics and Language*, Oxford, Oxford University Press, 1986, pp. 209–226.

⁹ G.W. Leibniz, *Fifth Writing*, §47 (GP VII, 401). "This passage is of the utmost importance for understanding Leibniz's philosophy", commented Russell. Leibniz was near "to realize that relation is something distinct from and independent of subject and accident". But "he thrusts aside the awkward discovery, by condemning the third of the above", that is to say: Leibniz rejected the independence of the relations. Thus they became "something purely ideal" (B. Russell, CEPL, §10, p. 15).

unequal magnitudes " L is greater than M ", the relation is likely to be rewritten thanks to the assertion of complex adjectives:

- (1) L has for predicate "... is greater than M ",
- (2) M has for predicate " L is greater than..." .

Besides, (2) is equivalent to (3), where the converse relation is associated to M at the place of the subject.

- (3) M has for predicate "... is less great than L ".

Russell did not miss to underline the relational character of the complex adjective "greater than". If the relations are avoided at the syntactical level, this is not semantically the case. Russell therefore drew the following conclusion in the *Principles of Mathematics*:

"An adjective involving a reference to M is plainly an adjective which is relative to M , and this is merely a cumbrous way of describing a relation".¹⁰

The so-called monistic reduction would hold for several forms of philosophic systems, from Spinoza and Hegel to Bradley. From such an outlook, L and M fuse in a whole subject whereof it would be asserted:

- (4) " $(L$ and M) differs with the respect of magnitude".

As the sense of a relation is only really relevant in the case of the asymmetric relations, the reduction works for the symmetric relations, where this sense does not matter. Considering " L is so great than M ", this relational proposition receives the monadistic reduction as well as the monistic one, because of the override of the sense of the relation:

- (5) L has for predicate "... is so great than M ",
- (6) M has for predicate "... is so great than L ",
- (7) " $(L$ and M) have a same magnitude".

Curiously, the fact that the complex adjective "so great than" is neither grammatically nor logically more or less relational than "greater than" does not

¹⁰ B. Russell, *Principles of Mathematics* [PoM], Cambridge, Cambridge University Press, 1903, §214.

seem to be an issue. And that the reduction works in this case becomes a feature of the relations of equivalence in general.

For Rescher, Leibniz would even carry out successfully in reducing the asymmetric relations. According to this commentator, the theory of the inter-nomadic relations must be considered as a whole piece of metaphysics, an essential clue for explaining how one possible world may diverge from the others. Thus a form of reality must be attributed to relations, even if they do not consist in separate entities. Relations are no more and no less actual than any "well-founded phenomena".

What is understood under "reducing a relation" must be brought forward. According to a strong meaning, a relation R between a and b is reducible if there exists some predicates P_1, P_2, \dots, P_n , all these P being predicates of a ; then if there exists some predicates Q_1, Q_2, \dots, Q_n , all these Q being predicates of b , the truth of aRb may be *equivalent* to this conjunction:

$$(8) P_1a \wedge P_2a \wedge \dots \wedge P_na \wedge Q_1b \wedge Q_2b \wedge \dots \wedge Q_nb$$

This rewriting is supposed to be logically equivalent to the initial relational proposition:

$$(9) (\forall a)(\forall b)aRb \equiv P_1a \wedge P_2a \wedge \dots \wedge P_na \wedge Q_1b \wedge Q_2b \wedge \dots \wedge Q_nb$$

$$(10) (\forall a)(\forall b)aRb \equiv P_Ra \wedge Q_Rb$$

The formula " aRb " is thus restricted in order to express a conjunction of non relational predications, and that works eminently for the case of the symmetric relation of similitude:

$$(11) \text{"A est similis B"} \equiv \text{"A est ruber"} \wedge \text{"B est ruber"}$$

But now, what about the reduction of the asymmetric relation? In an excerpt published by Couturat, related to the project of a universal language, Leibniz resolved the question in this way:

$$(12) \text{"Titius est magis doctus Caius"}.^{11}$$

$$(13) P_Ra = \text{"Caius is somewhat wise"}$$

$$(14) Q_Rb = \text{"Titius is very wise"}$$

¹¹ "Titius is wiser than Caius" (L. Couturat, Op & Fr., p. 280).

This twofold ascription is yet synthesized with a proposition of comparison, namely:

- (15) "Quatenus Titius est doctus, et Caius est doctus, eatenus Titius est superiori et Caius inferior".¹²

During the late scholasticism, the *expositio* of a proposition meant a paraphrase of this proposition via another kind of proposition. Here the exposition of a comparison, for its rewriting, requires a non truth-functional connective, expressed through a complex connecting link as *quatenus* (insofar), *eo ipso* (by the virtue of), *eatenus* (inasmuch as, hold as). Noted "@" by Rescher and some others logicians, this connective does not only mean a conjunction but also the reason-why of an attribubility.¹³

Any sentence involving an idea of comparison, such as "Pâris amat Helenam", reveals a similar example of an asymmetric relational proposition. It would then be insufficient to make the expression of an underlying link between Pâris and Helen equivalent to the conjunction of these predicative propositions:

- (16) "Paris est amans" \wedge "Helena est amata".

What would be needed would precisely be the reason-why of the attribubility, the *eo ipso* which indicates that it is exactly because Helen is beloved that Pâris is a lover. So these relations may be written under the following formulas, the second one simplifying the first one:

- (17) $(\forall a)(\forall b)aRb \equiv P_Ra \wedge Q_Rb \wedge (P_Ra@Q_Rb)$
 (18) $(\forall a)(\forall b)aRb \equiv P_Ra \wedge (P_Ra@Q_Rb)$

Under the circumstances making a property $P \notin a$, or a property $Q \notin b$, these two substances a and b become impossible. Rescher draws the conclusion that Leibniz required in some ways to trust in the reality of relations in order to construe his doctrine of the impossibility of possible worlds. Amazingly, such a belief may be linked with a project of reducibility. It is yet obvious that Rescher does not understand the reduction in a

¹²"Insofar as Titius et Caius are wise, inasmuch Titius is superior and Caius inferior" (L. Couturat, Op & Fr., *ibid.*). See, concerning Leibniz's medieval roots, Massimo Mugnai, *Leibniz's Theory of Relations*, Stuttgart, F. Steiner, 1992, pp. 97–107.

¹³Curiously, Rescher did not seem to be aware that the expression of the comparative proposition would also need such an intensional operator (*op. cit.*, p. 61).

Russellian sense, namely as the disappearing of all kind of relations under the judgements of predication; on the contrary, by appealing to the patterns of the *lingua rationalis*, he explains how Leibniz maintained the particularity of the relations.

Seemingly, Mates and Rescher do not agree on the meaning of "reduction".¹⁴ The former denies to the latter the right to put a sign of equivalence in the formulas of the so-called *expositiones*: even on the favourable ground of symmetric relational propositions, it may no longer be question of anything but a weaker form of reduction, under the form of a mere *implication* of the initial sentence from the sentences which analyze the first one, instead of an exact equivalence.

Let the exposition of a new comparative proposition be:

- (19) "Thaetetus is taller than Socrates",
- (20) "Thaetetus is 6 feet tall",
- (21) "Socrates is 5 feet tall".

B. Mates remarks that (20) and (21) are not at all equivalent to (19) so long as we neglect to mention:

- (22) "6 is greater than 5".

In order to restore the equivalence, it is necessary to add:

- (23) $(\forall x) \wedge (\forall y) [x \text{ is taller than } y \text{ if and only if } (\exists m, n)(x \text{ is } m \text{ feet taller} \wedge y \text{ is } n \text{ feet taller} \wedge m > n)]$.

From the very fact of having had to mention an order asymmetric relation " $m > n$ " inside the preceding formula (23), Mates contends that the relation "to be greater than" is not strictly reduced to the properties of the related entities. Thus Mates reckons that the achievement of the reduction may not hinge upon a logical equivalence, but is merely determined by an entailment. So it happens that:

- (24) $[(20) \wedge (21)] \Rightarrow (19) \text{ or } \neg \diamond \{[(20) \wedge (21)] \text{ Truth} \wedge (19) \text{ False}\}$.

¹⁴B. Mates, *op. cit.*, chap. xii, §4, "Reducibility and Dispensability". See also his paper "Nominalism and Evander's Sword", *Studia Leibnitiana*, Supplementa xxi, Wiesbaden, F. Steiner, 1980, Teil 3.

As we see, Mates does not propound any specific intensional connective. Thus it is Lewis's strict implication which provides the suitable tool for a successful though weak reduction. Again this may be checked through the exposition of an asymmetric relation:

$$(25) \text{ (Paris amat Helenam)} \Rightarrow \text{(Helena est amata)} \\ \equiv \neg\Diamond[(\text{Paris amat Helenam}) \wedge \neg(\text{Helena est amata})].^{15}$$

If we now inquire into symmetric relations, as the one of similitude, the same translation of equivalence may be done:

$$(26) \text{ ("A est ruber"} \wedge \text{"B est ruber"} \Rightarrow \text{"A est similis B"}) \\ \equiv \neg\Diamond[(\text{"A est ruber"} \wedge \text{"B est ruber"}) \wedge \neg(\text{"A est similis B"})].^{16}$$

B. Mates blames Rescher for having asserted that Leibniz would admit the reality of the relations. Remember that his aim consists in underlining all the nominalistic features of Leibniz's thought. The ground of this nominalism here is nothing else than the thesis that only individual substances and modifications of them may exist. In the respect of such a reading and in conformity with what Aristotle wished, relations must not constitute a third order of reality.¹⁷ But it is quite obvious that Leibniz has never sought to wander out of the ontological tradition about relations.

As far as reducibility is concerned, some commentators, like Hidé Ishiguro and Jaakko Hintikka, followed by Fred d'Agostino or, more recently, Mark Kulstad, dissent from the "old-fashioned" view opened by Russell.¹⁸ This fresh tendency to impugn the reducibility thesis has principally invoked

¹⁵L. Couturat, *Op & Fr*, p. 287.

¹⁶One even may wonder whether this sort of proposition on similitude, according to Rescher's outlook, could avoid a rewriting with the help of the connective @: is not the respect under which *A* may be said being similar to *B* to be clarified?

¹⁷Some authors qualify these views as "particularism" (see, for example, Frédéric Nef's views in his paper "Accidents et relations individuelles chez Leibniz", in *Leibniz et les puissances du langage*, ed. by D. Berlioz et F. Nef, Paris, Vrin, 2005, p. 132). Then a transition to nominalism goes on further by eliminating all abstract terms. So did Leibniz intend to do. However, we must carefully notice that nominalism also entails a reduction of the concepts to the words. From this side, Leibniz's philosophy seems to be more conceptualistic than nominalistic.

¹⁸Jaakko Hintikka, "Leibniz on Plenitude, Relations, and the 'Reign of Law'", in *Leibniz. A Collection of Critical Essays*, ed. by Harry G. Frankfurt, New York, Anchor Books, 1972, pp. 155–190; Fred d'Agostino, "Leibniz on Compossibility and Relational Predicates", in *Leibniz: Metaphysics and Philosophy of Science*, ed. by R.S. Woolhouse, Oxford, Oxford

a well-known passage of the *Discours de métaphysique*, where it looks unavoidable to recognize that some predicates of the individual substance actually are relational: God, seeing the individual notion of Alexander, sees in it at the same time the reason and the foundation of all the predicates which can truly be ascribed to him, "as, for example, that he is the conqueror of Darius and Porus".¹⁹

The reductionist view takes seriously the connection of metaphysics with the Leibnizian project of building a *lingua philosophica*, for which predicates would irreducibly be non relational. For any doctrine of substance, nominally determined as the last subject of attribution, relationship certainly represents a stumbling block. Indeed a relational predicate keeps a tension between the inherence (*in-esse*) and the reference to another subject, its "adity" (*ad-esse*). However, considering the famous definition of the complete notion of a substance in the *Discours*, is it indubitable that referring to relational predicates is enough to disqualify any commitment to the task of reduction? Speaking of "the conqueror of Darius and Porus", Leibniz could use an ordinary language without asserting that its syntactic structures express any ultimate metaphysical truth about substances and their accidents.

J.A. Cover and J. O'Leary-Hawthorne underline the reductionist flavor of the §viii of the *Discours*.²⁰ The supporters of the non reductionist thesis read that all relational facts are contained in the complete notion of the individual substance. According to Cover and O'Leary-Hawthorne, saying that a complete notion is "the ground and the reason" from which it is possible to "deduce" the set of its predicates, amounts to the oft-repeated adage "there is no purely extrinsic denomination which is not grounded on an intrinsic denomination"; or, quoting Aristotelian categories, all relations have their roots in qualities of substances:

"[...] and so they are merely relations which demand a foundation derived from the category of quality, that is, from an intrinsic

University Press, 1981, pp. 89–103; Mark Kulstad, "A Closer Look at Leibniz's Alleged Reduction of Relations", in *Southern Journal of Philosophy* 18, 1980, pp. 417–432.

¹⁹ G.W. Leibniz, *Discours de métaphysique*, §viii, GP IV, 443.

²⁰ J.A. Cover and J. O'Leary-Hawthorne, *Substance and Individuation in Leibniz*, Cambridge, Cambridge University Press, 1999.

accidental denomination" ("ex praedicamento qualitatis seu denominatione intrinseca accidentali").²¹

We may further notice that Leibniz surely extended the complete notion of an individual substance up to the extrinsic denominations, while he clearly distinguished the two levels of denominations, and while he linked the extrinsic denomination to the fact of the interconnectedness of the things and to the theory of expression:

"[. . .] I say that the concept of an individual substance involves all of its changes and all its relations, even those which are commonly called extrinsic (that is to say, which pertain to it only by virtue of the general inter-connection of things, and in so far as it expresses the whole universe in its own way), since there must always be some foundation for the connection of the terms of a proposition and this is found in their concepts".²²

Further, we cannot draw a non-reductionist argument from the very fact that first-order predicate logic is decidable while calculus of relations is not. Correctly, the authors of *Substance and Individuation in Leibniz* explain that this objection would bear on logical truths, whilst Leibniz is here dealing with contingent truths.

Nevertheless, despite all what distinguishes reductionism and non-reductionism, it is noteworthy that the contemporary model of supervenience finally provides the opportunity of an agreement, even if one reaches it by some opposite ways. We precise that relations or relational properties supervene on the ground of monadic properties if, for all relation R or relational property $R(y)$, necessarily if x is R to y or has the property $R(y)$, then there are monadic properties F, G, \dots of x and y , and necessarily, if x and y have

²¹L. Couturat, *Op & Fr*, p. 9 (1696). Translated by G.H.R. Parkinson under the title "On the Principle of Indiscernibles" in *Leibniz, Philosophical Writings*, Everyman's Library, London-Melbourne, 1984, p. 134.

²²Leibniz to Arnauld, 14th July 1686, GP II, 56 (see again *Monadologie* §56, for a revival of this thesis in the last Leibniz).

F, G, \dots , then x is R to y or x has $R(y)$.²³ Supervenience gives the advantage of making explicit an asymmetry: it happens that some facts which are relational, are obtained from others which are not, and however, the converse is not true.

So referring to supervenience, i.e. non-reductionist reliance, these interpretations strengthen Mates' reading by discarding the logical equivalence which Rescher believed to be unavoidable. The main relevance in the use of such a notion holds to a new confluence of the logico-syntactic analysis with metaphysics of substance and further with the particular conception of causality which is required by the doctrine of the pre-established harmony.

3. *Russell and Moore: two kinds of criticism on the relevance of the internal relations*

Russell's book *A Critical Exposition of the Philosophy of Leibniz* arose from some lectures uttered at Trinity College during a replacement of McTaggart, who had been invited in New Zealand in 1899.²⁴ Against Lotze' *Metaphysics*, McTaggart defended the non reducibility of relations.²⁵ Indeed Lotze rejected the existence of relations, and then McTaggart condemned his confusion of relations, which are between the things, with qualities of the things.²⁶ Surely a lot of qualities depends on relations, but McTaggart thought that it was possible to deal with pure qualities.²⁷

²³ J.A. Cover and John O'Leary-Hawthorne, *op. cit.*, p. 83. Hidé Ishiguro may even more ask for supervenience as Leibniz himself used the Latin verb *supervenire* about relations: "Relation [...] is an accident which is in multiple subjects; it is what results without any change made in the subjects but supervenes from them" (unpublished ms. quoted by Ishiguro, *Leibniz's Philosophy of Logic and Language*, Cambridge, Cambridge University Press, 2nd ed., 1990, p. 217). Before, in her paper "Leibniz's Theory of the ideality of Relations", this topic was sketched by an analysis of "presupposition" (in Harry G. Frankfurt, *Leibniz. A Collection of Critical Essays*, New York, Ancor Books, 1972, pp. 191–213).

²⁴ On the exact circumstances of the writing, see Walter O'Briant, "Russell on Leibniz", *Studia Leibniana*, Bd. xi, Heft 2, Wiesbaden, F. Steiner, 1979, pp. 159–222.

²⁵ Here Russell could give his approval, as noticed by F. Nef in *Qu'est-ce que la métaphysique?* (Paris, Gallimard, 2004, p. 570).

²⁶ According to Lotze, relations did not exist because they can be neither in one subject as in this case they are not relating, nor in many *relata*, because they can not be in two places at the same time.

²⁷ As for example, kindness, happiness, redness, sweetness (F. Nef, *ibid.*, p. 555).

The issue of setting sheer qualities apart from any touch of relationship, had yet been reckoned by Leibniz, who had written in the *New Essays on Human Understanding*:

“But no term is so absolute or so self-sufficient that it doesn’t involve relations, and whose the perfect analysis does not lead to another things, even to all the others. But we can say that some terms are relative and others are not by classifying as ‘relative’ only the ones that *explicitly* indicate the relationship that they contain.”²⁸

If it is so, the notion of a sheer quality bare of the slightest relationship would be a crude abstraction for Leibniz. Perhaps he had in his mind a continuous scale of predicates according to relatedness, with at the top — at the highest degree — the very relational predicates whose irreducibility, according to Russell, demonstrated the failure of a reduction of the asymmetric relations. The question is, for instance, whether “to be father” may be considered as a mere one place predicate or as an uncompleted two-place term with a bounded variable.

Russell’s and Mates’s criticisms have no care of such a difficulty to distinguish sheer absolute predicates, but not for the same reasons. From a strict nominalistic point of view, there is no room for the so-called relational properties as soon as we admit that only substances and their modifications exist:

“The paternity of David, in all its particularity, and the filiation of Salomon, similarly particular, are enough to make ‘David is the father of Salomon’ true; we don’t need, in addition to these two substances and their individual accidents, relations between them”.²⁹

Mates’s nominalism brings to completion the Aristotelian view on relations as the lowest degree of being in an ontology of substances. Russell’s criticisms, at the opposite, inaugurated the converse tendency, which started from the prevalence of the mathematical logic of relations, dealing with the definition of cardinal and ordinal numbers, and reached a form of Platonism, or

²⁸ G.W. Leibniz, *New essays on human understanding*, II, 25, §10 (GP V, 211). A narrow mechanistic philosopher of nature, in the XVIIth, would ascribe hardness as an absolute quality to a thing. However, mineralogists now use Mohr’s scale, where minerals are ordered by virtue of their capability to scratch the previous stuff just before. So, one cannot attribute an absolute hardness, but only a relative degree of hardness in relation with the dispositions of a substance.

²⁹ B. Mates, “Nominalism and Evander’s Sword”, p. 223. See also *The Philosophy of Leibniz*, p. 215.

at least some ontology of relations, in 1911–1912, because relations become the only true universals.

The axiom of internal relations did not yet constitute the core of Russell's criticisms in *A Critical Exposition of the Philosophy of Leibniz*, but we find the axiom at the leading place later in *My Philosophical Development* (1959) where Russell expressed his sound reluctance for this mode of thought. The focus of the doctrine is laid with the problem of a change about a man who became father as he lived in China and his wife still in Europe. This kind of paradox brings to the fore the unacceptable consequences of the idea of a whole organic truth reflecting a whole organic reality. In his paper on "The Monistic Theory of Truth" (1906), Russell listed two possible expressions of the axiom to be distinguished.

- 1) Either every relation is actually *constituted* by the nature of its terms;
- 2) or every relation has its *grounds* in such a nature.

Under the first version, we must recognize the monistic reduction of relations, each relation involving the properties of an inclusive whole; whilst the second version conceals the monadistic view. I don't know whether Russell was aware that, for the late scholastics, the word *fundamentum* about a relation was indeed a technical term exactly expressing that, as we saw above, there is no extrinsic denomination which is not grounded on an intrinsic one, i.e. no relation which is not grounded on a quality.³⁰

In 1906, Russell emphasized on the link between the axiom of internal relations and the principle of sufficient reason. Indeed, if the axiom is held, it follows that the relation of a subject to its predicate may no longer be a mere matter of fact. In any way, this must explain why Leibniz understood inherence as a containment relation. Given that a sufficient reason holds at the root of the reduction of all relations, Russell reached the modal issue of the problem, writing against Bradley: "[...] if two terms have a certain relation, they cannot but have it, and if they did not have it, they would be different".³¹

G.E. Moore's prevalent merit, in his brilliant paper "External and Internal Relations" (1919), was to bring the modal issue up to a sketch of formalization.³² At the beginning, Moore scrutinized the sort of meaning which is implied by Bradley's assertions in *Appearance and Reality* such as: every relation "affects" the related terms or "passes into its terms" or "modifies"

³⁰ L. Couturat, *Op & Fr*, p. 9.

³¹ B. Russell, *Philosophical Essays*, 1910, London, Routledge, 2000, p. 143.

³² G.E. Moore, "External and internal relations", in *Proceedings of the Aristotelian Society*, 1919–1920, collected in *Philosophical Studies*, 1922, London, Routledge, 1965, IX, pp. 276–309.

them. Obviously, there are some modes of settling a relation, for instance to bring a bit of wax close to a candle, where it would be right to say that some relations modify its terms. But Moore did not agree that this was always the case and denied that relations always modify their terms. The axiom of internal relations, understood as meaning "every relation modifies its terms", would undermine the objectivity of knowledge. Sooner in the paper on "The Nature of Judgement" (1899), Moore claimed, against Kant, that relations could not be only "the work of mind". According to this view, the Russelian theory of acquaintance will just apply the converse axiom of external relations to knowledge.

Differing from those of Russell, Moore's attacks against the axiom of internal relations did not work in support of an antagonist principle of external relations. Indeed a lot of relations are unquestionably internal, as, for instance, to be between red and yellow in the case of the orange colour. Moore's purpose amounts to reject that all relations are internal, and moreover that a relation may be twofold oriented, internal in one sense and external in the other. This is exactly what we find in Whitehead's theory of abstraction concerning the ingression of the eternal objects into the actual entities, even if Moore's ground, of course, was quite less speculative.

"In order to get an example, we have only to consider the relation which the red patch has to the whole patch, instead of considering as before that which the whole has to it. It seems quite clear that, though the whole could not have existed without having the red patch for a patch, the red patch might perfectly well have existed without being part of that particular whole".³³

For a whole, to have a part consists in an internal relation, while, in the other way, to be part of a whole consists in an external relation.

As far as I know, this tactfully way of dealing with relations disappears with Russell. Further, Moore shows in what a relational predicate works differently than a mere relation. A relational predicate gives the whole assertion as "to be father of *a*". If the same father has some children *a*, *b*, *c*, the corresponding relational predicates "to be father of *a*", "to be father of *b*", "to be father of *c*" would differ from each others, while the fatherhood relation remains the same. Thus at the two levels, relations and relational properties do not seem to modify their terms and reciprocally. A relation remains stable through its very function of relating, to the extent that relational properties and their terms are so correlated that it is no more possible to settle the sense of a modification. If one claims that every relation modifies its terms, one

³³G.E. Moore, *op. cit.*, p. 288.

insures, at the opposite, that fatherhood, as we are father of a, b, c , is specific for each occurrence.

The axiom of internal relations eventually pretends that a thing A *necessarily* would be different if A did not have a property as Φ . However this difference may be understood either as a numerical difference or as a qualitative difference. So there is a sophism denounced by Moore at the deep basis of the axiom: Moore agrees with the common sense that the fact for A of not having Φ makes A qualitatively different, but he notes a sophism as soon as the axiom substitutes to the qualitative difference a numerical one.

The root of the axiom holds on a mistake, and from now on, we will have to distinguish these two statements:

1) if a particular as A has a property Φ and x has not Φ , then A *must* be different from x . This true proposition alludes to a *qualitative* difference between A and x , so that it is not necessary that x is not A .

2) If a particular as A has a property Φ , it *follows* that, for every x which has not Φ , x would be *necessarily* different from A , this definitely referring to the idea of a numerical difference.

The dogma of internal relations blurs a groundless inference of the second proposition from the first. If we do not confuse them, it becomes again possible to pretend that King Edward VII might not have a relational property such as "to be father of Georges V" and still be Edward VII.

In the second part of his paper, Moore comes back to the nature of the connective needed by the attribution of an internal relational property as it is claimed by the dogma. He explains that the inference from the property "not having Φ " to "not being A " is of the same kind as the inference from "being a right angle" to "being an angle" or from "being red" to "being coloured". This link of inference, which Moore intended to separate from Russell's formal implication (to imply), may be symbolized with "ent." as "entailment", allowing this writing of the axiom of internal relations:

$$(27) \Phi a \rightarrow (\forall x)[(\sim\Phi x) \text{ ent. } (x \neq A)]$$

For two variables, the formula of the axiom yet needs the note of Russell's material implication:

$$(28) (\forall x)(\forall y)[(Fx) \text{ ent. } (\sim Fy \supset y \neq x)]$$

Here we meet Lewis' strict implication under Moore's entailment, Moore himself noting this relevant explanation. Then Hughes and Cresswell have strengthened the same reading: "entailment" here names the converse of the relation of following logically from, inasmuch as p entails a proposition q

amounts to saying that q follows logically from p .³⁴ So it is not amazing that Moore also paraphrases his formula, as well as its negation, in the words of a possible worlds-semantics before its time:

“The proposition, with regard to a given thing A and a given relational property P , which A in fact possesses, that P is *internal*. The required expression is $\sim Px$ entails $x \neq A$. [...] And this proposition is, of course, logically equivalent to $x = A$ entails Px . [...] This last proposition again, is, so far as I can see, either identical with or logically equivalent to ‘anything which were identical with A , would, in any conceivable universe, necessarily have P ’ or by ‘ A could not have existed in any possible world without having P ’; just as the proposition by ‘In any possible world a right angle must be an angle’, is, I take it, either identical with or logically equivalent to the proposition ‘ $(x$ is a right angle) entails $(x$ is an angle)’ ”.³⁵

In a following passage, Moore explicitly refers to Leibniz’s theory of eternal truths. To put what he says briefly: if all relations are internal, all properties become essential and all truths about them are eternal. The axiom of internal relations bears on the whole properties of a particular.

Moore’s moderate principle of external relations, namely that some relations are external and some others internal, remains compatible with a form of essentialism avoided by Russell’s hard principle. After Moore and Russell, we may suppose that the reject of the internal relations is overtaken by a commitment to a form of essentialism. We remember Quine’s reluctances against modal logic, suspected to be the Trojan horse of essentialism. And Leibniz’s case makes worth in the sense that he could be a “superessentialist” more than being only an essentialist philosopher.

Indeed, essentialism means that for any individual substance x , there is a property P of x such as, necessarily, if x exists, then x has P (for example, humanity for Caius):

$$(29) \forall x, \exists P(x) \rightarrow \Box(\exists x \rightarrow P \in x)$$

However Leibniz’s doctrine of the complete notion claimed something stronger: for any individual substance x , and for any property P of x , necessarily,

³⁴ G.E. Hughes and M.J. Cresswell, *A New Introduction to Modal Logic*, London, Routledge, pp. 202–203, 208.

³⁵ G.E. Moore, *op. cit.*, p. 293.

if x exists, then x has P (for example, wearing a white tunic for Caius):

$$(30) \forall x, \forall P(x) \rightarrow \Box(\exists x \rightarrow P \in x)$$

The logical conception of substance implies that every individual bears such a complete notion that it contains the set of all its monadic predicates as well as its relational predicates.

We have seen that, in his criticisms of F.H. Bradley's *Appearance and Reality*,³⁶ Moore, at the beginning, followed Russell. Bradley will later take these criticisms into account in his *Essays on Truth and Reality* (1909). The point whether Bradley actually was a proponent of the axiom of internal relations becomes more dubious nowadays. Bradley held it not specially against external relations but against the self-contradictory nature of all relations, following the style of Kant's antinomies and the Eleatist tradition, whose form of argument — namely the infinite regress, or, as it is sometimes said nowadays, a "supertask" — he remembered.

This mode of refutation by the infinite regress argument has been also used by Russell in the *Principles of Mathematics*. In the second book of *Appearance and Reality*, Bradley expounded the famous regress inside a dilemma. From one side, if a relation is taken as "independently real [...], a relation standing alongside of its terms is a delusion".³⁷ This is the side of the dilemma which exactly corresponds to Leibniz's third mode of dealing with relations in the correspondence with Clarke, *Fifth Writing*, §47. From the other side, relation becomes the adjective of its terms; then relation loses its power of relating.³⁸ The Bradlean regress consists in noticing that if there were a relation R between such terms as a and b , we would have to conceive a further relation R' to stick a to R and R to b , and so on *ad infinitum*.

$$(31) \begin{aligned} &R(a, b) \\ &R'[(a, R), (R, b)], \\ &R''\{[(a, R), R'], [(R, b)R']\} \dots \text{etc.} \end{aligned}$$

³⁶ *Appearance and Reality*, Oxford University Press, 1893, Bk I, chap. iii, 2nd ed. 1897, Appendice B.

³⁷ *Op. cit.*, 16th impression, Oxford University Press, 1978, chapter ii, p. 18.

³⁸ *Op. cit.*, note 1, p. 27.

Like Zeno's paradoxes, Bradley's regress has nowadays been rehabilitated and is sometimes held for a worthy argument.³⁹ But Russell himself denied that this regress actually was vicious. Indeed, did not the theory of series provide full examples of regress *ad infinitum* which do not carry any risky contradiction? So it becomes expedient to separate vicious and harmless regresses, as Russell in the *Principles of Mathematics* (§§55, 59):

- An "external regress" or "regress of implications": p implies p' which implies p'' *ad infinitum*. Such a regress is not necessarily vicious and Russell judged that the Bradley regress about relations belonged to this species.

- An "internal regress" or "regress of meaning": p is analysed by virtue of a proposition p' more complex than p itself, then p' by virtue of p'' , *ad infinitum*. This later regress suggests that one could not understand in a finite quantity of time a meaning, which is absurd if p may actually be understood in a finite period of time.

Near the end of his life, Bradley, again writing about the problem of relations in an unfinished draft, iterated his original thesis.⁴⁰ Against William James, he asserted that an "experienced interrelation" was never given, that every relation is secondary in comparison with a "direct and non relational awareness", which is "relation-free".⁴¹ When we ask for a "fact of relatedness", we focus on the plurality of the terms, losing the relating unity, the "between" inhering to any relation. Obviously, the relational view is still unavoidable in the practical way of life, but metaphysically, it is a sheer "makeshift" because relation and its terms constitute a whole reality such that modifying a term shifts the relation itself and modifying the relation changes in return its terms.⁴² Indeed practical life feeds on abstractions.

So, speaking in Whiteheadian terms, Bradley reacted to a kind of "fallacy of misplaced concreteness": he denied to Russell the right to root out a relation as an ultimate fact. All relations are grounded on a "relational situation" which could never be a simple juxtaposition of "and". The real issue becomes: where and how far may we make the choice of bounding a particularity into the whole experience? And this simple consideration is enough

³⁹ Richard Gaskin, "Bradley's Regress, the Copula and the Unity of the Proposition", *Philosophical Quarterly*, 1995, vol. 45, pp. 161–180; William F. Vallicella, "Relations, Monism, and the Vindication of Bradley's Regress", *Dialectica*, 2002, vol. 56, n° 1, pp. 3–35.

⁴⁰ F.H. Bradley, "Unfinished Draft of an Article on Relations", in *Collected Essays*, XXXI, vol. II, Greenwood Press Westport, 1935, Oxford, Clarendon Press, 1970, pp. 630–676.

⁴¹ *Ibid.*, p. 656.

⁴² "[...] with a relation and its terms, a change made on one side makes also a change on the other side" (*Collected Essays*, p. 639).

for dooming all "hard distinction" between what belongs to an essence and what is a pure "matter of fact".

According to these views, to distinguish between external and internal relations becomes of no use, so that this distinction is frankly said "untenable".⁴³

- In the case of the external relations: the "coming-in-relation" happens without any reason in the relation or in its terms. The relational fact escapes from the principle of sufficient reason in order to become, as Hume said, simple "matter of fact". However, for an individual or a particular, to enter in some relation must both depend on its nature and on some circumstances.⁴⁴

- In the case of the internal relations: there is a contradiction following from the isolation of the terms (held as real) and their fusion by a relation, so that such a doctrine may be qualified as "ludicrous". Nevertheless Bradley attached particular importance to the definition of the internal relation, remembering that all relations rest on a unity taking as a whole and that every change in the terms must alter the relation.⁴⁵

Yet Bradley was ready to acknowledge as relatively true the distinction between the internal and the external relations, distinction whose relevance rests on the cleaving of a term, with what is essential on one side and its accidental determination on the other.⁴⁶ This reasoning might have led to three kinds of relations: essence/essence, essence/accident, and accident/accident. But a new trouble arises, underlined by Bradley. How are related the relational part and the essential part of a term related? Thus the acceptance of a relative truth does not lead to the Absolute, where all contradictions would be appeased.

In a first stage, roughly speaking, Russell esteemed that if the glue did not stick directly without any further glue, no act of sticking would be possible any more. But at a deeper level, with the analyses of the §54 of *Principles of Mathematics*, Russell ran into the difficult problem of the unity of the proposition, which involved the puzzle of the linkage for the case of a relation. Moore explicitly revived this topic, as he started his paper in 1919 by noting that a relation does not amount to a mere list of its components. According to Russell himself, we reach here the blind spot of the philosophical analysis from 1903 to the logical atomism. Given that once the proposition is resolved, the *nexus* of the proposition is lost, a proposition essentially lies

⁴³ *Ibid.*, p. 641.

⁴⁴ *Ibid.*, note X, p. 667.

⁴⁵ *Ibid.*, p. 665.

⁴⁶ *Ibid.*, p. 645.

on a unity supported by the double nature of the verb, the verb properly said and the verbal name. And there is a parallel distinction to do about relations: the relation *per se* is not the same as the "relating relation", and precisely this is what is lost in Bradley's regress argument.

In his demonstration that a monist theory of truth doomed all kinds of relational propositions, Russell insisted on the directional feature of asymmetric relations. Bradley's gave another version, as any form of relation would be fateful for the Absolute.⁴⁷ And even before relational propositions, simple monadic propositions, lying on inherence, soon imply relations. We can no more escape to the same regress as when relations were considered as independent realities. A *p*-ness property indeed required that the predicate *p* would be connected to its subject *S* in order to be inherent. From a monistic point of view, the major point was not that all truths could be interpreted as predications of qualities to a totality, but further that all predications are in fact unreal.

So, by insisting on the directional feature of asymmetric relations, Russell perhaps neglected an anterior step, because before being oriented, a relation must be relating, for the very reason that the attribution of a predicate to a subject must itself be relating. So Bradley was not wrong when suggesting that the problem of relation was no more puzzling than the question of inherence which seemed to be preliminary. In a letter addressed to Bradley written in 1914, Russell again recognized that the matter of the unity of the proposition actually was a "vital" problem, using the same adjective in the §55 of the *Principles of Mathematics*, where the theory of relations was the focus.⁴⁸

4. Whitehead: Towards a Rehabilitation of the Internal Relations

For the ones who, from the Stoics to German idealism, intended to build a philosophy of nature, the main principle that all things are interrelated (*sumproia panta*, as Leibniz wrote) in accordance with a universal sympathy constituted a minimal claim. Nowadays we think that what Whitehead had in mind with the "bifurcation of the nature", seen in an historic context, corresponds to the victory of the Galilean-Cartesian mechanization of

⁴⁷ *Ibid.*, note XIII, p. 670.

⁴⁸ Quoted by Sébastien Gandon and Mathieu Marion, in "L'idéalisme britannique: histoire et actualité", <http://www.er.uqam.ca/nobel/philuqam/philomath/doc/Idealisme.pdf>, p. 28, which will be soon published in *Philosophiques*, review of the Philosophical Society of Québec. See also PoM, §439.

the world picture at the end of Renaissance, discarding the revival of the Hermetic tradition during the XVIIth century.

Yet Whitehead himself warns us that his own endeavours in *The Concept of Nature* should not be confused with a *Naturphilosophie* in the same sense as Schelling's, because that would imply a leap over his own field of ability.⁴⁹ Whitehead understands the excitement of overtaking the platitude of empiricism — to use a typical Hegelian formula —, without yielding to this temptation. Concerning, at least, these Tanner Lectures, the main discordance with the German idealism was that Whitehead did not intend to decide about the ultimate nature of things, given that he only wished to circumscribe his own discursive reasoning to the "factors of the nature" according to the "sensible awareness".⁵⁰ However, after the analysis of the relations of space and time, he drew the conclusion that if "[...] nature is a system [...] there is no mere localisation".⁵¹ So, to denounce the false obviousness of mere localisation upholds the thesis of an interconnected nature. Whitehead himself quotes Faraday in support of this view: for the electromagnetic theory, "an electric charge is everywhere". Even before special relativity, field theory provided him a suitable tool to overtake a simpleminded mechanism. Maybe Leibniz's drama was that he foretold a new physics beyond the mechanistic view of world, but could only find his own starting science, the *Dynamica*, which was first grounded on the "vis viva", then on the "actio". He had then to stress upon the superseding of the mechanism by his new science, to which he attributed some features of the future field physics.

In Whitehead's tract *On Mathematical Concepts of the Material World* (1905–1906), the use of relations consisted in applying Russell's theory of polyadic relations to the necessary and sufficient geometry needed by the physical theory, keeping in mind the nominalistic imperative of the Ockham's razor: reducing the number of these relations to the bare minimum.⁵²

By the way, Whitehead disclosed the kinship of the dynamical concepts in a Leibnizian style with some resources coming from projective geometry, vectorial calculus and, of course, Maxwell's electromagnetism. The entities of the physical world, generated thanks to some polyadic relations, become the elements of the field — in a logical meaning — of a determined relation.

⁴⁹ Alfred North Whitehead, *The Concept of Nature* [CN], The Tanner Lectures delivered at Trinity College, Cambridge University Press, 1920, p. 47.

⁵⁰ CN, p. 151.

⁵¹ CN, p. 146.

⁵² Alfred North Whitehead, "On Mathematical Concepts of the Material World", in *Philosophical Transactions*, Royal Society of London, series A, 1906, v. 205, pp. 465–525.

Under the expression "material concept", Whitehead does not thus designate the "ultimate existents" of the natural world, but a whole set of axioms satisfying some fundamental relations. The ultimate existents themselves refer to the instants of the time and to the "objective reals" which involve two others classes of non temporal entities, points of space and particles of matter. The composition of time by instants being yet indisputable, Whitehead here adheres to an absolute thesis concerning this topic.

Following the terminology settled in the *Principia Mathematica*, every axiom has to be discerned from a proposition. Given that an axiom keeps undetermined what class of entities is relevant, axioms are rather bare propositional functions. And because geometry may determine the same entities by the mean of diverse sets of relations, Whitehead wrote at the beginning of his work on *The Axioms of the Projective Geometry* that geometry amounts to the "science of crossed-classifications".⁵³ According to MCMW, the determination of the stuff in the universe needs three characteristic features:

1) an "essential" relation " $R ;$ ", which holds a connexion between a finite number of terms, for instance

$$(32) R ; (abc)$$

This short formula denotes a linear order between three non interchangeable points a, b, c . This essential relation appears triadic in the case of the material concepts I and II, which are classical punctual concepts. More exactly, the essential relation here consists in an asymmetric relation tying three spatial entities, following Oswald Veblen's axiomatic of projective geometry.

This essential relation becomes tetradic in the only case of a punctual concept qualified as "Leibnizian" (concept III: between three points and one instant), and even pentadic in the case of linear concepts, similarly Leibnizian (concepts IV and V: between four linear reals and one instant).

2) Then Whitehead considers a serial dyadic relation representing the temporal order, having as its field the whole set of instants, and this relation may be either external or integrated to the essential relation as for the case of the Leibnizian concepts.

3) Regarding the relations said "extraneous", in the simplest case, they deal with a triadic relation of occupation about a point of space in an instant of time by a particle. A triadic relation is in question, whereas, under the control of Ockham's razor, due to a fusion of points of space with particles of material stuff, the initial triadic relation becomes dyadic with the concept II, then tetradic, in order to introduce a system of kinetic coordinates, with

⁵³This first chapter was available in a French translation (*Revue de métaphysique et de morale*, t. xv, n° 1, 1907, pp. 34–39) under the title: "Introduction logique à la géométrie".

the concept III. The concept IV involves two versions, since it repeats, on the level of the linear concepts, the monist revolt of the concept II against the dualist concept I.

Concept I	Concept II	Concept III	Concept IV	Concept V
<p><dualistic punctual CLASSICAL concept: separation between geometry and physics></p> <p>Lines and planes are some classes of points.</p> <p><i>Objective reals:</i> points of space and particles of matter.</p> <p><i>Essential relation:</i> non symmetric relation between 3 spatial terms. "$R ; (abc)$" means a linear order between non interchangeable points a, b, c</p>	<p><monistic punctual CLASSICAL concept: separation between geometry and physics></p> <p>Lines and planes are some classes of points.</p> <p><i>Objective reals:</i> only points of space</p> <p><i>Essential relation:</i> non specified, it goes back to the concept I.</p> <p><i>Extraneous relations:</i> dyadic between instants and points, they are of an undefined, if not infinite, number in order to determine the positions of particles.</p>	<p><LEIBNIZIAN and monistic punctual concept: any separation between geometry and physics due to the temporalization of geometrical propositions of the concept I></p> <p>Lines and planes are some classes of points, or even better, moving particles; points are disintegrated at each instant.</p> <p><i>Objective reals:</i> moving material points (those of Cartesian physics or modern aether)</p>	<p><LEIBNIZIAN linear concept></p> <p>Dualistic concept IV_A while concept IV_B is its monistic version.</p> <p>Lines are simple entities; points or even better, moving particles, lines and planes are disintegrated at each instant.</p> <p><i>Objective reals:</i> linear entities + loaded particles positively or negatively in IV_A; monism of linear entities in IV_B (let's note the analogy I/II and IV_A / IV_B).</p>	<p><LEIBNIZIAN monistic linear concept></p> <p>Lines are simple entities; points or even better, moving particles, lines and planes are disintegrated at each instant.</p> <p><i>Objective reals:</i> exclusively linear entities (as in IV_B)</p> <p><i>Essential relation:</i> between 5 terms, "$R; (abcdt)$": linear reals bcd are intersected in t by a.</p> <p><i>Extraneous relations:</i> tetradic relations in a single number in order to measure the velocity of particles.</p>

<p><i>Extraneous relations:</i> in an undefined, if not infinite, number between 3 terms: a particle, a spatial position, an instant of time. (defective in relation to the Ockham's principle)</p>	<p>(defective in relation to the Ockham's principle)</p>	<p><i>Essential relation:</i> between 4 terms: "R; $(abct)$" or "R_t; (abc)". <i>Extraneous relations:</i> tetradic relation (t and spatial coordinates u, v, w) alone, to determine only some kinetic axes in order to measure the velocity of particles. (agreeing with the Ockham's principle)</p>	<p><i>Essential relation:</i> between 5 terms, "R; $(abcdt)$": linear reals bcd are intersected in t by a. <i>Extraneous relations:</i> · analogous to I in IV_A, triadic relation between an instant, a point and a particle of an undefined, if not infinite, number in order to determine the position of particles. The laws of nature bear on particles as well as linear reals. · In IV_B, dyadic relation between a point and an instant, substituting a relation to a particle as, punctually, in II. (defective in relation to the Ockham's principle)</p>	<p>(agreeing with the Ockham's principle)</p>
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This logical apparatus extended by Whitehead through a calculus of relations was imagined by Russell, developing some of Peano's ideas in his paper composed in 1901, *The Logic of relations with some Applications to the Theory of Series*. Behind the mathematical purpose, Russell concealed a philosophical involvement which, eventually, particularly in his address to the Aristotelian Society *On the Relations of Universals and Particulars* (1911),⁵⁴ led him to reverse Aristotle's initial decision on the status of relations, as it is stated in the *Metaphysics* that "relation is, among all categories, the one which is the less determined reality or substance" (N, 1, 1088^a 20) since it is derived from other properties. Russell, however, did not discard the leading principle of keeping predication as implying a logical main difference between two kinds of terms, particulars, i.e. subjects of predicates, and universals, with, among them, predicates and relations. Russell acknowledged that Berkeley and Hume, decrying abstract ideas, could not avoid using a likeness relation, for instance, colour-likeness between two instantiations of the same colour. Here the demonstration of the irreducibility of relation to a twofold predication was still useful in order to underline the precedence of at least one relation, this likeness-relation.⁵⁵

According to modern philosophy, this reversal happened during what we could now name the quarrel over relations, opposing Russell, James, Bradley and Bosanquet. Later on, writing about Hume — and inspired by his master Jean Wahl —, Gilles Deleuze will emphasize on the empiricist roots of the advent of the external relations, recovered after the neo-Hegelianist interlude.⁵⁶ Without taking seemingly part in this quarrel, did Whitehead deal, at the beginning of his philosophical career, with relations because of some metaphysical ulterior motive?

Again, Whitehead's first important philosophical work is often credited to be a sketch touching the ontology of relations.⁵⁷ At Russell's difference, Whitehead did not care to insist on the crucial divergence of the symmetric and asymmetric relations, but he took advantage of the order asymmetric relations coming from Veblen's axiomatization. As seen above, he highlighted

⁵⁴ In Bertrand Russell, *Logic and Knowledge. Essays 1901–1950*, ed. by R.C. Marsh, London, George Allen and Unwin, 1988, p. 103–124.

⁵⁵ Perhaps Jules Vuillemin, even esteeming that Russell have held relations as substances, went too far concerning this point (*La logique et le monde sensible*, Paris, Flammarion, 1971, p. 49).

⁵⁶ Gilles Deleuze, *Empirisme et subjectivité*, Paris, P.U.F., 1953, under the inspiration of Jean Wahl's *Les philosophes pluralistes d'Angleterre et d'Amérique*, Paris, F. Alcan, 1920.

⁵⁷ See Luca Gaeta, *Segni del Cosmos. Logica e geometria in Whitehead*, Milan, Il Filarete, pubblicazioni della Facoltà di Lettere e Filosofia, 2002.

an essential relation as well as some extraneous relations, which had to be reduced for satisfying the nominalistic claim. From the concept I to the concept II, classical mechanics is meanwhile rewritten, in conformity with a Russellian insight for the chapter LIII of the *Principles of Mathematics* on "Matter".

Among the things being likely to exist, under the lead of rational dynamics, explains Russell, one finds: 1) instants; 2) points; 3) terms occupying instants but not points; 4) terms occupying points and instants together.⁵⁸ Russell holds "occupying" as an undefinable and therefore fundamental relation. Matter is nothing but a "many-one" correlation relating all the instants of time and some points of space, time being a one-dimensional series, whilst space may be a *n*-dimensional series. Russell's fundamental and undefinable relation of occupation translates Whitehead's extraneous relations for the concept II, which is less wasteful than the concept I, but still defective in relation to the Ockham's principle. Then with the concept III, Leibniz's relational theory of space defined as the order of coexisting things is brought to the fore, this becoming possible from the reducibility of the extraneous relations. Instants of time integrate the essential relations, with a moving world as an effect:

$$(33) R ; (abct) \text{ or } R_t ; (abc)$$

The Leibnizian material world corresponds to the suppression of the dualism of points and particles. The fact that time is involved into the essential relation yet does not imply any physical relativity theory. Here Whitehead's skyline was still bounded by Maxwell's and Maxwellians' electromagnetism. Something stronger concerning the relationship between *abc* and *t* would otherwise be required in order to understand mass as a function of velocity. Of course this comment remains very theoretical, considering that *On Mathematical Concepts of the Material World* was written just at the time of the birth of special relativity.

The best the extraneous relations have to do, in order to respect Ockham's razor principle, is to provide kinetic axes for measuring the velocity of particles. At the time of the *Inquiry Concerning the Principles of Natural Knowledge* (1919), Whitehead will use similar kinetic axes to make possible the deduction of Lorentz's equations following the reconstruction of a Minkowskian space-time.⁵⁹ The 1905 work reaches its end as Whitehead

⁵⁸ Bertrand Russell, PoM, §438, p. 465.

⁵⁹ See our paper, "La dérivation des transformations de Lorentz par Whitehead", in *Chromatikon IV. Yearbook of Philosophy in Process*, Presses universitaires de Louvain, Louvain-la-Neuve, 2008, pp. 9–20.

elaborates the concept v , which goes on to assume a realistic thesis regarding lines of force, as Faraday finally seemed to have upheld.⁶⁰

Qualifying as "monist" his more refined concept v , Whitehead certainly did not allude to a philosophical monism. However it comes then that physical field theory, on which, as it was noticed by Keith Campbell in his book *Abstract Particulars*, the prevalent concept of material world lies, did not particularly promote any pluralism.⁶¹ Physical reality consists in the overlapping of different fields, each corresponding to one of the fundamental forces of the universe. And as soon as field comes to prevail over particles, parts of a field are not actually separable. If the great unification of the four kinds of force would be achieved, this could be even less possible. At once, each field possesses its own intrinsic properties; however, all the fields are interconnected, internally linked with the others. Thus the monist ontology could have a broad future, or at least a pure atomistic insight of the universe is out of game. After the results of the experimentations concerning interactions between correlated particles from a distance, quantum mechanics appears to lead to the same outcome.

From his early cosmological works, Whitehead drew another conclusion than Russell's, about the criticism of the prevalent judgement of predication. According to Russell's first mathematical philosophy, the irreducibility of relations led to preserve the absolute space and time. Whitehead inquired into the nature of space and time foremost through the mereological relation, which he first applied to space thanks to an early version of the method of abstractive extension in his French essay "La théorie relationniste de l'espace" (1914).⁶² That the whole-and-part relation is as well spatial as temporal then arose from *The Anatomy of Some Scientific Ideas* (1917).⁶³ First applied to generate the points of space, the generality of the method was such that

⁶⁰ "The inquiry in now entered upon of the possible and probable physical existence of such lines" (*On the Physical Lines of Magnetic Forces*, 1852; quoted by Bertrand Saint-Sernin, "Morphogenèse mathématique du monde matériel", in *Les études philosophiques*, octobre-décembre 2002, Paris, P.U.F., p. 435).

⁶¹ Keith Campbell, *Abstract particulars*, Oxford, Blackwell Publishers, 1991; quoted by François Clementz in "Réalité des relations et relations causales" (*La structure du monde: objets, propriétés, états de chose. Renouveau de la métaphysique dans l'école australienne de philosophie*, Paris, Vrin, 2004, pp. 517–518).

⁶² Alfred North Whitehead, "La théorie relationniste de l'espace", in *Revue de métaphysique et de morale*, 1916, t. XXIII, pp. 423–454.

⁶³ Alfred North Whitehead, "The anatomy of some scientific ideas", first published in *The Organisation of Thought, Educational and Scientific*, London, Williams & Norgate, 1917, chapter vii, then in *The Aims of Education and Other Essays*, London, Williams & Norgate, 1929, chapter ix.

Whitehead might also obtain instants of the time by starting from W. James's "specious present". In the case of time, the mereological interlocking by an "enclosure relation" must be associated with an order relation which defines the direction of time. Indeed the rebuilding of Minkowski's space-time must include such relations of order on time. The whole-part relation must be distinguished from the logical belonging. Against Russell, space can no longer remain a set of points.⁶⁴

Whitehead did not care of any controversy with idealism and monism though he started as an empiricist for whom the stubborn facts, in a theory of knowledge, consist in the perception of the *membra disjecta* which are to be put in relation through space and time orders. Nevertheless, with PNK, Whitehead accessed the concept of organism, explaining why even an electron could share some properties of an organism. This bold approach imported into the analysis of the concepts of the material world the Hegelian Trojan horse of the whole organic unity. And not amazingly, Whitehead carried on a plea in favour of the universal interconnectedness of the whole events, for this time in unison with the physical relativity.

"Significance' is the relatedness of things. To say that significance is experience, is to affirm that perceptual knowledge is nothing else than an apprehension of the relatedness of things, namely of things in their relations and as related".⁶⁵

Afterwards Whitehead expressed more and more his doubts about the ability of the external relations to account for the system of nature.⁶⁶ Nature was referred to "a complex of related entities", analysed by our ideas of space and time.⁶⁷ However CN is placed under the patronage of Russell due to the fact that Whitehead, at the beginning, uses Russell's theory of denoting and asserts that "evidently the relations holding between natural entities

⁶⁴ B. Russell, PoM, p. 443.

⁶⁵ Alfred North Whitehead, *An Enquiry Concerning the Principles of Natural Knowledge*, Cambridge, Cambridge University Press, reprint New York, Dover Publications Inc., 1982, §3.5, p. 12.

⁶⁶ "Thus the origin of the doctrine of matter is the outcome of uncritical acceptance of space and time as external conditions for natural existence" (CN, p. 20).

⁶⁷ CN, chap. i, p. 13.

are themselves natural entities".⁶⁸ If thought deals originally with individuals, the primary factors of sense-awareness are relations, and secondarily thought deals with all the "this" and "that", that is to say, Russell's logical proper names which denote without meaning.⁶⁹ That what is called "bifurcation" corresponds also to the genesis of the judgement of predication, given that the *terminus* of the sense-awareness slips in the background.⁷⁰ So we find in Whitehead the same Russellian struggle against the prevalence of predication, indeed a "confuse notion" hiding the whole diversity of relations among entities.⁷¹ It is still worth noting that Whitehead seemed to tend towards realism about the relations, but surely more about internal relations than in a Russellian way, though the Russellian Leibniz pervaded in depth CN, for instance, as Whitehead wrote:

"Some schools of philosophy, under the influence of the Aristotelian logic and the Aristotelian philosophy, endeavour to get on without admitting any relations at all except that of substance and attribute. Namely all apparent relations are to be resolvable into the concurrent existence of substances with contrasted attributes. It is fairly obvious that the Leibnizian monadology is the necessary outcome of any such philosophy. If you dislike pluralism, there will be only one monad."⁷²

Brought to the fore by the predicate-subject relation, every substance is recognized from the criteria of not being predicabile to anything else, that which constituted, according to Leibniz in the *Discourse of Metaphysics* (§viii), only its "nominal definition", the definition by the complete notion being the actual one. Transformed into a concrete thing underlying to all what is disclosed to the sense-awareness, substance leads straight to the aether of the modern physics. Then matter is thrown into space and time,

⁶⁸ CN, p. 14.

⁶⁹ "The 'it' for thought is essentially a relatum for sense-awareness" (CN, p. 8).

⁷⁰ "The entity has been separated from the factor which is the terminus of sense-awareness. It has become the substratum for that factor, and the factor has been degraded into an attribute of the entity. In this way a distinction has been imported into nature which is in truth no distinction at all. A natural entity is merely a factor of fact, considered in itself." (CN, p. 16).

⁷¹ "Personally, I think that predication is a muddled notion confusing many different relations under a convenient common form of speech." (CN, p. 18).

⁷² CN, chap. vii, p. 150.

and space and time become the attributes of the substance or, that amounting to the same, external relations.

But precisely Bradley tried to show in which way a spatial relation, often classically chosen as the very case of external relations, could be understood as an internal one. Only "for working purpose", the human mind may keep what is perceived as "relatively external". According the common-sense, changing in space is as well external as the relation of comparison, which does not look to modify the compared terms. However, if a comparison would not imply the inner nature of the terms, the relation which it involves would be arbitrary. And about spatial modification, Bradley considered that all spatial organization takes on a qualitative dimension. Writing that a sheer space grounded on external relations becomes an inconsistent abstraction, Bradley met a main feature of Leibniz's theory of space, as linked to the principle of sufficient reason. Space is but an order of external relations resolvable to internal relations. Thus to be part of a fresh whole cannot happen without any reason. Without any qualitative difference, there is no more distinction in space. Where Bradley claimed that "nothing in the world is external so except for our ignorance",⁷³ to dispose of a perfect knowledge of the universe, according to Leibniz, amounts to see the reason of all the relations, that which allows to deduce the universe from the minor detail, namely an internal feature of any term. In a whole, nothing may be external. The point where monadism met up with monism is thus: a whole which is less than the Universe always constitutes an abstraction.

However, Whitehead esteemed that the prominence of the substance ontology in Leibniz impeded space and time to be actually internal relations. Because relations for Leibniz stay depending on attributes, Leibniz in fact is closer to the Newtonian absolutes that he has imagined. Spatial and temporal relationism cannot be performed before events have superseded substances. Then space and time could be relations of attributes or, even better, relations of the factors of the events.

In *Science and the Modern World*, Whitehead comes to a resolution which invalidated Russell's kind of pluralism, upon which henceforth weights the suspicion of materialism. Indeed, the main feature of the materialistic conception of nature consists in connecting portions of matter with external relations. A mereology of events conceivable for a philosophy of organism is grounded on the internal relations tying events and eternal objects. Furthermore, as we saw at the beginning, a double sort of relation ties together eternal objects and events. Then it appears that if there were any internal relations, there could be no process. In the same sense, a famous Leibnizian

⁷³ F.H. Bradley, *Appearance and Reality*, p. 519.

argument against Descartes' mechanisms shows that a set of external relations is deprived of any criterion of distinction.⁷⁴ In the longest analysis which Whitehead devotes to Leibniz in SMW at the end of the chapter ix, the British philosopher precises his paradoxical views, that true internal relations have become incompatible with the notion of substance.

Finally Whitehead denies to Leibniz a right understanding of the meaning of internal relationships, notwithstanding that "it is obvious that the basing of philosophy upon the presupposition of organism must be traced back to Leibniz".⁷⁵ But if we consider the scholastic background of Leibniz's theory of relations (i.e. Suarez), namely that relations draw their reality from a *fundamentum* which comes under the category of quality, Whitehead has correctly advanced: "Accordingly for him there was no concrete reality of internal relations". There he put forward a sort of a dilemma:

"He had therefore on his hands two distinct points of view. One was that the final real entity is an organising activity, fusing ingredients into a unity, so that this unity is the reality. The other point of view is that the final real entities are substances supporting qualities. The first point of view depends upon the acceptance of internal relations binding together all reality. The latter is inconsistent with the reality of such relations".⁷⁶

So the Leibnizian ideality of internal relations may turn out exclusive of the actual entity of *Process and Reality* (1929). While Russell ascribed a reality to his external relations, Whitehead ends to attribute a reality to his internal relations, this becoming then a theory of prehensions, in the extent that prehensions of the past entities by the actual ones are nothing but actual connexion.

Leibnizian internal relations seemed too strong according to Russell, constituting a tight network, as he imagined that the death of a woman in India could affect her husband staying in Europe. Russell gave to the Leibnizian internal relations the same strength as the one of his own external relations, and then monadology could be the equivalent of a mere monism. From a Whiteheadian side, Leibniz's internal relations looked, at the opposite, to be rather weak, with regard to the fact that they were nothing more than mere

⁷⁴ G.W. Leibniz, *De ipsa natura*, §xiii, 1698 (GP IV, 512–514).

⁷⁵ A.N. Whitehead, SMW, Mentor Book, 20th printing, 1960, p. 140. In the next page, Whitehead refers to Russell's book CEPL "for the suggestion of his line of thought".

⁷⁶ SMW, pp. 140–141.

abstractions according to Leibniz himself, even if they had a *fundamentum in re*. The way was broadly opened for Whitehead, to hold his strong internal relations as the prehensions of *Process and Reality*.

5. Conclusion

As a mathematician, Whitehead started up in understanding relations from a universal algebra. Leibniz himself intended to classify all the mathematical relations inside the *Mathesis universalis*, expecting some progress together with the improving of the mathematical notation.⁷⁷ His doctrine of the complete notion compelled him to ascribe to the relations in his metaphysics an ontological importance not exactly suitable to the Aristotelian tradition of the *ens rationis* to which, however, he explicitly wished to be faithful. As a result, it was not amazing that the problematic of the reductibility of the relations was brought to the fore. Here Whitehead had noticed an overwhelming tension, in the meaning that the importance of relations was contradictory with the very notion of a substance supporting its qualities. *Process and Reality* was nothing but a demonstration of the necessity to reject the ontology of substance in order to assume the internality of the relations on the level of the relatedness of the actual entities.

Whitehead avoided the difficulties inherent to the position of what we call nowadays "bare particulars" and grounded the continuity of the nature. About *Process and Reality* in comparison with Bradley, Leemon B. McHenry has noticed that for explaining this continuity, pluralism must concede to Bradley, in the way of William James in *A Pluralistic Universe*, upholding that parts of the world must be "in some ways connected, in some other ways not connected with the other parts" and adding that this "way can be discriminated".⁷⁸ It would still be worth to study how the old opposition between external and internal relations was superseded in the philosophy of organism by the beautiful disjunction of the two modes of perception described in Whitehead's *Symbolism, Its Meaning and Effect* and reiterated in

⁷⁷ See G.W. Leibniz, "Specimen geometriae luciferae", in *Mathematische Schriften*, published by C.I. Gerhardt, Halle, 1849–1863, reprint by Georg Olms, Hildesheim, 1971, vol. vii.

⁷⁸ William James, *A Pluralistic Universe*, New York, Longmans, Green and Co., 1909, p. 79. See Leemon McHenry's comment in *Whitehead and Bradley. A Comparative Analysis*, Albany, State University of New York Press, 1992, pp. 78–93.

the crowning work PR.⁷⁹ Indeed, since the actuality is externally related under the way of presentational immediacy, the power of the past, still active along the chain of causal events, manifests itself under the way of causal efficacy. The actualities are externally related, but the power of the past remains active through the process of concrescence. In the same slice of a Minkowskian cone, relations are homogeneous, namely external. When a subject prehends an object, the relation is internal from the point of view of the prehending subject but it falls into externality from the point of view of the prehended object.⁸⁰ So the interrelatedness of the things in Whitehead's cosmology would must less promote the image of a confuse network than a clear cleavage between this two modes of perception, which enriches the use of internal relations in the analysis of the relational essence of eternal objects.

Then there were two directions for those who intended to revolutionize the metaphysical tradition of the *ens rationis*, which indeed consisted in seeing relations as an *ens diminutum*. Russell's way consisted in ascribing reality to external relations, becoming the true universals, whereas he preserved the traditional asymmetry of the judgement of predication — Ramsey will go further in inquiring about predication —, once he had recognized the irreducibility of the relational propositions to this form. It is finally dubious that Whitehead would intend to radically impugn an intellectual schema so suitable for the practical life. He rather gave a genesis of this schema along PR. The main default of Russell's radical reject of internal relations boiled down to bring all the kinds of relations on the same level, where James' or Dewey's pragmatisms rendered conceivable all degrees and modes of connexions in the nature. Akin with this line of thought, Whitehead's refined resolution in the quarrel over relations, attributing efficacy to the internal relations through a theory of perception in accordance with the relativity physics, combined with the logical use of the relations on the level of the eternal object as instantiated universals, appears nowadays to sketch a form of a moderated Platonic solution, elaborated by the modern logic of relation, in the problem of the status of the universals, beside the trope theory of the analytical metaphysics.

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⁷⁹ Alfred North Whitehead, *Symbolism, Its Meaning and Effect*, New York, The Macmillan Co., 1927.

⁸⁰ Alfred North Whitehead, PR, 309.

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