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CHARACTERIZING NEGATION TO FACE DIALETHEISM

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Abstract
It has been said that, when some paraconsistent logicians supporting dialetheism assert: "For some sentence α, both α and not-α are true", therefore claiming that the Law of Non-Contradiction (LNC) fails, we should wonder what "true" and "not" mean here. After surveying two classical paraconsistent approaches to negation (provided by da Costa's *positive-plus* systems and Graham Priest's Logic of Paradox), I describe a negation with the following features: (1) its definition does not make reference to the controversial concept *truth*; (2) it has strong pre-theoretical appeal and motivation, because it performs an indispensable expressive function in language and communication; and (3) it is accepted by dialetheists, too, since it is based on a very deep metaphysical intuition they also show to fully share: this intuition I call the one of *material exclusion*. If my characterization is sufficient to confer a determinate meaning to the negation in question, we can conveniently formulate via *this* negation a version of the LNC which I take, therefore, to be indisputable also from the dialetheist's point of view. Such a result, however, does not constitute a quick and easy success against supporters of true contradictions. It may simply show that the versions of the LNC dialetheists most convincingly attack are those that were not to be defended, and that supporters of consistency have been historically confused in assimilating them to the indisputable one.

> Quine's famous argument, that to change the logic is to change the subject, may be right to this extent: classical negation and non-classical negations have different meanings. But the substantial issue that Quine never addressed is why we should suppose that the meaning of the vernacular negation is classical.
>
> *Graham Priest*

1. *Introduction*

Dialetheists say: "For some sentence α, both α and not-α are true", therefore arguing that the Law of Non-Contradiction (LNC) fails. One wonders what is meant by "true",[1] and, of course, by "not":

> The fact that a logical system tolerates A and $\sim A$ is only significant if there is reason to think that the tilde means 'not'. Don't we say 'In Australia, the winter is in the summer', 'In Australia, people who stand upright have their heads pointing downwards', 'In Australia, mammals lay eggs', 'In Australia, swans are black'? If 'In Australia' can thus behave like 'not' [...], perhaps the tilde means 'In Australia'?[2]

Philosophers often disagree on the content of basic logical and metaphysical concepts (such as *identity*, *existence*, *necessity*, etc.), or on the validity of some very basic principles of inference (such as Contraposition, *reductio ad absurdum* or Disjunctive Syllogism). It is well known that this kind of discussion often faces an impasse, or seems to turn into a hard conflict of intuitions (this may be due, among other things, to the fact that we cannot examine such concepts as *predication*, *negation*, etc., without using them). It is very difficult to establish when some party or other begins to beg the question, and it is not an easy issue whether a non-standard explanation of a basic notion involves a real disagreement with a classical account of *that* notion, or its principles simply describe a different thing using the same name or symbol. Is intuitionistic negation the real negation, or does it simply mean, though typographically identical, something else than the classical one? Do non-truth-functional theories of conjunction and disjunction, such as supervaluationism[3] and non-adjunctive logics,[4] describe *conjunction* and *disjunction*? Authors like Michael Resnik consider at least some of these puzzles simply unsolvable.[5]

[1] See e.g. Slater [1995], whose position I shall examine later.

[2] Smiley [1993], p. 17.

[3] See e.g. van Fraassen [1966], Fine [1975].

[4] See e.g. Rescher and Brandom [1980], Varzi [1997], [2004].

[5] "I take a dim view of the idea that revising our logic entails using so-called logical words with new meanings. Suppose that until now my mathematical proofs used non-constructive principles, but now I announce that I will restrict myself to constructively acceptable proofs. Have I revised my logic, while continuing to mean the same by 'not' and 'or' or have I

In this paper I provide a short survey of the treatment of the two aforementioned basic notions, *truth* and *negation*, in two (types of) paraconsistent logics.[6] I focus my attention on the *positive-plus* systems due to da Costa, *et al.*, and Graham Priest's Logic of Paradox (LP). My aim is primarily descriptive: to show how meaning-variance issues and battles of intuitions quickly arise in discussions on the two approaches. Such a survey, though, should also direct us to the positive view I am then going to propose: if we want to have a non-question-begging debate on the validity of the LNC, we should not concentrate on truth, but on negation. This is not to mean that there is a unique good description of negation. It is sometimes said, in the spirit of so-called "logical pluralism", that we do not have one conditional, but many, and different accounts (material truth-functional conditional, C.I. Lewis' strict implication, subjunctive conditionals, the entailment of relevance logics, etc.) describe different connectives which entertain family resemblances. Similarly, we may have distinct intuitions on different sentential negations, which may be characterized by different theories. This does not entail, though, that no non-question-begging debate is feasible. On the contrary, I think it is possible to characterize a negation (I shall label it "NOT") with the following pleasant features: (1) its definition does not refer to the contentious concept *truth*; (2) it has a strong pre-theoretical motivation, because of its indispensable expressive function in language and communication; and (3) it is fully accepted also by dialetheists, because it is based on a deep metaphysical intuition they show to fully share: the intuition of *material exclusion*. If such a characterization is sufficient to confer a determinate meaning to the negation in question, we can conveniently phrase the LNC via *this* negation, providing a formulation which I take, therefore, to be indisputable also from the dialetheist's point of view.

Such a result does not, however, constitute a quick and easy triumph on supporters of true contradictions. It may simply show that the versions of the LNC dialetheists rightly attack are those that were not to be defended, and that supporters of the LNC have been confused in assimilating them to the indisputable one.[7]

decided to use those words with a different meaning? I don't perceive a fact of the matter here." (Resnik [2004], p. 180).

[6] It is quite obvious that a paraconsistent logician may well not be dialetheist (that is to say, a supporter of true contradictions), and probably most are not. Nevertheless, the paraconsistent logics I am discussing in the following have all been considered suitable for so-called strong paraconsistency or dialetheism. For a general introduction to paraconsistent logics, dialetheism and their philosophical motivations, see Berto [2006].

[7] As Graham Priest appropriately observed, "whether or not dialetheism is correct, a discussion of the questions it raises, concerning fundamental notions like negation, truth and

2. *Three battlefields for intuitions and the Italics Argument*

I suggest that there are three major (variously overlapping) battlefields for conflicting intuitions, when we are dealing with the above sketched subjects. As we shall see, the three kinds of clash are all displayed in paraconsistent discussions on negation.

First, the interplay between truth and negation unfolds a very general issue of standard formal semantics. As it is well known, the recursive clauses expressing truth conditions for sentences may be taken as explicating (given that Tarski was right) the concept of truth for the object-language. Alternatively, the formal recursion may be considered as giving the meanings of the logical vocabulary. But it seems that we cannot have it both ways. By paying in an independent understanding of the logical vocabulary, we may buy a characterization of truth. Conversely, by paying in an independent understanding of the concept of truth, we may buy a characterization of the logical vocabulary. Such a cross-dependence, of course, is one of the first sources of equivocation and question-begging cross-charges in discussions on deviant logics that seem to provide a non-standard account of logical vocabulary: did they begin with an alteration of the meaning of connectives and/or quantifiers, or did they move from a change in the notion of truth (probably supported by different metaphysical intuitions)?[8]

rationality — questions that have been little asked for two millennia — can hardly fail to deepen our understanding of these notions" (Priest [1993], p. 35).

[8] Some authors reject the idea that we can go from truth conditions to the meanings of the logical constants. For instance, Michael Tye observes that homophonic semantic clauses presuppose that we grasp the meaning of the connective used in the metalanguage, in order to understand truth conditions. He expresses his point with reference to disjunction: "It is [...] a mistake to suppose that the truth-conditions for disjunctive sentences analyse the meaning of the term 'or'. Rather it is because 'or' means what it does, that the truth-conditions obtain. [fn. 24:] One who lacks the concept of disjunction, for example, will not come to understand it by being shown the truth-conditions for disjunctive sentences. Rather, the purpose of a formal statement of truth-conditions is to explain rigorously how the truth-value predicates are to be applied..." (Tye [1990], p. 547). One may simply reply that recursive clauses are not necessarily homophonic: a so-called choice negation can be defined via two clauses,

(C1) "$\neg\alpha$" is true if and only if "α" is false

(C2) "$\neg\alpha$" is false if and only if "α" is true,

and, as we can see, "not" does not appear in the metalanguage here. On the contrary, one may conjecture that to understand how "$\neg$" is characterized by (C1) and (C2) for the object-language we have to know something about what "true" and "false" mean in the metalanguage. This point will be particularly relevant when we come to consider Priest's LP.

A second source of disagreement comes from the fact that we currently have (at least) two competing characterizations of the logical vocabulary itself: the one in terms of truth values (via Tarskian clauses or truth tables), and the Gentzen-style presentation in terms of introduction/elimination rules. So, one may consider the fact that some commonly accepted inference involving a given connective fails within a system as a decisive sign that something has gone wrong with it; while her *opponens* may reply that the fact that a truth-table presentation yields the "intuitively expected" truth values provides sufficient evidence that we have hit the target, or even question the inferential approach in general (*das Tonkproblem* easily comes to mind).[9]

A third source of discussion is the issue, whether sentential connectives should be truth-functional. Supervaluational treatment of disjunction is paradigmatic. Suppose both α and β come out neither true nor false in some evaluation, because different ways of filling the gaps due to vague predicates or non-denoting singular terms yield different truth values. Then, it may be the case that $\alpha \vee \beta$ is truth-valueless, but it may also be the case that it comes out true — particularly, if β is $\neg\alpha$. Thus, Kripke [1975] has argued that such a failure of truth-functionality is a sign that supervaluationism misses the point of the meaning of disjunction. When you say "$\alpha \vee \beta$", I am perfectly entitled to ask: "Ok, which one then, if not both?". Adapting Tappenden [1993] and Varzi [2004], one may react by calling this the *Italics Argument*: "you claim that 'either α or β' holds, so *either* α *or* β [stamp the foot, bang the table] must hold!":

> In a way, this sort of objection can be dismissed on the grounds of its unfair appeal to intuition. Change of semantics, change of subject — says the objection. Fair enough. But who got the semantics right in the first place?[10]

It is fairly clear that the three kinds of clash are variously intermingled. For instance, these features of supervaluationism can produce a battle of intuitions, not on the concept *disjunction*, but, again, on the concept *truth*. Since supervaluational semantics validates the Law of Excluded Middle while dismissing the Principle of Bivalence, it has to reject the inference from the

[9] See Prior [1960], Belnap [1962]. On these issues, also see Haack [1978], chapters 3 and 11. It is sometimes said that the inferential account provides the constructive meaning of logical vocabulary, while the truth-table account gives us the classical meaning. But, as it is well known, we may have perfectly acceptable natural deduction presentations of classical logic and, *vice versa*, truth-table presentations of non-classical (e.g., many-valued) logics.

[10] Varzi [2004], p. 103.

former to the latter. This, given standard rules of inference, amounts to rejecting a half of the T-schema,

(T) "α" is true if and only if α.

Now critics of supervaluationism like Timothy Williamson can easily put forth the(ir) intuition according to which, even though "it is not claimed that a Tarskian theory tells the whole truth about truth", nevertheless "it tells an essential part of the truth". So "without a disquotational schema, it is doubtful that one has a truth predicate at all".[11] This amounts to saying that a concept of truth that does not fully satisfy (T) is not *truth* — and so, the Italics Argument strikes again.

3. *Negation in da Costa's* positive-plus *systems*

One can be a paraconsistent logician without believing in true contradictions (this was Anderson and Belnap [1975] position, for instance, since the paraconsistent features of their systems were considered a side-effect of the search for a relevant implication). But if you are a dialetheist, you are supposed to embrace some paraconsistent logic. Classical logic is, as Priest and Routley [1989a] say, *explosive*, i.e., its consequence relation is such that $\{\alpha, \neg\alpha\} \vDash \beta$. So admitting just one contradiction leads to *trivialism*, that is, the view according to which everything is true. In the so-called *positive-plus* systems,[12] of which da Costa's systems $\mathrm{C}_n (1 \leq n \leq \omega)$ are probably the most representative,[13] explosion is avoided by significantly altering the treatment of negation. While the positive fragment of these systems is nearly classical (e.g., they retain the *a fortiori* principle, $\alpha \rightarrow (\beta \rightarrow \alpha)$, which is irrelevant by the standards of Relevant logics), truth conditions for negation are (typically) spelt this way:

[11] Williamson [1992], p. 268fn.

[12] See Carnielli and Marcos [2002].

[13] But see also, e.g., the system PI in Batens [1980], or the one labelled CLuN, prominently used in the adaptive approach of Diderik Batens *et al.* (Batens [2000]; Bremer [2005], Ch. 7).

(C1) If "α" is false, then "$\neg\alpha$" is true.

(C2) If "$\neg\neg\alpha$" is true, then "α" is true.[14]

While (C2) does the job of validating Double Negation Elimination, (C1) ensures that at least one of α and $\neg\alpha$ is true, thereby validating Excluded Middle. Such systems avoid explosion,[15] but they also turn on all the three main clashes of intuitions mentioned above.

First, two typical laws/rules involving negation that hold even in minimal logic, Contraposition and Double Negation Introduction, fail for most (though, admittedly, not all) da Costa systems. Those who maintain that inferential features are the source of meaning for logical constants have good reasons to claim that da Costa negation is not *negation*.[16] For instance, according to Richard Routley

> The weakened negation systems lack all forms of contraposition though surely some are correct, and indeed there is little basis for regarding the so-called negations of these systems as genuine negations at all rather than, say, positive modal connectives, e.g. weird truth or necessity connectives.[17]

[14] See e.g. da Costa [1974], da Costa and Alves [1976].

[15] Actually, some of these systems are called *gently explosive*. Typically, they include some sentential functor (say "$^{\circ}$") which should allow to express the consistency of a formula within the object language: "$^{\circ}\alpha$" is true if and only if α is not paradoxical. Then in such systems we have:

$^{\circ}\alpha, \alpha, \neg\alpha \vdash \beta$,

that is, if a formula that is assumed as consistent turns out to be paradoxical, *this* produces explosion (see Carnielli and Marcos [2002], pp. 28 and 33; Bremer [2005], pp. 108ff).

[16] Sometimes even less is required: Hazen [1994] investigates what he calls *subminimal negation*, for which only Contraposition holds. In some *positive-plus* systems we have a Restricted Contraposition which uses the consistency operator:

$^{\circ}\beta, \alpha \rightarrow \beta \vdash \neg\beta \rightarrow \neg\alpha$

(see Carnielli and Marcos [2002], p. 39).

[17] Routley [1979], p. 305. See also Mortensen [1980], and Lenzen [1996], where the point is discussed entirely from an inferential point of view. Lenzen distinguishes "*dispensable* principles which, though they are valid principles of classical negation, need not necessarily be satisfied by arbitrary other negations"; and "*indispensable* principles which a logic L always has to satisfy if its monadic operator $\sim$ is to count as a genuine negation" (p. 40). And

But a paraconsistent logician like Diderik Batens replies that

> Routley's intuitions on the matter are wrong [...]. I must confess that I have (distinct) intuitions on several kinds of "negation", that I consider it interesting to study some of them, that I presume that some are useful in certain context whereas others are useful in other contexts...[18]

Second, da Costa negation is not truth-functional but, at most, partially truth-functional: given that α is false, $\neg\alpha$ is true; but given that α is true, $\neg\alpha$ may be true as well as false. According to Routley and Priest, such a failure of extensionality is sufficient to certify that such a connective "is not our friendly neighbourhood extensional negation, but a radically intensional functor of some sort".[19]

Third, and probably most interesting for our purposes, da Costa negation makes $\neg(\alpha \wedge \neg\alpha)$ fail, while validating the Excluded Middle, $\alpha \vee \neg\alpha$. Because of this, da Costa's systems are again accused of missing the very point of the meaning of negation:

> The law of non-contradiction has traditionally been seen as a central property, if not a defining characteristic, of negation. [...] That an account of negation violates the law of non-contradiction therefore provides *prima facie* evidence that the account is wrong. This is [a] piece of evidence that *da Costa negation is not negation*.[20]

Priest and Routley stress that it is *essential* to the characterization of negation that "negation is a contradiction forming functor, not a subcontrary forming functor".[21] Now, according to the traditional story, α and β are contraries if and only if "$\alpha \wedge \beta$" is logically *false*, sub-contraries if and only if "$\alpha \vee \beta$" is logically *true*, and contradictories if and only if they are both

failure of Contraposition is considered by itself a sufficient reason not to count an operator as a negation in any way (see pp. 43ff.).

[18] Batens [1980], p. 212.

[19] Priest and Routley [1989a], p. 164.

[20] Priest and Routley [1989a], pp. 164–165. For a defence of non-truth-functional features of negation in a (good) paraconsistent logic, see Batens [1980], pp. 228–9, da Costa and Marconi [1989], pp. 18–21.

[21] Priest and Routley [1989a], p. 165.

contraries and sub-contraries. The fact that in da Costa's systems $\neg(\alpha \wedge \neg\alpha)$ fails while $\alpha \vee \neg\alpha$ holds is then taken as the final evidence that "$\neg$" is a pseudo-negation. Such a criticism is thus based on a traditional account of contrariness and sub-contrariness in terms of *truth* and *falsity*.

4. *Negation in Priest's LP*

Priest's treatment of negation in *Logic of Paradox* and in *In Contradiction* seems to make as little changes as possible with respect to classical negation. Truth conditions for negated sentences are spelt this way:

(C1) "$\neg\alpha$" is true if and only if "α" is false

(C2) "$\neg\alpha$" is false if and only if "α" is true.

Priest and Routley claim that "the truth conditions [for such a negation] look very familiar. Indeed, they are just like the classical ones".[22] Actually, (C1) and (C2) make it look more like choice negation than like classical negation, whose truth conditions are spelt by a homophonic clause. The underlying intuition, anyway, is clear: negation is the operator that switches truth and falsity. Furthermore, (C1) and (C2) make "$\neg$" truth-functional, and Double Negation Introduction is validated.[23] So, Priest concludes that his logic "is exactly the same as classical logic, except that one does not make the assumption, usually packed into textbooks of logic without comment, that truth and falsity in an interpretation are exclusive and exhaustive".[24]

But how much does the textbook assumption on the concepts *truth* and *falsity* weigh? (C1) and (C2) characterize negation also for a semantics which admits truth value gaps, but no truth value glut at all.[25] Therefore, it is

[22] Priest and Routley [1989a], p. 168.

[23] Actually, Contraposition is a tricky issue here, but this seems due more to the features of conditional(s), than to those of negation. This makes quite a difference with da Costa systems, whose conditional seems to behave in a standard fashion, e.g., by allowing irrelevant inferences such as $\alpha \rightarrow (\beta \rightarrow \alpha)$. The point is that — as is observed, e.g., by Bremer [2005], pp. 45ff — LP should be considered as a basic paraconsistent logic, to be *extended* by a (usually Relevant) conditional. Now, Priest [1987], Ch. 6, discusses a possible failure of Contraposition, but such a failure is due exactly to the choice of a non-contraposible intensional conditional.

[24] Priest [1998], p. 413.

[25] See e.g. Parsons' "comfortable negation", which works for gappers or for glutters (or for both) depending on the interpretation (Parsons [1990]).

exactly the interpretation of "true" and "false" that changes here: are they exclusive, exhaustive, both, or neither?[26] Now, according to some authors Priest has simply shifted the illicit paraconsistent trick from the meaning of "not" to the meaning of "true". If da Costa's "not", according to Priest himself, is not negation because $\neg(\alpha \wedge \neg\alpha)$ fails for it, whereas "the law of non-contradiction has traditionally been seen as a central property, if not a defining characteristic, of negation"; then Priest's "true" is not truth because it does not rule out *false*, whereas it is a central property, if not a defining characteristic, of *truth* to exclude falsity. B.H. Slater, thus, has claimed that Priest "nominally remedied" the defect in da Costa's systems, but

> The remedy is only a face-lift. [...] In this terminology, 'logically false' does not rule out 'true'. Hence, 'A' and '¬A' are *still* only subcontraries, for the same reason as before. [...] As a result, while 'truth' and 'falsity' are only subcontrary in Priest's language, that does not show, in any way, that *truth* and *falsity* are only subcontraries. For no change of language can alter the facts, only the mode of expression of them, as we saw before. And one central fact is that *contradictories* cannot be true together — by definition.[27]

This way, we are back to clashes of intuitions and the Italics Argument: is Priest's truth *truth*? Are Slater and Priest dealing with two different concepts? Of course, neither would argue that they are (although both would claim to deal with the only right one).

Now, a popular reply to such puzzles adopts the Wittgensteinian jargon and suggests that, to grasp the meaning of some conceptual word (be it a logical or a descriptive one), we should look at its actual *use* in language — at the linguistic practices within which it receives its intended meaning. If such problems cannot be solved by a *fiat*, via stipulative definitions or language regimentations, we need a phenomenology of ordinary language. But this is a difficult path, and it does not seem to lead to a solution of our

[26] Of course, this is where the transition from "pure" to "applied" semantics should take place. This terminology is due to Plantinga [1974], pp. 126ff. Dummett [1973] talks of, respectively, "merely algebraic" and "properly so called semantic" notions. According to Copeland [1986], p. 479, "The assignment of meanings to expressions of a language [is] available only when the second stage of development has been successfully completed". Looking at a pure formalism is not sufficient to decide whether a notation represents a logic, or the behaviour, e.g., of electrical circuits, with "true" meaning *on*, and "false" meaning *off* (see Haack [1978], p. 189).

[27] Slater [1995], pp. 452–453.

issue. I think it is uncontroversial that we often use "true" in order to immediately *rule out* "false", thereby applying the concepts *truth* and *falsity* as exclusion-expressing devices. But is it always so? If an analysis of our linguistic business with the word "true" can provide an account of our intuitive semantics, it seems that dialetheists, not supporters of the LNC, have a point here. For Extended Liar paradoxes like "This sentence is not true" are spelt in ordinary English. Their paradoxical characteristics, as dialetheists stress, are due exactly to intuitive features of ordinary language: unavoidable self-reference; failure of metalinguistic hierarchies producing only languages that are expressively weaker than English; and the obvious presence of a truth predicate, "is true", which is characterized (at least extensionally) by (T), the Tarskian equivalence.[28] And dialetheism claims to provide the most natural (if not the unique) real "solution" to semantic paradoxes: "accept them and learn to come to live with them".[29]

Furthermore, Priest [2000] has also argued that, besides the next man's intuitions on truth, also the most popular *theories* of truth do not constitute a real challenge for dialetheism. We all know that deflationist, semantic, correspondence, coherence, pragmatist theories provide competing accounts of the concept *truth*. But Priest has claimed that none of them speaks against the view that the LNC fails and, on the contrary, some speak in favour of it (e.g., the deflationist and semantic theories do, because of their basing themselves on some version of (T)).[30]

[28] Regarding all these points, see Priest [1987], ch. 1.

[29] Priest [1979], p. 219.

[30] It would have taken too much space to debate here a third, well developed and much discussed paraconsistent approach to negation: the one provided by the famous "star semantics" for relevant logic, due to Routley and Meyer (see Routley and Routley [1972], Routley and Meyer [1976], Routley [1979], Meyer and Martin [1986]). For our purposes, it may suffice to observe that many authors have criticized it by calling into question, again, the underlying intuitions on *truth*. Both van Benthem [1979] and Copeland [1979], [1986], point out the illicit shifts from "true" to "told true" or "believed true" in the (rather scattered) remarks on the intuitive reading of the semantic notions. This is particularly evident, for instance, when Meyer and Martin [1986] talk about the truth values assigned in the "Australian plan" as "ontic" values at page 319, after having described them as "holding that A" and "not denying that A" at page 309. Now, if the intuitive reading of the valuation function of the semantics points into an interpretation in terms of membership of a set of beliefs, or a theory, or a fictional story, it seems that the admission of true contradictions is nothing but what Bertrand Russell would have called "a case of the fallacy of verbalism — the fallacy that consists in mistaking the properties of words [or beliefs] for the properties of things" (Russell [1923], p. 62). As this critical view goes, just like vagueness, inconsistency can only belong to representations of the world, such as language and thought, not to the world itself. So there is no *de re* inconsistency, just as there is no *de re* vagueness — at least, according to the vast majority of the theorists of vagueness: it is fair to say that a few of them (e.g., Tye [1990], Parsons and Woodruff [1995]), support the view that the world itself can be vague, that is

5. *Material exclusion*

Since the question whether the concepts *truth* and *falsity* are exclusive or incompatible is so contentious, we may look for the characterization of a negation that does without them. I think the simplest move is to start from *the very notion of exclusion, or incompatibility*. At first sight, there is nothing new in such an approach. But we must be careful about how to proceed: we cannot express exclusion or incompatibility via the traditional concept of *contrariness*, for such a concept typically depends upon those of truth and falsity. There is no point in defining α and β as contraries if and only if "$\alpha \wedge \beta$" is logically false, because, as Huw Price [1990] observes, "it clearly depends on our knowing that truth and falsity are incompatible", so that "if we do not have a sense of *that*, the truth tables for negation give us no sense of the connection between negation and incompatibility".[31]

But some intuitive notion of exclusion, in my opinion, is itself inescapable and so primitive that it *can* be taken as an intuitive basis for the definition of a negation:

> The apprehension of incompatibility [is] an ability more primitive than the use of negation. The negation operator is being explained as initially a means of *registering* (publicly or privately) a perceived incompatibility. [...] For present purposes, what matters is that *incompatibility be a very basic feature of a speaker's (or proto-speaker's) experience of the world*, so that negation can plausibly be explained in terms of incompatibility.[32]

Precisely, I shall talk of *material exclusion* or, equivalently, of *material incompatibility*, and label it with a symbol logicians know very well: "$\perp$". It can be expressed in terms of concepts, properties, states of affairs, or worlds, depending on one's metaphysical preferences. For instance: it is the relation that holds between a couple of properties P_1 and P_2, if and only if the very

to say, there can be ontologically vague objects, sets, and/or states of affairs. Of course, we may reply by denying that a realist or "ontic" theory of truth actually explicates *truth* (Italics Argument). One may provide an independent anti-realist account of "true" in terms of "told true" or "believed true", and claim that *truth* is nothing more than this. And some dialetheists have — for instance, see J.C. Beall's *constructive methodological deflationism*, in Beall [2004]: "After all, if truth is a mere (human) construction, introduced to play a given expressive role, then it is not surprising — indeed, it is likely — that the construction should turn out to be inconsistent" (p. 208).

[31] Price [1990], p. 226.

[32] Price [1990], pp. 226–228, my italics.

having P_1 by an object a precludes the possibility that a simultaneously has P_2, and *vice versa* ($\perp$ is obviously supposed to be symmetric, i.e., if $P_1 \perp P_2$ then $P_2 \perp P_1$). We may also say that material incompatibility holds between two *concepts* C_1 and C_2, if and only if the very instantiating C_1 by a excludes the possibility that a also instantiates C_2, and *vice versa*. Or we may say that it holds between two states of affairs s_1 and s_2, if and only if the holding of s_1 (in world w, at time t) precludes the possibility that s_2 also holds (in world w, at time t), and *vice versa*.

So, $\perp$ is a deeply metaphysical notion: it is rooted in our experience of the world, rather than in semantics or pragmatics. It is also a strongly modal one: material exclusion does not hold between two merely different properties, like *being red* and *being circular*, which can be instantiated by the same object, even though sometimes they are not. It holds between two properties, such that an object instantiating one of them has dismissed any chance of simultaneously instantiating the other.

I propose, then, the following description of a negation via material exclusion. Such an account adapts (by avoiding reference to truth and truth conditions) the idea, developed by J.M. Dunn [1996], that "one can define negation in terms of one primitive relation of incompatibility [...] in a metaphysical framework".[33] Dunn refers to the Birkoff-von Neumann-Goldblatt definition of *ortho negation*, a notion originally developed within quantum logic. What makes this characterization interesting is that it uses precisely a relation of incompatibility (also called "orthogonality", or simply "perp").[34] In most presentations it holds between *states*, or *worlds*, but we may rephrase it in terms of properties (very small fine-tunings would equally allow us to express it in terms of concepts, or states of affairs). Take an ordered couple $\langle \mathsf{S}, \perp \rangle$, where S is a set of properties, and $\perp$ is our binary relation of material exclusion, defined on S. Then we have:

(DfNOT) NOT-$P_1(x) =_{\text{df}} \exists P_2(P_2(x) \wedge P_1 \perp P_2)$.

To say that something is NOT-P_1 is to say that it has some property P_2, which is materially exclusive with respect to P_1. Such a partial indeterminacy in the information conveyed by an expression containing "NOT" reflects a very simple fact of ordinary language. When we say "The car is red", this is not the weakest, or less informative, sentence incompatible with the sentence "The car is blue" (provided that, for the sake of the argument, we believe the concepts *red* and *blue* to be exclusive). The weakest sentence incompatible with "The car is blue" is "The car is NOT-blue", which, given

[33] Dunn [1996], p. 9.

[34] See Birkoff and von Neumann [1936], Goldblatt [1974].

(DfNOT), merely says that the car has some other (incompatible) property than that of being blue, not specifying which one. "The car is red" specifically says which other, incompatible colour the car has (a property need not exclude *one* other, but it may exclude a whole assortment of alternative properties; Patrick Grim [2004] talks about the *exclusionary class* of a given property). If the set of properties incompatible with *being blue* is not an infinite one, of course, "The car is NOT-blue" is nothing but a long disjunction: "The car is red, or orange, or yellow, or...". It is clear that such a distinction is the heir of the one traditionally made between *contraries* and *contradictories*, which as we know was defined by reference to truth and falsity.

Now, it seems to me that "NOT" may have the three pleasing features I promised in the beginning.

(1) It is not defined via the concept *truth*. It is defined via the concept *exclusion*, whose primitiveness is now clear: it is entailed, for instance, by our experience of the world as agents, facing choices between performing some action or other — something we think non-linguistic animals as well do every day. And to face a choice is to perceive an incompatibility. But it may also be entailed by the simple and basic capacity to recognize the boundary (even a blurred one) between something and something else, between an object and another one. It is fair to say, as Grim does, that exclusion is such "a very basic term", that "without some fundamental grasp of precisely that notion to begin with it seems quite possible that it cannot later be specified [...]. If exclusion is not understood to begin with, what possible exposition could we rely on to nail it?".[35]

(2) "NOT" has a strong pre-theoretical appeal and motivation as an expressive tool, because what we often need as speakers — even as dialetheists — in order to convey determinate information is precisely an exclusion-expressing device. One could hardly improve Huw Price's description of what a conversation between me and you would be if we had no means to exclude (via negation, rejection, falsity, or whatever) the possibility of Fred's being simultaneously in the kitchen and in the garden:

> *Me*: 'Fred is in the kitchen.' (Sets off for kitchen.)
> *You*: 'Wait! Fred is in the garden.'
> *Me*: 'I see. But he is in the kitchen, so I'll go there.' (Sets off.)
> *You*: 'You lack understanding. The kitchen is Fred-free'.
> *Me*: 'Is it really? But Fred's in it, and that's the important thing.' (Leaves for kitchen.)[36]

[35] Grim [2004], p. 70.

[36] Price [1990], p. 224.

If you could tell me a simple: "Look, Fred is NOT-in the kitchen" (that is to say: "Fred is somewhere else — in the garden — *and* his being there excludes his being in the kitchen"), life would definitely be easier.

(3) Finally, of course paraconsistent logicians and dialetheists do grasp the notion of exclusion. What supporters of inconsistency ask us is to stop using "not" or "true" as exclusion-expressing devices, because "not-α" is insufficient by itself to rule out α, and "α is true" is insufficient by itself to rule out that α is also false. Priest and Routley explicitly admit that "we [as dialetheists] cannot use content-exclusion as a way of defining the sense, or content, of negation. But then there are plenty of other ways of doing this, for example, through a semantic account".[37] Now, of course they *can* give a semantic account of negation — such as the one of LP, described in the previous paragraph. But it is, by the admission we have just heard them making, not strong enough to support content-exclusion. Therefore, now they need *some* other linguistic tool, in order to express their basic concepts, to exclude rival positions, and to convey through their theories determinate information, if they want to avoid ending up like you and me in the Fred-dialogue.[38]

As an exclusion-expressing device, Priest chooses the notion of *rejection*, and argues that accepting $\neg\alpha$ is different from rejecting α, and *vice versa*. A dialetheist can accept $\neg\alpha$ while failing to reject α, exactly if she thinks that α is paradoxical. So, although the dialetheist cannot exclude α by simply saying "$\neg\alpha$", she can *reject* α. But dialetheists' account of *acceptance* and *rejection* shows that they do believe in the impossibility of couples of states of affairs to simultaneously obtain, and they assert that some properties materially exclude some others. For instance:

> Someone who rejects A cannot simultaneously accept it any more that a person can simultaneously catch a bus and miss it, or win a game of chess and lose it. If a person is asked whether or not A, he can of course say 'Yes and no'. However this does not show that he both accepts and rejects A. It means that he accepts both A and its negation. Moreover a person can alternate between accepting and

[37] Priest and Routley [1989b], p. 513.

[38] That dialetheism faces the risk of ending up inexpressible, has been pointed out by many authors: see e.g. Parsons [1990] — who nevertheless essentially embraces Priest's way out through the pragmatic notions of *rejection* and *denial*; Batens [1990], who advocates the necessity of admitting a classical, exclusive negation against "global paraconsistency"; and Shapiro [2004], who directly challenges the dialetheist's capacity to provide a coherent notion of *exclusion*.

> rejecting a claim. He can also be undecided as to which to do. But do both he can not.[39]

So, when in *In contradiction* Priest says: "The rational acceptability and rejectability of something, though not exhaustive, are certainly *in*compatible"; or: "it is *im*possible jointly to accept and reject the same thing"; or "joint rational acceptance and rejection are *not* possible";[40] here the "in-" and "not" must be taken as tools used in order to express the reciprocal exclusion of the concepts *acceptance* and *rejection*. Could Priest's position be meaningful otherwise?

I have called $\perp$ *material* exclusion, to stress the fact that it is not a merely logical, in the sense of *formal*, notion: it is based on the material content of the involved concepts. Neil Tennant call such concepts *antonyms*, and observes that

> Here the antonyms A and B are so simple and primitive that there cannot be any question of their 'dialetheically' holding simultaneously. Such antonyms A and B are antonymic not on the basis of their logical form, but on the basis of their primitive non-logical contents. The tension between them — their mutual exclusivity — is a matter of deep metaphysical necessity.[41]

Tennant's examples are: phenomenological colour incompatibilities, such as *being (solidly) Red* and *being (solidly) Green*; concepts that express our categorization of physical objects in space and time, such as x *being here right now* and x *being way over there right now*, for a suitably small x. Other cases provided by Grim are x *being less than two inches long* and x *being more than three feet long*.[42] We may also take Priest's above x*'s catching the bus* and x*'s missing the bus*. But this immediately leads to the following important point. We must keep in mind that the characterization of $\perp$ does not entail any particular commitment on which are the specific properties, or concepts, or states of affairs, between which it holds. This may sound somewhat disappointing, but such a merely formalistic description is exactly what we should expect when dealing with purely metaphysical notions that

[39] Priest [1989], p. 618.

[40] Priest [1987], p. 128 and p. 142, my italics.

[41] Tennant [2004], p. 362.

[42] Grim [2004], p. 63.

leave our epistemic troubles just where they are. We claimed that material exclusion is based on the content of concepts, or properties, but how do we *know* what the content of a concept is, or which are the actual fields of applications of a property? Given two properties P_1 and P_2, the question whether they are exclusive can involve broadly empirical matters, difficult analyses of our conceptual toolkit and/or of our use of ordinary language expressions. And, of course, this may produce battles of intuitions: are *young* and *old* actually exclusive? *Blue* and *green*? *True* and *false*? *Circular* and *square*? This is exactly the kind of *a posteriori* disquisition one should avoid when dealing with the claim that there are true contradictions, or that a sentence can be both true and false.

What I have suggested is that the notion of material exclusion is itself inescapable. After characterizing a negation which is very similar to the one proposed here, Grim observes:

> The outline above uses various forms of negation, including the English 'not', prominently and repeatedly in trying to get the idea across. If these forms of negation can be understood a particular way, it seems inevitable that ['NOT'] can be understood a particular way. Given a dialetheic interpretation of all the various forms of negation in the outline, then, one might well end up with a dialetheic interpretation of ['NOT']. The result could be that every claim made above is allowed but without the concept of exclusion that is their main intent [...]. All I can say is that those forms of dialetheism seem less interesting to me: I don't see how the prospect of impasse is then to be avoided, and such forms don't seem to me to promise any deeper understanding of notions as central to our conceptual toolkit as is the notion of contradiction.[43]

But we can advance more compulsory considerations. The dialetheist does not believe, of course, that anything is compatible with anything, or that all states of affairs obtain, or that anything can be anything (else?). This may be a characterization of the aforementioned trivialism. In Priest's [1999] words:

> One cannot choose between this and that if one believes that this and that are the same thing, which the trivialist does. Of course, the trivialist believes that this and that are distinct too. But, as before,

[43] Grim [2004], pp. 69–71.

> for the trivialist, two things being distinct does not rule out their being identical.[44]

Regardless of the transcendental-phenomenological argument Priest uses to criticize the trivialist's position (properly: to show that "our opponent does not exist"),[45] it is fairly clear that the paradigmatic dialetheist is not a trivialist.[46] So, we do *not* need to undertake the challenge Priest proposes to the defender of the LNC in the quotation at the beginning of this paper: given that change of logic is change of subject, and so classical and nonclassical negations have different meanings, try to demonstrate that the vernacular negation is classical (and not paraconsistent). We can take the couples of properties, or concepts, or states of affairs Priest himself assumes as materially exclusive (*acceptance* and *rejection*, or *x's catching the bus* and *x's missing the bus*) as instances of a primitive, intuitive notion of exclusion, $\perp$, holding between couples of properties, or concepts, etc. Then we define via $\perp$ a sentential operator, NOT, which works as an exclusion-expressing device for our language. As we have seen, after dismissing the ordinary "not" and "true" of classical logic and semantics, because $\neg\alpha$ does not rule out α, and "α is true" does not rule out "α is false", the dialetheist herself realizes she needs some new exclusion-expressing-device — something to say that something excludes something else — if her own position is to be expressible. So, there is no point with the dialetheist refusing our procedure: NOT does exactly the expressive job rejection is supposed to do in the dialetheic framework.

Now, the final step: express the LNC via "NOT". Take Aristotle's traditional formulation of the LNC, in Book Γ of *Metaphysics*, and just put in it our NOT:

> Evidently then such a principle is the most certain of all; which principle this is, let us proceed to say. It is, that the same attribute canNOT at the same time belong and NOT belong to the same subject and in the same respect.[47]

[44] Priest [1999], p. 194fn.

[45] Priest [1999], p. 195.

[46] Priest's argument against trivialism has been criticized by Kroon [2004], who suggests that a *realist* dialetheism entails a slippery slope to the trivialistic position.

[47] Arst. *Met.* 1005b18–21.

"P_1 does NOT belong to x" should be a short form for "to x belongs some property P_2, which is materially incompatible with P_1". And this does not seem to be questionable by the dialetheist anymore, provided she has understood "NOT" — and to understand "NOT" is to understand exclusion (what the dialetheist does, as we have seen). If the dialetheist refuses to subscribe the characterization of NOT via the intuitive notion of exclusion, she seems to actually end up as unable to express the exclusion of any position (is she trying to exclude exclusion?). Our sense of exclusive possibilities (beginning from our capacity of recognizing that an object is separated by a boundary from what that object cannot be) seems to be *a priori*: it is, to use the Kantian jargon, a condition of the possibility of our having any experience of *a world* at all. And a dialetheism without the LNC stated in terms of "NOT" looks very much like a trivialism. Such a LNC, to use Aristotle's words, is "a principle which every one must have who knows anything about being".[48]

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