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## TRUTHMAKING AND THE ALLEGED NEED FOR RELEVANCE

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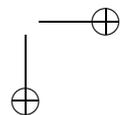
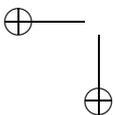
### *Abstract*

Since 1969, when Bas van Fraassen wrote ‘Facts and Tautological Entailments’, it has been assumed that if facts, or states of affairs, exist at all, they can only play the rôle of truthmakers for propositions if the truthmaker relation is defined in a relevantist revision of classical logic. Greg Restall revived this notion in 1996, and it has since been discussed positively by Stephen Read. I argue in this paper that this was always a mistake. The truthmaking relation between facts and propositions can indeed be made sense of — but no relevantist revision of classical logic is required. The correspondence theory of truth can thus be shown to have been essentially correct all along.

Do we need relevance logic to make sense of truthmaking? Greg Restall has argued that we do and Stephen Read has recently agreed.<sup>1</sup> I argue that this is incorrect; that Restall’s argument was fallacious, and that classical entailment is quite sufficient to make sense of truthmaking for contingent truths. But Read is right in claiming that necessities are not handled well by this approach. I therefore describe a more constructivist solution which agrees with the “standard view” on contingencies, but which gives the intuitively correct solution for necessities.

Thus we are able to defend the correspondence theory of truth, while fending off claims that non-classical logics are required. Later in the essay I will defend the correspondence theory against some venerable objections that have taken root in contemporary philosophy.

<sup>1</sup> Greg Restall ‘Truthmakers, Entailment and Necessity’ *The Australasian Journal of Philosophy*, 74, 1996, pp. 331–40; and Stephen Read ‘Truthmakers and the Disjunction Thesis’ *Mind*, 109, Jan. 2000, pp. 67–79.



1. *Truthmaking Postulates*

We give, following Read, a brief statement of the truthmaking postulates and their simple consequences. However, for reasons that will become clear in §.3, (p. 352). I will use the symbol ' $\nearrow$ ', rather than ' $\models$ ', to denote the truthmaking relation: thus,  $s$  is a truthmaker for  $A$  will be written as  $s \nearrow A$ , rather than, as in the Restall-Read notation,  $s \models A$ . (Likewise, we leave discussion of the characterization of the truthmakers,  $s$ , till §.7.)

*Postulate 1.1: (Truthmaker (TA)) For all propositions  $A$ , if  $A$  is true then there is some  $s$  such that  $s \nearrow A$ .*

*Postulate 1.2: (Factive Condition (FC)) For all  $A$ , if there exists a truthmaker  $s$  for  $A$ , then  $A$  is true.*

*Postulate 1.3: (Expressibility (Ex)) For all  $s$ , there is some  $A$ , such that  $s \nearrow A$ .*

*Postulate 1.4: (Truthmaker Fusion (TF)) If  $s \nearrow A$  and  $t \nearrow B$  then  $s + t \nearrow A \& B$ , where  $s + t$  is the mereological fusion of the truthmakers  $s$  and  $t$ .*

*Postulate 1.5: (Entailment Thesis (ET)) For all truthmakers  $s$  and propositions  $A$  and  $B$ , if  $s \nearrow A$  and  $A \models B$  then  $s \nearrow B$ . (Where ' $\models$ ' expresses the classical entailment relation.)*

These postulates have a number of important, highly desirable, consequences. The most significant is the following — simply a combination of TA and EX — which Read calls *the correspondence intuition*:

*Proposition 1.1: (CI) For all  $A$ ,  $A$  is true iff there exists some  $s$  such that  $s \nearrow A$ .*

This expresses the core of the idea of the correspondence theory of truth: *every truth is made true by some part of reality — some state of affairs, or fact — and every part of reality makes some proposition true.* That CI is not all that is required of a correspondence theory of truth is a point to which we will return.

Another significant consequence of the postulates is the following — which I will simply call the *Universal Truthmaking of Necessity Thesis*:

*Proposition 1.2: (UTN) If  $A$  is a necessary truth then, for all  $s$ ,  $s \nearrow A$ .*

This follows from ET: since every proposition entails every necessary truth, every *true* proposition entails every necessary truth. But then every truthmaker will, by ET, be a truthmaker for every necessary truth. To some this will have the air of a false consequence: surely necessary truths — the objection might run — cannot be ‘made true’ by a contingent state of affairs, let alone all of them? Surely they are not *made* true at all? I will argue that this is half-right, half-wrong.

I have called this the Universal Truthmaking of Necessity Thesis, whereas Read called it the *Global Supervenience of Necessity Thesis*. But can that be right? Is it really a supervenience thesis at all? Recall what it means to say that the mental supervenes on the physical (our paradigm case of supervenience): it means that there can be no change in the mental without there being a corresponding change in the physical — note: *not* vice versa. But how could this apply here? We can’t say there could be no change in the set of necessary truths without some change in the truthmakers for contingent truths; for, quite apart from the fact that we can make no sense of talk of changes to the necessary truths, that is not what UTN says. A change at the level of the necessary truths would — at least as far as UTN goes — require *no* change at the level of truthmakers. If the set of necessary truths *were* different then UTN merely says that the same truthmakers would make that *new* set true.

Considered the other way round, UTN says that the truthmakers for necessary truths are redundant: they have no effect at all. Were the world different in the states of affairs that obtain, there would be no difference at all in the necessary truths. The oddity of UTN is that necessities have a lot of truthmakers, not one of which does any making. It is counterintuitive, perhaps, perhaps even plainly wrong, but it is not supervenience.

More precisely, we might say that supervenience is a form of backtracking counterfactual dependence, in David Lewis’ sense. If the mental *were* different the physical *would have to be* different as well. But UTN is not a form of backtracking counterfactual dependence, it is a form of (ordinary) counterfactual dependence with gross overdetermination. UTN is analogous to causation where no single cause accomplishes anything because of the presence of the many other causes.

But Read’s belief that UTN is a supervenience thesis has repercussions for his claims that Relevant Entailment is required as a replacement for ET.<sup>2</sup> We consider this in the next section.

<sup>2</sup> Strictly, it is difficult to tell what Read’s real commitments are here: it is possible to read him as merely defusing an objection to the use of Relevant Entailment, without actually committing himself to its employment.

2. *Relevant Entailment*

Before going on to this, however, we mention another consequence of the above postulates. This essentially says that states of affairs make true what their parts make true.

*Proposition 2.1:* (TP) *Suppose  $s$  is a proper mereological part of  $t$  and consider any  $A$  which is such that  $s \nearrow A$ ; then  $t \nearrow A$  also.*

This is intuitively obvious — if what is immediately before me makes true the proposition *there is a cup on the desk*, then so does the entire room which contains me, the cup, the desk, and other things besides. But even though it is fairly obvious, it is worth having the proof in front of us for future reference. Thus if  $s$  is a mereological proper part of  $t$ , then  $t$  is the fusion of  $s$  and some  $r$ . Suppose  $s \nearrow A$ , and  $r \nearrow B$ ; then  $(s + r) = t \nearrow A \& B$ ; but  $A \& B \models A$ , and so  $t \nearrow A$ .

Read claims that even if the entailment thesis ET were replaced by the more restrictive Relevant Entailment, a local form of, what he called the Global Supervenience of Necessity, would still hold. The proof goes as follows (and is derived from Restall):

For suppose  $p$  is logically true, and  $q$  is not, but nonetheless true. Then  $p \& q$  is true, and  $(\exists s)(s \nearrow p \& q)$  by TA. Since  $p \& q \models p$ ,  $s \nearrow p$  by ET. So any logical truth is made true by some contingent truthmaker.<sup>3</sup>

Read appears to think that this is a more restricted form of the UTN, giving local rather than global dependence, but that is not true. Logical truths are not ‘made true by *some* contingent truthmaker’, they are made true by *all* of them, just as with the UTN. This is obvious from the fact that  $q$  was chosen arbitrarily. Thus it will, once again, be the case that all truthmakers will be truthmakers for all necessary truths.

But Read further thinks that the above reasoning is ‘sophistical’, so that UTN doesn’t actually follow after all. The argument is complex, so I quote it in full.

What makes  $p \& q$  true does indeed make  $p$  true. But what makes  $p \& q$  true? Only if we assume that the supposed logical truth  $p$  has no special truthmaker does the supervenience thesis follow — that is, the argument is circular.

This is odd on the face of it, because we have made *no* such assumption in the above argument — and since we haven’t been told what a ‘special’

<sup>3</sup> This is directly from Read (2000), p. 70, with only notational changes.

truthmaker would be, it is not clear how the assumption would figure. But Read continues

The idea of supervenience is that the truth of logical necessities should follow from what makes contingent truths true *whatever* those truths are. Suppose  $p$  is a logical truth and  $q$  contingent, and suppose  $[s \nearrow p]$  and  $[r \nearrow q]$ . Then  $[s + r \nearrow p \& q]$ , so  $[s + r \nearrow p]$ . But we can infer that the truth of  $p$  supervenes on contingent truth only if the existence of  $s + r$  supervenes on that of contingent truthmakers. (p. 70)

But claims about supervenience are muddling the argument here. What Read wants to point out — perfectly correctly — is that we cannot infer that  $s$  is a contingent truthmaker merely from the fact that  $s$  is a truthmaker for  $p \& q$ . But what he has overlooked is that  $p \& q$  is a contingent truth and *ex hypothesi*  $s$  is a truthmaker for *it*. So if we say that every truthmaker for a contingent truth is a contingent truthmaker then the ‘sophistical’ argument goes through after all:  $s$  is a contingent truthmaker for a necessary truth. Moreover I will argue in §4 that it is a very desirable feature of a truthmaker theory that the necessary truths are made true by contingent truthmakers. What is undesirable is that they should be made true by all of them.

But postulating ‘special truthmakers’ for necessities would not have helped anyway. To see this, suppose that  $p$  does have a special truthmaker, and denote it by  $\alpha$ . Then it will still be perfectly true, as the above argument says, that for any contingently true,  $q$ ,  $(\exists s)(s \nearrow p \& q)$  — since  $p \& q$ , recall, is contingent. And since  $p \& q \models p$  then  $s \nearrow p$ . Thus whatever special truthmaker  $p$  has, it will still be true that every truthmaker for contingencies will be a truthmaker to the necessities. Necessities will have *both* the special and the contingent truthmakers.

We may conclude from this that Read’s defence of the use of Relevant Entailment as a way of escaping from UTN fails. So how should we respond to UTN? I take this up in the next section.

### 3. Spreading Truthmakers

A good conservative principal is that we should stick to classical logic until we are actually *forced* to abandon it. The reason is that a move to a weaker logic will be felt throughout the entire corpus of our scientific and practical reasoning, past and present; and that represents a large number of inferences that we will have to re-check to see that they are still secure. For the simple fact is that if a more restrictive logic than the one that we have used for so long *were* correct, it would be little short of miraculous that we hadn’t discovered it long ago — for our use of classical inference would have been

continually overshooting.<sup>4</sup> This is an argument against the wholesale replacement of classical logic by a weaker system — but it does not, by itself, forbid us to look for a non-classical set of rules in some particular circumstance for a particular purpose.

I want to offer a way around UTN that involves denying the principle that has taken us there: ET.

We may first note that ET is a strange principle to want in a truthmaking theory, for all it does is pass truthmakers along from one truth to another. But why should we want to do that? By TA every truth has a truthmaker already; why give it the truthmaker of every proposition that entails it? We might want to do this in some circumstances, but surely not all.

I suggest that Truthmaker theory be reformulated to include postulates 1.1–1.4: TA, FC, EX, and TF — but not ET. But once we exclude ET, the other four postulates are no longer sufficient and we must add more. Here are the additions.

*Postulate 3.1: (Conjunction Thesis (CT)) For all truthmakers  $s$  and propositions  $A$  and  $B$ ,  $s \nearrow A \& B$  iff  $s \nearrow A$  and  $s \nearrow B$ .*

*Postulate 3.2: (Left-Disjunction (LD)) If either  $s \nearrow A$  or  $s \nearrow B$  then  $s \nearrow (A \vee B)$ .*<sup>5</sup>

Note that with this set of postulates — TA, FC, EX, TF, CT, and LD — we have the resources to see that the tautology  $A \vee \neg A$  does indeed have a truthmaker. Suppose  $A$  is true and it has the truthmaker  $s$ , then by LD,  $s \nearrow A \vee \neg A$ . On the other hand, if  $\neg A$  is true then, for some  $r$ ,  $r \nearrow A \vee \neg A$ . We can also see that the different tautologies  $A \vee \neg A$  and  $B \vee \neg B$  may have different truthmakers.

But tautologies may be written in a form other than  $p \vee \neg p$ . How can we see what the truthmaker for a statement like  $A \supset A$  is if we don't have ET available? The answer is that we may proceed using syntactic replacement rules, of the kind defined in elementary logic texts. (I will use the rules

<sup>4</sup>Of course the defenders of Relevance Logic have long argued that the so-called paradoxes of implication are precisely evidence of such overshooting but the case was not, I think, well-made by Anderson and Belnap — nor has it been subsequently improved upon. I argued this in my *The Art of Necessity: Deductivism, Modality, and the Limiting of Reason*, ch. 5, (forthcoming). When this point is combined with the untenable aspects of relevant entailment — the loss of disjunctive syllogism, for example — the case for retaining classical logic is strong.

<sup>5</sup>The failure of the converse is the explicit subject of Read's paper. We will look at the argument for this in §.5.

defined in Copi's *Symbolic Logic*, chapter 3, pp. 39–40.<sup>6</sup>) Thus, to take our example, we note that there is a rule (Imp) which allows us to go from a statement of the form  $p \supset q$  to  $\neg p \vee q$ . Thus we may reason as follows: There is a rule that allows us to transform  $A \supset A$  to  $\neg A \vee A$ , and another rule (Com) that allows us to go from  $\neg A \vee A$  to  $A \vee \neg A$ ; and so the truthmaker for  $A \vee \neg A$  passes back, by this syntactic transformation, and is the truthmaker for  $A \supset A$ .

Note that this process will not allow us to give the truthmaker for  $A \vee \neg A$  to  $B \vee \neg B$ , because there is no replacement rule that will take us from the one to the other (there is no generalised substitution of logical equivalents). Likewise there is no way of transmitting every truthmaker for contingent statements to arbitrary necessary truths. We have thus blocked the derivation of UTN given in §.2.

We should not consider the employment of the Replacement Rule as on a par with the truthmaking postulates. The components are auxiliary rules that facilitate the comparison of some logically equivalent expressions. With them we are able to see that if  $s \nearrow A \& B$  then  $s \nearrow B \& A$  — something which is intuitively obvious but doesn't follow from the postulates alone. Moreover with the Replacement Rule we are able to see that, for example, if  $s \nearrow A \& (A \supset B)$  then  $s \nearrow B$ . Simply note that by an application of the rule Imp on  $A \supset B$  to get  $\neg A \vee B$ , and then using Distribution to obtain  $(A \& \neg A) \vee (A \& B)$ ;  $s$  could not be a truthmaker for  $(A \& \neg A)$  because it is false, so it must be a truthmaker for  $(A \& B)$  — whence it is a truthmaker for  $B$ . We can apply the same procedure to show that if some  $s$  is a truthmaker for the conjunction of the premises of, what Copi calls, an *elementary valid argument*, then it will be a truthmaker for the conclusion also.

In particular we note that if  $s \nearrow (A \vee B) \& \neg A$  then  $s \nearrow B$  — i.e. Disjunctive Syllogism holds — and therefore our way around *ET* is not equivalent to replacing it with a notion of Relevant Entailment.

Our use of Copi's Replacement Rule gives us, I maintain, all of the intuitively desirable (truthmaking) relations between logically equivalent expressions, and none of the undesirable ones.<sup>7</sup> But we have not replaced classical logic. Logic defines the implicational relations between propositions, whatever the truth values of the propositions. In defining how truthmakers should spread to other propositions we are looking at the relations between *true* propositions and considering the conservative spread of those states of affairs to other truths. The fact that truthmaking involves a necessary conditional dependency should not mislead us into thinking of this as a form of entailment, for truthmaking is a relation between the wrong kind of things to

<sup>6</sup> Irving Copi *Symbolic Logic*, fifth edition, (New York: Macmillan, 1979)

<sup>7</sup> A proof of this claim will be given elsewhere.

be considered entailment. Truthmaking is a relation between states of affairs and propositions and thus one property of the entailment relation makes no sense in this context: entailment is reflexive. (It was for this reason that I changed the symbol for the truthmaking relation: retaining the entailment symbol will only continue to cause confusion, of the kind that we have already seen.)

#### 4. *Necessity Regained*

But we have here a rather significant philosophical result, that goes beyond our immediate concerns with the correspondence theory of truth. For we have found specific truthmakers for a significant group of necessary truths, the tautologies, that are not supernatural, indeed that do not go beyond the states of affairs of *this* world. Moreover, from within truthmaker theory we have an explanation for why they are necessary.

To see this, let us reflect for a moment on the origins of our concept of necessity. In Medieval Philosophy the principal contrast was between the contingent and the necessary — the necessary being whatever was not contingent. However, somewhere between Scotus and Leibniz, this primary contrast was lost and the necessary was instead defined in terms of its conjugate, possibility — as that which was true in all possible worlds. The formalization of this idea in the Twentieth Century caused the concept of contingency to all but fall from sight — in logic, if not in metaphysics.<sup>8</sup> But truthmaker theory allows us to recapture the concept of necessity in its original form. For we can note that tautologies do indeed have truthmakers — as they should, by FC — but they would still be true even if the states of affairs that constitute their truthmakers had *not* obtained.

To see this note that there are two ways that  $A \vee \neg A$  could be made true, either by some  $s$  making  $A$  true or by some  $r$  making  $\neg A$  true. Consider the first option, where  $s \nearrow A \vee \neg A$ . The state of affairs  $s$  obtains, but what if  $A$  had been false? Then, obviously,  $s$  would not have obtained, and neither would any other state of affairs that would be sufficient to make  $A$  true. But then  $\neg A$  would have been true, instead. And so  $r$  would have been the case, and hence the truthmaker for  $A \vee \neg A$ .<sup>9</sup>

<sup>8</sup> The definability of contingency within modal logics is rarely discussed. Two notable exceptions are, H.A. Montgomery and F.R. Routley 'Contingency and Non-Contingency Bases for Normal Modal Logics', *Logique et Analyse*, 9, 1966, pp. 318–28, and M.J. Cresswell 'Necessity and Contingency', *Studia Logica*, 47, 1988, pp. 145–9.

<sup>9</sup> We should not be misled by grammatical form into thinking that negations are made true by 'negative facts' or 'absences'. Only existent states of affairs can do any truthmaking. This means that in the above argument one should not be misled into thinking that if  $s$  had

The necessary truth  $A \vee \neg A$  is thus made true by some ordinary state of affairs, either  $s$  or  $r$ . What about other necessary truths, such as *everything that's red is coloured*? Is that also, perhaps, made true by some state of affairs? The answer is surely that it is not made true by some particular state of affairs but by the structure of any state of affairs. Because being red is simply a (one) precisification of being coloured. It is thus necessary because it is, Quine notwithstanding, analytic. And it is analytic because of the way we (correctly) talk about what is.

>From this we can see that a necessary proposition is made true by an ordinary state of affairs. But it differs from a contingent proposition in that, *whatever* state of affairs obtains, it could not have been false. For if the state of affairs that made it true had not obtained then some other state of affairs would have. We thus have a non-supernatural explanation of necessary truth.

### 5. Right-Disjunction

Consider the converse principle to Left-Disjunction, which we will call Right-Disjunction.

*Definition 5.1: (Right-Disjunction (RD)) If  $s \nearrow (A \vee B)$  then either  $s \nearrow A$  or  $s \nearrow B$ .*

In his paper 'Truthmakers, Entailment and Necessity' Greg Restall urged that RD should be accepted, even though in combination with some other plausible principles it led to, what he called, 'Truthmaker Monism'.

*Definition 5.2: (Truthmaker Monism (TMon)) For all  $s$ , and all true propositions  $A$ ,  $s \nearrow A$ .*

Obviously, TMon would be a disaster for truthmaker theory, if allowed to stand — for it says that all states of affairs are truthmakers for any and all true propositions. Here is the argument that will take us from postulates 1.1–1.5 to TMon.

We have already seen that postulates 1.1–1.5 will allow us to derive UTN: if  $A$  is a necessary truth then, for all  $s$ ,  $s \nearrow A$ . Now consider the arbitrary tautology  $B \vee \neg B$ . By UTN, every  $s$  is such that  $s \nearrow B \vee \neg B$ . But then, by RD, either  $s \nearrow B$  or  $s \nearrow \neg B$ . Now only one of these can be true, because if  $B$  is true then  $\neg B$  would be false,

not made  $A$  true, then its absence — the negative or 'null truthmaker' — must make  $\neg A$  true. The form of propositions is not isomorphic to the form of states of affairs.

and *vice versa*. Suppose that it is  $B$  that is true; then every  $s$  is a truthmaker for  $B$ . But  $B$  was an arbitrarily chosen true proposition, and so we have TMon: every truthmaker is a truthmaker for every truth.

I have already recommended a change that will block this particular derivation: not including the postulate ET. But it is worth seeing why RD should be rejected quite apart from that.

The central definition of Truthmaker theory came from Mulligan, Simons and Smith.<sup>10</sup> They defined a truthmaker as follows:

*Definition 5.3:* (MSS)  $s \nearrow A$  iff  $s$  obtains and  $\neg\Diamond(s$  obtains  $\& \neg A)$ .

(I have used the expression ‘ $s$  obtains’ rather than their ‘ $s$  exists’ (E!) because it better reflects the way we speak about states of affairs. We say that objects exist, but that states of affairs obtain. In future we will abbreviate ‘ $s$  obtains’ to  $\mathcal{O}s$ .)

This was the definition that was adopted by Restall and, through him, by Read.<sup>11</sup> It makes truthmaking a species of necessary dependence — and therefore essentially modal in character. But if we try to expand RD, with this definition, then we find that we cannot.

Suppose  $s \nearrow A \vee B$ , then, by MSS,  $\mathcal{O}s \& \neg\Diamond(\mathcal{O}s \& \neg(A \vee B))$  (1)

$$\equiv \mathcal{O}s \& \Box(\mathcal{O}s \supset (A \vee B))$$
 (2)

$$\equiv \mathcal{O}s \& \Box((\mathcal{O}s \supset A) \vee (\mathcal{O}s \supset B))$$
 (3)

But we cannot get from this last line to

$$\mathcal{O}s \& \Box(\mathcal{O}s \supset A) \vee \mathcal{O}s \& \Box(\mathcal{O}s \supset B)$$

This is because it is a modal fallacy to infer  $\Box p \vee \Box q$  from  $\Box(p \vee q)$ . Therefore if we keep definition 5.3 — or anything with a similar modal character

<sup>10</sup>Mulligan, K., Simons, P., and Smith, B., ‘Truth-makers’, *Philosophy and Phenomenological Research*, 44, 1984, pp. 287–321.

<sup>11</sup>Curiously, however, Restall did not cite the Mulligan *et al* paper as the source — though it was the source — and that paper did not appear in his bibliography.

— we should not accept RD. RD not only leads to the collapse of truthmaker theory, it is incompatible with the motivating definition.<sup>12</sup>

The error underlying RD was first pointed out in print by Read in his article — indeed pointing this out was really the article's main purpose. Read adds an illustrating example which strongly suggests that truthmaker theory *should* adhere to definition 5.3 and that RD should, therefore, be rejected. He also shows that if the modality is dropped from MSS then RD would follow. The main difference between Read's and the present proposal is that Read believes that in order to escape the consequence UTN, some non-classical logic must be adopted. But where did Restall stand on these matters?

To begin with we must note that at the beginning of Restall's paper MSS is called '*the classical entailment thesis*' — and it is this that is meant to be the explicit target of his discussion. In fact, Restall derives UTN directly from MSS and RD as follows. First we have:

It is well known that if  $A$  is necessary (so  $\neg\Diamond\neg A$ ) then *anything* 'entails'  $A$ ... As a result we have the following consequence.

*Consequence 4:* If  $\Box A$  is true, then any existing  $s$  is a truthmaker for  $A$ . (p. 333)

And then

We assume that every instance claim of the form  $A \vee \neg A$  is a necessary truth. By one fact we have already seen, every  $s$  is a truthmaker for each instance of  $A \vee \neg A$ . Let  $A$  be a truth. So, any  $s$  either makes  $A$  or  $\neg A$  true, by the disjunction hypothesis [*i.e.* RD]. Given that  $A$  is true, then nothing makes  $\neg A$  true. So  $s$  is not a truthmaker for  $\neg A$ . Hence, it must be a truthmaker for  $A$ . (p. 334)

Calling MSS '*the classical entailment thesis*' was, however, rather perverse, since it is not a part of classical logic at all — it is a definition of the truthmaker relation in terms of necessary conditional dependence. But in the course of his article Restall calls no less than three other propositions '*the classical entailment thesis*' as well — including ET — engendering a great deal of confusion in the process.

Moreover, given that Restall declares it his aim to solve the problem of the derivation of TMon, while keeping RD, we would expect that at the end of his article there would be an alternative truthmaker definition to replace MSS. But in fact in his concluding two pages, where his positive solution is sketched, there is no mention of an alternative to MSS at all. Despite the stated intention of the article, we never get an answer to the question of

<sup>12</sup> In Arnold Koslaw's *A Structuralist Theory of Logic*, (Cambridge: Cambridge University Press, 1992) pp. 246 ff, the failure of this inference is taken to be the defining characteristic of a modal logic.

what the relation between truthmakers and truths is. Instead Restall speaks merely of a 'map' between truthmakers and truths and the modal character of the relation is quietly passed over. But since he retains RD, and derives ET, he cannot solve the problem that he started with: TMon must still be derivable.<sup>13</sup>

One of the virtues of Read's article is that he quietly tries to tidy this situation. In one respect, however, he falls victim to it as well. Read gives the impression that the route to TMon is through ET:

$$\{TA, FC, Ex, TF, ET, CT, LD\} \Rightarrow UTN, \quad \{UTN, RD\} \Rightarrow TMon.$$

But this is only one way: the shorter way is, as Restall's earlier argument has it, through MSS.

$$\{MSS, RD\} \Rightarrow TMon.$$

We have already seen that RD should be rejected, and therefore both routes to TMon are blocked — but note that the reason for rejecting RD was the modal nature of the truthmaking relation expressed in MSS. So if one doesn't have a truthmaker definition which expresses the *necessity* of the dependency of truths on truthmakers then one will not have a reason for rejecting RD.

Thus the dialectical situation is rather heavily constrained: if we want to reject TMon we should reject RD; but we can only reject RD if MSS is retained, or if it is replaced by something that retains its modal character. But to block the derivation of UTN we must modify ET and MSS.

Undoubtedly the simplest way out of this situation would be simply to retain MSS and ET and accept UTN. Over time perhaps our intuitions will be schooled to accept that every state of affairs is a truthmaker to every necessary truth. (And certainly we must acknowledge that our current intuitions are neither strongly opposed, nor fixed by constraints inherited from some other problem.) Let me call this the Standard View.

But since we do have some contrary intuitions here it is worth seeing whether they can be accommodated. It is for this reason that I have been seeking out a more constructivist model. On that view we deny ET and supplement the remaining truthmaker axioms with replacement rules that allow

<sup>13</sup> By the end of his paper his purpose seems to have turned upside-down: he seems by then to want to show that starting with a primitive truthmaker relation one can recapture classical entailment, and, 'with a more fine-grained' analysis, something close to relevant entailment. His purpose thus seems closer to van Fraassen's in 'Facts and Tautological Entailments'. I discuss this shortly — and draw a negative conclusion.

us to see *when* two logically equivalent expressions should share the same truthmakers.

But we must also change MSS so that, on the one hand, it doesn't lead directly to UTN, but so that it still supports the blocking of RD. To this end I suggest emending MSS to apply only to contingent truths. But first let us define a purely contingent proposition

*Definition 5.4: (Pure Contingency)* A pure contingent proposition is one that cannot be written in conjunctive normal form so that any conjunct is a necessary truth.

This is intended to screen out contingent propositions such as  $A \& (B \vee \neg B)$  that led to trouble before, but it will also screen out cases where the necessity is atomic — such as *seven is prime*. With this we can define the truthmaker relation for these propositions.

*Definition 5.5: (MSS\*)* For all pure contingencies  $A$ ,  $s \nearrow A$  iff  $\mathcal{O}s$ , and  $\neg \diamond (\mathcal{O}s \ \& \ \neg A)$ .

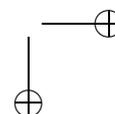
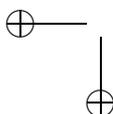
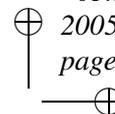
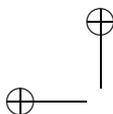
And now that we have a truthmaker definition for simple contingencies we can use LD to spread these conservatively to primitive tautologies of the form  $p \vee \neg p$ , where  $p$  is a pure contingency.

*Definition 5.6: (Inherited Truthmaker)* If  $s \nearrow A$  then  $s$  is the inherited truthmaker for  $A \vee \neg A$ .

We then use the replacement rules mentioned in §3 to spread the inherited truthmakers for  $A \vee \neg A$  to logically equivalent tautologies — *i.e.* equivalent under the replacement rule. In this way we constructively extend the truthmaker relation so that all and only the truths that should have truthmakers actually have them.<sup>14</sup>

We can use this constructivist truthmaking definition to show the weakness in the idea of tautological entailment, that developed from the work of von

<sup>14</sup>It is not guaranteed by this method that *all* necessary truths will have truthmakers: consider an assertion of self identity for something that does not exist, say Mars (the God). Mars = Mars is true and a necessary truth, but it is not obvious what its truthmaker would be in the absence of the thing itself. This may be considered to be a possible argument in favour of the Standard View, since there every state of affairs is a truthmaker for all such truths.



Wright, Geach, Smiley, Dunn and van Fraassen.<sup>15</sup> To do this it is necessary that we adapt a proof of van Fraassen's to our account.

*Theorem 5.1:* (Tautological Entailment) "If  $\forall s(s \nearrow A \supset s \nearrow B)$  then  $A$  tautologically entails  $B$ " fails.

*Proof:* (Our replacement rules are the same as Anderson and Belnap's, so, like them, we need only consider entailments in normal form.) Let us consider the contrapositive:  $A$  does *not* tautologically entail  $B$ , and let us write the inference out as

$$\phi_1 \vee \dots \vee \phi_n \rightarrow \psi_1 \& \dots \& \psi_m \tag{4}$$

Inference (4) decomposes into  $n \times m$  primitive tautological entailments of the form

$$p_1 \& \dots \& p_r \rightarrow q_1 \vee \dots \vee q_s \tag{5}$$

(4) will be invalid when one of these primitive tautological entailments say,  $\mathcal{A}_1 \& \dots \& \mathcal{A}_r \rightarrow \mathcal{B}_1 \vee \dots \vee \mathcal{B}_s$ , is invalid, which occurs when  $\mathcal{A}_i \neq \mathcal{B}_j$  for all  $i$  and  $j$ . But note that it is not possible to conclude from this that no truthmaker for one of the  $\mathcal{B}_j$  is not also a truthmaker for one of the  $\mathcal{A}_i$ s because we can make *no* claims about what make these true.

The problem is that the validity and invalidity of the primitive tautological entailments is not a matter of truth conditions: it is a simple matter of inclusion: one atomic statement in the atomic premise list being included as a disjunct in the conclusion. It is this that makes the property of primitive tautological entailment decidable. Thus if one of the  $\mathcal{A}_i$  was the same as one of the  $\mathcal{B}_j$  we could conclude that they have the same truthmaker and that the entailment is valid, but we can conclude nothing where they aren't equal.

This has double significance. Van Fraassen had wanted to show that there was an explanatory rôle for relevance logic that went beyond seeking a cure for the "paradoxes of implication", and also that sense could be made of the notion of facts making statements true. His semantics for the fragment of relevant entailment that was marked out by the concept of tautological entailment (called  $E_{fde}$ ) was meant to secure both results. (It was this same

<sup>15</sup> And given an axiomatic formulation by Anderson and Belnap, see *Entailment: the Logic of Relevance and Necessity*, (Princeton: Princeton University Press, 1975) §§. 15 and 20. Bas van Fraassen offered a fact, or state of affairs, semantics for tautological entailments in 'Facts and Tautological Entailments' in *The Journal of Philosophy*, 66, 1969, pp. 477–487.

aim that Restall was resuscitating at the end of his paper.) But, as we have seen, the goal is illusory.<sup>16</sup>

Truthmaking is a semantic property *par excellence* — it is not syntactic. There is no close connection between truthmaking and tautological entailment, or indeed relevance logic in general, despite the claims of recent tradition.

### 6. Minimal and Maximal Truthmakers

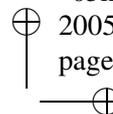
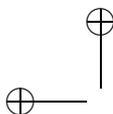
We can get a better understanding of the truthmaker relation if we define the notion of a *minimal truthmaker*. This is intended to capture the idea that there is usually a very localised state of affairs that makes a proposition true. Thus 'the cat is on the mat' is made true by a truthmaker with the length and breadth of a mat, and the height of a cat; whereas at present any state of affairs of which that localised state of affairs is a part counts as a truthmaker for the truth.

*Definition 6.1: (Minimal Truthmakers)* If  $s$  is a truthmaker for some proposition  $A$  then it is a minimal truthmaker if no proper part of it is also a truthmaker for  $A$ . If a proposition has a unique minimal truthmaker then we speak of that as the truthmaker for the proposition.

Some true propositions have minimal truthmakers, others do not. Some have unique minimal truthmakers, while others do not. 'Jinx (the cat) is on the mat' has a unique minimal truthmaker, while 'some men are more than six feet tall' does not — it has multiple minimal truthmakers. Some propositions do not have minimal truthmakers at all, unique or otherwise. 'There is an interval that is smaller than a centimetre long', has no minimal truthmaker.

Because of Truthmaker Fusion, the global state of affairs, the fusion of all states of affairs — which we will denote as  $\Omega$  — will be a truthmaker for all truths. A large number of truths, however, have no truthmakers that are less than  $\Omega$ . For example the proposition 'there are no unicorns' is not made true by this or that patch of unicornless space-time — it is made true by all such patches. Similarly, for other negated universal generalisations — such as, there are no Higgs Bosons; if true, it is only *everything that is* that makes it true.

<sup>16</sup>For van Fraassen's original proof see Anderson and Belnap (*op cit*) p. 229. In my view Restall was too scant in his acknowledgement of Van Fraassen's earlier effort.



But it would be wrong to conclude that every negative existential proposition has  $\Omega$  as a minimal truthmaker. 'There is no apple on this table' has, as unique minimal truthmaker, this appleless table. The state of affairs that makes true the proposition, 'this table is bare', also makes true the first proposition.

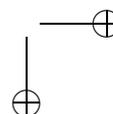
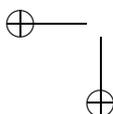
Unique minimal truthmakers are thus important but are not the norm; where they exist their importance is more epistemological than ontological, for they represent states of affairs that we are often able to take in a glance, rather than by some painstaking induction. However, too narrow a focus on such states of affairs in the post-*Tractatus* discussions on the Correspondence Theory led to the mistaken idea that we were looking for a 1:1 correspondence between facts and truths. Thus when the correspondence-as-picturing model was rejected it was replaced by an equally limited correspondence-as-1:1 correlation model that was also unrealistic.<sup>17</sup>

But it is worth noticing that even when we just restrict ourselves to the set of statements with unique minimal truthmakers, and the set of minimal truthmakers, that the relation between these two sets is not a 1:1 correspondence. It is of course true that to every truth in the set there is a unique truthmaker in the other set, by definition, but it is not true that to every minimal truthmaker there corresponds a single statement. For example the cat, Jinx, being on the mat is a unique minimal truthmaker for the two statements (1) 'either Jinx is on the mat or two and two is five,' and (2) 'either Jinx is on the mat or New York is the capital of Greenland'. Hence the mapping from the set of minimal truthmakers to the set of statements that have minimal truthmakers is not even a function, let alone 1:1. The inverse mapping *is* a function, however, and is **onto** — *i.e.* it is a surjection.

This is better than the general situation, however: the mapping from truthmakers to propositions is not a function, and neither is the mapping from propositions to truthmakers. The same holds even if we restrict ourselves to true propositions.

This fact, of the frequent non-uniqueness of minimal truthmakers, helps us to be clear about the relata in a T-Schema formulation of truth, such as that given by William Alston. Alston formulates his minimal theory in the following way:

<sup>17</sup> It is worth pointing out that when J.L. Austin so ably argued for correspondence-as-correlation, as against Wittgenstein's correspondence-as-picturing ('Truth' in *Proceedings of the Aristotelian Society, Supp. Vol. XXIV*, 1950) he did not tie himself to 1:1 correlation — indeed he urged against it. However, when the paper was reprinted in *Truth*, edited by George Pitcher (Englewood Cliffs: Prentice-Hall, 1964), pp. 18–31, the editor, in his introduction *did* paraphrase it in this way (see pp. 10–15). The resulting confusion did much to damage the correspondence theory.



(*p*): The proposition that *p* is true *iff* it is a fact that *p*.<sup>18</sup>

But note that this encourages the mistaken belief that propositions and facts correspond 1:1. If we substitute in the proposition that some man is more than six feet tall then the fact on the RHS is simply: some man being more than six feet tall. But the specific truthmakers may be Abe, Jack, George, *etc* being more than six feet tall, and the relation of *these* specific truthmakers to the proposition is many-to-one. Too narrow a focus on schema such as the above can lead us to think that the correspondence relation is either trivial or inexplicable, when it is far from either.

In the next section, §.7, I consider other aspects of the philosophical background to the correspondence theory.

### 7. *States of Affairs Justified*

Those who object to the correspondence theory often do so by declaring the notion of a fact, or state of affairs, unintelligible — as if it were metaphysics-mongering of the worst kind. Defenders of truthmakers often aid and abet this attitude by thinking of ‘truthmaker’ as a preferable, metaphysically neutral, term that by-passes the problems with facts and states of affairs.

I think this is a mistake. We should see ‘truthmaker’, ‘fact’, and ‘state of affairs’ as synonymous, where no term is to be avoided, since they have the same metaphysical weight. (We should do this until such time as we need to make them terms of art, and distinguish one from another.) But this requires me to say something about some of the arguments that have been leveled against them — to deflect this sense of illegitimacy.

Complaints about the notion of facts originated in Peter Strawson’s response to Austin, originally made to *The Aristotelian Society* in 1950.<sup>19</sup> Strawson gave two arguments that have carried some weight ever since. The first is that facts cannot be individuated except by stating the sentences that speak of them — thus making them essentially linguistic in character. And

<sup>18</sup> William P. Alston *A Realist Conception of Truth* (Ithaca: Cornell University Press, 1996) p. 38. Of course we could just as easily have chosen some other minimalist account — for example Horwich’s — to make the point.

<sup>19</sup> ‘Truth’ reprinted in *Truth*, ed. George Pitcher, (*op cit*) pp. 32–53.

second: facts are not elements of a respectable science; the world consists of things, not facts.<sup>20</sup>

Let us take the individuation problem first. Strawson does not express the point clearly, but the intention is plain enough. Thus commenting on Austin's claim that true statements "fit the facts", he says,

But what could fit more perfectly the fact that it is raining than the statement that it is raining? Of course, statements and facts fit. They were made for each other. If you prize the statements off the world you prize the facts off it too; but the world would be none the poorer. (You don't also prize off the world what the statements are about — for this you would need a different kind of lever.) (pp. 38–9)

Strawson avoided answering Austin's lethal point that the facts vastly outrun the statements that are available to state them, contenting himself with this counter-assertion. So, is his claim true? It is hard to see that it could be. Even if facts *were* individuated solely by statements that would not be a sufficient reason to identify them: how one identifies something does not constitute what it is. But, also, it is not obvious that facts *are* thus individuated. Facts have causal rôles that are distinct from the causal rôles of statements: the fact that the bank was robbed has a different causal rôle than the statement that the bank was robbed. Thirdly, Strawson omits to say that it is *true* statements that "fit the facts"; but this omission is significant because he cannot add in 'true' without circularity.

So this first part of Strawson's case failed. What then of his second point, that the world consists of things rather than facts — facts having no part in a scientific world-view?<sup>21</sup> Here is Strawson's own statement of his position:

My objection goes farther. It is that there is no thing or event called "a statement" (though there is the making of the statement) and there is no thing or event called a "fact" or "situation" which stand to one another in any, even a purely conventional, relation. . . The facts (situation, state of affairs) cannot, like the chessboard and pieces, have coffee spilled on them or be upset by a careless hand. (p. 41)

<sup>20</sup>In Kenneth Russell Olson's excellent *An Essay on Facts* (CSLI Lecture Notes, No. 6) there is a consideration of non-Strawsonian arguments against facts, that stem from arguments from Frege, Church and Gödel (the basis for what Barwise and Perry called the *Slingshot Argument*). But the formal working out of truthmaker theory was always — whether particular authors made it clear or not — directed toward the solution of the slingshot problem. The first important step was the recognition that the relation between facts and propositions was not 1:1.

<sup>21</sup>Of course, Strawson did not put the point in terms of science, but later philosophers, like Quine, making the same point, did. It is convenient to roll Strawson in with those he influenced.

But though Strawson repeatedly charged Austin with making category mistakes, here he makes some of his own. That one cannot spill coffee on a fact is hardly a reason for thinking that are no facts: is the spilt coffee not a fact, in itself? But let us pass over this and just consider Strawson's claim that the world consists of things, not facts or states of affairs.

Surely the appropriate test of this claim is to consider a world which has the very same things in it as our world but arranged differently: say with all the modish Italian furniture in London, somewhere close to the star Rigel, the Oxford colleges somewhere in the Horsehead Nebula, *etc.* According to Strawson this must be an identical world to our world. But it is plainly not. Our world consists not merely of things, but things with properties, standing in particular relations to other things. These complexes are what we call facts and states of affairs. If two worlds had the same states of affairs then it is plausible to think them identical, and if not, not. An identical inventory of dry goods falls a long way short of guaranteeing sameness of worlds.<sup>22</sup>

Finally, is it true that facts, or states of affairs, are not recognised by science?

It is certainly true that the *individuals* postulated consist of quarks, leptons, gluons, space-time manifolds, *etc.* But it is also the case that physicists speak of the *states* that those entities have. These are represented by trace class operators on a Hilbert space. But just as single individuals have states, so a larger 'state of affairs' is represented by the combined states of an ensemble of particles in what is called, a Fock Space. This is a tensor product of the Hilbert spaces for each species of particle  $S_i$ :

$$\mathcal{H}_{S_1, \dots, S_N} = \mathcal{H}_{S_1} \otimes \mathcal{H}_{S_2} \otimes \dots \otimes \mathcal{H}_{S_N}$$

The states so represented are perfectly real, are everyday quantified over, and enter into, and indeed are the primary explanatory apparatus for, the events that take place in our world. Only a nominalism that has hardened irretrievably into dogmatism could fail to acknowledge them as parts of the world.<sup>23</sup> Moreover they are parts in the sense misunderstood by Strawson:

<sup>22</sup> Behind Strawson's article there was an unargued-for Nominalism that seems as implausible as the consequences that he draws from it. One may say the same of Quine's similar position. *c.f. Word and Object* (Cambridge: MIT Press, 1960) §50.

<sup>23</sup> I should note that this sketch is not affected by interpretational issues having to do with the measurement problem in quantum mechanics. Those problems fall elsewhere. For note that even on the Copenhagen Interpretation — which is the interpretation of QM that comes closest to pure Idealism — every system possesses a state at every instant, whether one is looking or not.

for, contrary to Strawson, *things* are items *in* the world, whereas states (of affairs) are genuine mereological parts *of* the world.

Of course there is some distance between the physicist's conception of the world and the ordinary person's but the difference favours the physicist. My point is that states of affairs are not pieces of recondite metaphysics, they are an ordinary way of speaking about something for which there exists a more precise speaking. What is passing strange is that in all the disparagement that they have received on both sides of the Atlantic for being un-scientific, no one has thought to look at the science to see whether it was so.

### 8. *Concluding Remarks*

We have described two truthmaker theories that differ only in the states of affairs that are assigned to necessities. On the standard solution, based on the Mulligan, Simons and Smith definition (MSS), we have all states of affairs standing as truthmakers for all necessary truths. But this may seem counterintuitive to many, and so I have also given a more constructivist solution, adapting MSS so that, in the first instance, we only give truthmakers to contingencies; but then use disjunction to give them to tautologies expressed in standard form; and finally use replacement rules to give them to all tautologies expressed using the familiar connectives.

There is still more work to be done, however. One important question is whether we can extend the truthmaker theory to deal with sentences rather than propositions. What we would like, of course, is to "chain" the necessary connection between states of affairs and propositions with the contingent connection between propositions and sentences. It is intuitively obvious that this can be done but we leave the discussion to another time and place.

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