

A NOTE ON ARISTOTELIAN THEORIES

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Abstract

This paper examines the formal nature of Aristotle’s principle: if a theory T does not entail the negation of a proposition, then according to T that proposition is possibly true. Aristotle’s principle is shown to have some elegant and surprising features. It is also argued that every ideal metaphysical theory is closed under Aristotle’s principle.

1. *Introduction*

In [4], Tim Maudlin argues that David Lewis’ modal realism commits Lewis to holding that there are qualitatively identical but numerically distinct possible worlds. Maudlin then goes on to argue that this consequence is untoward. In his argument that Lewis must accept indiscernible worlds, Maudlin appeals to a principle that he extracts from the following passage from Aristotle:

I use the terms ‘to be possible’ and ‘the possible’ of that which ... being assumed results in nothing impossible. (*Prior Analytics* 32^a 17–20)¹

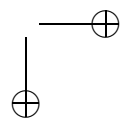
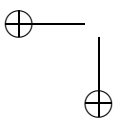
In Maudlin’s formulation, *Aristotle’s principle*, as we shall now call it, says that if a proposition does not entail any impossible propositions, then it is possible ([4] p. 671).

In his reply to Maudlin [3], Lewis interprets Aristotle’s principle in terms of theories. On Lewis’ reading, Aristotle’s principle says that

whatever cannot be refuted in [a theory] T is possibly true [according to T] ([3] p. 683).

In this paper, I examine Lewis’ version of Aristotle’s principle. I look at formal theories that are closed under Aristotle’s principle. That is, we discuss

¹The ellipsis here is in place of the phrase “not being necessary but”. Like most modern philosophers and unlike Aristotle, Maudlin does not take only contingencies to be possible.



theories T of the language of modal logic that satisfy the following rule:

$$\frac{\neg A \notin T}{\Diamond A \in T} \text{ (AP).}$$

Theories that satisfy AP are said to be *Aristotelian* and are called "A theories".

It is not the aim of the present paper to adjudicate the debate between Maudlin and Lewis, nor do I make it my business to engage in Aristotle scholarship. Rather, I think that AP is interesting in ways that Maudlin and Lewis do not acknowledge. In this paper, I undertake a purely formal examination of AP with the aim of showing that every *ideal* metaphysical theory must be Aristotelian. A metaphysical theory, as I use the term here, is not about the contingent features of the world, but is about the nature of all possible worlds. Accordingly, 'necessity' and 'possibility' as they are used in metaphysical theories are supposed to be universal and existential quantifiers over the set of all metaphysically possible worlds. I show that every complete metaphysical theory (in a sense specified below) of a set of possible worlds is Aristotelian. Moreover, in the "natural" model of any Aristotelian theory — based on the set of its maximal consistent extensions — its modal operators act as unrestricted quantifiers over worlds. In fact, we will see that given any possible worlds model of an Aristotelian theory, we can convert it into a model in which the modal operators are unrestricted quantifiers over the set of worlds, and the resulting model validates the same set of formulae as the original model.

2. Language, Schemes, and Logics

We begin by covering some very familiar ground. Since the theories that we are talking about are modal theories, we need to know of which modal logics that are theories. The logics that will concern us are the base normal logic K and its extensions KT , $K4$, $K5$, and $S5$ ($= KT45 = KT5$).

Before we can set out the logics formally, we need a formal language. Our language has propositional variables, parentheses, the binary connective \supset (the material conditional), and the unary connective \Box (necessity), and the zero-place connective \perp (the falsum). We use the standard formation rules. We also utilize the usual defined connectives: $\neg A =_{df} A \supset \perp$; $A \wedge B =_{df} \neg(A \supset \neg B)$; $A \vee B =_{df} \neg A \supset B$; $\Diamond A =_{df} \neg\Box\neg A$.

We take a logic to be identical to the set of its theorems. Thus, K is the smallest set of formulae such that it contains all substitution instances of PC tautologies, all instances of the scheme K ($\Box(A \supset B) \supset (\Box A \supset \Box B)$), and

is closed under the rules

$$\frac{A \supset B}{A} \text{ (MP)}$$

and

$$\frac{A}{\Box A} \text{ (N)}.$$

We will use the following fact about K:²

Lemma 1: $\vdash_K (\Box A \wedge \Diamond B) \supset \Diamond(A \wedge B)$.

Proof.

1. $\Box(A \supset \neg B) \supset (\Box A \supset \Box \neg B)$ K
2. $\neg(\Box A \supset \Box \neg B) \supset \neg\Box(A \supset \neg B)$ 1, PC
3. $(\Box A \wedge \neg\Box \neg B) \supset \neg\Box \neg(A \wedge \neg\neg B)$ 2, def \wedge
4. $(\Box A \wedge \Diamond B) \supset \Diamond(A \wedge B)$ 3, def \Diamond , PC

□

The logics that we use are created by adding all instances of some or all of the following schemes to K and closing under MP and N. We include also variations on each scheme that are easily proved from the scheme by principles of PC, which variations we use in our argument.

Name	Scheme	Variation
T	$\Box A \supset A$	$A \supset \Diamond A$
4	$\Box A \supset \Box \Box A$	$\Diamond \Diamond A \supset \Diamond A$
5	$\Diamond A \supset \Box \Diamond A$	$\Diamond \Box A \supset \Box A$

The logics KT, K4, and K5 result from the addition of T, 4, and 5 to K respectively. The logic S5 plays a special role in our discussion. It is the logic that results from adding all of these schemes to K, although the addition of 4 is redundant.

3. Theories

Theories are defined in terms of consequence relations. For a logic S , we define the consequence relation \vdash_S such that for a set of formulae Γ , $\Gamma \vdash_S B$

²We also tacitly appeal throughout this paper to the fact that in the logics concerned we can replace probably equivalent formulae for one another in any context.

if and only if there are A_1, \dots, A_n in Γ (for some $n \in \omega$) such that it is a theorem of S that $(A_1 \wedge \dots \wedge A_n) \supset B$. Γ is a *theory of S* if and only if, for all formulae B such that $\Gamma \vdash_S B$, $B \in \Gamma$, that is, Γ is closed under the consequence relation for S .

For the logics that we are considering here, every theory of S contains all the theorems of S . Now let S be one of the logics that results by adding zero or more of the schemes listed above to K . It is clear that it is equivalent to say that Γ is an S theory and that it is a K theory that contains all the theorems of S .

We use the standard definitions of S -consistency and maximality. A set of formulae Γ is said to be *S -inconsistent* if $\Gamma \vdash_S \perp$. Γ is *S -consistent* otherwise. Γ is said to be *maximal* if and only if for all formulae A either $A \in \Gamma$ or $\neg A \in \Gamma$. It is easily shown that every maximal S -consistent set of formulae is a theory (although the converse is not true).

We will make heavy use in our argument of Lindenbaum’s extension lemma, viz.:

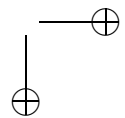
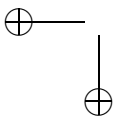
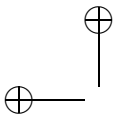
Lemma 2: (Lindenbaum) Let Γ be an S -consistent set of formulae. Then there is a maximal S -consistent set of formulae containing Γ .

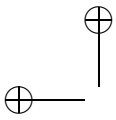
It is an easy corollary of Lindenbaum’s lemma that every S -consistent S -theory is the intersection of its maximal S -consistent extensions. For suppose that T is an S -theory and that $A \notin T$. Then $T, \neg A \not\vdash_S \perp$. So the closure of $T \cup \{\neg A\}$ under the rules of S (i.e. the smallest S -theory containing $T \cup \{\neg A\}$) is consistent. We then know, by Lindenbaum’s lemma, that there is a maximal S -consistent set, Γ , that contains $T \cup \{\neg A\}$. Clearly, $T \subseteq \Gamma$. Generalizing, for each $A \notin T$, there is a maximal S -consistent set that does not contain A . So, T is the intersection of its maximal S -consistent extensions.

Before we leave the topic of theories, we need two more definitions. Given a theory T , the *de-necessitation* of T (written ‘ $\Box^{-1}T$ ’) is the set of formulae A such that $\Box A \in T$ and the *de-possibilization* of T (written ‘ $\Diamond^{-1}T$ ’) is the set of formulae B such that $\Diamond B \in T$.

4. Aristotle’s Principle

Aristotle’s principle (AP) is not an inference rule in any standard sense. We cannot merely add AP to a system or merely state that theories are closed under it. For there are not always minimal Aristotelian closures of theories.





Here is a brief argument for this fact. Let T be a consistent KT-theory that is not Aristotelian (e.g. the set of theorems of KT)³. By Lindenbaum’s lemma, T is the intersection of the maximal KT-consistent sets that extend T . Let us suppose that there is more than one maximal KT-consistent set that extends T . Then there is no unique smallest Aristotelian theory that extends T . For every maximal KT-consistent set is Aristotelian and there is no theory that is a subset of every maximal KT-consistent set that extends T and is also bigger than T , since T is the intersection of all these maximal consistent sets.⁴

If there is no unique smallest Aristotelian theory that extends T , it makes no sense to talk about *the* closure of T under Aristotle’s principle. Hence, Aristotle’s principle is not an inference rule in any normal sense. But this does not mean that Aristotle’s principle is not interesting. As we shall see in what follows, theories that are closed under Aristotle’s principle are very interesting indeed.

5. Categoricity

Aristotelian theories of some modal logics satisfy what we call *categoricity properties*. An S -theory T is said to be \Box -categorical if for any maximal consistent extensions Γ and Δ of T , for all formulae A , $\Box A \in \Gamma$ if and only if $\Box A \in \Delta$. Similarly, T is \Diamond -categorical if for any maximal consistent extensions Γ and Δ of T , for all formulae A , $\Diamond A \in \Gamma$ if and only if $\Diamond A \in \Delta$.⁵

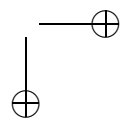
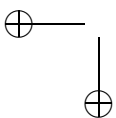
Lemma 3: Every Aristotelian K5-theory is \Diamond -categorical.

Proof. Suppose that T is an Aristotelian theory. Let Γ and Δ be maximal consistent extensions of T . Assume for the sake of a reductio that $\Diamond A \in \Gamma$

³ It is obvious that the set of theorems of KT is not Aristotelian. For $\neg p$ is not a theorem of KT, for any propositional variable p , but $\Diamond p$ is not a theorem either. Since the set of theorems of KT is closed under uniform substitution, if $\Diamond p$ were a theorem, then all instances of $\Diamond A$ would also be theorems.

⁴ As an anonymous referee pointed out, the problem here is a symptom of a more general phenomenon. $A \notin T$ occurs negatively in the definition of an Aristotelian theory. Since these theories are not characterised by a positive inductive definition there is no guarantee of a minimal (or any) fixed point.

⁵ Note that we do not have to specify that the extensions of T are S -consistent, since T contains all theorems of S so do its extensions. If they are S inconsistent, then they will be K inconsistent as well.



and $\diamond A \notin \Delta$ for some formula A . Then, $\diamond A \notin T$, i.e. $\neg \Box \neg A \notin T$. But, since T is Aristotelian, this implies that $\diamond \Box \neg A \in T$. Thus, by 5, we have $\Box \neg A \in T$. Thus, $\diamond A \in \Gamma$ and $\Box \neg A \in \Gamma$ and so, by lemma 1, we have $\diamond(A \wedge \neg A) \in \Gamma$. But, $\neg \diamond(A \wedge \neg A)$ is a theorem of K and so Γ is inconsistent. Thus, by reductio, T is \diamond -categorical. \square

Lemma 4: Any Aristotelian K4-theory that is \diamond -categorical is also \Box -categorical.

Proof. Suppose that T is \diamond -categorical. Assume for the sake of a reductio that it is not \Box -categorical. Then there are maximal consistent extensions of T , Γ and Δ and a formula A such that $\Box A \in \Gamma$ and $\Box A \notin \Delta$. Thus, $\Box A \notin T$ and so $\neg \diamond \neg A \notin T$ and, because T is Aristotelian, $\diamond \diamond \neg A \in T$. By 4, we obtain $\diamond \neg A \in T$. Therefore $\Box A \in \Gamma$ and $\diamond \neg A \in \Gamma$, hence by lemma 1, $\diamond(A \wedge \neg A) \in \Gamma$. Therefore, T is \Box -categorical. \square

We will use these categoricity results later to prove the central theorems of this paper. But they are interesting in their own right. For they tell us that Aristotelian S5 theories are both \Box and \diamond -categorical. To use Lewis' apt phrase, these theories do not allow for modal mysteries — they answer all questions about what is necessary and what is possible.

6. Models

In this section we set out the semantics we will use in subsequent proofs and our definitions that we use later. We will discuss two sorts of models for our theories: relational and absolute models.

A *relational model* is a triple $\langle W, R, v \rangle$ such that W is a non-empty set (of worlds), $R \subseteq W^2$, and v is a function from propositional variables to subsets of W . Given a relational model $\langle W, R, v \rangle$, we define a satisfaction relation, \models_v between worlds and formulae such that for all worlds w , all propositional variables p , and all formulae A and B ,

- $w \models_v p$ if and only if $w \in v(p)$;
- $w \models_v A \supset B$ if and only if either $w \not\models_v A$ or $w \models_v B$;
- $w \not\models_v \perp$;
- $w \models_v \Box A$ if and only if, for all $w' \in W$, if wRw' , then $w' \models_v A$.

An *absolute model* is a pair $\langle W, v \rangle$ such that W is a non-empty set and v is a function from propositional variables into $\wp(W)$. Given an absolute model $\langle W, v \rangle$ we define a relation \models'_v between worlds and formulae such that for all worlds w , all propositional variables p , and all formulae A and B ,

- $w \models'_v p$ if and only if $w \in v(p)$;

- $w \models'_v A \supset B$ if and only if either $w \not\models'_v A$ or $w \models'_v B$;
- $w \not\models'_v \perp$;
- $w \models'_v \Box A$ if and only if, for all $w' \in W$, $w' \models'_v A$.

The only difference between the satisfaction relation in relational models and that in absolute models is in their truth clauses for necessity.

We say that a relational model $\langle W, R, v \rangle$ is a model for a theory T if and only if, for all formulae A in T and all $w \in W$, $w \models_v A$. Similarly, an absolute model $\langle W, v \rangle$ is a model for T if and only if, for all formulae A in T and all $w \in W$, $w \models'_v A$.

We also use the following definitions. Given a model $\langle W, R, v \rangle$ (or $\langle W, v \rangle$) and a world $w \in W$, $t(w)$ is the set of formulae A such that $w \models_v A$ (or $w \models'_v A$). And we define

$$\tilde{W} =_{df} \bigcap_{w \in W} t(w).$$

7. Metaphysical Theories

The theories that we are most interested in here are theories about metaphysics. A metaphysical theory is not about what is contingently true of the world, but what is necessarily true, in the sense of metaphysical necessity. It is generally accepted among philosophers that the logic of metaphysical necessity is S5, so we can say that

A theory T is a *metaphysical theory* if and only if (i) T is closed under the rule $A \in T \Rightarrow \Box A \in T$ (for all formulae A) and (ii) T includes all the theorems of S5.

A metaphysical theory is an "M theory" and an Aristotelian metaphysical theory is an "AM theory".

Theorem 5 below shows that every absolute model characterizes an AM theory.

Theorem 5: Let $\langle W, v \rangle$ be an absolute model. Then, \tilde{W} is an AM theory.

Proof. (a) $\langle W, v \rangle$ is a model for S5. Thus, every S5 theorem is true at every world. Hence, every S5 theorem is in \tilde{W} .

(b) Suppose that A is in \tilde{W} . Then $A \in t(w)$, for every world w in W . Thus, $\Box A \in t(w)$ for every $w \in W$. So, $\Box A \in \tilde{W}$.

(c) Suppose now that $\neg A \notin \tilde{W}$. Then there is some world $w \in W$ such that $\neg A \notin t(w)$. Thus, for all $w' \in W$, $\Box \neg A \notin t(w')$ and so, $\neg \Box \neg A \in$

$t(w')$. Thus, by the definition of \diamond , $\diamond A \in t(w')$, for all $w' \in W$. Hence $\diamond A \in \tilde{W}$.

By (a) and (b) \tilde{W} is an M theory, and by (c) \tilde{W} is an A theory. \square

The preceding theorem shows that the theory that captures all the truths in a set of worlds is an AM theory. If W is the (actual? real?) set of possible worlds and v is the intended interpretation of our language, then, \tilde{W} is the *complete* theory of metaphysical necessity and possibility. And \tilde{W} is an AM theory. Thus the true and complete theory of metaphysics is an AM theory.

8. Canonical Models

In this section we show that every consistent AM theory T determines an absolute model $\langle W, v \rangle$ such that $T = \tilde{W}$.

We begin with the canonical model for S5. It is a triple $\langle W_{S5}, R_{S5}, v_{S5} \rangle$ such that W_{S5} is the set of maximal S5-consistent sets (henceforth, 'MCS'), $R_{S5} \subseteq W_{S5} \times W_{S5}$ is such that $wR_{S5}w'$ if and only if $\Box^{-1}w \subseteq w'$, and $v_{S5}(p) = \{w \in W_{S5} : p \in w\}$ for all propositional variables p . Using standard methods (see [2] theorem 6.5), it can be shown that, for each $w \in W_{S5}$, $w = t(w)$. Thus, in this section we will dispense with talk of $t(w)$ and merely discuss the contents of w .

As is shown in the standard completeness proofs for S5, R_{S5} is an equivalence relation. Since R_{S5} is an equivalence relation, we can divide W_{S5} into *R-clusters*. For each world w in W_{S5} , $[w]_{R_{S5}}$ (henceforth, abbreviated as ' $[w]$ ') is an *R-cluster*. It is defined so that, for all worlds $w' \in W_{S5}$, $w' \in [w]$ if and only if $wR_{S5}w'$.

We need the following lemma:

Lemma 6: For all $w \in W_{S5}$, $\diamond A \in w$ if and only if there is a $w' \in W_{S5}$ such that $wR_{S5}w'$ and $A \in w'$.

Proof. Suppose first that $\diamond A \in w$. Then $\neg\Box\neg A \in w$. Since w is consistent, $\Box\neg A \notin w$. Then, $\Box^{-1}w \cup \{A\}$ is consistent and so, by Lindenbaum's lemma, there is a $w' \in W_{S5}$ such that $wR_{S5}w'$ and $A \in w'$.

Suppose now that there is a $w' \in W_{S5}$ such that $wR_{S5}w'$ and $A \in w'$. Then, $\Box^{-1}w \cup \{A\}$ is consistent and so $\Box\neg A \notin w$. Thus, since w is maximal, $\diamond A \in w$. \square

It can also be shown that R_{S5} is symmetrical, transitive, and reflexive (see, e.g., [2] theorems 6.7–6.11). This implies the following lemma:

Lemma 7: $[w] = [w']$ if and only if $w \in [w']$.

We now define a *canonical model for an S5 theory T*. The canonical model for T is a triple $\langle W_T, R_T, v_T \rangle$ such that W_T is the set of MCS that extend T , R_T is $R_{S5} \upharpoonright W_T$, and $v(p) = \{w \in W_T : p \in w\}$ for all propositional variables p . By Lindenbaum's lemma, we know that T is the intersection of W_T .

Theorem 8: If T is an AM theory, then $W_T = [w]$, for all $w \in W_T$.

Proof. Let T be an AM theory and w and w' be MCS that extend T . By lemmas 3 and 4, T is \Box -categorical, hence $\Box^{-1}w = \Box^{-1}w'$. Moreover, since w' contains all instances of \top , $\Box^{-1}w' \subseteq w'$, so $\Box^{-1}w \subseteq w'$. Thus, wR_Tw' and so $w' \in [w]$. Therefore, by lemma 7, $[w'] = [w]$, Generalizing, $[w] = W_T$. \square

Theorem 9: If T is an M theory and W_T is an R-cluster in $\langle W_{S5}, R_{S5} \rangle$, then T is an A theory.

Proof. Suppose that T is metaphysical and that W_T is an R-cluster in $\langle W_{S5}, R_{S5} \rangle$. Also assume that $\neg A \notin T$. Since T is metaphysical, it contains all instances of \top , and so $\Box\neg A \notin T$. Thus, $T \cup \{\Diamond A\}$ is S5-consistent. By Lindenbaum's lemma, there is an MCS $w \in W_T$ that contains $T \cup \{\Diamond A\}$. Thus, by lemma 6, there is a $w' \in W_T$ such that $A \in w'$. But then, since W_T is an R-cluster, by lemma 6, for all $w'' \in W_T$, $\Diamond A \in w''$. Therefore, since T is the intersection of W_T , $\Diamond A \in T$. Hence, T is Aristotelian. \square

We can think of the set of MCS that extend T as a natural model for T . Thus, the model operators work as unrestricted quantifiers over the set of worlds in the natural model for T .

9. Absolute Models for AM Theories

Let T be an AM theory and $\langle W, R, v \rangle$ be a relational model for T . Then we will show that $\langle W, v \rangle$ is an absolute model for T .

Lemma 10: For all $w \in W$, $w \models_v \Box A$ if and only if, for all $w' \in W$, $w' \models_v A$.

Proof. \implies Suppose that $w \models_v \Box A$. Let w' be an arbitrary world in W . Since $\langle W, R, v \rangle$ is a model for T , for all $B \in T$, $w \models_v B$ and $w' \models_v B$. Thus, $t(w)$ and $t(w')$ are MCS that extend T . By lemma 4, $\Box^{-1}t(w) = \Box^{-1}t(w')$. Since \top is in $t(w')$, $\Box^{-1}t(w) \subseteq t(w')$. Thus, $w' \models_v A$.

\Leftarrow Suppose that, for all $w' \in W$, $w' \models_v A$. Then, for all w'' such that wRw'' , $w'' \models_v A$. Thus, by the truth condition for necessity, $w \models_v \Box A$. \square

Lemma 11: For all formulae A and all $w \in W$, $w \models_v A$ if and only if $w \models'_v A$.

Proof. By induction on the length of A . The cases in which A is a propositional variable, a material conditional, or \perp are trivial. The case in which $A = \Box B$ is only slightly more difficult:

$$\begin{aligned} w \models'_v \Box B & \text{ iff } \forall w'(w' \models'_v B) && \text{by def of } \models'_v \\ & \text{ iff } \forall w'(w' \models_v B) && \text{by inductive hypothesis} \\ & \text{ iff } w \models_v \Box B && \text{by lemma 10} \end{aligned}$$

\square

It follows directly from lemma 11 that theorem 12 holds:

Theorem 12: If T is an AM theory and $\langle W, R, v \rangle$ is a model for T , then $\langle W, v \rangle$ is also a model for T .

10. Completeness of AM Theories

Perhaps the strangest feature of AM theories is that every model for an AM theory characterizes that theory. This means that if an AM theory T is true at every world in a model $\langle W, v \rangle$, then $\tilde{W} = T$. This is proven in the following manner.

Theorem 13: Let T be an AM theory. If $\langle W, v \rangle$ is a model for T then $\tilde{W} = T$.

Proof. Suppose that T is an AM theory and that $\langle W, v \rangle$ is a model for T . Then, $T \subseteq t(w)$, for every $w \in W$. So, $T \subseteq \tilde{W}$. Thus, it suffices to show that $\tilde{W} \subseteq T$. Suppose that $A \in \tilde{W}$ and assume, for the sake of a reductio, that $A \notin T$. Thus, $\neg \neg A \notin T$. Since T is Aristotelian, $\Diamond \neg A \in T$, hence $\neg \Box A \in T$. Since $T \subseteq t(w)$, for every $w \in W$, $w \models'_v \neg \Box A$ for all $w \in W$. Thus there is a $w' \in W$ such that $w' \not\models'_v A$. But then $A \notin \tilde{W}$ contradicting the assumption of the reductio. Therefore, $\tilde{W} \subseteq T$. \square

Theorem 13 says that if T is sound over $\{\langle W, v \rangle\}$, then T is semantically complete over $\{\langle W, v \rangle\}$. This is a very surprising result indeed!

11. Conclusion

Starting from a set of worlds, we end up with an AM theory. As we have seen, every absolute Kripke model characterizes an Aristotelian metaphysical theory. Thus, the theory of the set of possible worlds, in which necessity and possibility act as unrestricted quantifiers over the set of worlds, is an AM theory. Therefore, our ideal metaphysical theory — the one true metaphysical theory — is an AM theory.

If we start with an AM theory, like any theory, it defines a set of models. In its natural model — the set of MCS that extend it — the modal operators act like unrestricted quantifiers over worlds. In fact, we can convert any model of an AM theory into an absolute model merely by removing the accessibility relation, and this removal does not alter the set of formulae that are true at any world.

Moreover, if we have an intended model and an AM theory, then (if this model is a model of our theory) we have a complete theory of our intended model.

Thus, AM theories have several virtues. The only problem with them is that a satisfactory Aristotelian metaphysical theory will clearly be rather difficult to construct!

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