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GENERALIZED QUANTIFIERS, AND BEYOND

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In recent decades there has been considerable advance in our understanding of natural language quantification. Following a series of works, natural language quantifiers that are one-place determiners are now analyzed as a subset of generalized quantifiers: binary restricted ones. I think, however, that this analysis has failed to explain what is, according to it, a significant fact about those quantifiers; namely, why one-place determiners realize only binary *restricted* quantifiers, although the latter form just a small subset of the totality of binary quantifiers. The need for an explanation has been recognized in the literature; but, I shall argue, all the explanations that have been provided fail.

By contrast to this analysis, a Geachean analysis of quantification, according to which common nouns in the grammatical subject position are logical subject-terms or plural referring expressions, does explain this fact; in addition, it also explains the successes and failure of the generalized quantifiers analysis. I present the Geachean analysis below and develop these explanations. In this way, the fact that only specific quantifiers of all logically possible ones are realized in natural language is emphasized as one that should be explained; and the ability of competing theories to supply an explanation should be a criterion for deciding between them.

1. *Desiderata*

Many natural language quantifiers are one-place determiners; i.e., they combine with a general noun, simple or compound, to form a noun-phrase. For instance, the quantifier 'all', when followed by the general noun 'clever students', forms the noun-phrase 'all clever students'. Let us list a few examples of such quantifiers, which we shall use below:

all, some, three, at least seven, infinitely many, most, many.

All these quantifiers function in grammatically similar ways in English, a fact constituting a *prima facie* good reason for thinking that roughly the same semantic principles govern their meaning.

Moreover, the grammatical resemblance between these quantifiers is preserved in translation between historically and grammatically remote languages; to the best of my knowledge, this might be a universal of natural languages. Accidental coincidence is therefore highly unlikely, and we have another good reason for assuming that the grammatical uniformity reflects a semantic one.

Lastly, all these quantifiers can be used, in all languages, to answer the questions 'How many S's?' or 'How many of the S's?'. It may thus seem obvious that they are all used to specify the quantity of something mentioned in the course of discourse. Indeed, first appearances might mislead; but we need a good reason for thinking so.

We therefore have good reasons for maintaining that these quantifiers are governed by the same semantic principles.

Comparative quantifiers, e.g. 'more' and 'twice as many', form a different class. Syntactically, they can function as *two-place* determiners but not as one-place ones: they can combine with *two* general nouns to form a noun-phrase; as, for instance, 'more boys than girls' in 'More boys than girls came to the party'. And this syntactic property is preserved across languages. Again, they cannot be used straightforwardly in reply to a 'How many?' or 'How many of...?' question. All these features should make us presume that the semantic principles governing their use are different from those governing the use of quantifiers that are one-place determiners. All the same, since comparative quantifiers are determiners across languages, and, moreover, they are clearly about quantities (hence their name) it seems the semantic principles governing their use should be related in some intelligible way to the latter principles.

We have thus specified a group of quantifiers — those that are one-place determiners in natural language — which should be given a uniform semantic analysis, an analysis that should also distinguish them from other natural language quantifiers and one-place determiners. We shall now turn to examine how their analysis as generalized quantifiers succeeds in this task.

2. *Natural language quantifiers as binary monadic generalized quantifiers*

In Frege's *Begriffsschrift*, quantifiers are second order functions that operate on a single first order sentential function. Since they take a *single* function as their argument, his quantifiers are *unary* quantifiers. Frege used only the universal quantifier in his calculus, but many other unary quantifiers can be introduced into it. While '$\forall x \varphi x$' is defined as true iff *every* individual in the domain is in 'φ's denotation, '$\exists x\varphi x$' can be defined as true iff *some* individuals in the domain are in 'φ's denotation, '$\exists_3 x\varphi x$' iff three are, and '$\exists_\infty x\varphi x$' iff infinitely many.

Turning to the relation of the *Begriffsschrift*'s quantifiers to natural language ones, Frege supplied reasonable translations of sentences containing 'every' and 'some' into his calculus (1879, §12). Equally reasonable translations can be supplied to those containing 'three', 'at least seven', 'infinitely many', and many others. Only *reasonable*, since while his '$\forall x(Sx \rightarrow Px)$' and '$\exists x(Sx\&Px)$' are, respectively, true and false when nothing in the domain is S, it is doubtful whether 'Every S is P' and 'Some S are P' then are (do plural subject-terms in natural language presuppose reference to denotata?). But we will not elaborate this point here. Also significant is the fact that while the sentences 'Every S is P' and 'Some S is P', for instance, have the same grammar, they are translated into the calculus by sentences with different structures: the translation of the former uses an implication, of the latter a conjunction. Natural language seems to supply no justification for this divergence in translation.

More influential in fostering the conviction that some kind of revision is necessary was the fact that the natural language quantifier 'most' cannot be introduced as a unary quantifier into Frege's calculus (a fact first noted by Rescher (1962)), as well as 'many' and most other *proportionality* quantifiers.[1]

The reason for the *Begriffsschrift*'s failure in translating proportionality quantifiers by means of unary quantifiers can be clarified as follows. Using 'q' to stand for any unary quantifier, the structure of a quantified sentence is '$qx\varphi x$'. If our domain is D, we circumscribe by means of this sentence two areas in the domain, φ and D-φ (see Figure 1).

We can therefore say by means of '$qx\varphi x$' anything about the quantity of φ, of D-φ, or of their relative quantities. However, when we use a proportionality quantifier, say 'most' as in 'Most S are P', we say something about the relative quantities of the S that are P and the S that are not P (i.e., that there are more of the former). To say that, the *two* areas $S\&P$ and $S\&\neg P$ should be circumscribed in the domain (see Figure 2).

This, however, cannot be done with unary quantification, which circumscribes only a *single* area.

We have explained in the previous section why there are good reasons for thinking that 'most' and 'many' are governed by the same semantic principles that govern 'all', 'some', 'three' and 'infinitely many'. Therefore, since 'most' and 'many' are *not* unary quantifiers, we have good reasons for thinking that neither are 'all', 'some', 'three' and 'infinitely many'. We therefore

[1] For a proof see (Kolaitis and Väänänen 1995).

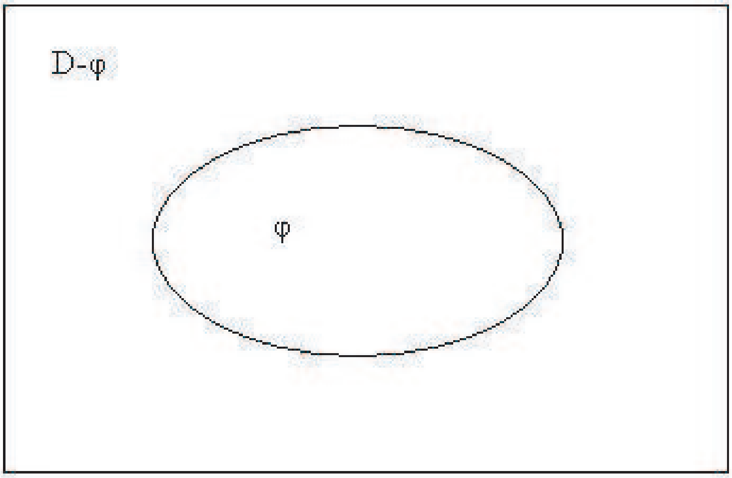

Figure 1

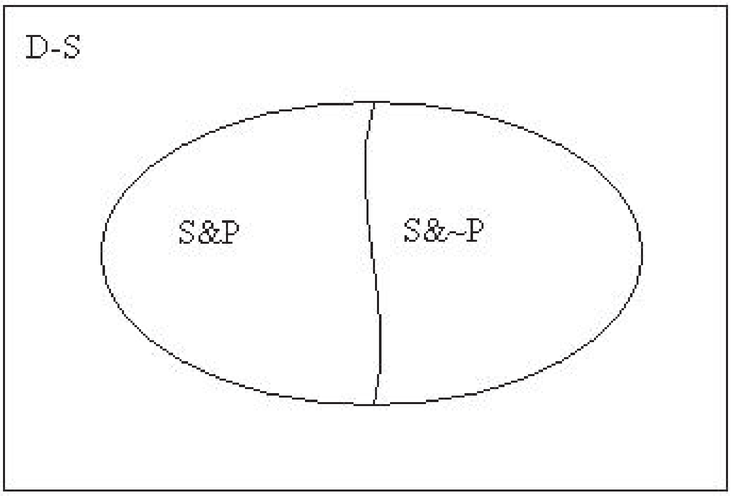

Figure 2

have good reasons for maintaining that Frege's original treatment of quantifiers in his *Begriffsschrift* does not capture the semantics of natural language quantifiers.

That was one reason why both philosophers and linguists welcomed the development, by Mostowski (1957) and Lindström (1966), of generalized quantifiers. Although we shall restrict our attention below to unary and binary monadic generalized quantifiers, we shall first present the general idea.[2]

The Fregean unary quantifier $\exists$, for instance, can be thought of as a function operating on a subset of a domain — 'φ's denotation in '$\exists x\varphi x$' — and assigning to this subset the truth-value True iff it is not empty. Similarly, $\exists_\infty$ can be thought of as operating on a subset of the domain — 'φ's denotation in '$\exists_\infty x\varphi x$' — assigning to it the truth-value True iff it contains infinitely many members. And so on.

But now, there is no justification for taking quantifiers as functions operating only on a *single* subset. They can equally well operate on two, three or on as many subsets as one wishes. Moreover, they may operate not just on *subsets* of the domain, but also on *relations* on the domain (thought of extensionally). The latter quantifiers, however, will not interest us here; only generalized quantifiers that operate on a domain's subsets are relevant to our discussion. These are called *monadic* generalized quantifiers. Since we shall discuss below only monadic quantifiers, we shall generally omit the adjective 'monadic' and call them simply 'generalized quantifiers'.

Fregean quantifiers are type <1>, i.e., they operate on a single subset; they are therefore called *unary* generalized quantifiers. Quantifiers that operate on an ordered pair of subsets are type <1, 1> and are called *binary* (monadic) generalized quantifiers. Most binary generalized quantifiers cannot be defined by means of unary ones. The expressive power of a language with binary generalized quantifiers is therefore greater than that of the same language with unary ones alone. Consequently, it is possible that binary generalized quantifiers can translate some natural language quantifiers that cannot be translated by unary ones.

Let us, for instance, define the binary generalized quantifier-symbol 'most' as follows: 'most $x(\varphi x, \psi x)$' is true iff most individuals in the domain that are in 'φ's denotation are also in 'ψ's denotation. It can easily be verified that this formula translates natural language's 'Most S are P'. Other proportionality quantifiers can also be translated into this enriched language.

Natural language quantifiers that were earlier translated by unary generalized quantifiers can also be translated by binary ones. For instance, define 'some $x(\varphi x, \psi x)$' as true iff some individuals in the domain that are in 'φ's denotation are also in 'ψ's denotation, and then translate 'Some S are P' by it. The analogous translations of all other quantifiers are straightforward.

[2] Detailed presentations, which also discuss additional issues irrelevant to our purposes, can be found in (Westerståhl 1989) and (Keenan 2002). The most detailed and updated account is (Peters and Westerståhl 2006).

Moreover, translating natural language quantifiers by means of binary generalized quantifiers does not require any divergence in translation structure of sentences with the same natural language syntax. While the copulative structure of 'Every S is P' was translated by an implication and that of 'Some S are P' by a conjunction when we used unary quantifiers, all sentences of the form 'q S are P' are translated by sentences of the form '$q\ x(Sx, Px)$'. As was argued above, this is also an advantage of the latter translation over the former.

For these reasons the analysis as binary generalized quantifiers of natural language quantifiers that are one-place determiners is a significant improvement over their former analysis as unary generalized quantifiers.

3. *The problem of non-restricted quantifiers*

Generalized quantifiers are a very powerful logical device. Extensionally, there are $2^{4x4} = 65536$ different binary generalized quantifiers on a domain with just two members.[3] Some of these, however, are not what we would ordinarily consider quantifiers: quantifiers are about *quantities*, so the *identity* of individuals in a predicate's domain should not matter for the truth-value of a sentence in which something is said about their quantity.

Formalizing this constraint and putting it on binary generalized quantifiers[4] considerably reduces their number. On a domain with two members there are only $2^{3x3} = 512$ differing binary generalized quantifiers that comply with it. Nevertheless, many of these generalized quantifiers still have no parallel among natural language quantifiers that are one-place determiners. Consider, for instance, the following binary generalized quantifier:

'$\mathsf{more}\ x(\varphi x, \psi x)$' is true iff more individuals are in 'φ's denotation than in 'ψ's denotation.

'more' can be used to translate natural language sentences using the *two*-place determiner 'more'. For instance, the two sentences

There are more students than professors.
More boys than girls came to the party.

[3] See the references in footnote 2 for the calculation.

[4] 'ISOM'; see the works mentioned in footnote 2.

Can be translated, respectively, by

more x(Student x, Professor x)
more x(Boy x & Came-to-the-party x, Girl x & Came-to-the-party x)

Next, let us define the binary generalized quantifier blik as follows:

'blik $x(\varphi x, \psi x)$' is true iff there are three individuals in the domain that are *neither* in 'φ's denotation nor in 'ψ's.

blik is an unexceptional binary generalized quantifier, but it is doubtful whether there is *any* natural language determiner or construction that it can be used to translate.

So the situation according to the contemporary view on natural language quantifiers is as follows. Natural language quantifiers that are one-place determiners are binary generalized quantifiers. However, there are binary generalized quantifiers that are not realized by one-place determiners in natural language (we have mentioned more and blik). The questions then arise, *which* binary generalized quantifiers are realized by one-place determiners, and *why* these and not others?

Contemporary semantic theory supplies us with an answer to the *Which?* question. One-place natural language determiners realize only binary generalized quantifiers that meet the following two independent conditions, *conservativity* (CONS) and *extension* (EXT). A binary generalized quantifier q satisfies CONS iff the truth-conditions of 'q $x(\varphi x, \psi x)$' do not depend on the number of individuals that are in 'ψ's denotation but not in 'φ's. A binary generalized quantifier q satisfies EXT iff the truth-conditions of 'q $x(\varphi x, \psi x)$' do not depend on the number of individuals in the domain that are neither in 'φ's denotation nor in 'ψ's. As can easily be checked, more does not satisfy CONS while blik does not satisfy EXT. The combined effect of CONS and EXT is called *Domain Restriction*, and the binary generalized quantifiers satisfying these two conditions are called *restricted*. It is consequently claimed that natural language quantifiers that are one-place determiners are *restricted* binary generalized quantifiers.

We are therefore left with the second question, i.e., *why* do one-place determiners in natural language realize, of all binary generalized quantifiers, only restricted ones? I shall claim that contemporary semantic theory has failed to answer this question.

The fact that non-conservative binary generalized quantifiers are not realized in this way is *contingent* according to contemporary semantics, and the grounds for maintaining it are *empirical*. Glanzberg, for instance, writes:

> It appears that all natural-language quantification is restricted quantification. This is not a conceptual or a logical matter [...] there are perfectly intelligible non-conservative quantifiers. Rather, it appears to be an empirical fact about human languages that though logically speaking they could have non-conservative determiner denotations, they do not. We thus have a proposed *linguistic universal*: a non-trivial empirical restriction on possible natural languages. (2006: 801)[5]

From the point of view of the binary quantification construal, '[a] priori, nothing would seem to prevent some quantirelation in some language from denoting a quantifier' that is not restricted (Peters and Westerståhl 2006: 138). Empirical observation, and not theoretical considerations, brought philosophers and linguists to the conclusion that all natural language quantifiers that are one-place determiners are restricted. It *could* be otherwise, but it isn't.

So why isn't it otherwise? Keenan and Stavi tried to account for this fact by supplying a 'psycholinguistic interpretation' of it:

> The language learner does not have to seek the meaning of a novel [determiner] among all the logically possible ways in which [common noun phrase] denotations might be associated with [noun phrase] denotations. He only has to choose from among those ways which satisfy conservativity. Thus the conservativity constraint partially accounts for how children learn languages (including their meaning systems!) readily, with limited exposure, etc., since it limits the class of meaning systems they must choose from in deciding which system is the one the language they are learning has. (1986, §2.7)

But I think this explanation fails. First, since non-conservative binary quantifiers cannot be reduced to conservative binary quantifiers, omitting non-conservative binary quantifiers from natural language would reduce its expressive power: we would not be able to say things that otherwise could be said, and probably should occasionally be said. And although making language simpler for children to acquire is important, it does not seem to justify such a reduction. It seems unlikely that our linguistic capabilities would evolve to be limited in this way.

Moreover, if making language easy to learn were the reason for this reduction in expressive power, why not make language even simpler by reducing

[5] See also (Keenan 2002: 636), (Westerståhl 2001: 456), (Forbes 2004: 14), (Peters and Westerståhl 2006: 138–9).

expressive power further still? Why stop at domain restriction? Keenan and Stavi note that if the set of one-place determiners is to be closed under Boolean operations, then if it contains all the determiners they classified as basic it has to contain all conservative determiners (ibid.: 292). But why should it contain all these so-called basic determiners? If more is not included among them, why should most be? Or, to look at the matter from the opposite direction, why shouldn't more be included among basic determiners? The set of so-called basic determiners seems arbitrary: Keenan and Stavi's explanation could equally well justify *any* set of determiners closed under Boolean operations, and it therefore justifies *none*.

Lastly, to facilitate learnability we may initially limit the set of quantifiers we use while teaching young children their language, and only later, after basic linguistic knowledge has been acquired, introduce the more demanding ones. In fact, children do learn the use of some quantifiers — 'an even number of', for instance, or 'infinitely many' — only after they had learned to use simpler ones; it seems they could as well learn the use of non-conservative quantifiers only at a later stage. Learnability constraints cannot explain why quantifiers that are one-place determiners are restricted.

Keenan has later realized perhaps some such difficulties with his 'psycholinguistic interpretation': this may account for the fact that in his publications from the last two decades he does not mention it, but instead suggests an alternative explanation, to which we now turn. In his 1996 paper he writes:

> The combined effect of CONS and EXT, namely Domain Restriction, is a kind of "logical topicality" condition. It says in effect that the head noun determines the relevant universe for purposes of the statement we are concerned with. Worth noting here is that mathematical languages such as those used in Elementary Arithmetic, Euclidean Geometry or Set Theory are special purpose in that the range of things we can talk about is fixed in advance (numbers, points and lines, sets). But natural languages are general purpose — speakers use them to talk about anything they want, and common nouns in English provide the means to delimit "on line" what speakers talk about and quantify over. (p. 56)[6]

The picture this quotation draws is as follows. While we talk, the context determines a universe or domain of discourse. And then, when we use a common noun as an argument of a determiner, this common noun further

[6] Cf. (Keenan and Westerståhl 1997: 852).

determines a sub-domain of discourse, about which we wish to say something.

Yet I think that this explanation, despite its initial appeal, is unsatisfactory. No doubt such a linguistic tool, as common nouns were claimed by Keenan (and Westerståhl) to be, should be useful. But there is no need for it to be *exclusive*. I.e., there is no reason why we could not have both (1) quantifier-words that would indicate that the common noun they bind into a noun-phrase is used in the limiting way described above; and *in addition* (2) quantifier-words that would not indicate any such restrictions — natural language's analogues of 'more' and 'blik', for instance. Moreover, if the analysis of natural language quantifiers as generalized quantifiers were correct, then, as was said above, the exclusion of non-restricted quantifiers would limit the expressive power of language: restricted quantifiers are but a fraction of binary generalized quantifiers. And such limitation would be disadvantageous. So we should have quantifiers of both sorts: in this way, the functional role described by Keenan would not be lost, and we would also gain expressive power.

If CONS and EXT as constraints on determiners were a local phenomenon, characterizing only this or that natural language, then Keenan's explanation might perhaps be acceptable. But apparently, CONS and EXT characterize *all* natural languages. Such a universal diminution of expressive power therefore seems unlikely.

I am not acquainted with any other attempt in the literature to explain why, of all binary generalized quantifiers, only restricted ones are realized by one-place determiners in natural language. In fact, the recurring reference to observation, as the basis on which natural language quantifiers are deemed restricted, already suggests the lack of any adequate theoretical explanation.

I think this lack of an acceptable account is a serious fault. First, the failure to explain a non-trivial and puzzling semantic universal is obviously a significant weakness of the theory. Moreover, the criteria that have shown the analysis of natural language quantifiers by means of Frege's original unary quantifiers to be unsatisfactory apply in essence to the analysis of the former by means of binary quantifiers as well. In both cases we do not have correspondence between the classes of quantifiers in the logic theory and those in natural language. Let us see that.

Unary quantifiers had once been held to capture the semantics of universal and existential quantification in natural language, as well as of numerical quantification. But then it was noticed that they could not explain the semantics of 'most' and other proportionality quantifiers. The situation was consequently as follows: according to the theory, some natural language quantifiers that are one-place determiners corresponded to unary quantifiers, some did not but required a different logical analysis. Philosophers and linguists alike deemed that unexplained mismatch implausible, as is attested by their later adoption of the binary analysis.

The state of affairs is analogous if quantifiers in natural language are analyzed as binary generalized quantifiers: according to the theory, some binary quantifiers correspond to natural language one-place determiners, while others (e.g. 'more') do not. We again have an unexplained mismatch between classes of quantifiers. Accordingly, if the mentioned lack of correspondence in the unary analysis case was deemed a serious problem for that analysis, then so it should be for the binary one.

The fact that this lack of explanation is indeed a significant difficulty for the binary quantifiers' analysis would be underscored by the availability of an explanation on a different analysis of quantification in natural language. We turn to such an analysis in our next section.

4. *Quantification with common nouns as plural referring expressions*

In this section I present an alternative analysis of quantification in natural language and explain why, according to this analysis, natural language quantifiers that are one-place determiners *have* to be "restricted". I shall also use it to explain a few other facts about natural language quantifiers, and the successes and failure of the generalized quantifiers approach. My presentation of this alternative analysis or system, however, will be rudimentary, for several reasons. First, a full development cannot be supplied within the bounds of a journal article; and this alternative system has been fully developed elsewhere.[7] Secondly, the primary purpose of this paper is to raise and discuss the issue of restricted quantification in natural language, and a detailed development of an alternative semantic system would distract attention from this issue. Still, as many readers would not be familiar with my system, I start with a concise presentation of its main ideas.

Frege thought that reference is always singular: a referring expression always refers to a *single* particular. This position has been accepted, usually unreflectively, by the great majority of later linguists and philosophers. In recent decades, however, the notion of *plural reference* has reappeared in

[7] The alternative semantics, together with a deductive system, are developed in detail in (Ben-Yami 2004). In (Lanzet 2006) a formal system with model-theoretic semantics is constructed, on the basis of the semantics and deductive system of (Ben-Yami 2004); Lanzet proves there that this formal system is sound and complete, and that it contains, in a sense specified in his work, the first-order Predicate Calculus.

the philosophical literature, and has recently gained much attention; I shall assume below familiarity with it.[8]

The plural referring phrases mentioned in contemporary philosophical literature are plural pronouns, plural demonstrative phrases, plural definite descriptions and conjunctions of referring expressions. I agree that these are often used as plural referring expressions. However, unlike most authors, I maintain that *common nouns as well* are often used as plural referring expressions.

Strawson (1950, 1952) and Geach (1962) are, to the best of my knowledge, the only philosophers in the analytic tradition who have held a similar position (yet see (Klima 1988)). Strawson's few remarks on the subject were, however, ambiguous. Geach, by contrast, has clearly claimed, in explicit contrast to Frege, that common nouns can function as plural referring expressions:

> The defect in Frege's reasoning, it appears to me, was his unquestioned assumption that a name [...] cannot name more than one thing and that in consequence a general term is not a name but is essentially predicative. [...] So far from following Frege in the view that a word following [a quantifier] is thereby shown to be a word for a *Begriff* and not a logical subject standing for individual things, we shall hold that a general term in such a position has the role of a name, of a logical subject. (1962: 179)

Following Aquinas, Geach then maintained that the quantifier has 'the role of showing the way the predicate goes with the subject, *ordinem praedicati ad subjectum*' (ibid.: 180). As Geach wrote, 'Aquinas's naïve-seeming statement [...] is a philosophical thesis whose value becomes clear only through studying the various miscarrying attempts to set up an alternative thesis.' (ibid.: 188)

Although I depart from Geach in various points, my position below can be seen as an adaptation and extension of his position, which is in fact the old Aristotelian one: we both maintain, like Aristotle and unlike Frege, that not only singular terms, but general terms as well, can function as logical subject-terms. Common nouns, when used as logical subject-terms, are plural referring expressions, designating more than a single individual.

I shall now introduce my Geachean analysis of quantification, starting with a few words on the nature of quantification generally. To avoid irrelevant

[8] For recent developments, clarification and defense of the idea of plural reference see, for instance, (Ben-Yami 2004, Part I), (Yi 2005–6, Part I), (McKay 2006, Chaps. 1–2).

complications I limit the discussion to quantified subject-predicate sentences containing only one-place predicates.

What is involved in quantified assertions? We refer to a plurality and specify by means of a quantifier to how many items of this plurality a predicate applies. Let us first see how this is realized in natural language, according to my Geachean analysis. Consider for instance the sentence 'Some students have failed the exam'. 'Students', I claim, is here used as a plural referring expression, not as a predicate. In a specific use of this sentence, the term 'students' may refer, say, to my students in a specific course. Notice that it *does not* refer to *some* of these students, but to *all* of them. The quantifier 'some' then determines that the predicate 'failed the exam' should apply to some of the students referred to.

Let us generalize this account. In a sentence of the form 'q S are P', where 'S' is a common noun, 'S' is used to refer to a plurality, and the quantifier 'q' determines to how many items of this plurality the predicate 'P' should apply. (Mutatis mutandis if 'S' is a definite noun phrase — e.g., 'these students', 'my children' — and the sentence is then of the form 'q of S are P'.)

Notice that I did not mention any universe or domain of discourse in my explanation. This is not due to any lack of rigor. *Natural language has no domain of discourse*, in the technical sense in which this term is used in predicate logic semantics. The plurality over which we quantify is determined by the plural referring expression we use. No plurality in the form of a domain of discourse is required for any semantic reason. Different plural referring expressions can specify different pluralities, without any recourse to the mediation of a discourse domain.

By contrast to natural language, the predicate calculus has no plural referring expressions. The plurality over which we quantify cannot therefore be determined by any expression in the calculus sentence. Consequently, this plurality has to be *presupposed* by the quantified construction. And different sentences cannot specify different pluralities. In the predicate calculus, a plurality, which is unspecified by the sentence, is introduced by presupposing a domain of discourse — a semantic constituent that has no parallel in natural language.[9]

Let us now try to explain why natural language has, according to my Geachean analysis, the quantifiers it actually has. When one utters a sentence of the form 'q S are P', one determines by means of 'S' a plurality about which something is being said. Now this plurality can be divided into

[9] It might be thought that the predicate calculus on its many-sorted logic version both has plural referring expressions and at least approximately fits my analysis above of quantification in natural language. For a comparison and contrast of my approach with many-sorted logic, see (Lanzet and Ben-Yami 2004, §4).

two sub-pluralities (one of which may be empty): those that are both S and P, and those that are S but not P. Apart from the particulars referred to by means of 'S', no other particulars have been mentioned in the sentence or presupposed to exist in any domain of discourse, so this division of the particulars referred to is exhaustive. See Figure 3; notice also that by contrast to the earlier figures, this time we did not draw an additional domain of discourse:

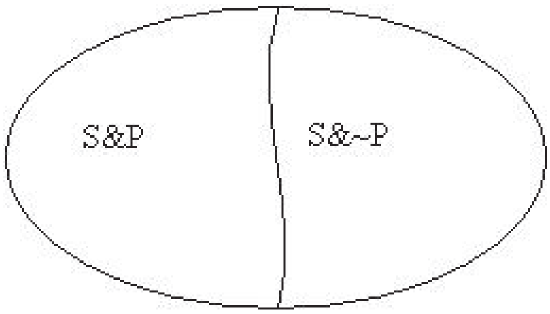

Figure 3

The quantified sentence can now make a claim either about the quantity of the items in the first sub-plurality (the S's that are P, the *intersective* quantifiers), or about their quantity in the second (S's that are not P, the *co-intersective*), or about the relation between these quantities (*proportionality*). However, since no particulars that are not S are specified by the sentence 'q S are P', no claim *can* be made by such a sentence about particulars that are not S; more specifically, no claim can be made (1) about particulars that are P but not S, or (2) about particulars that are neither S nor P. Consequently, no quantifier such as more or blik *can* be introduced as a one-place determiner. Quantifiers that are one-place determiners are therefore *necessarily* restricted: they simply have no particulars but the S's over which they could quantify.

We see that a claim that was a *contingent* result of *empirical observation* according to the analysis of natural language quantifiers as generalized quantifiers — a result it failed to explain — is a *necessary conceptual consequence* of my Geachean analysis. My analysis derives this universal of natural languages a priori. This result was first noted by Klima (1988: 69).

Moreover, we can understand why comparative quantifiers are *two*-place determiners. We use comparative quantification to make claims about relative quantities of *two* pluralities. When we say, for instance, 'More boys than girls came to the party', we refer to two pluralities: on the one hand, the boys, on the other, the girls; and we compare the relative quantities of those of the two pluralities to which the predicate 'came to the party' applies. We therefore *have* to apply the comparative quantifier to two referring expressions, and it therefore *has* to combine syntactically with two referring expressions. Comparative quantifiers could not be one-place determiners, binary generalized quantifiers approach notwithstanding, but they have to be two-place determiners.

Lastly, it is clear why natural language has no quantifier that does not obey Extension — either one-place determiner or not — such as blik. Suppose we had a sentence of the form 'blik S are P'. blik essentially depends on the quantity of particulars in the *domain of discourse* that are not S or P. But if there is no domain of discourse, then no such quantifier is possible.

On the other hand, the greater success of the analysis of natural language quantification by means of binary generalized quantifiers, compared to that by means of unary ones, is also explainable by means of the alternative Geachean analysis. The fact that the binary generalized quantifier-symbol operates on *two* predicates, makes it possible to use the first predicate as a distorted referring expression and only the second one as really a predicate. In this way the quantifiers of natural language that are one-place determiners can be translated into the modified calculus.

This alternative analysis can also explain the *failure* of the generalized quantifiers approach. The general structure of a basic sentence with the binary quantifier 'q' is '$q\ x(Sx, Px)$'. This sentence therefore circumscribes *four* areas in the domain of discourse (see Figure 4).

Accordingly, if natural language quantifiers that are one-place determiners were binary generalized quantifiers, then any fact about the quantities in one or more of the four areas circumscribed above could be represented by some such quantifier. For instance, the quantities in S&¬P and (¬S)&P could be compared; i.e., one-place determiners could be comparatives quantifiers. The binary generalized quantifiers approach cannot explain why only the *first* predicate in the sentence '$q\ S$ are P', a sentence which according to it is equivalent to '$q\ x(Sx, Px)$', should be used as a referring expression; from its point of view, both predicates can be used in the same way. The binary generalized quantifiers approach cannot therefore explain why only restricted quantifiers are realized by one-place determiners in natural language.

But can't one raise against my Geachean analysis a criticism analogous to the one I raised against the generalized quantifiers analysis? I claimed that according to the latter, the fact that quantifiers are conservative is unexplained — a brute fact, so to say. But then, the fact that, according to the Geachean analysis, the subject-term in quantified sentences is used as a plural referring expression is equally unexplained: why should *any* term be

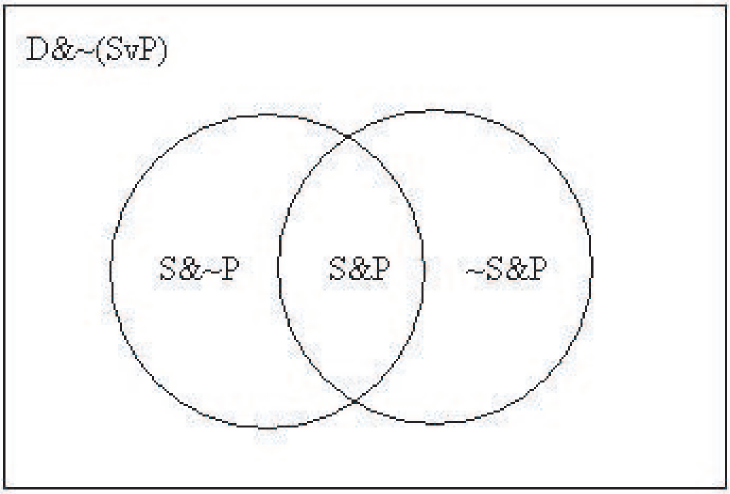

Figure 4

used as such? Why shouldn't *both* subject and predicate be used this way? Why should it be the subject-term, and not the predicate? On *any* analysis, so would this criticism continue, we eventually get to brute facts; perhaps I have displaced the unexplained, but I did not eliminate it.

This criticism relies on a misunderstanding of the nature of quantified assertions. Any quantified assertion makes a claim about quantities of a certain plurality. Now since Fregean logic did not have plural subjects, which are needed to determine a plurality, it had to introduce the plurality needed for quantified assertions as a plurality *unspecified* by the sentence — a domain of discourse (the universe, in Frege's own case). However, once we acknowledge plural referring expressions, it is obvious that an expression specifying the plurality should be included in a quantified assertion (apart from some special, 'elliptic' cases, as in the second contribution to the following exchange: 'Have all students arrived?' – 'No. Some are missing.'). So a quantified assertion should of course contain a plural referring expression. In addition, if any claim is to be made about quantities of this plurality, it should contain a predicate as well; so not *both* terms in 'q S are P' could be used as plural referring expressions or logical subject-terms. Lastly, since we call 'subject-term' the term occupying the grammatical position usually responsible for the referential function, it is analytic that the grammatical subject-term, and not the grammatical predicate, is the term specifying the plurality.

That is, an analysis of the nature of quantified assertions justifies the Geachean analysis; and, together with the observation that Fregean logic lacks plural referring expressions, it explains the weaknesses of the generalized quantifiers approach.

We have seen that the Geachean analysis of common nouns in subject position as plural referring expressions or logical subject-terms can explain various facts about natural language quantification better than can their analysis as logical predicates and of natural language quantifiers as generalized quantifiers. Moreover, the mentioned successes and failure of the latter analysis can also be explained by this alternative Geachean approach. This of course supports the Geachean analysis; but it also corroborates our claim in the previous section, that the failure of the binary generalized quantifiers approach to explain what is according to it an empirical fact of human languages, namely, that natural language quantifiers are all restricted, is a serious flaw in the theory.

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