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LEITGEB, "ABOUT," YABLO*

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Abstract

Leitgeb (2002) objects against the clarity of the debate about the alleged (non-)circularity of Yablo's paradox, arguing that there actually are at least two notions of self-reference and circularity at play. One, on which Yablo's paradox is not circular, is defined *via* the reference of the constituents of a sentence, and another, on which the paradox is circular, is defined *via* syntactic mappings and fixed points. More importantly, Leitgeb argues that both definitions aren't satisfactory and that before we can undertake a serious debate about the circularity of Yablo's paradox we first need to clarify the notions involved. I will focus on Leitgeb's criticism of the first definition[1] and will argue that the problems arise not as much on the level of our definition of circularity as on the level of our definition of reference of sentences (aboutness). Leitgeb's main worry is the failure of a requirement called 'Equivalence Condition', which says that if a formula is self-referential, any formula logically equivalent to it should also be self-referential. I will argue that preservation under logical equivalence is unreasonable with respect to self-reference, but is indeed needed with respect to aboutness. Since Leitgeb's own tentative notion of aboutness doesn't satisfy the requirement, I will suggest another approach which fixes this problem. I also explain why the intuitions that circularity should satisfy the equivalence condition are misled. Next, I argue that the new notion of aboutness is not susceptible to slingshot arguments. Finally, I compare it with Goodman's notion of absolute aboutness, emphasizing those features of Goodman's approach that make his notion inapplicable in the present discussion.

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[1] Thus, the present discussion will not be strongly related to (Leitgeb 2005).

1. *Leitgeb's self-referentiality*$_1$

There is a sense in which sentences are said to refer to objects. As Leitgeb puts it:

> This usage of 'refers to' and 'says of' seems to presuppose that the usual reference relation ref, which holds between (singular or general) terms and their referents, is extended or complemented by a reference relation holding between *sentences* and objects (but where the referents of the sentences are not the truth values of the sentences). (Leitgeb 2002: 4)

Say we allow individual variables range over both objects and sentences (we reserve ϕ and similar variables for sentences only), and abbreviate 'x is a sentence' by '$Sen(x)$', 'x is a singular term' by '$Sin(x)$', and 'x (syntactically) contains y' by '$Con(x, y)$'. Basically, if ref is a relation between terms and objects, Leitgeb defines ref_1 between sentences and objects as follows:

$$ref_1(x, y) \Leftrightarrow Sen(x) \wedge \exists_z(Sin(z) \wedge Con(x, z) \wedge ref(z, y)) \quad (1)$$

Then he defines self-referentiality by:

$$selfref_1(x) \Leftrightarrow ref_1(x, x) \quad (2)$$

and takes $ref_1^\star$ to be the transitive closure of ref_1, that is, the least superset of ref_1 with the property:

$$ref_1^\star(x, y) \wedge ref_1^\star(y, z) \rightarrow ref_1^\star(x, z)$$

For instance, let Tr be the truth predicate. Consider:

$$a = \text{`}Tr(a)\text{'} \qquad b = \text{`}\neg Tr(b)\text{'}$$

Clearly on this notion of self-reference, both $selfref_1(a)$ and $selfref_1(b)$. Define circularity by:

$$circular_1(x) \Leftrightarrow ref_1^\star(x, x) \quad (3)$$

Now consider:

$$c = \text{`}\neg Tr(d)\text{'} \qquad d = \text{`}\neg Tr(c)\text{'}$$

Then, even though we neither have $selfref_1(c)$ nor $selfref_1(d)$, we get $circular_1(c)$ and $circular_1(d)$.

So far we defined self-referentiality and circularity in terms of the occurrence of singular terms only. What about purely quantified statements? Leitgeb introduces these notions also for sentences of the form:

$$\forall_x (A(x) \rightarrow B(x))$$

by saying that:

$$ref_1(`\forall_x (A(x) \rightarrow B(x))', x) \Leftrightarrow A(x) \tag{4}$$

2. *Yablo's paradox and Leitgeb's objections*

Here's a version of Yablo's paradox. Consider an infinite sequence of sentences $s_0, s_1, s_2, \ldots$ such that:

$$\begin{aligned} s_0 &= `\forall_x (P_1(x) \rightarrow \neg Tr(x))', \\ s_1 &= `\forall_x (P_2(x) \rightarrow \neg Tr(x))', \\ s_2 &= `\forall_x (P_3(x) \rightarrow \neg Tr(x))', \ldots \end{aligned} \tag{5}$$

Assume that the extension of every P_n, for $n = 1, 2, 3, \ldots$, is $s_n, s_{n+1}, s_{n+2}, \ldots$. So every s_i says that all s_j's with $j > i$ are not true. Now ask yourself: is s_0 true? If yes, then for any $k > 0$ the sentence s_k is false. But this also means that for any $k > 1$ the sentence s_k is false. But this is exactly what s_1 says and hence s_1 is true, which falsifies s_0. Suppose then that s_0 is false. This means that there is a $k > 0$ such that s_k is true. But we can repeat the reasoning we led about s_0, this time about this s_k to show that s_k can't be true. Hence the paradox. Note that no s_n is $selfref_1$ or $circular_1$.

The first of Leitgeb's worries is that the notions of self-referentiality and circularity are defined for very specific sentences only, and no definition that applies to any sentence of a given first-order language is given. I agree, this is a serious flaw and Leitgeb's requirement is methodologically sensible.

Another worry will be best put in Leitgeb's own words:

> Consider '$selfref_1(x)$' and '$circular_1(x)$': what is conspicuous about them is that they do not satisfy the following Equivalence Condition (EC): if A is self-referential/circular, and if B is logically equivalent to A, then also B is self-referential/circular. EC is

> plausible because logically equivalent sentences are not only extensionally equivalent in the actual world, but indeed in every logically possible world, and thus indistinguishable in terms of the semantics of first-order predicate logic. If self-reference is to be defined by extending the usual reference relation for terms, i.e., a semantical relation, it is certainly strange if EC is invalidated. If EC is not true, then self-referentiality or circularity of a sentence does not only depend on what the sentence says, but also in which way its content is being expressed. (Leitgeb 2002: 9)

Why does EC fail here? Take for instance:

$$b' = \text{`}(P(a) \vee \neg P(a)) \vee \neg Tr(b')\text{'} \tag{6}$$

Clearly, b' is $selfref_1$ and yet a formula logically equivalent to it:

$$P(a) \vee \neg P(a)$$

isn't. Similarly, Leitgeb claims that c' introduced by:

$$c' = \text{`}\forall_x((A(x) \vee \neg A(x)) \rightarrow (A(x) \rightarrow B(x)))\text{'} \tag{7}$$

is $selfref_1$ because it satisfies $A(x) \vee \neg A(x)$. However, the logically equivalent '$\forall_x(A(x) \rightarrow B(x))$' is not self-referential.

Leitgeb's third worry is closely related to the second one. If we want to avoid the failure of EC, we might try to define:

$$selfref'_1(x) \Leftrightarrow \exists_y(Sen(y) \wedge \\ y \text{ is logically equivalent to } x \wedge ref_1(y,y)) \tag{8}$$

But the conclusions that (8) leads to are too strong. For instance, by the same reasoning as above '$P(a) \vee \neg P(a)$' and '$\forall_x(A(x) \rightarrow B(x))$' would come out self-referential.[2]

[2] Martin Bentzen asked an interesting question: what would happen if we replaced existential quantification in (8) with universal quantification? Here is the answer. Say we do that, thus obtaining:

$$selfref''_1(x) \Leftrightarrow \forall_y(Sen(y) \wedge y \text{ is logically equivalent to } x \rightarrow ref_1(y,y)) \tag{9}$$

The problem is now that too many self-referential sentences would be excluded. Consider:

$$a = \text{`}\neg Tr(a)\text{'} \qquad b = \text{`}\neg Tr(a) \wedge \neg Tr(a)\text{'}$$

Leitgeb also puts forward a suggestion that seems to strengthen his case for EC. Again, it's best to quote his own words:

> Indeed, we should even liberalize the notion of logical equivalence in the definiens furthermore to 'equivalent in the standard model of arithmetic (under arbitrary interpretations of 'Tr')', or even to 'equivalent according to (i.e., derivable from) Peano arithmetic or some proper fragment of the latter', since otherwise no philosopher may any longer argue in the following way: "By Gödel's diagonalization lemma, we know that there is a sentence A such that A is equivalent to '$\neg Tr(\underline{A})$' in arithmetic. Thus there is a self-referential sentence, that is, A." (Leitgeb 2002: 9)

3. *Defining aboutness*

First, observe that problems arise already on the level of our notion of aboutness. It seems that already ref_1 itself doesn't get things right. The problem is that given the way ref_1 is defined, for any object x, every sentence ϕ is logically equivalent to a sentence ϕ' such that $ref_1(\phi', x)$. To produce such a ϕ', it is enough to introduce a constant 'b' for x and define:

$$\phi' = \text{`}\phi \wedge (P(b) \vee \neg P(b))\text{'} \tag{10}$$

In this sense, if we require that two logically equivalent sentences should be about the same things, we may reach the conclusion that any sentence is about everything, because for any object this sentence is logically equivalent to a sentence that's also about that object.

But this doesn't seem right. *Prima facie*, there is something fishy about how we can practically for free introduce arbitrary objects that a sentence is about. The trick that we used is simple: we just conjoin a sentence with another, logically redundant claim which contains a constant referring to that object. This however indicates that the bare fact that a constant that names an object occurs in a sentence is not sufficient for that sentence to be about that object.

This motivates certain Wittgensteinian intuitions about what sentences are about ("6.11 Therefore the propositions of logic say nothing."). In a sense, to be about x, ϕ' would have to say *something* about x, provide us with some *contingent* information about it. Otherwise it's just stating a logical triviality

Clearly, a is self-referential. Yet, it is logically equivalent to b, which is not self-referential. Hence, according to (9), a is not $selfref_1''$.

without providing us with any real information about x. Let's try to capture this idea more formally.

Say we have a formula ϕ which contains a constant a and an extra-logical predicate P but contains neither a constant b nor a predicate R. Let the result of replacing a (P) in ϕ with b (R) on all occurrences be $\phi(a/b)$ ($\phi(P/R)$). We say that a (P) *occurs in ϕ informatively* iff there are b/R such that ϕ is not logically equivalent to $\phi(a/b)$ (to $\phi(P/R)$).[3] Take a first-order model $\langle S, I_P, I_c\rangle$ where S is a set of objects that may include the formulas of our language itself, I_P is a function that maps predicates into appropriate subsets of Cartesian products over S and I_c maps individual constants into S.

Definition 1: A sentence ϕ is about x because of a constant a relative to $\langle S, I_P, I_c\rangle$ iff a occurs in ϕ informatively and $I_c(a) = x$. A sentence ϕ is about x because of a constant iff there is a constant a such that ϕ is about x because of a. □

According to this definition, for instance, (10) is not about an object to which b refers in a model unless ϕ already is about that object. Nor is (6) about b' itself, because (6) is logically equivalent to (6)(b'/c) for any constant c.

Things get a tad more complicated when it comes to defining the relation of aboutness that arises from the predicates that occur in ϕ. Here is a fairly simple, but also quite a strong reading.

Definition 2: A sentence ϕ is about$_i$ x because of an extra-logical predicate P relative to $\langle S, I_P, I_c\rangle$ iff P occurs in ϕ informatively and $x \in I_P(P)$, if P is an unary predicate, and x is in an n-tuple in $I_P(P)$, if P is an n-ary predicate ($n > 1$). A sentence ϕ is about$_i$ x because of a predicate iff there is a predicate P such that ϕ is about$_i$ x because of P. A sentence ϕ is about$_i$ x relative to $\langle S, I_P, I_c\rangle$ iff ϕ is about x because of a constant, or ϕ is about$_i$ x because of a predicate.[4] □

On this reading we part with Leitgeb's intuition that a universally quantified conditional:

$$\forall_x(A(x) \rightarrow B(x)) \tag{11}$$

[3] Perhaps in some context this might be strengthened to mathematical equivalence.

[4] The expression 'about$_i$' can be read as 'informatively about'.

is only about A's. On the present view, it's about the A's, but also about the B's. One might say that it equally well says about all the A's that they're included among B's, as about the B's that they include the A's (note that it is not taken to be about anything but individuals, though).

The reason why the definition contains a restriction to extra-logical predicates is that if we included the identity symbol, then $a = a$ would come out as being about any object whatsoever since any pair $\langle x, x\rangle$ is in the extension of this predicate.[5]

Definition 3: We take $\mathsf{about}_i^\star$ to be the transitive closure of $about_i$, and say that a sentence ϕ is $\mathsf{self\text{-}referential}_i$ iff $about_i(\phi, \phi)$, and that it is $\mathsf{circular}_i$ iff $about_i^\star(\phi, \phi)$. $\Box$

Note that this notion of aboutness is fairly coarse-grained: a sentence will come out as being about more objects that we sometimes intuitively think it is. A few examples might help to notice this fact.

- If both a and b are P, then $P(a)$ is about exactly the same things as the things that $P(b)$ is about.
- $\neg P(a)$ is about the same things as $P(a)$.
- $\exists_x \neg C(x)$ is about the things that are C, not about a thing that's not C.
- What a sentence is about is also sensitive to our choice of predicates. If we have two predicates P and P' such that $\forall_x(P'(x) \equiv \neg P(x))$ in a given model, then $P'(x)$ will be about all the objects in the model that aren't P, whereas $\neg P(x)$ will be about all the objects in the model that are P, even though the sentences are *materially* equivalent.
- Even if there are immaterial things, 'Everything is material' will come out as being about all and only material things, not about simply everything (immaterial things included).

Now let's see how aboutness$_i$ deals with Leitgeb's worries.

4. *Aboutness and Leitgeb's qualms*

First off, observe that aboutness is defined for any sentence of a given language, so Leitgeb's methodological qualm doesn't arise in this case.

[5] I thank Hannes Leitgeb for this observation.

Next, we were worried that reference is not preserved under logical equivalence. Aboutness$_i$ is.

Fact 1: *If a formula ϕ is about$_i$ an object x w.r.t. a model, then for any formula ψ, if ψ is logically equivalent to ϕ, then (i) ψ is about$_i$ x w.r.t. to that model, and (ii) ψ is not about$_i$ any object that ϕ is not about$_i$.*

Proof sketch. Indeed, suppose that ϕ and ψ are logically equivalent but aren't about the same things with respect to a certain model $\langle S, I_P, I_c\rangle$. We may safely suppose that it is ϕ that is about an object x such that ψ is not about x. This implies that either ϕ is about x because of a name constant that occurs in ϕ or because of a predicate that occurs in ϕ. If the former is the case, ϕ contains a constant a such that $I_c(a) = x$ and there is an interpretation $\langle S, I_P, I'_c\rangle$ where I'_c differs from I_c only on a, such that:

$$\langle S, I_P, I_c\rangle \models \phi \text{ iff } \langle S, I_P, I'_c\rangle \not\models \phi$$

but ψ is logically equivalent to ϕ and therefore:

$$\langle S, I_P, I_c\rangle \models \psi \text{ iff } \langle S, I_P, I'_c\rangle \not\models \psi$$

But the only difference between I_c and I'_c is that they assign different objects to a. So ψ must contain a and in fact it has to be about x.

Similarly, if the latter is the case, ϕ contains a predicate P such that x is a constituent of $I_P(P)$ (i.e. is a member of $I_P(P)$ if $I_P(P)$ is a subset of S, and a member of a tuple in $I_P(P)$ otherwise) such that there is a mapping I'_P that differs from I_P at most on P and:

$$\langle S, I_P, I_c\rangle \models \phi \text{ iff } \langle S, I'_P, I_c\rangle \not\models \phi$$

By the logical equivalence between ϕ and ψ we get:

$$\langle S, I_P, I_c\rangle \models \psi \text{ iff } \langle S, I'_P, I_c\rangle \not\models \psi$$

which gives us the desired conclusion: ψ contains P and is about x. □

Further, we have to explain how EC can fail for circularity$_i$ even though it holds for aboutness$_i$ and why this is a desired result. Consider the following:

$$a = \text{‘}\neg Tr(a)\text{’}$$

$$b = \text{‘}\neg Tr(a) \wedge (P(c) \vee \neg P(c))\text{’}$$

$c =$ '$P(b)$'

It is easy to verify that a and b are logically equivalent and that b is about the same object as a (b mentions c but doesn't provide any contingent information about it). However, a is clearly self-referential whereas b isn't. This is not an undesired result. Actually, the way the liar arises in natural language is quite often determined not only by what a sentence states but also what the context of its production is. This is especially true for descriptive liars, where a sentence becomes self-referential only because it happens to satisfy a certain description it contains. Two different sentences can be about the same sentence, but only one of them can be this very sentence, and there's nothing wrong about it. Interestingly, b is still circular in yet another sense: it is about a which is about itself.

In this sense, Leitgeb's claim that "if EC is not true, then self-referentiality or circularity of a sentence does not only depend on what the sentence says, but also in which way its content is being expressed" needs to be clarified. It's not just any way that a content of a sentence is being expressed that can make a sentence self-referential. Rather, it's the sentence's being or not being identical with one of the objects that it is about. Once we put formulas in a model itself it's no longer obvious that this is not a semantical fact.

What about Leitgeb's claim that logically equivalent formulas are indistinguishable in first-order semantics? Well, this is true, but *only* if we don't allow the formulas themselves to occur in the model itself. In the present case, however, to even make it possible to talk about self-reference, we did the opposite. We assumed that formulas can belong to the model. And in the above-mentioned example, even though a and b are logically equivalent, they can still be distinguished simply because it is false that $a = b$. We also need to check the following:

Fact 2: *Not every sentence is logically equivalent to a self-referential*$_i$ *sentence.*

Argument. For instance, if a sentence ϕ is not about$_i$ any sentence at all, no sentence ϕ' logically equivalent to it will be self-referential$_i$, because to be self-referential$_i$ it would have to be about$_i$ at least one sentence, but then ϕ itself would be about$_i$ a sentence (aboutness$_i$ is preserved under logical equivalence), which contradicts the assumption. □

What should we make of Leitgeb's complaint that we won't be able to say that the diagonalization lemma allows us to construct self-referential sentences? Well, the answer is simply that on this notion of aboutness, the diagonal construction doesn't give us a self-referential sentence, and that's it. This, however, doesn't mean that we cannot use the phrase 'self-referential' in a weaker sense, which allows us to call a formula self-referential if it is equivalent (*modulo* syntactic encoding and Peano arithmetic) to a formula in which its code number occurs (or what have you, the details aren't crucial here).

One thing we need to ask now is whether Yablo's paradox is circular under this reading. Recall that s_0 is not only about$_i$ P_1's but also about$_i$ those objects that satisfy $Tr(x)$. Then, it would seem, if it's among true formulas, it's circular$_i$. Now, is s_0 true? This heavily depends on what your solution to Yablo's paradox is. If, for instance, you think that s_0 is both true and false, then yes, s_0 will even come out self-referential$_i$.

However, there still is a version of Yablo's paradox which is not circular$_i$ independently of whether one takes s_0 to be true, even on this fairly strong notion of circularity$_i$. First, define a sequence of predicates $Untrue_n(x)$ for $n = 1, 2, \ldots$ by saying:

$$Untrue_n(x) \Leftrightarrow \neg Tr(x) \wedge \exists_{i>n} x = s_i \tag{12}$$

Then to get a non-circular$_i$ version of Yablo's paradox take:

$$\begin{aligned} s_0 &= \text{`}\forall_x(P_1(x) \rightarrow Untrue_0(x))\text{'}, \\ s_1 &= \text{`}\forall_x(P_2(x) \rightarrow Untrue_1(x))\text{'}, \\ s_2 &= \text{`}\forall_x(P_3(x) \rightarrow Untrue_2(x))\text{'}, \ldots \end{aligned} \tag{13}$$

It is important to remember what notion of logical equivalence is being used here. Two formulas are logically equivalent iff they are satisfied in exactly the same models. Models taken into considerations here don't obey the definitions introduced: they just assign completely arbitrary subsets of the domain to unary predicates and so on, which means that definitions are not purely logical equivalences (in other words, definitions aren't true in virtue of their logical form). As a result, replacing a *definiendum* with its *definiens* doesn't have to preserve aboutness$_i$. A good example[6] is the difference between s_0 in (13) and:

$$\forall_x(P_1(x) \rightarrow \neg Tr(x) \wedge \exists_{i>1} x = s_i) \tag{14}$$

given the definition (12), both sentences are equivalent. Aboutness$_i$, however, is not preserved between them. The equivalence, however, is not logical, because there are models which assign to $Untrue_0(x)$ a different set than the set of objects which satisfy $\neg Tr(x) \wedge \exists_{i>1} x = s_i$, because as far as our notion of model goes, models treat different predicates independently.

[6] I owe it to Christian Strasser.

5. *What about the slingshot?*

A question arises as to whether this notion of aboutness will not be susceptible to slingshot-style arguments. Let's make sure that it won't. Slingshot arguments (as I use the notion) intend to show that any two true sentences denote the same thing. Is it the case that we can *mutatis mutandis* run a slingshot argument to show that two arbitrary true sentences are about the same things? Let's take a look at (a formulation of) the slingshot argument first.

The argument uses the notion of denotation (D) as applied to both sentences and terms. It relies on two principles: that logically equivalent sentences denote the same thing(s) (ED), and that substitution of co-denoting constituents of a sentence does not change what a sentence is about (SD). Suppose now that ϕ and ψ are both true. Let '$\Leftrightarrow_l$' stand for logical equivalence. Observe first that:

$$\phi \Leftrightarrow_l a = (\iota x)(x = a \wedge \phi) \tag{15}$$

$$\psi \Leftrightarrow_l a = (\iota x)(x = a \wedge \psi) \tag{16}$$

The reason why the former holds is that $(ix)(x = a \wedge \phi)$ will pick an object if and only if ϕ is true, and that if this description does pick an object, it picks the same object as a. The rationale for the latter claim is analogous. So by ED we get:

$$D(\phi) = D(a = (\iota x)(x = a \wedge \phi)) \tag{17}$$

$$D(\psi) = D(a = (\iota x)(x = a \wedge \psi)) \tag{18}$$

But both ϕ and ψ are assumed to be true, so thanks to (15–16) we also get:

$$a = (\iota x)(x = a \wedge \phi) \tag{19}$$

$$a = (\iota x)(x = a \wedge \psi) \tag{20}$$

The transitivity of identity gives us:

$$(\iota x)(x = a \wedge \phi) = (\iota x)(x = a \wedge \psi) \tag{21}$$

So we can apply SD to (17), (18) and (21) and obtain:

$$D(\phi) = D(\psi) \tag{22}$$

Now, there are at least two interesting ways one can try to run the slingshot argument to prove that two arbitrary true sentences are about the same. The first one requires us to extend the notion of aboutness to terms, and to modify the principles slightly. Roughly speaking, a term t is about$_t$ an object (or, we can say, involves an object) x w.r.t. a certain model iff it contains either a constant a that refers to it or a predicate P such that x is a constituent of the reference of this predicate (i.e. is a member of this predicate's denotation in a given model, or is a member of a tuple in the predicate's denotation, if that predicate is relational), and this constant/predicate occurs in the term informatively, that is if we replace a (P) with a constant b (a predicate R) that doesn't occur in t, thus obtaining a term t', it won't be logically necessary that $t = t'$. Aboutness$_t$ coincides with aboutness$_i$ on sentences. We assume that two logically equivalent sentences are about$_t$ the same (EA) and that the substitution of terms that are about$_t$ the same things doesn't change what the whole expression is about$_t$ (SA). If χ is a term or a sentence, let $A(\chi)$ be the set of objects that χ is about$_t$. Let's see where the argument fails if we replace D with A. Appropriately changed versions of (15–21) resulting from this replacement hold. A problem arises when we need:

$$A((\iota x)(x = a \wedge \phi)) = A((\iota x)(x = a \wedge \psi)) \tag{23}$$

in order to apply SA, because (23) doesn't follow from (21). Worse still, (23) is in general false. Just because two terms refer to the same object, it doesn't mean that the individuals that they are about$_t$ are the same. If, for instance it is the case that both $P(c)$ and $R(d)$, it doesn't have to be the case that:

$$A((\iota x)(x = a \wedge P(c))) = A((\iota x)(x = a \wedge R(d)))$$

even though both descriptions pick the same object. The individuals that the first description involves are: the object named by 'a', the constituents of the reference of 'P', and the individual named by 'c'. The individuals involved in the second description are: the object named by 'a', the constituents of the reference 'R', and the individual named by 'd'. But these can differ.

So here's another stab. Maybe instead of using a uniform notion of aboutness$_t$ we should use an operator $A'(\chi)$ which gives the set of objects that χ is about$_i$ if χ is a sentence, and just the *reference* of χ if χ is a term?[7] Can we run the slingshot argument with A'?

[7] Here's an example that should help one to get clear on the difference between A and A'. If we take $(\iota x)(x = a \wedge \phi)$, then $A'((\iota x)(x = a \wedge \phi))$ will be just the referent of the definite description, that is, a itself (assuming that ϕ is true). $A((\iota x)(x = a \wedge \phi))$ however, will include not only a, but also those objects that ϕ is informatively about.

If we replace 'D' with 'A'', the steps from (15) to (21) hold. We even can use (21) to obtain trivially:

$$A'((\iota x)(x = a \wedge \phi)) = A'((\iota x)(x = a \wedge \psi))$$

The problem is, however, that now, instead of SA we need a slightly different principle, SA$'$ which says that substitution of terms χ_1 and χ_2 such that $A'(\chi_1) = A'(\chi_2)$ in an expression ζ doesn't change $A'(\zeta)$. But this principle is clearly invalid. If ζ is a sentence and χ_1 and χ_2 are terms that involve (are about$_t$) different objects, then even if χ_1 and χ_2 refer to the same thing, $A'(\zeta)$ might change if we replace χ_1 with χ_2.

6. *Comparison with Goodman's 'about'*

Goodman (1961) suggested a slightly different definition of aboutness, which also stems from the idea that in order to say something about an object a sentence has to say something contingent about it. We'll be interested in his notion of *absolute aboutness.*[8] A few preliminary definitions first.

For Goodman, an individual constant designates an object that it refers to, and a predicate designates its extension:

> ... a sentence mentions what a predicate in it designates (the whole extension of a predicate) but not necessarily what the predicate denotes (the several things the predicate applies to).
> (Goodman 1961: 4)

A generalization of a sentence Q with respect to an expression E is arrived at by putting an appropriate variable for E everywhere in Q and prefixing to the result a universal quantifier governing that variable.

A statement T follows from S differentially with respect to k, if T contains an expression designating k and follows logically from S, while no generalization of T with respect to any part of that expression also follows logically from S. Then, S is absolutelyabout k iff some statement T follows from S differentially with respect to k.

A few points are important when we compare the notion of absolute aboutness with the notion of aboutness$_i$. First off, Goodman defines what a sentence is about in terms of implication. To be about an object, a sentence has to imply a certain sentence, but also it cannot imply certain other sentences.

[8] His notion of relative aboutness boils down to absolute aboutness relative to some other sentences. This notion is irrelevant for our present consideration.

This entails that if a sentence is contradictory, it is not about anything, because it doesn't imply differentially any sentence. Also, Goodman takes a fairly strong sense of implication here. It's not only logical consequence but some sort of 'analytical entailment'. For instance, for him, the liar sentence is self-contradictory, even though, technically speaking, it is not logically contradictory without additional premises.[9] This makes Goodman's notion of aboutness not very useful when we want to talk about what paradoxical sentences are about. For if they indeed are contradictory, they simply aren't about anything. On the other hand, consider:

$$a = `\neg Tr(a)'$$

Observe that a is not logically equivalent with $a(a/b)$ (that is, with $\neg Tr(b)$). Moreover, 'a' refers to '$\neg Tr(a)$', and hence, a is about$_i$ itself. In a sense, the feature that allows paradoxical sentences to be about$_i$ something is that aboutness$_i$ is defined in terms of equivalence, whereas Goodman's notion is defined in terms of implication. Even though a contradictory sentence (classically speaking) implies logically any sentence, it is hardly the case that a contradictory sentence is logically implied by any sentence.

Another difference is that on Goodman's account, sentences are not about individuals because of predicates. It is the set designated by a predicate, not individuals denoted by the predicate that a sentence can be about because of a predicate. Even more strikingly, a sentence is not only about the sets designated by some predicates that occur in it. As Goodman puts it, "a statement absolutely about any class or classes is absolutely about each Boolean function of them." For instance, 'All crows are black', on Goodman's view, is equivalent to 'Everything is either a black crow or a black non-crow or a non-black non-crow' and hence it is absolutely about "the class of non-crows, the class of non-black things, the class of black non-crows, the class of things that are either black crows or non-black non-crows, and so on." (Goodman 1961: 12)

So it seems that this notion not only is not very useful when we want to talk about paradoxical sentences, but also when we want to talk about sentences that might be self-referential or circular in virtue of the predicates that occur in it. For instance, consider:

$$a = `\forall_x (P(x) \rightarrow Tr(x))'$$

[9] "*if S happens* to be (20) ['Statement S is false'], it is self-contradictory and so not absolutely about anything." (Goodman 1961: 13)

where it indeed is the case that a lies in the denotation of P. On Goodman's approach, a, even if not inconsistent, is still not self-referential: it is about Boolean combinations of the sets of P's and Tr's, and that's all to it.

Notice also that we can't expand Goodman's account by postulating that if a sentence is absolutely about a set, it is absolutely about all its members, because if a sentence is absolutely about a set, it's also absolutely about its complement, and hence, if it were absolutely about any member of a set it is absolutely about, it would be absolutely about anything if only it were about at least one set.[10]

7. *Summary*

I started with explaining how, according to Leitgeb, there are two different notions involved in the debate surrounding the self-referentiality of Yablo's paradox. One notion is motivated by intuitions about sentences being or not being about objects due to the semantical role of expressions that occur in it. The other notion stems from certain ideas about arithmetized syntax, syntactic mappings and fixed points.

I focused on the first notion. I presented Leitgeb's attempt to define it and his qualms about it. There were two main worries. One was that the definition was only partial, that is, it only told us what sentences are about only for a fairly narrow class of sentences. The other one was that self-reference wasn't preserved under logical equivalence.

I then moved on to giving a fairly strong definition of aboutness$_i$. This notion is defined for all sentences of a given language (which deals with the first qualm). Aboutness$_i$ is preserved under logical equivalence, which is a desired result. But, even this being the case, even if a sentence is self-referential, there can be sentences logically equivalent to it that aren't. I argued that this is not an undesired result and explained how it can happen. I also discussed other minor qualms about the notion raised by Leitgeb. Next, I showed where slingshot-style arguments given for the notion of aboutness$_i$ fail, compared the notion of aboutness$_i$ to Goodman's notion of absolute

[10] Goodman attempts to explain how his notion of absolute aboutness can be thought of in nominalistic terms, but the way he deals with predicates still doesn't give us the desired reading. He simply says that relative to any predicate P, instead of saying that ϕ is absolutely about the class of P's, we should introduce a separate operator, 'P-about', and say that ϕ is P-about, giving the claim the same truth conditions. In the same way he wants to deal with fiction. For instance, instead of saying that a sentence is about Pickwick, he says that we need to introduce a separate predicate 'Pickwick-about'. However, it seems that there is a crucial difference between, say, Yablo sentences being about other sentences because of predicates that occur in them on one hand, and pieces of fiction on the other. This difference is not mirrored in the way Goodman handles predicates.

aboutness and explained why aboutness$_i$ is more fit to be used in discussions about the reference of paradoxical sentences.

The notion of aboutness$_i$, however, is still at odds with some of our intuitions regarding what sentences are about. Whether these intuitions are intuitions stemming from a single notion of aboutness and whether a formal notion of aboutness that captures all those intuitions can be sensibly defined is still an open question.

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REFERENCES

Goodman, N. (1961). About. *Mind*, 70:1–24.

Leitgeb, H. (2002). What is a self-referential sentence? critical remarks on the alleged (non-)circularity of Yablo's paradox. *Logique & Analyse*, 177-178:3–14.

Leitgeb, H. (2005). What truth depends on. *Journal of Philosophical Logic*, 34:155–192.