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RELEVANT LOGIC, PROBABILISTIC INFORMATION, AND CONDITIONALS*

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Abstract

This paper sets out a theory of relevant conditional probability. The theory is motivated as a way of incorporating probabilistic inference into the theory of situated inference of [12]. The theory is then adapted to provide a theory of relevant conditional subjective probabilities and this latter theory is then used to provide a basis for a theory of indicative conditionals.

1. *Introduction*

In [12], I set out a theory of "situated inference" in order to give an interpretation of the model theory for relevant logic. According to that theory, an implication $A \to B$ is true in a situation s if and only if in s there is the information that if there is a situation in the same world as s in which A obtains, then there is also a situation in that world in which B obtains.[1]

In situated inferences, one is allowed to manipulate information using logical rules such as being allowed to take the available information in whatever order one wants, and to "special" pieces of information that tell us about connections between situations. The pieces of information that tells us about these sorts of links between situations in worlds are called *informational links*. A paradigm informational link is a law of nature. For example, if true in our situation, Newton's law of universal gravitation would tell us that if there is a situation in the same world in which two pieces of matter exist, then we could infer that in our world there is a situation in which these two pieces of matter attract one another.

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[1] The theory of situated inference is one interpretation of relevant logic. There are other informational interpretations of relevant logic, such as those in [2], [16], and [19].

The informational links that are utilized in [12] are all perfectly reliable. They must be in order to provide truth-makers for relevant implications. For, in relevant logic, the arrow is read as telling us that the antecedent in some way forces the consequent to come true. But much of our information about the relationship between propositions (and between situations in worlds) is not perfectly reliable. Much of it is at best probabilistic.

Consider a simple example. A die is thrown. When the die is in the air, if it is fair, the probability that it will land a six is $\frac{1}{6}$. Moreover, the throwing of the die makes it $\frac{1}{6}$-likely that the die will land a six. Thus, we can say that a fair die's being thrown implies to the degree of a sixth that it will land 6, or semi-formally, *A fair die is thrown* $\rightarrow_{\frac{1}{6}}$ *the die will land 6*.

Suppose now that when the die is in the air, one is considering flipping a coin. Given the independence of the flipping of the coin and how the die will land, on the classical theory of probability we have that the conditional probability of the die landing on a number greater than 2 is $\frac{2}{3}$. But we would not want to say that the flipping of the coin (relevantly) implies that the die will land on 3-6 is $\frac{2}{3}$. On the theory that we present, the conditional probability of the die landing on 3-6 is near zero. The fact is that the flipping of the coin does not make the die land on 3, 4, 5, or 6 and this should be reflected in a relevant theory of probability.

It is the aim of this paper to set out a semantics for relevant probabilistic implication and to use that semantics as a basis for a theory of conditionals.

2. *Situated Inference*

We begin with a distinction between worlds and situations. A world is a possible world in the sense of contemporary modal logic and metaphysics. It is a complete universe. To borrow a phrase from Barwise and Perry [3], worlds "decide every issue". More formally, worlds support the principle of bivalence — every statement is either true or false at a world. Situations, on the other hand, support information about worlds, and they usually do so in a partial manner. Consider, for example the situation that incorporates all and only the information currently available in my study as I write this paragraph. That situation contains information about me and my dog, but not about, say, the weather in Brussels or Bari.

Thus, situations capture partial information about worlds. The information that they capture need not be about one place or time in a world, but can be about information that is widely distributed over time or space. Moreover, a situation need not accurately characterize only one world. Two or more worlds might contain the same information, and so have the same situation "in" them. Situations can be in more than one world, since (in the sense being used here) situations are abstract entities and, like properties and other abstracta, can exist in more than one place at one time.[2] Note also that the information in a situation need not accurately characterize any world. In this case the situation is not in any world. In this case, the situation is said to be an impossible situation.

On the theory of situated inference, an implication $A \to B$ holds in a situation a if and only if in a there is the information that, if there is a situation b in the same world as a in which A is true, then there is also a situation c (perhaps distinct from a and b, but perhaps not) in the same world in which B is true. We can formalize this as

$$Ia|A||B|,$$

where $|A|$ and $|B|$ are the propositions expressed by the statements A and B respectively.[3] A proposition is just a set of situations. Thus, $IaXY$ says that a contains the information that if there is a situation in X which is in the same world as a then there is also a situation $c \in Y$ in that world.

3. *Probabilizing Situated Inference*

In order to treat probabilistic relevant implication, we develop here a theory of probabilistic situated inference. Instead of just perfectly reliable links, we add ones that are probabilistic. The most obvious examples of such links are probabilistic laws such as the laws of quantum theory, but these are not the only ones that we need. Consider again the throwing of a die. Suppose for a moment that we are in a world in which the laws of nature are all deterministic. Then, given all the information in the world it is determined when the die is thrown how it will land. But suppose also that we are considering a situation in this world in which not enough information is given to determine how the die will land. Then, there still may be enough information (say that the die is not loaded and that it is symmetrical) to allow us to infer that the probability of a die's landing on any given side is $\frac{1}{6}$. How exactly probabilities supervene on partial information is a difficult matter, but what is clear is that this sort of supervenience is common.

[2] Of course, on some theories, individuals can be in more than one world. But this is controversial and it is not a controversy that I can to enter into in this paper.

[3] In [12] I skipped this step and went right to the relations of the form $Iab|B|$. Now I think that it is rhetorically better to start with relations between situations and pairs of propositions.

On the face of it, the semantic theory seems easy to produce. We merely add an extra argument to our implication relation, that is, for any real number r between 0 and 1,

$A \rightarrow_r B$ is true at a if and only if $I_r a|A||B|$.

This seems straightforward enough. We now admit probabilistic informational links into our theory, and so our semantics recognizes this with the addition of a parameter in its implication relation between propositions.

4. *From I to R*

Let's make this all a bit more formal. In the early 1970s, Richard Routley and Robert Meyer produced a model theory for relevant logic that uses an accessibility relation on situations in order to model implication (see [17] and [18]). In [12] (chapters 2 and 3), I motivate the Routley-Meyer model theory using the theory of situated inference. We will not reproduce that motivation here. The upshot is that we can replace our relation I between situations and pairs of propositions with a ternary relation, R, between situations such that, for any situations a, b, and c,

$Rabc$ iff, for all propositions X,Y, if $IaXY$ and $b \in X$, then $c \in Y$.

Having done this, the truth condition for implication now reads as follows:

$A \rightarrow B$ is true at a
iff $\forall b \forall c((Rabc$ & A is true at $b) \supset B$ is true at $c)$.

This move to talk about a ternary relation on situations has great value. We have a good grasp on the mathematics of relations and we know how to manipulate them. As the work of Routley and Meyer shows, the ternary relation semantics is very powerful and flexible. It can be used to provide models for a large range of logical systems and it can be used to prove interesting and otherwise difficult results about them ([18] and [4]).

Our problem is now to integrate a semantics for probabilistic relevant implication into the Routley-Meyer Model theory for relevant implication. Their model theory places an important constraint on the semantics for probabilistic implication. Given that the relevant arrow represents perfectly reliable connections between situation types, it seems clear that if $A \rightarrow B$ is true at a then so is $A \rightarrow_1 B$. Thus, if the truth condition for a relevant implication obtains at a situation, the truth condition for the corresponding $\rightarrow_1$ formula must also obtain there.

The following is one way of satisfying this constrain and giving an intuitive meaning to $\rightarrow_r$. The idea here is that we look at the set of situations that are R-related to a and situations in which A is true and find that the proportion of them that are $|B|$. If this proportion is r, then the probabilistic implication $A \rightarrow_r B$ is true at a.

In order to understand all of this in more depth, we will examine the precise definition of a Routley-Meyer frame and of our conditional probability measures.

5. *Routley-Meyer Frames*

An R-frame is a structure $\mathcal{F} =< S, P, R, * >$ such that S is a non-empty set (of "situations"), P is a non-empty subset of S (of "world-like situations"), R is a ternary relation on S, and $*$ is a unary operator on S, which satisfy the following definition and postulates:

$$a \leq b \ =_{df} \exists x(x \in P \ \& \ Rxab).$$

F1 if $a \in P$ and $a \leq b$, then $b \in P$;

F2 $\leq$ is transitive and reflexive;

F3 if $Rabc$, then $Rbac$;

F4 if $\exists x(Rabx \ \& \ Rxcd)$, then $\exists x(Racx \ \& \ Rxbd)$;

F5 $Raaa$;

F6 if $Rabc$, then Rac^*b^*;

F7 if $Rbcd$ and $a \leq b$, then $Racd$;

F8 $a^{**} = a$.

Where X and Y are sets of situations, we define $X \Longrightarrow Y \ =_{df} \ \{a : \forall b \forall c((Rabc \ \& \ b \in X) \ \supset \ c \in Y)\}$ and $-X =_{df} \{a : \ a^* \notin X\}$. We say that a set $X \subseteq S$ is closed upwards if for any $a \in X$, if $a \leq b$, then $b \in X$. A set *Prop* is a set of closed upwards sets that is closed under $\cap$, $-$ and $\Longrightarrow$. It is easy to show that *Prop* is also closed under $\cup$.

In what follows, we will also need the following definition: Where X is a set of situations, let RaX be the set of situations, $\{c : \ \exists b(b \in X \ \& \ Rabc)\}$. The reader should be careful not to confuse the set RaX with an arbitrary set of situations Y such that $IaXY$. Rather, it is the intersection of these sets, i.e. $RaX = \cap_{Y \in Prop} IaXY$. Thus, RaX is the set of situations that satisfy all the consequents of all of all the conditionals true at a such that the antecedents of those conditionals represent the proposition X.

Our base language is a standard propositional language with the connectives $\wedge$, $\neg$, and $\rightarrow$, propositional variables and parentheses. It has the standard formation rules. Later we will add "psuedo-connectives" $\rightarrow_r$ for each real number r in the closed interval $[0, 1]$. The subscripted arrows are not real connectives, since we will not allow them to be nested in formulae. That is, a formula containing a subscripted arrow will be well-formed only if the subscripted arrow is the main connective. But for now, we will deal only with the base language.

A general R-frame is a pair $< \mathcal{F}$,*Prop*$>$, where $\mathcal{F}$ is a Routley-Meyer frame and *Prop* is a set of propositions over $\mathcal{F}$. A valuation over a general frame $< \mathcal{F}$,*Prop*$>$ is a function from the propositional variables into *Prop*. Each valuation v determines a satisfaction relation $\models_v$ between situations and formulas such that the following truth clauses obtain:

- $a \models_v p$ iff $a \in v(p)$, for all propositional variables p;
- $a \models_v A \wedge B$ iff $a \models_v A$ and $a \models_v B$;
- $a \models_v \neg A$ iff $a^* \not\models_v A$;
- $a \models_v A \rightarrow B$ iff $\forall b \forall c((Rabc \,\&\, b \models_v A) \supset c \models_v B)$.

We also set $|A|_v = \{a \in S : a \models_v A\}$. By an easy but tedious induction we can show that, for any formula A, $|A|_v \in$*Prop*. And we can show that $|A \wedge B|_v = |A|_v \cap |B|_v$, $|A \rightarrow B|_v = |A|_v \Longrightarrow |B|_v$, and $|\neg A|_v = -|A|_v$. A model is a structure $< \mathcal{F}$,*Prop*$, v >$, where $< \mathcal{F}$,*Prop*$>$ is a general R-frame and v is a valuation. A formula A is valid on a model $< \mathcal{F}$,*Prop*$, v >$ if and only if $P \subseteq |A|_v$. A formula is valid on a general R-frame if it is valid on models based on that frame and it is valid on the class of R-frames if it is valid on every frame in that class.

The logic R is characterized by the class of general R-frames, as we have defined them here. The set of propositions plays no role in the soundness or completeness proofs for R. We add *Prop* because it is needed for the definition of relevant probability functions that we present in the next section.

6. *Relevant Probability Functions*

Now that we have defined a class of frames, we can define probability functions on them. The sort of probability function that we use here is adopted from the paraconsistent theory of probability given in [11] (also see [15]). In that paper I generalized the standard Komologorov axioms for probability theory to fit with Dunn's logic D4. The relevant logic R, in a certain sense, results from the addition of the implication connective to D4. And, because of this, the generalization of probability theory seems to fit equally well with R.

Before we can get to the definition of a probability function itself, we need to define a lattice of subsets. A *lattice of subsets* over a set X is a structure $\mathcal{L} = < L, \cap, \cup, \subseteq >$ such that $L \subseteq \wp X$, where $X \in L$, $\emptyset \in L$, L is closed under $\cap$ and $\cup$, and $\mathcal{L}$ is ordered by $\subseteq$.

A *relevant probability function* $\Pr$ is a function from a lattice of subsets L over a set X into the closed interval of real numbers, $[0, 1]$ such that the following conditions are met. Where Y and Z are any members of L,

- $\Pr(X) = 1$; $\Pr(\emptyset) = 0$;
- $\Pr(Y \cup Z) = (\Pr(Y) + \Pr(Z)) - \Pr(Y \cap Z)$;
- If $Y \subseteq Z$, then $\Pr(Z) - \Pr(Y) \geq 0$.

The lattice of sets over which our probability function is to be defined is the closure of *Prop* $\cup \{RaX : a \in S \ \&\ X \in$*Prop*$\}$ under $\cap$ and $\cup$.

For each situation, we define a conditional probability function $\Pr_a$ such that

$$\Pr_a(|B|_v, |A|_v) = \frac{\Pr((Ra|A|_v) \cap |B|_v)}{\Pr(Ra|A|_v)}.$$

The idea is that the probability of B on A in a is the proportion of situations in $Ra|A|_v$ that are also in $|B|_v$.

Now we add the subscripted arrows, $\rightarrow_r$ (for each $r \in [0, 1]$) to our language. We use this conditional probability function to give a truth condition for our psuedo-connective $\rightarrow_r$, viz.,

$$a \models_v A \rightarrow_r B \text{ iff } \Pr_a(|B|_v, |A|_v) = r.$$

Let's return to the example from the introductory section above. Suppose that a is a situation in which a die is thrown and that an agent in a is considering whether to flip a coin. The conditional probability of the die landing on a number greater than 2 (G) given that the coin is flipped (C) is $\Pr_a(|G|_v, |C|_v) = \frac{\Pr((Ra|C|_v) \cap |G|_v)}{\Pr(Ra|C|_v)}$. The situations in which the coin is flipped (i.e. the situations in $|C|_v$) are not restricted here to ones in which the die is also thrown. They include situations in which the die is not thrown and ones which contain no information about whether the die is thrown. Unless in a there are informational links that connect the coin's being flipped with the die's landing, there will be a very small percentage of the worlds in $Ra|C|_v$ that actually have the die landing on any number. Thus, $\mathrm{Pr}_a(|G|_v, |C|_v)$ will be very low.

7. *Probability and Conditionals*

In the majority of the remainder of this paper, we provide a version of the theory of conditionals due to David Lewis and Frank Jackson. The difference between our version and theirs is that whereas theirs is based on classical logic and classical probability theory, ours is based on relevant logic and relevant probability theory.

The Lewis-Jackson theory (henceforth, the 'LJ theory') claims that the truth condition for indicative conditionals is the same as for the corresponding material conditional (see [6]). They do not hold, however, that indicative conditionals are *merely* material conditionals. The material conditional has properties that the indicative conditional does not seem to share. For example, material conditionals contrapose, whereas indicative conditionals do not always do so. Here is an example due to Frank Jackson:

$$\frac{\text{If Mary makes a mistake, she will not make a big mistake.}}{\therefore\ \text{If Mary makes a big mistake, she will not make a mistake.}}$$

Clearly, the conclusion of this argument is absurd. Similarly, the indicative conditional is not in general transitive and does not allow for strengthening of the antecedent.

Thus, the LJ theory holds that when one states an indicative conditional, the truth condition of her utterance is the same as the corresponding material conditional, but she is also expressing that she holds corresponding conditional probability to be high. Thus, for example, when Susan says 'if Mary makes a mistake (M), she will not make a big one ($\neg B$)' she is expressing that her subjective probability $P(\neg B, M)$ is high. Just because $P(\neg B, M)$ is high, it does not follow that its "contrapositive" $P(\neg M, B)$ will also be high. Thus, although the LJ theory claims that the argument given above is valid (it preserves truth), it holds that the premise may be *assertible* by a speaker when the conclusion is not. Thus, the LJ theory does not quite identify the indicative conditional with the material conditional. They have the same truth conditions, but different pragmatic properties.

Despite the fact that the pragmatic filter avoids some of the problems of identifying the indicative and material conditionals, it does not avoid all. For example, as we have already seen, if one knows that a proposition p is true, then her conditional probability $P(p, A) = 1$ if $P(A) \neq 0$. Thus, the LJ theory inherits a paradox of material implication: If one knows that p is true, then 'If A, then p' is assertible for any sentence A such that $P(A) \neq 0$. Thus it is assertible that 'if Brazil wins the next World Cup, 2+2=4'. Similarly, if one's probability for p is high, $P(q) \neq 0$, and p and q are independent, then 'if q, then p' is assertible. So, to return to our example from the introductory section above, on the LJ theory, when a die is in the air the conditional 'if I toss a coin, the die will land on a number greater than two' is assertible. Therefore, it would seem that the LJ theory is in need of further emendation.

8. *Subjective Probability*

One key feature of the LJ theory is that the assertibility conditions of conditionals are conditional *subjective* probabilities. So far we have dealt with objective probabilities that supervene on the information available in situations. In order to produce a theory of subjective probability in our framework, we treat the content of an agent's belief state as a set of situations. In doing this, we are adapting the treatment of contents of intentional states from doxastic logic — in the semantics for doxastic logics, one takes a content to be a set of possible worlds. Each of these worlds corresponds to what is possible given one's beliefs. Here we change only the view that contents are sets of worlds to the claim that they are sets of situations. The reason that we need to use a set of situations, rather than a single situation, is to deal with unresolved disjunctions. On our semantics, a disjunction $A \vee B$ is true in a situation if and only if at least one of A or B is true in that situation.[4] But we do tend to have disjunctive beliefs even in cases in which we do not believe either disjunct. Taking the content of our beliefs to be sets of situations allows unresolved disjunctions. In addition, the use of sets of situations to model contents allows us to treat ambiguity of reference and vagueness.

The changes that we suggest to the LJ theory is to take an indicative conditional to have the same truth condition as the corresponding relevant implication and to have as an assertibility condition that the corresponding relevant subjective conditional probability be high. Thus, we need a theory of subjective conditional probability. To represent the degrees of beliefs of an (ideal) agent i, we assume that she has a probability function P_i over the closure under $\cap$ and $\cup$ of the set of propositions together with the set of $X \Longrightarrow_r Y$, which is the set of situations a such that $\mathrm{Pr}_a(Y, X) = r$, for all $r \in [0, 1]$. The function P_i is monadic, that is, it takes propositions as arguments and

[4] We are assuming that $A \vee B$ is defined as $\neg(\neg A \wedge \neg B)$. Given this definition, we can derive the standard truth condition for disjunction.

returns values in the unit interval. The task here is to create a binary subjective probability function (a conditional probability function), which takes pairs of propositions as arguments.

I take a subjective relevant conditional probability Pr_i to be a statistical average of the probabilities of $X \Longrightarrow_r Y$. When we have two probability functions — a first order function and a second order function — the idea behind taking a statistical average is to give a weighted average of the first order probabilities. Here we have two probability functions — a relevant conditional probability function and a subjective probability function. The conditional relative probability function acts here as the first order function and the subjective probability function P_i acts here as the second order function. In order to define the subjective conditional probability function, let $\mathbb{R}(X, Y)$ be the set of $r \in [0, 1]$ such that $P_i(X \Longrightarrow_r Y) \neq 0$. If $\mathbb{R}(X, Y)$ is finite, then we set

$$\mathrm{Pr}_i(Y, X) = \sum_{r \in \mathbb{R}(X,Y)} (P_i(X \Longrightarrow_r Y) \times r) \ (\ddagger).$$

(compare [8]). If $\mathbb{R}(X, Y)$ is countable, then given a discrete ordering on $\mathbb{R}(X, Y)$, we can apply (‡). If $\mathbb{R}(X, Y)$ is uncountable, then we can use approximation techniques and take a countable partition on $\mathbb{R}(X, Y)$, and choose one number from each element of the partition and, given a discrete ordering on the resulting set, apply (‡).[5] For formulas A and B, we set $\mathrm{Pr}_i(B, A)_v = \mathrm{Pr}_i(|B|_v, |A|_v)$.

9. *Relevant Conditionals*

There are some close connections between natural language conditionals and corresponding implications. In mathematics in particular, but in ordinary speech as well, we often use 'if ... then' and 'implies' interchangeably. The relevant logic R, I claim, captures the notion of implication well. But relevant implication is transitive, it allows for strengthening of the antecedent, and it contraposes.

[5] Note that this method will return a value for $\mathrm{Pr}_i(Y, X)$ only when $\lim_{n \to \infty}((P_i(X \Longrightarrow_{r_1} Y) \times r_1) + ... + (P_i(X \Longrightarrow_{r_n} Y) \times r_n))$ is defined. The possibility of undefined conditional probabilities does not, however, affect my theory.

So, I suggest here[6] that we take the truth condition of indicative conditionals to be the same as their corresponding relevant implications and their assertibility condition to be that the corresponding subjective relevant conditional probability to be high. To make the theory more precise, let's formalize the indicative conditional with the arrow, $\rightsquigarrow$. Thus, we set $A \rightsquigarrow B$ is assertible for i on v iff $\mathrm{Pr}_i(B, A)_v$ is high.

Apart from avoiding the problems already cited, the relevant theory of conditionals has some advantages over the LJ theory. First, consider a conditional with a antecedent that is known to be impossible. On the LJ theory, such conditionals are never assertible. But just because a proposition is necessarily false, need not mean that it is always irrational to assert conditionals with it as an antecedent. For example, consider the conditional 'if Fermat's last theorem is false, then Wiles' proof is wrong' is assertible (and true). Similarly, 'if Fermat's last theorem is right, then Wiles' proof is wrong' is not assertible (and false). Second, the present theory does better with nested conditionals. Where P is a classical probability function, the conditional probability $P(q \supset p, p)$ is always 1, where $P(p) \neq 0$. Thus, the LJ theory is stuck with a paradox of material implication, that is, they have to accept 'if p, then if q, p' whenever the probability of p is non-zero. But, where P_i is a relevant subjective conditional probability function, $P_i(q \rightsquigarrow p, p)$ (i.e. $P_i(q \rightarrow p, p)$) need not be high (regardless of the value of $P_i(p)$).

It would also be interesting to combine relevant logic with the *logic of being informed* (see [5]). Instead of looking at the set of the situations that matches one's overall belief state, we could look at the set of situations that is the content of one's information. That is, we could look at the set of situations s that is such that if one is informed that A, then A is true in s. We could then combine our relevant logic with the logic of being informed. If, as is argued in [5], this logic is the modal logic KTB, then this task is straightforward, for it is not difficult to add a binary accessibility relation to our semantics that is reflexive and symmetrical (see, e.g., [10]).[7]

10. *Concluding Remarks*

We have set out a theory of probability based on the semantics for relevant logic. We have motivated this theory as a basis for probabilistic inference

[6] I'm not claiming that this is the *right* theory of conditionals. In [12], chapter 7, I develop a non-probabilistic theory of indicative conditionals. I still prefer that other theory, but the present theory is an interesting alternative that deserves to be explored.

[7] It would be more difficult, but also very interesting, to combine relevant logic (and relevant probability), with the adaptive logic given in [1].

from partial information. We have also used it as basis for providing a relevant version of the LJ theory of conditionals.

There is clearly much more work to be done. In particular, it would be interesting to investigate whether it is possible to allow nested probabilistic implications. The problem here is to set out a theory of nested conditional probability that does not collapse into triviality. As David Lewis has shown, in classical probability theory, allowing nested conditionals the probability of which is equal to their corresponding conditional probabilities leads to a form of triviality — probability functions of this sort do not allow formulas to take more than a small number of values [7]. Moreover, some non-classical probability functions that obey quite weak conditions trivialize as well given the assumption that the probability of a conditional is the same as the corresponding conditional probability (see [14]).

This work should be of interest both to philosophers working on conditionals and those interested in a theory of information. The theory fixes certain problems in the LJ theory of conditionals — by making it relevant. The theory of situated inference of [12] is a theory about how we use partial information to make inferences about the world. The current paper extends the theory of situated inference to handle probabilistic inference. There is also more work to be done on the theory of inference. We need a theory that also treats defeasible (and perhaps other non-monotonic) inference, where specific probabilities are not available.

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