

## THE WORLDS OF LOGIC AND THE LOGIC OF WORLDS

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*“Все современное движение в логике  
есть восстание против Аристотеля,  
медленно, шаг за шагом идет это  
восстание то в одном пункте, то в  
другом. Трудно предсказывать будущее”.*

*“Every contemporary movement in logic is a  
revolt against Aristotle. Slowly, step by step,  
this revolt progresses now here, now there.  
It is difficult to predict the future”.*<sup>1</sup>

1. *Introduction: A Man and His Work*

The Russian philosopher and logician Nikolai Alexandrovich Vasil’ev (1880–1940) lived in turbulent times, both politically and scientifically. The end of the nineteenth century and the beginning of the twentieth century in the field of logic marked a period of transition from ‘traditional’ to ‘modern’ logic. Frege, Russell and many others were debunking the old paradigm of Aristotelian logic and replacing it with a new kind of logic, a process that had already started earlier in the nineteenth century. In this evolution the ‘fundamental logical laws’ that are present in the Aristotelian paradigm were not left untouched.

This paper presents an in-depth study of the main (logical) ideas of N.A. Vasil’ev in the light of their evolution and of the general logical framework they are to be situated in, more specifically w.r.t. the relation between ‘modern’ and ‘traditional’ logic. At the same time, the aim is to point out that the common conception (in both Russian and non-Russian work on Vasil’ev) of

<sup>1</sup> Vasil’ev 1912–1913, p. 123. All references to pages in Vasil’ev’s work are to the edition of V.A. Smirnov (Vasil’ev N.A. 1989). The year is each time the year the paper was published (see in this respect the bibliography of Works of Vasil’ev in this paper).

Vasil'ev as a proponent of Psychologism (in logic) is, in this respect, completely misdirected.

In a number of papers published between 1910 and 1913 Vasil'ev has tried to set up an alternative for traditional Aristotelian logic, an alternative he calls 'Imaginary Logic'. We try to show how the ideas Vasil'ev develops there are indeed based on a thorough criticism and rethinking of the classical logical laws and present an alternative to Aristotelian tradition 'from within'. In that sense, Vasil'ev was not influenced by Frege, Russell a.o. We show how the concept of negation is pivotal in his re-interpretation of the Aristotelian paradigm. Vasil'ev questioned the basic principles of Aristotelian logic and the basis of his criticism is somewhat different from the semantical or technical limits of the Aristotelian paradigm that led to what we are calling 'Modern Logic'. Though one may say that in this sense Vasil'ev's work is nothing but an armchair revolution in logic, this does not mean that his work does not and cannot have a broader philosophical and logical impact. We show this in an analysis of Vasil'ev's alleged 'Psychologism in logic' and by indicating how Vasil'ev's ideas are still remarkably relevant to modern logic in a 'Vasil'evian' analysis of Cantor's Diagonal Procedure. In order to stimulate further discussion, this issue of *Logique et Analyse* also contains our English translation of Vasil'ev's most important paper, *Imaginary (non-Aristotelian) Logic*.<sup>2</sup> Up to now, this text has only been available in Russian and ours is the first ever translation of this work by Vasil'ev.

## 2. Nikolai Alexandrovich Vasil'ev: A Biography

Nikolai Alexandrovich Vasil'ev was born on 29 June, 1880 in Kazan. His father, Aleksandr Vasil'evich Vasil'ev (1853–1929), was a well known mathematician who begun his academic career as 'privatdozent' of mathematics in Kazan University (at the time of his son's birth) and was gradually promoted to ordinary (1887) and extraordinary professor (1899) at the same university, later holding the same position at St. Petersburg University (where he moved after being elected to the State Council, *Gosudarstvennyi Sovet* in 1907), and thereafter at Moscow University (1923). Besides his teaching and research he was known as an administrator and promulgator of new scientific ideas: in Kazan University he was one of the founders and then chairman of the local Physical-mathematical society (1890–1905). Having moved to St. Petersburg, he published (between 1912 and 1915) a series of books under the title "New Ideas in Mathematics," in which Russian translations of the most

<sup>2</sup> Vasil'ev 1912a.

important recent works on mathematics and the philosophy of mathematics appeared. Aleksandr Vasil'ev had close contacts with leading European mathematicians of the time (at the very beginning of his career, in 1879 he attended lectures by Karl Weierstrass, Leopold Kronecker and Felix Klein in Berlin, and Charles Hermite in Paris) and took active part in many International 'Congresses of Mathematics' from the inaugural meeting in Paris (1899) onwards.

Vasil'ev's family was rather rich in prominent names whose legacy is found throughout Russian political and intellectual history. The father of Aleksandr Vasil'ev — Vasilii Pavlovich Vasil'ev (1818–1900) — was a famous sinologist, professor at Kazan, and later St. Petersburg University, member of the St. Petersburg Academy of Sciences. In 1855 he became a hereditary nobleman. His wife — Sof'ia Ivanovna Simonova — was a daughter of the rector (vice-chancellor) of Kazan University — Ivan Mikhailovich Simonov (1794–1855). The roots of a family tree of N. Vasil'ev's mother's, Aleksandra Pavlovna Maksimovich, go back to a certain Baron von Ikskül, a Prussian nobleman, who immigrated to Russia in 1545. Some of his Russian descendants were active in political life, such as Alexei Sokovnin, an accomplice in the failed conspiracy against Peter the Great, and his sisters — Evdokiia (Princess Urusova) and Fedos'ia (Boyarynia Morozova) — tragic figures, who became symbols of the resistance against the Nikon Church reform in the 17th century. Her father (Vasil'ev's grandfather), Pavel Pavlovich Maksimovich was a famous activist for public education in the region of Tver.

During his childhood, up to his seventh year, Vasil'ev's father — with whom Nikolai always had a good relationship — took upon himself the intellectual development of his son. Indeed, in Vasil'ev's own words, it was due to his father that “all that is usually acquired with difficulty, came easily to me.”<sup>3</sup> At the age of four, for example, his father showed him how to construct an equilateral triangle. At eight Nikolai spoke German fluently; French and English he mastered later. He could also read Greek, Latin and Italian. The influence of his father, who was also an editor of Lobachevski's works, author of papers on Lobachevski and of his biography, is undoubtedly manifest in Vasil'ev's interest in the phenomenon of non-Euclidian (imaginary) geometry, which he considered a parallel and additional argument for the existence of 'imaginary logic'.

Vasil'ev already developed an interest in logic, philosophy and psychology at high school. So e.g., he studied the works of the Russian logician, M.I. Vladislavlev, and made excerpts from works on psychology. In 1897, a

<sup>3</sup>Bazhanov 1988a, p. 17: Bazhanov's biography is a primary source for our biographic note on Vasil'ev.

new section, “Mathematical logic” appeared in Vasil’ev’s diary, under which he compiled a detailed synopsis of a paper by Charles S. Pierce on the logic of relatives<sup>4</sup>.

Eager to devote himself to psychology, but understanding that mastering this discipline demanded fundamental knowledge of medicine Vasil’ev, after leaving secondary school in 1898, became a student at the department of medicine of Kazan University. During his student years, he continued his study of philosophy and psychology. It is worth noting that at some moment during his university years, Vasil’ev was active in student movements. In 1904 he graduated from University, married Ekaterina Stepanovna Zav’ialova and opened a practice as a general practitioner in the countryside. He did not, however, interrupt his philosophical studies. In one of his letters to his wife he wrote: “Now I am avidly reading Hegel. I like it. To derive all logic and the entire world from a unique concept is, to say the least, a brave idea.”<sup>5</sup> Vasil’ev’s work as a physician did not last long. Interest in philosophy and psychology superseded his devotion to medicine, and in 1906 he passed his final exams at the department of history and philology.

In 1907 he became affiliated with the University for a period of three years, in qualifying for a professorial position. Two topics received particular attention in Vasil’ev’s courses on logic: the study of Aristotle’s syllogistics (in which *The Prior Analytics* was mentioned as a source) and the logic of J.S. Mill (and his *System of Logic*). Parallel with his study and research, Vasil’ev taught philosophy and psychology at the Women’s College in Kazan (where his father was at the same time teaching mathematics); his lectures on psychology were published as a pamphlet in 1908 (2nd edition in 1915).<sup>6</sup>

In 1904–1909, Vasil’ev revealed himself as a poet and translator of poetical work. His first book, *Longing for eternity* (1904) included his own lyrical verses and Russian translations of J.W. Goethe, Ch. Baudelaire and P. Verlaine.<sup>7</sup> In 1907, he published translations from the Belgian (Flemish) poet (1855–1916) Emile Verhaeren’s book, *Les Campagnes Hallucinées*,<sup>8</sup>

<sup>4</sup>Ch. S. Pierce, *The logic of Relatives: The Monist*, Vol. 52, n. 2, p. 161–217, 1897.

<sup>5</sup>Bazhanov 1988, p. 27.

<sup>6</sup>Vasil’ev 1908b, 1915.

<sup>7</sup>Vasil’ev 1904.

<sup>8</sup>Vasil’ev 1907. At that time Verhaeren was in vogue in Russia. His poetry was translated into Russian by poets as Aleksandr Blok, Maksimilian Voloshin and Valery Briusov. Briusov had good personal contacts with Verhaeren and was in correspondence with him; he sent Verhaeren the book with Vasil’ev’s translations.

followed in 1909 by translations of and essays on O.Ch. Swinburne's poetry.<sup>9</sup> Vasil'ev also wrote a critical paper on N. Gogol on the occasion of the centenary of his birth (1909).<sup>10</sup> His translation of the 20th Ode of the Third Book by Horatius (written around 1910) remained unpublished. Vasil'ev's poetical style possessed characteristic features of so called "symbolism," a literary trend which flourished in the beginning of the 20th century in Russia.

The year 1908 was a turning point in the inner development and scientific career of Vasil'ev. At that time he realized that the value of his psychological scholarship consisted in serving as a prolegomenon to a serious study of logic and philosophy. He stopped his work in psychology and completely devoted himself to logic<sup>11</sup>.

The summer and autumn of 1908 Vasil'ev spent in Germany absorbed in the study of new logic literature. In September, he took part in the Third International Congress of Philosophy in Heidelberg (he compiled a review of this Congress).<sup>12</sup> It is during his stay in Germany that the idea of an 'imaginary logic', as completely distinct from traditional logic, first emerged. In his last letter from Germany addressed to his wife Vasil'ev wrote: "I have conceived a project, which I hardly believe can be realized, but which for this very reason is very attractive, and I am more and more enthusiastic to carry it out."<sup>13</sup> No doubt, Vasil'ev was here writing about imaginary logic. From another source we know about the circumstances of its conception: the idea of imaginary logic came to Vasil'ev in Berlin during a game of chess with another Russian logician, G. Itelson.<sup>14</sup>

After returning back home on May 18 1910, Vasil'ev delivered a test lecture at Kazan University, which he devoted to the exposition of the new logical theory (an expanded version of this lecture was published separately).<sup>15</sup> In the autumn of the same year he was appointed as 'privatdozent' of philosophy and began his teaching career with the course, "Major logical problems including a concise historical review." At the end of 1910, he applied at the

<sup>9</sup> Vasil'ev 1909b, 1909c, 1909d, 1913b.

<sup>10</sup> Vasil'ev 1909a.

<sup>11</sup> He returned to his psychological studies only in 1920.

<sup>12</sup> Vasil'ev 1909.

<sup>13</sup> Bazhanov 1988, p. 63.

<sup>14</sup> Bazhanov 1988, p. 65.

<sup>15</sup> Vasil'ev, 1910.

University for another sabbatical to Germany in order to continue his studies on the new logic. He was granted one year's leave and in the summer of 1911 Vasil'ev, with his wife and son, Julian<sup>16</sup> (who was born in 1907), departed for Germany. The first half of his German sojourn was spent in Berlin, the second half in Munich. His stay in Germany was useful and fruitful. A report written by Vasil'ev upon his return,<sup>17</sup> contained an impressive list of names and works of logicians and philosophers he had studied. Consequently, the idea of Vasil'ev being a provincial thinker or an outsider to the European logical community, should be ruled out. Vasil'ev was always eager to test his ideas against the background of contemporary logical research. Thus, for example, in his report he gives a list of ten treatises on logic in which, contrary to the established tradition but in accordance with his own conception, the particular propositions of Aristotle were treated as having the form “some (but not all) A are B.” As a result of his stay in Germany two important works appeared viz. *Imaginary (non-Aristotelian) Logic*,<sup>18</sup> and *Logic and Metalogic*.<sup>19</sup>

After returning to Kazan, Vasil'ev continued to teach. In 1913 he taught the course, “A reading of fragments from Aristotle's ‘Organon’,” and in 1914, with two other professors, he taught a course on “Problems on the boundary between logic and the philosophy of mathematics.”

His teaching and research activity was interrupted in the autumn of 1914 owing to the beginning of the First World War. Vasil'ev, being a trained physician, was enrolled in the military service. In 1915 he was decorated with the order of St. Stanislav of the third grade. However, his experience of war, witnessing first hand the large-scale suffering of the soldiers he treated, was to have a severe effect on Vasil'ev's mental health. An emotional crisis gradually developed into mental illness and in 1916 he was admitted to a sanatorium for the mentally ill and discharged from the military. It took him a long time to recover his health, which was nonetheless never fully restored.

Vasil'ev was in Moscow around the time of the October Revolution in 1917, practically in the thick of the Bolsheviks' rise to power. In his letters he remarked upon the superior organization and general military skills of the Bolsheviks. Nikolai Vasil'ev, like his father, accepted Soviet rule, and after returning to Kazan at the end of 1917, continued teaching, first as a ‘dozent’, and from 1918 onwards, as a professor of philosophy at Kazan University.

<sup>16</sup> Named after Julian the Apostate.

<sup>17</sup> Vasil'ev 1912.

<sup>18</sup> Vasil'ev 1912a.

<sup>19</sup> Vasil'ev 1912–1913.

The summer of 1918 was a hard time for Vasil'ev and his family. They happened to find themselves in the middle of the White-Chzech offensive, witnessing the cruelty and the incessant artillery bombardment first hand. Anxiety for his family provoked a new psychological crisis in Vasil'ev and his health deteriorated severely.

In 1920, Vasil'ev turned again to his studies in psychology, the discipline which he gave up in 1908 in favor of logical-philosophical investigations. He delivered a series of lectures in psychology at the University, which, according to his students' recollections attracted great interest. Indeed, Vasil'ev played an instrumental role in the establishment of a new department of psychology at the University. In the spring of 1921, he delivered courses on logic and methodology, social psychology and the history of *Weltanschauung*. Later on, he conceived and delivered courses on the history of Russian philosophy and German Idealism, while conducting seminars on Aristotle's poetics. When asked as to his profession, by this time Vasil'ev would respond first in pointing to logic and then to psychology. In 1921 he published a paper he wrote in 1906 as a student, entitled "On the Question of the Fall of the Western Roman Empire and of ancient Culture in Historiography and the History of Philosophy in Connection with the Theory of the Decline of Nations and of Mankind."<sup>20</sup> Concluding this paper, he remarks there that, "considering the history of mankind as a history of the human species, and historical evolution as a reflection of a biological evolution, I am forced to draw the conclusion that the historical evolution of culture consists in the accumulation of harmful biological variations leading to degeneration."<sup>21</sup> According to Vasil'ev, the role of revolutions consists in their being a natural process of the renewal of a social organism. This pertains equally to both Russian revolutions of 1917, which Vasil'ev assessed as "the biological movement of fresh social strata toward culture."<sup>22</sup>

The year 1922 was a period of structural reorganization at Kazan University. The department of social sciences, in which Vasil'ev held his position, was abolished. According to the plans of the administration, he was to have taken on the responsibilities as the head of a research group for child psychology at the newly founded Institute of Philosophy, Psychology and Pedagogy<sup>23</sup>. Unfortunately, these plans were never realised.

<sup>20</sup> Vasil'ev 1921b.

<sup>21</sup> Bazhanov 1988, p. 36.

<sup>22</sup> Bazhanov 1988, p. 36–37.

<sup>23</sup> In 1921 Vasil'ev wrote a concise note on the education of the blind (Vasil'ev 1921a).

In the summer of 1922, another psychological crisis brought an end to Vasil'ev's career. He was admitted to the Kazan University Hospital and subsequently retired from his teaching position. Vasil'ev was diagnosed with manic depression. He himself believed that at the roots of his illness was his experience of a fire which consumed the Vasil'evs' home when he was four years old (after which Vasil'ev apparently suffered severe sleep disorders). He also mentioned the severe emotional strain of 1918 when his family was caught in the middle of the war. Since Vasil'ev required permanent observation, he was finally transferred to a special psychiatric clinic near Kazan. There, he was provided with an office in which he could work during periods of remission. Thus, in 1924 he composed an abstract of his paper, "Imaginary (non-Aristotelian) Logic" for the Fifth International Congress of Philosophy held in Naples in 1925. The abstract (in English) of his paper (which he did not deliver) was published in the Proceedings of the Congress (where, incidentally, a paper by his father was also published).<sup>24</sup> This was Vasil'ev's last publication.

During his stay at the clinic, Vasil'ev endeavoured to pursue his studies further. Around 1926–27 he wrote to his wife that he was studying logic and mathematics, that he "had discovered a predicate calculus (mathematical logic) of content" and was preparing a paper on the subject, which he intended to deliver in the Kazan physical-mathematical society. Other letters testify that he was interested in and meditated on Einstein's theory of Relativity and the ideas of Lorentz<sup>25</sup>. In 1933, in one of his letters, he also wrote about his reading of K. Marx.

Meanwhile, his health deteriorated and Vasil'ev could now join his family and friends only on rare occasions. He especially suffered from the loss of contact with his wife to whom he wrote in one of his letters: "I am not wanted here, I am not wanted here. . . , oh, if you could imagine how it hurts. . . But at the same time. . . how I want to live, how high my interest is in contemporary history, how confident I am in my ideas and their paramount importance."<sup>26</sup>

Vasil'ev died on December 31, 1940.

During his lifetime Vasil'ev's publications did not go unnoticed.

His paper "On Particular Propositions, the Triangle of Oppositions and the Law of Excluded Fourth" (1910) was reviewed by S.I. Gessen (twice)<sup>27</sup>

<sup>24</sup> Vasil'ev 1925.

<sup>25</sup> Bazhanov 1988, p. 41–42.

<sup>26</sup> Bazhanov 1988, p. 42.

<sup>27</sup> Gessen 1910, 1910a.

and K.A. Smirnov.<sup>28</sup> In his review, Gessen indicated that although the criticism of traditional logic was becoming commonplace in contemporary logic texts, only one logician, namely Sigwart<sup>29</sup>, had proposed a “detailed and immanent” analysis. In his view, Vasil’ev’s work constitutes another example of a matter of fact criticism of traditional logic. Relating the main ideas of Vasil’ev, Gessen reproached him for his lack of philosophical argumentation in defense of his logical conception. This rebuke by Gessen was justified and Vasil’ev took it seriously: in both of his later papers — “Imaginary (non-Aristotelian) Logic” (1912a) and “Logic and Metalogic” (1912–1913) — his logical ideas were supported by a substantial philosophical analysis.<sup>30</sup>

A thorough discussion of Vasil’ev’s ideas took place in January 1911, during a meeting of the Kazan Physical-Mathematical Society where Vasil’ev delivered a paper under the title “Non-Euclidean geometry and non-Aristotelian logic.” In a published report of this meeting, it was remarked that the meeting was attended by an unusually large number of people and that Vasil’ev’s paper provoked a major discussion. The situation was compared to that of the discovery by Lobachevski of non-Euclidean geometry. The discussion lasted almost until midnight and was reported in the local media (the materials of the meeting were published in four issues of the newspaper *Kamsko-Volzhskaia rech’*).<sup>31</sup>

However, it should be noted that in spite of the large number of sympathisers of Vasil’ev’s ideas (which was manifest, e.g., in the discussion at the Society meeting) very few, if any, of those who attended his lecture ever understood the scope and importance of the logical reform he proposed. When Vasil’ev fell ill, there were no scholars who could carry on his logical investigations and his illness probably also prevented him to work out his ideas as a formal rigorous system. There are many reasons for this. The most important is that traditional logic at the time was thought to be outdated and essentially superseded by mathematical logic. Based on predicate calculus,

<sup>28</sup> Smirnov 1911.

<sup>29</sup> On the Relation between Sigwart and Vasil’ev, see Raspa and Vergauwen 1997.

<sup>30</sup> In his book, Bazhanov (Bazhanov 1988) indicates an interesting fact, worthy of further investigation. In 1913, a paper by S. Ginsberg was published, the content of which is close to the first publication by Vasil’ev concerning the criticism of Particular Propositions and the Law of excluded Middle (S. Ginsberg, “Note sur le sens équivoque des propositions particuliers”: *Revue de Métaphysique et de Morale*, 1913, Vol. 20, n 1, p. 101–106.). It is, however, not clear whether Ginsberg was familiar with Vasil’ev’s work (Bazhanov 1988, p. 79–80).

<sup>31</sup> Ivanovski 1911.

mathematical logic could not only adequately express the specifics of reasoning in modern mathematics, but it also claimed to be a means of proving the consistency of mathematical theories, and in particular of solving the paradoxes of set theory. Thus, the attention of the best logicians of the time was focused on the development and investigation of predicate calculus in relation to problems in the foundations of mathematics. Another reason for the neglect of Vasil'ev's ideas, especially in the Soviet Union in the first half of the 20th century, was that Vasil'ev's project was often interpreted as an attempt to apply formal logic to real contradictions; contradictions, which, according to Hegel, constituted the object of dialectics and its logic. Considered as one of the sources of Marxism, Hegel's 'Dialectical Logic' had adopted at that time the place of a philosophical paradigm. One of the few Soviet scholars of the time who understood the novelty and importance of Vasil'ev's ideas was a mathematician with a strong interest in philosophy, Nikolai N. Luzin, who in reference to Vasil'ev's work, compared his criticism of the law of Excluded Middle with that of the intuitionists.<sup>32</sup>

In the first half of the 20th century in the Soviet Union 'Formal Logic' was mostly considered as a purely 'formal' discipline without applicability to reality. The combination of 'Formal' Logic and 'Dialectics' was at times an uneasy one in Soviet times.<sup>33</sup> However, in the second half of the 20th century the tide turned and the interest in Vasil'ev's ideas started to grow again.

### 3. A Logician's Progress

In a letter to William James in 1909, C.S. Peirce writes the following about his discontent with traditional Aristotelian logic and the laws prevailing there; "I have long felt there is a serious defect in the existing logic that it takes no heed of the *limit* between two realms. I do not say that the Principle of Excluded Middle is downright *false*; but I *do* say that in every field of thought whatsoever there is an intermediate ground between *positive assertion* and

<sup>32</sup> Luzin 1927.

<sup>33</sup> Bocheński 1950, p. 133ff., Wetter 1956, pp. 539–566. The main problem was the relation between 'Logic' and Marxist Dialectics. Put briefly, during the first half of the 20th century formal logic under the Soviet regime was first considered as foreign to dialectics, subsequently it was considered as an essential part of it and then again it was considered as a discipline in its own right, but with no links to dialectics. Writing in 1956 on the situation of logic in the Soviet Union, Wetter has the following prophetic words: "Dies sind einige Ansätze, die in die Richtung einer geplanten 'dialektischen Logik' weisen. Etwas Ganzes und Abgeschlossenes darüber liegt noch nicht vor" (p. 566).

*positive negation* which is just as Real. Mathematicians have always recognized this, and seek for that limit as the presumed lair of powerful concepts; while metaphysicians and old-fashioned logicians — the sheep & goat separators — fail to recognize this. This recognition does not involve any denial of existing logic, but rather a significant addition to it.”<sup>34</sup> At the turn of the twentieth century, the notion of a non-Aristotelian logic was still somewhat vague and only the abstract possibility of its construction was entertained. In particular, the work by Boole, Frege, Schröder and Russell had also cast doubt on the viability and applicability of Aristotelian logic and the idea of a non-Aristotelian logic became increasingly plausible, as indeed, it was felt, it became increasingly necessary. In 1910, P. Carus writes: “The world has seen many inventions. We can talk over the telephone at almost unlimited distances, and some of our contemporaries fly like birds through the air. Radium has been discovered which is often assumed, with a certain show of plausibility, to upset the law of physics, but the invention of non-Aristotelian logic would cap the climax.”<sup>35</sup>

Vasil’ev, who was at the time a professor at Kazan University, was familiar with Peirce’s work<sup>36</sup> and even more familiar with the work of Lobachevski who had been at Kazan University and in whose work on non-Euclidian geometry he found an important source of inspiration. It is Lobachevski who is explicitly mentioned and discussed in his paper on *Imaginary (non-Aristotelian) logic*.<sup>37</sup> Between 1910 and 1913, Vasil’ev published several papers representing the bulk of his research on Imaginary non-Aristotelian Logic. In developing his ideas, however, he was not only influenced by C.S. Peirce and Lobachevski. Other influences are worth mentioning too. Symbolist poetry, with the theme of ‘another world’, and Charles Darwin’s ideas on the evolution of life are cases in point here.<sup>38</sup> Thus, e.g., Vasil’ev would affirm that in ‘our world’ only positive sensations are possible that are at the basis of qualitatively different types of propositions.

In order to be able to appreciate Vasil’ev’s contribution to and divergence from certain ideas that were accepted in his time and that may clarify his

<sup>34</sup> Quoted in Fish and Turquette 1966, p. 81.

<sup>35</sup> Carus, 1910, p. 45.

<sup>36</sup> Cfr. Bazhanov 1992.

<sup>37</sup> Vasil’ev 1912a.

<sup>38</sup> Bazhanov 1998, p. 18 and 2001, p. 209–210; Bazhanov mentions several other influences such as Psychologism in logic. Let us note right away that we do not agree here, as we will prove later.

own ideas on logic, it is necessary to indicate the main ideas in his most important papers pertaining to the development of what he himself called Imaginary Logic.

In 'On particular propositions, the Triangle of Oppositions, and the law of excluded Fourth'<sup>39</sup>, Vasil'ev attempts to show that the 'traditional' classification of propositions in Aristotelian logic is incomplete because the usual particular propositions are ambiguous and one should, furthermore, distinguish between propositions about facts and propositions about concepts. The usual particular propositions are ambiguous because they may have two possible interpretations, viz. 'only some (not all) S are P' (strong particular propositions) and 'some (possibly all) S are P' (weak particular propositions). It follows that such a proposition may be true when either only some S are P or when every S is P. For Vasil'ev such a proposition is, then, some kind of 'indefinite proposition' which is not a real proposition since it ambiguously expresses two different ones. For science and natural language the only particular proposition that is important is the one that expresses 'Only some (not all) S are P'. In his view, this proposition says that 'some but not all S are P' and that the remaining S are not P. Therefore, the subject is taken in its full extension and as a consequence it is some kind of universal proposition.

It is reasonable to already assume here that Vasil'ev is diverging from the Aristotelian paradigm of classical syllogistics — though the divergence is syntactic rather than semantic — since his interpretation of particular propositions comes down to making a kind of pragmatic difference between two kinds of particular propositions. He does attack the (semantic) law of excluded third (replacing it with the law of excluded fourth in certain cases) but the other 'laws' (the law of identity, the law of (non)-contradiction and the principle of sufficient reason) remain unchanged. This attitude he will, however, change a few years later.

In 1910 Vasil'ev also introduced the difference between propositions about concepts ('rules') and propositions about facts, which shows that at that time he was not yet ready to fundamentally question basic logical law such as the law of 'excluded third (middle)'.<sup>40</sup> Vasil'ev explains the difference between propositions about concepts and propositions about facts as follows: "In the former case the singular proposition is a proposition about a concept, [considered as] a rule, and therefore the triangle of oppositions and the law of Excluded Fourth are applicable. So it is a rule that Caesar was a Roman, a genius and so on. It is, furthermore, a rule that he was not a Gaul and so on.

<sup>39</sup> Vasil'ev 1910.

<sup>40</sup> See also Stelzner 2000, p. 134 and 2001, p. 257.

It is also a rule that he was sometimes ill and that he was in Gaul, in Britannia. In the latter case, the singular proposition is a proposition about a fact, e.g. 'Ivan Ivanovich is now drunk' or 'yesterday morning at five o'clock NN died'. In such propositions the copula always contains a precise indication of a moment in time since the subjects of such propositions — perceptions and representations — are related to a definite moment in time".<sup>41</sup> It is with respect to these propositions of facts that the law of excluded middle remains valid: "We apply the law of excluded third only to reality (perception and representation). The perception of the sky at this moment, the sky as something real and not as a concept, can be either blue or not blue and a third predicate, a third proposition, is (in this case) not possible. But if we take the sky in the sense of a concept and we give it an enduring temporal character, then the predicate 'blue' only accidentally applies to the sky, which means that here a third possibility holds. Therefore, singular propositions are as ambiguous as are particular ones. The subject of a singular proposition such as Caesar, Goethe and so on, can be a concept. Such a proposition, then, symbolizes the whole of Caesar's or Goethe's life and subsumes the set of moments of the lives of Caesar or Goethe under one concept".<sup>42</sup> For propositions about facts, the law of excluded third is valid, but not for propositions about concepts. For propositions about concepts, according to Vasil'ev, "the law of excluded fourth" holds. This law is formulated in different ways, the most important of which<sup>43</sup> states that relative to each concept and predicate we can formulate three different propositions: one about the necessity of the predicate for the concept, another about its impossibility, and finally a third about its possibility. Just one of these propositions is true, and we can not formulate a fourth one.

The fact that for Vasil'ev the law of excluded third is only applicable to propositions about facts, and therefore constitutes some kind of an "empirical law" (not a law of thought), will be the basis of a further radicalization in his thinking which is prominent in his paper on "Imaginary (non-Aristotelian) Logic".<sup>44</sup> Here, Vasil'ev sets himself the task of describing and (partially) constructing an alternative to classical Aristotelian logic. This 'new logic' differs significantly from the kind of logic that was envisaged by Vasil'ev in 1910, even though no real system was worked out there. Vasil'ev now wants to considerably change the semantic basis of logic. This new

<sup>41</sup> Vasil'ev 1910, p. 51.

<sup>42</sup> Vasil'ev 1910, p. 50–51.

<sup>43</sup> Vasil'ev 1910, p. 50.

<sup>44</sup> Vasil'ev 1912a.

logic should be founded on different principles and on a fundamentally revised logical ground. This ground is to be found through a careful consideration of certain laws and principles underlying traditional logic: “The aim of this paper is to show the possibility of a logic and of logical operations different from those we use and to show that our Aristotelian logic is only one of the many possible logical systems. This new logic will not be a novel account of the old one. It differs from it not as an account, but in the very train of its logical operations; this is a “new logic” and not a new treatise on logic. Different treatises on logic differ in their contents, but all have the same subject matter: our logical world, our logical operations. Imaginary (non-Aristotelian) logic is different from our logic — which I call ‘Aristotelian’ after its first systematist — in its very subject matter.”<sup>45</sup>

This logic, which he calls ‘Imaginary Logic’ by analogy to the ‘Imaginary Geometry’ of Lobachevski is to be an instrument of knowledge of ‘imaginary’ worlds, i.e. worlds constructed by our mind. In attempting to show the feasibility of such a logic, Vasil’ev starts from the idea that the soundness of the laws implicit in Aristotelian logic depends on the structure of the world around us, and is therefore in a sense empirical. Furthermore, since logic is the synthesis of several independent axioms, replacing one or more of these axioms (laws) by different ones should result in a different system that can still be called a logic in its own right. Whether one considers logical laws to be descriptive-psychological, normative, ‘ideal truths’ or generalizations from experience, from each of these points of view the construction of a different (‘imaginary’) logic follows, albeit for different reasons. So e.g. the idea of logical laws as ideal truths, interpreted by Vasil’ev as a kind of formalist approach, is seen as a possibility to create ‘imaginary objects’ and a logic thereof in the same way as in mathematics: “In such a conception, the laws of logic come close to the axioms of mathematics. But then it is completely impossible to defend the unicity of logic. Precisely mathematics provides us with rigorous scientific instances of ‘imaginary disciplines’, as, for example, non-Euclidean geometry. At each step (in its development) mathematics involves a generalization of its operations and the extension of the field of its objects. In this way it moves, for example, from the real numbers to the imaginary numbers. It is impossible to extract the square root of a negative number, since every squared real number is positive; but mathematicians, by introducing imaginary numbers, can extract the square root of a negative number and [thus] give a more general characterization of the operation of ‘taking the root of’. Exactly in the same way as mathematical operations can be generalized, logical operations can be generalized too, and

<sup>45</sup> Vasil’ev 1912a, p. 53–54.

in both cases this generalization may lead to the creation of imaginary objects.”<sup>46</sup> From the laws that underlie traditional logic, Vasil’ev, now, picks out *the law of (non)-contradiction* and explicitly defines imaginary logic as a logic without this law. In Vasil’ev’s view, this law essentially expresses the incompatibility between an affirmation and a negation, i.e. *A is not not-A* or, *an object cannot have a predicate which contradicts it or is incompatible with it*. However, the only logical foundation for negation is precisely the incompatibility of predicates that express properties. Since negation is founded on the incompatibility of predicates, and the law of contradiction expresses this incompatibility, the law of contradiction is already implicit in the definition of negation. The law of contradiction can not, then, be denied in Aristotelian logic. Negative propositions are, furthermore, obtained as inferences (by means of some syllogism) from propositions about the incompatibility of properties, since we do not have an immediate sense-perception of ‘negativity’: *“All negative propositions about objects and perceptions of our world are obtained as inferences derived from propositions about the incompatibility of two properties. I cannot see in a direct way that a given object is not white. We have no negative perceptions, as e.g. the perception of [being] “not white”. I can have only definite positive perceptions of e.g. red, blue, black, etc.. When I assert that a certain object is not white, I have undoubtedly made an inference. I saw that a certain object was red, and I have inferred — knowing that red cannot be white — that the object was not white.”*<sup>47</sup>

Constructing a ‘different’ or ‘imaginary’ logic, then, implies constructing a logic with a negation different from our negation. This different kind of negation is epistemologically founded on the possibility of ‘directly’ perceiving negative states of affairs or, alternatively, of having instantaneous negative sense-perceptions in the same way as we have instantaneous affirmative sense-perceptions in our real world. Every negative proposition has both a material and a formal aspect. The first one, the logical foundation of negation, is that a negative proposition such as ‘S is not P’ is founded upon the incompatibility of predicates and is therefore empirical, while the second one pertains to a property of negation to the extent that a negative proposition ‘S is not P’ expresses the falsehood of an affirmative proposition ‘S is P’.

<sup>46</sup> Vasil’ev 1912a, p. 56–57.

<sup>47</sup> Vasil’ev 1912a, p. 60–61.

Denying the law of contradiction implies (merely) changing the material (empirical) aspect of negation, while denying the formal aspect of negation would mean violating what Vasil’ev calls *the law of Absolute Difference between Truth and Falsehood*, which he also calls ‘the law of Non-Self-Contradiction’ (‘one and the same proposition cannot be both true and false’). It is a law that prevents the subject from contradicting him/herself and therefore acts as a coherence-preserving principle that structures logical activity. This law belongs to what Vasil’ev calls Metalogic, which is the science of structures valid for every logical system. “*That is how matters stand in our world and in our logic. But let us imagine a different world where negative propositions would be just as immediate as affirmative propositions [are] for us, where experience itself would convince us without any inference that “S is not P”*”. Such propositions would remain negative, since they would retain the formal property of our negative propositions, which consists in stating the falsehood of the affirmative ones. But this negation would be different from ours, since it would be based on immediate perception, and not on propositions about incompatibility or inferences from them. It would have a different material aspect. In other words, in our world immediate perception provides us with only one kind of propositions — the affirmative ones — but we can image a logical world and a logic in which immediate perception generates two kinds of propositions: affirmative ones and negative ones. [...] It is not difficult to see that the law of contradiction and the law of absolute difference between truth and falsehood are not two formulations of one and the same law at all, but two completely different laws. The law of absolute difference between truth and falsehood applies to the cognizing subject and forbids him/her to contradict him/herself; [it] indicates that a true proposition is always true, and a false one is always false, and that therefore he/she cannot declare one and the same proposition now true, now false. This law forbids self-contradiction; [it] imposes ‘self-consistency’, the coherence of propositions of the cognizing subject. Therefore, it could be called the law of ‘non-self-contradiction’ ”.<sup>48</sup> In imaginary logic the law of contradiction may be rejected but not the law of Absolute Difference between Truth and Falsehood.

The existence of incompatible predicates (and properties) is the basis of affirmative and negative propositions (in Aristotelian logic). However, if one imagines a world in which ‘direct negative perceptions’ do exist, a third kind of propositions may be distinguished. Vasil’ev calls these “indifferent propositions”, which assign to a subject contradictory properties ‘S is P and not P (simultaneously)’. In these worlds the simultaneous occurrence of two incompatible facts a and b may form the foundation for an affirmative and

<sup>48</sup> Vasil’ev 1912a, p. 63–64.

a negative proposition 'S is P' and 'S is not P' respectively. Because of the formal aspect of negative propositions, this would imply that 'S is P' and 'S is not P' are simultaneously true and false, which is impossible because of the law of non-self-contradiction. Therefore, there must be a third kind of proposition 'S is P and not P', which is true in this case.<sup>49</sup> Priest and Routley, though they consider Vasil'ev's words as rather obscure, try to give an example of such a proposition along the following lines: "That is, where  $\alpha$  is ground for the affirmative judgment "S is P" and  $\alpha^*$  ground for the negative judgment "S is not P", both  $\alpha$  and  $\alpha^*$  may obtain. In this case the indifferent judgment "S is P and not P" is true. What exactly this means is obscure. Vasil'ev allows substitutions of colour predicates for P. Presumably he has in view situations where the light or the glass (S) is, for example, green and not green, because positive sense perceptions inform us it is green and negative ones it is not green. More familiar examples such as the bent oar help make such scenarios quite intelligible and even a little tempting: the oar is bent, so visual sense-perception informs us (or seems to), and is also not bent, so tactual perception informs us. The fairly accessible claim "The oar is bent and not bent" might be taken as a working example of Vasil'ev's "S is P and not P."<sup>50</sup> This example is intuitively appealing and shows how even 'in our world' indifferent propositions are possible. These three kinds of propositions are, then, the three *qualitatively distinct* propositions by means of which Vasil'ev subsequently attempted to reformulate traditional syllogistics, and to develop a new theory of the syllogism.

Though that theory, in its practical exposition, may be wrought with difficulties<sup>51</sup> — one reason being that it is not fully worked out by Vasil'ev — it illustrates in Vasil'ev's view a general mechanism of enlarging syllogistics (in fact the same could be said about predicate logic) by adding extra 'dimensions'. These dimensions pertain to the number of qualitatively different propositions one is willing to accept in one's logic. The imaginary logic he envisages is of dimension 3, but could be extended to any dimension  $n$  larger than 3, provided of course it is accompanied each time by a law of exclusion of the  $(n+1)$ th quality (excluded third, fourth, fifth, and so on). The number of qualities one accepts would depend on the imaginary world under consideration (but there is also a general law of thought having as particular cases the law of excluded third, fourth, and so on). As was already hinted at in his 1910 paper, Vasil'ev now states that his imaginary logic may be realized as a logic of concepts, not of facts, in the same

<sup>49</sup> Arruda 1984, p. 478, Bazhanov 1990, p. 338.

<sup>50</sup> Priest and Routley 1989, p. 32.

<sup>51</sup> Cf. Arruda 1984, p. 479.

way as Lobachevski's (non-Euclidian) geometry is realized in the geometry of pseudo-spheres, once again stressing the close analogy that he considers there to be between his way of working and the method of constructing a geometry proposed by Lobachevski who, not surprisingly, called his geometry 'imaginary' (*voobrazhaemaia geometriia*) (the term 'non-Euclidian' became current somewhat later).<sup>52</sup>

Vasil'ev then goes on to give several interpretations of his imaginary logic (or, rather, the different kinds of propositions in it). That these interpretations need not be founded upon epistemological considerations but may e.g. also be founded upon a *logic of similarity and difference* is well illustrated by the following: "But other interpretations of imaginary logic could be given as well. Consider the following. Suppose an affirmative proposition designates the similarity of two phenomena. The affirmative propositions of our logic can be interpreted in that sense, too. The proposition "the rose is red," signifying the inclusion of the rose in the class of red objects, also expresses the rose's similarity to red objects. Let us assume, then, that negative propositions do not express an incompatibility, as with us, but difference, an absolute difference or an absolute dissimilarity. Then, clearly, propositions are possible that simultaneously express similarity and dissimilarity between two phenomena, i.e., indifferent propositions. In our world too, two similar objects are at the same time also different."<sup>53</sup> On the whole, the picture we now get is one of a distinction between at least two kinds of logic. On the one hand there is 'empirical logic' (including the several 'Imaginary Logics') the properties of which are affected by certain basic properties of the (imaginary) world, and which is (at least partly) determined by an empirical content.

However, on the other hand there is also some other kind of logic which was hinted at in our discussion of the law of Excluded Fourth, and which Vasil'ev calls *Metalogic*. Metalogic is not quite what we would nowadays understand by it, but the term was coined by Vasil'ev by analogy to Metaphysics. It is a logic beyond empirical considerations (even in 'imaginary logics' there are 'empirical' considerations, albeit not the ones that are prevalent in our world). Metalogic contains the minimum logical knowledge that is present in all possible logics (real and imaginary). The propositions belonging to metalogic are the ones that cannot be eliminated from logic without it ceasing to be logic. Metalogic is, then, the invariant 'minimum logic'

<sup>52</sup> Kline 1965, p. 315. So the name 'Imaginary Logic' does not originate in "the fact that Vasil'ev did not believe that contradictions do exist in our real world, but only in a possible world created by our mind" as put forward in Arruda 1989, p. 102.

<sup>53</sup> Vasil'ev 1912a, p. 87 (The same may done in terms of the difference between a relative and an absolute negation as Vasil'ev continues to explain).

that is common to all logics. Logics may be many but metalogic is one. Here is what Vasil'ev himself says about metalogic and its relation to empirical logic: “Metalogic is the knowledge of thought regardless of the conditions of experience. Metaphysics is the science of pure being. It constitutes an abstraction from the world of phenomena, and it is the knowledge of that, which is common to empirical things. Metalogic is a discipline of pure thought. It is an abstraction from everything in thought that is empirical. There may be many worlds, but the essence of being is one. Such is the basic premiss of metaphysics. There may be many logics, but they all have something in common, which is only one, viz. Metalogic. Metalogic, then, is the discipline of the formal aspect of thought regardless of its content. Therefore, the only formal logic is metalogic. Our so-called formal logic is, in fact, not formal, since it does not completely abstract from the content of thought. For example, the law of contradiction is a material principle. Therefore, we should carefully distinguish between metalogic and empirical logic. One could also define these two disciplines as follows: Metalogic is the science of propositions and inferences in general. Empirical logic is the science of those kinds of propositions and inferences that correspond to our world.”<sup>54</sup> This allows Vasil'ev to affirm that the method of construction for imaginary logic will greatly advance axiomatic investigations in logic (analogous to Lobachevski's contribution to geometry).

The importance of what Vasil'ev calls metalogic is even more prominent in his paper “Logic and Metalogic”<sup>55</sup> in which he, in addition to repeating familiar points on the nature of imaginary logic, tries to say more about the nature of his metalogic. An important element here is that he considers it to be a logic of exclusively affirmative propositions. In support of this, he shows that the difference between affirmative and negative propositions — usually taken for granted in logic — is in fact not valid for metalogic.<sup>56</sup> Beginning with negative propositions, it can be shown that they are of basically two forms, one with an ‘internal’ negation such as ‘S is non-P’ and one with an ‘external’ negation, such as ‘Not (S is P)’. According to Vasil'ev the first is clearly affirmative because of its form whilst the second may be paraphrased as ‘The proposition S is P is false’, and is therefore connected to the avoidance of error. Avoidance of error or misconception, however, relates to knowledge (of the world) and the possible lack thereof. Therefore: “Negative propositions presuppose knowledge because of their formal aspect

<sup>54</sup> Vasil'ev 1912a, p. 89–90.

<sup>55</sup> Vasil'ev 1912–1913.

<sup>56</sup> Vasil'ev 1912–1913, pp. 117ff.

and negative propositions may be considered as affirmative propositions because of their formal aspect. Since metalogic is a formal logic, it should deal only with affirmative propositions. Consequently, the only aspect of negative propositions that is important for metalogic is the formal aspect."<sup>57</sup> In other words, only where knowledge is involved (knowledge of the world), can there be 'real' negative propositions. Recall, however, that Vasil'ev takes these propositions to be affirmative. They are secondary forms of affirmative propositions. To quote Vasil'ev: "If we utter a negative proposition, we have reason to believe it is true. This belief may be expressed in the affirmative form which is equivalent to the given negative form. Let me explain this with an example: when I say 'The horse is of a non-black colour', I may have some reason for doing so, e.g. the white colour of the horse. Therefore, the negative proposition 'The horse is of a non-black colour' may be replaced by the affirmative proposition: 'The horse is white.'"<sup>58</sup> However, when one considers metalogic, no such (empirical) knowledge is required. Vasil'ev therefore also calls it 'divine logic, the logic of perfect knowledge, a logic without negative propositions.'<sup>59</sup> Metalogic is, thus, 'positive logic'. It is a logic without negation and it will, in Vasil'ev's view, only contain syllogisms with positive propositions. Since it has only one form of propositions, it will be characterized by the law of excluded second and, as Vasil'ev also states, by 'the Law of the Perfection of Knowledge and the Impossibility of Error'<sup>60</sup>. Without going into the 'metaphysics' of this point of view as to what exactly 'perfect knowledge' would be like or what is so divine about it, we can now see that Vasil'ev is able to generalize his *law of exclusion of the (n+1)th quality*.<sup>61</sup> Metalogic is a logic with the law of excluded second, Aristotelian logic will be a logic with a law of excluded third and, for any imaginary logic of 'dimension' (quality)  $n$ , there will be a law of excluded  $(n+1)$ th.

<sup>57</sup> Arruda 1984, p. 482. In connection with Vasil'ev Arruda, uses 'judgement(s)' whereas we would prefer 'proposition(s)' (as does, e.g., also Kline 1965, p. 318).

<sup>58</sup> Vasil'ev 1912–1913, p. 119.

<sup>59</sup> Vasil'ev 1912–1913, p. 118.

<sup>60</sup> Vasil'ev 1912–1913, p. 119. One may add here (cfr. Priest 2000, p. 142) also the subalternation relation between the two positive forms in the square of opposition, conversion of these two forms, the laws of identity and sufficient reason and the law of Absolute Difference between Truth and Falsehood that we discussed earlier.

<sup>61</sup> Cf. also Arruda 1984, p. 482.

We have not, up to now, paid much attention to the actual construction of imaginary logic which is, in Vasil'ev, closely related to the presuppositions of Aristotelian logic (albeit in a negative way in the form of a critique). We will, therefore, now go into more detail regarding the relation between Aristotle and Vasil'ev in order to subsequently both evaluate Vasil'ev's presumed Psychologism and his place in the history of logic and to suggest ways in which his ideas might, even today, still be of critical importance.

#### 4. *Vasil'ev and Classical Logic: Aristotle revisited*

Both the law of contradiction and the law of excluded middle are, for Aristotle, in the first place ontological, and only secondarily logical laws. They are primarily concerned with the forms of being itself, and only thereafter are they forms of speech (*logoi*). They are not laws applying to concepts as such, since there are in Aristotle no concepts as forms of thought separate from the forms of reality (such a viewpoint will appear later, in Kant).

According to Aristotle, the principal logical forms are the forms of affirmation ("A is B") and negation ("A is not B."). The meaning of affirmation consists in fixing — in the form of speech — a real relation between a subject (either an individual or a universal) and a predicate (a species, a genus or a property of the subject). The meaning of negation consists in fixing the absence of such a relation. In other words, affirmation is a form of speech in which the concrete being of the subject (as being such and such) is expressed by means of the copula "is"; negation is a form of speech, in which the non-existence of the subject (as being such and such) is expressed by means of the copula "is not." Since there is no intermediate between the existence of a thing as such and such and its non-existence, the pair of oppositions consisting of the affirmation "A is B" and the negation "A is not B" is subject both to the law of contradiction (they are contrary) and the law of excluded middle (they are contradictory). Aristotle writes: "But in the case of affirmation and negation, whether the subject exists or not, one is always false and the other true. For manifestly, if Socrates exists, one of the two propositions 'Socrates is ill', 'Socrates is not ill', is true, and the other false. This is likewise the case if he does not exist; for if he does not exist, to say that he is ill is false, to say that he is not ill is true. Thus it is in the case of those opposites only, which are opposite in the sense in which the term is used with reference to affirmation and negation, that the rule holds good, that one of the pair must be true and the other false."<sup>62</sup>

<sup>62</sup> Aristotle, Cat. 10, 13b27–35.

It follows from this that the law of contradiction and the law of excluded middle for the oppositions between affirmation and negation have absolute validity: no condition, even the condition of the mere existence of the subject of the proposition, is necessary for their being satisfied. If the subject of an affirmation and a negation exists, then it is such and such or it is not such and such, so that both laws are valid, though we may not know which one of the two propositions is true. If the subject does not exist, then it has no properties whatsoever and therefore negation (which negates its having a property expressed by the predicate) is true, and the subject is not such and such. Affirmation, stating that the subject possesses a particular property, is then false. Thus, even in the case of non-existence, the law of contradiction and the law of excluded middle are satisfied.

A "looser" kind of opposition, according to Aristotle, is the opposition between possession and privation. For example, "Socrates sees" ("Socrates has eyesight") is a (positive) proposition, expressing possession, while "Socrates is blind" is a proposition, expressing privation (of sight). These propositions fall under the law of contradiction — they are contrary to each other. But in contrast to affirmation and negation, propositions of possession and privation fall only conditionally under the law of excluded middle. Specifically, the condition of falling under the law of excluded middle consists in the possibility for the subject (according to its nature) of possessing the given property or being devoid of it. Socrates, according to his nature, should have sight; therefore one of the two opposites — whether he has sight or is blind — is true. But if he does not exist at all, then he is devoid of any property he may have according to his nature and therefore both propositions of him having sight and being blind are false. The same can be said of a subject that cannot possess a given property nor, correspondingly, lack it. Propositions such as "the stone has eyesight" and "the stone is blind" are both false. Thus, for propositions of possession and privation the law of contradiction is satisfied, but the law of excluded middle is satisfied only conditionally. Since there are no (formal) logical means to decide whether or not a subject may possess a given property by its nature, we should conclude that for propositions expressing the opposition of possession and privation, the law of excluded middle does not hold.

An important point to be made is that propositions of privation are, according to their form, affirmations as much as propositions of possession. Consequently, they are not opposed to each other as affirmation and negation, but, rather, as one affirmation to another. Let us dwell at some length on the question why propositions of privation are affirmations or, rather, why they cannot be considered as negations.

The meaning of the negation "A is not B" consists in negating the existence of the thing A as being such and such (under the property B). The meaning of a proposition of privation, which has the form "A is non-B," is different.

Such a proposition does not negate the existence of the thing itself (if the thing does not exist, then the proposition of privation is false!), but rather the possession by the thing of the ascribed property. The property in question is not arbitrary, but is that which can be possessed by the thing according to its nature. Consequently, the negative particle "non" is not associated with the copula "is," which expresses the being of the thing, but with the predicate. Kant called such propositions "infinite propositions." Affirmations and negations differ from propositions of possession and privation in that they have no intermediates. One of them is true and the other is false. The opposition of possession and privation does have such an intermediate. This is the very possibility of possessing a certain quality, and accordingly, of being devoid of it. Aristotle says of the predicate "to be in error," which belongs to the kind of predicates expressing privation (in this case the privation of knowledge): "What is generally thought to be in error is not that which has no knowledge, but rather that which has been deceived, and for this reason we do not talk of inanimate things or of children as 'erring.' 'Error', then, is not used to denote a mere privation of knowledge."<sup>63</sup> Another example of the opposition between propositions expressing privation and negations are the predicates "to be equal" and "to be unequal." Between the affirmation "these two things are equal" and the negation "these two things are not equal" there is nothing intermediate, so that these two propositions are contradictory. Contrariwise, there is an intermediate between the predicates "to be equal" and "to be unequal," and this intermediate is constituted by the possibility of possessing the predicate "to be equal." Aristotle writes: "... for everything is equal or not equal, but not everything is equal or unequal, or if it is, it is only within the sphere of that which is receptive of equality."<sup>64</sup> Furthermore: "for there is something underlying the one, viz. that which is not-equal, and this is the unequal, but there is nothing underlying the other. Wherefore not everything is either equal or unequal, but everything is equal or is not equal."<sup>65</sup> Therefore, in the case of possession and privation it is presupposed that an appropriate genus, and accordingly properties which can be ascribed to the individuals of this genus according to their nature, are fixed beforehand. If the propositions expressing possession and privation concern things which belong to this genus, then (and only then) are they contradictory, i.e. both the law of contradiction and the law of excluded middle hold there. Otherwise, if the universe of discourse is not fixed as an appropriate

<sup>63</sup> Aristotle, *Topics*, VI, 9, 148a2–10.

<sup>64</sup> Aristotle, *Metaphysics*, X, 4, 1055b7–11.

<sup>65</sup> Aristotle, *Anal. Pr.* I, 46, 51b26–27.

genus, only the law of contradiction holds, while the law of excluded middle is violated.

An important consequence of the violation of the law of excluded middle for propositions expressing possession and privation is that propositions of privation of the form "A is non-B" (we shall call them internal negations) and negations of the form "A is not B" (external negations) are not equivalent. Let us demonstrate this thesis using a conditional proof. Let us suppose that the propositions "A is non-B" and "A is not B" are equivalent. Then, since the propositions "A is not B" and "A is B" are contradictory (for they remain instantiations of negation and affirmation respectively), it follows that the propositions "A is non-B" and "A is B" are contradictory too. But the latter statement is false, since for those A, which fall outside the genus, i.e. whose individuals according to their nature may possess or be devoid of the property B, both the propositions "A is non-B" and "A is B" are false.

With respect to the proposition "A is non-B" we can construct a negation of the form "A is not non-B" which, as a negation, differs from the affirmation "A is B." Thus we have two pairs of propositions — the first being "A is B" and "A is not non-B," the other being "A is not B" and "A is non-B" — the elements of which are in various relations to each other. It is appropriate to discuss these relations using the propositions "a man is just," "a man is not unjust," and "a man is not just," "a man is unjust."<sup>66</sup> It follows from the discussion in *Anal. Pr. I, 46*, that these pairs are equivalent only because the subject of these propositions belongs to a genus ("man") which is supposed to possess or be devoid of justice (to be just, or to be unjust). If, however, a universe of discourse is not fixed at all or universe is fixed that is broader than the genus "man," such that things other than men fall under this genus (things that, according to their nature, cannot be just or unjust), then only one way implications are valid: "to be just" implies "not to be unjust" and "to be unjust" implies "not to be just". The converse implications do not hold, since that which is not unjust (or just), can be such that it is meaningless to say of it that it is just (or unjust).<sup>67</sup>

<sup>66</sup> Aristotle, *De Int.*, 10, 19b25–32.

<sup>67</sup> Another example from Aristotle concerns the proposition "this is good." This proposition has two different opposing propositions "this is not good" and "this is bad (non-good)." The affirmation "this is good" and the negation "this is not good" are contradictory. In contrast, the propositions "this is good" and "this is bad" are only contrary (and not contradictory). Since the proposition "this is bad" is an affirmation, and every affirmation has its negation, the negation of the affirmation "this is bad" will be "this is not bad": Therefore it is clear that "it is not-good" is not the denial of "it is good". If then every single proposition may truly be said to be either an affirmation or a negation, if it is not a negation clearly it must in a sense be an affirmation. But every affirmation has a corresponding negation. The negation then of "it is not not-good." (*Anal. Pr. I, 46, 51b31–35*). The latter proposition

Another, even "looser" kind of a non-contradictory opposition (in comparison with possession and privation) is represented by the opposition of the extreme species of the same genus, which Aristotle usually calls contrary opposites. The difference between the opposition of possession and negation on the one hand and that of extreme species on the other, consists in the following. In the case of possession and privation it is essential to fix a genus with respect to the subject of the proposition, within which assertions of possession and privation are meaningful. In the case of contrary opposites, both the genus of the subject (sometimes very broad) and the genus of the predicate (within which we speak about two extreme species) should be fixed.

In order to illustrate this latter kind of opposition, Aristotle often uses propositions like "this thing is white" and "this thing is black." Here two genera are fixed: "bodily substance" (which has color by its nature) with respect to the subject and "color" with two extreme species, viz. "white" and "black." "White," according to Aristotle, is a *piercing* and "black" is a *compressing* color.<sup>68</sup> Between these two extreme species there are intermediate species belonging to the same genus "color" but having different *differentia specifica* (specific differences). Between the propositions "this thing is white" and "this thing is black" there is a relation of contrariety, i.e. they cannot both be true at once: a white thing cannot be black and vice versa, such that the law of contradiction holds. But the law of excluded middle does not hold for contrary opposites, since it does not follow from the fact that a thing is not white that it is black, and vice versa; the thing in question can, e.g., be yellow. The cause of the violation of the law of excluded middle consists in the possibility of an intermediate species between the two extreme species of the same genus. These intermediate species consist of colors different from white and black. It may happen that a contrary opposition turns out to be a contradictory one. This happens when there are exactly two species (that is there are no other species in between) within the genus of which they are the extremes.

Therefore, for contrary opposites the condition of contradiction holds even more rarely than is the case of propositions of possession and privation. In the latter case, the fixation of the appropriate genus is sufficient. In the case

differs from the proposition "this is good." This means that the law of double negation does not hold either.

<sup>68</sup> Aristotle, *Metaphysics*, X, 7, 1057b7–18. In Plato's "Timaeus" (49b–e) a theory of vision is expounded, according to which particles emanate from bodies and cause different effects upon the (visual) flux issuing from the eye. These tiny particles pierce the flux, producing the perception of white color; those that consist of larger particles compress this flux, not allowing it to be dispersed, and thus produce the perception of black color.

of contrary opposites, the fixation of the appropriate genus to which the predicate of the proposition belongs is not sufficient. Whether the pair of the contrary opposites is contradictory or not depends on the specifics of the structure of the genus to which the predicates belong i.e., the contradiction holds only if this genus is constituted by exactly two (extreme) species.

The above-mentioned difference may also be expressed as follows. If in the case of possession and privation an appropriate genus is fixed, then within this genus for a given predicate B one can construct the negation non-B in a correct way, which is contradictory to the predicate B (which is impossible without fixing the genus). But for contrary opposites, even if an appropriate genus is fixed, the negation non-B (within this genus) may happen to be contradictory only by accident. As an example one can consider two ways of introducing a negation for the predicate “to be white.” Firstly, one can conceive of it in the form “to be non-white.” Then, within the genus “bodily substance” the properties “to be white” and “to be non-white” will constitute a specimen of possession and privation. Within this genus they are contradictory. Within a broader universe of discourse, which includes individuals that can be neither white nor non-white as, for example, numbers, these properties are not contradictory.

But one can also conceive of the property “non-white” differently, namely by defining it as the extreme species contrary to white within the genus “color,” that is, as “black.” In this case, the contradiction between the two predicates does not hold. Note that if the genus “color” is not fixed and one is allowed to choose a predicate of another genus, say, the predicate “dry,” as an opposite to the predicate “white,” then not only is the law of excluded middle violated, but the law of contradiction as well. A thing can be both white and dry. Aristotle calls such oppositions relative oppositions. “White” and “dry” belong to different genera and the genus to which the predicate “dry” belongs, is not subordinate to the genus “color,” and vice versa. Within its genus, “dry” is an extreme species with respect to “humid.”

We shall conclude our discussion on the difference between internal and external negation in Aristotle by one of his most convincing arguments in favor of such a differentiation, viz. the proposition “he is able to walk”: “In establishing or refuting, it makes some difference whether we suppose the expressions “not to be this” and “to be not-this” are identical or different in meaning, e.g. “not to be white” and “to be not-white”. For they do not mean the same thing, nor is “to be not-white” the negation of “to be white”, but “not to be white”. The reason for this is as follows. The relation of “he can walk” to “he can not-walk” is similar to the relation of “it is white” to “it is not-white.””<sup>69</sup> The external negation of this proposition is the proposition

<sup>69</sup> Aristotle, *Anal. Pr.*, I, 46, 51b5sq.

“he is not able to walk,” while the internal negation is “he is able not to walk.” The former negation belongs to a type of “not to be such (and such),” and the latter to the type “to be non-such (and such).” It is clear that the external negation is contradictory to the proposition “he is able to walk.” If, however, one supposes that the internal negation is equivalent to the external one, one is forced to admit that the internal negation is contradictory to the proposition “he is able to walk” too. At this point a contradiction arises. On the one hand, the propositions “he is able to walk” and “he is able not to walk” are contradictory and therefore cannot both be true. On the other hand, it is obvious that the same man is able to walk and is able not to walk. This contradiction shows that the identification of the external and internal negation is, generally speaking, incorrect.

To sum up what has been said about the kinds of oppositions according to Aristotle: only one of them — the opposition between affirmation and negation — falls unconditionally under both logical laws — the law of contradiction and the law of excluded middle. For the opposites of possession and privation, only the law of contradiction holds, while the law of excluded middle holds only if some non-formal conditions are satisfied (if a genus of the subject is fixed). For the contrary opposites of extreme species, only the law of contradiction holds; the law of excluded middle holds only accidentally, namely, when the genus of the predicate consists of exactly two species (and this condition is also substantial, i.e. non-formal). For relative opposites, when the subject is related to the predicates of different genera neither the law of contradiction nor the law of excluded middle hold. What is important is that the fact of validity or non-validity of these laws for the oppositions of the kind “A is B” and “A is non-B” heavily relies upon the way of defining the predicate “non-B” which in its turn relies upon the genus with respect to which the predicate B is ascribed, since within different genera we will have different predicates “non-B.” Anticipating our discussion of Vasil’ev’s ideas, let us notice that just this variety of the mode of negation (combined with the idea that external negation is derivative of internal negation) constitutes Vasil’ev’s principal motive for doubting the absolute value of the law of contradiction.

In the above exposition of Aristotle’s discussion of oppositions, we spoke about particular propositions in which subjects were always considered as wholes (individuals in the etymological sense of the word “in-dividua” — inseparable things). If we consider a proposition in which the predicate is predicated only of a part of the subject, then even for the oppositions of affirmation and negation we must — in order to preserve the validity of the law of contradiction and the law of excluded middle — additionally modify the forms of the propositions by discerning them not only according to quality (affirmative versus negative), but also according to quantity. That is, we have to divide propositions into universal and particular ones. If the

predicate "white" is predicated of a universal subject "man," then the propositions "man is white" and "man is not white" are, according to their form, affirmative and negative respectively; therefore, one of them should be true and the other false. However, none of these propositions is true with regard to the subject "man" as a whole. The indicated contradiction is resolved in Aristotelian logic by introducing the difference between universal and particular propositions. Let us, therefore, quickly repeat the main outlines of this theory.

In "On interpretation" (Part 7) Aristotle begins his analysis by indicating the difference between singular (individual) and universal terms. He writes: "Some things are universal, others individual. By the term 'universal' I mean that which is of such a nature as to be predicated of many subjects, by 'individual' that which is not thus predicated. Thus 'man' is a universal, 'Callias' an individual." Both universal and individual terms may constitute the subject of a proposition: "Our propositions necessarily sometimes concern a universal subject, sometimes an individual." Propositions concerning an individual subject are propositions in which a predicate is predicated of the whole of a given thing. Among the propositions about universal things there is a difference between those that are of a universal character and those that are not. As examples of propositions concerning a universal and having a universal character Aristotle gives "every man is white" (universal affirmative) and "no man is white" (universal negative). As examples of propositions concerning a universal and not of universal character Aristotle gives "man is white" and "man is not white."<sup>70</sup> Let us notice that in "On Interpretation" Aristotle speaks about the oppositions of two kinds of propositions — of universal and not of a universal character — concerning a universal subject; there is no mention of particular propositions.

In the "Prior Analytics," anticipating the exposition of his theory of syllogism, Aristotle proposes another classification of propositions which later became the standard account. All propositions concerning universal subjects are divided into three kinds — universal, particular and indefinite ones. Aristotle writes: "[A premiss] is either universal or particular or indefinite. By universal I mean the statement that something belongs to all or none of

<sup>70</sup> "If, then, a man states a positive and a negative proposition of universal character with regard to a universal, these two propositions are 'contrary'. By the expression 'a proposition of universal character with regard to a universal', such propositions as 'every man is white', 'no man is white' are meant. When, on the other hand, the positive and negative propositions, though they have regard to a universal, are yet not of universal character, they will not be contrary, albeit the meaning intended is sometimes contrary. As instances of propositions made with regard to a universal, but not of universal character, we may take the propositions 'man is white', 'man is not white'. 'Man' is a universal, but the proposition is not made as of universal character; for the word 'every' does not make the subject a universal, but rather gives the proposition a universal character." (De Int., 7, 17a38–b13)

something else; by particular that it belongs to some or not to some or not to all; by indefinite that it does or does not belong, without any mark to show whether it is universal or particular, e.g. 'contraries are subjects of the same science,' or 'pleasure is not good.'<sup>71</sup>

The division of propositions concerning universal subjects into three groups allows Aristotle to save both his axioms — the law of contradiction and the law of excluded middle — with respect to propositions in which the predicate can be distributed into parts of the subject.

The traditional point of view, going back to Aristotle (with an exposition of which Vasil'ev, incidentally, begins his paper from 1910<sup>72</sup>) consists in the belief that propositions concerning concepts (Aristotle calls them universals) are divided according to quality into affirmative and negative ones, and according to quantity into universal and particular ones. Thus there exist four kinds of propositions — universal affirmative ("all A are B"), universal negative ("no A is B"), particular affirmative ("some A are B") and particular negative ("some A are not B"). These four kinds of propositions — in scholastic logic they were denoted by letters A, E, I and O, correspondingly — are located in the vertices of a square which is called the Square of Oppositions. Aristotle himself indicates that the propositions "all A are B" and "no A is B" are contrary, but not contradictory. In other words, for them the law of contradiction holds (both propositions cannot be true at once), but the law of excluded middle does not hold (if one of them is false, it does not imply that the other is true). Contradictory are the following pairs of propositions (they are located along the diagonals of the square): "all A are B" — "some A are not B" and "no A is B" — "some A are B". One proposition in each pair is necessarily true and the other is false, which means that for them the law of excluded middle holds. Let us notice that though not all of the affirmative and negative propositions are contradictory, for every one of the propositions A, E, I, or O there is an proposition which is contradictory to it. In other words, for each of the propositions A, E, I, or O there is proposition such that the law of excluded middle holds with respect to this proposition and its opposite (its contradictory).

In once more anticipating our exposition of Vasil'ev's logic, let us note that Vasil'ev begins his criticism of the law of excluded middle by criticizing the traditional division of propositions into universal and particular ones. For Aristotle, both laws are primary and the forms of the propositions — universal and particular — are secondary, which preserves the validity of the aforementioned laws for propositions concerning universals.

<sup>71</sup> Aristotle, *Anal. pr. I*, 1, 24a16–21.

<sup>72</sup> Vasil'ev 1910, pp. 12–52.

Let us review some of the problems which are solved as a result of the introduction of the universal/particular distinction. They are well known: First of all, the distinction between universal and particular propositions together with the distinction between affirmation and negation allows us to formulate the negation of a proposition in such a way that it is contradictory to the proposition in question (we have already had cause to mention this fact). Secondly, this distinction allows us to convert the terms in propositions. For a logic which, in the form "A is B" purports to express a real relation between the thing A and the thing B and in the form "A is not B" the lack thereof, it is necessary to maintain the possibility of converting the terms in question, that is, of turning its predicate (thing) B into a subject, and its subject (thing) A into a predicate so as to obtain the (converse) relation. The difference between universal and particular propositions constitutes a necessary basis for conversion, be it 'simple conversion' or 'conversion by limitation'. For example, the proposition "man is an animal," expresses the real connection between the species "man" and its genus "animal." The converse relation of the genus "animal" to the species "man," which for Aristotle is not only logical, but ontological as well (according to Aristotle there is no difference between an animal and a man at the embryonic stage), can be expressed by the particular proposition "some animals are men." Finally, the contradictory negation together with the conversion of the premisses allows us to formulate of a large number of moods, which complete the theory of the syllogism (especially with the introduction of the moods of the fourth figure).

This purely technical aspect of Aristotelian logic, which is based on the idea that every concept is determined by its extension and vice versa (the extensional viewpoint), has been much appreciated by those schools of philosophy that were oriented towards the development and the external correspondence of conceptual forms.

However, such an approach to Aristotelian logic does not take into account that the difference between universal and particular propositions which preserves, within the framework of syllogistics, both Aristotelian axioms, is hardly compatible with another axiom of Aristotelian ontology, namely the genus-species structure of the universe.

The problem of the relation of syllogistics to the genus-species ontology is a rather difficult one. We will focus only on those aspects of the problem that are directly related to the logical reform proposed by Vasil'ev.

On the one hand, it is rather obvious that Aristotelian syllogistics grew out of Aristotle's genus-species ontology. The key text that can be referred to in support of this claim is the third part of the *Categories*. There, Aristotle considers two possibilities of predicating individuals of a given species by a genus, depending upon whether this species is subordinate or not to the

genus. This ontological consideration serves as a prototype for the first two moods of the first syllogistic figure, something also noticed by Hegel.<sup>73</sup>

On the other hand, the genus-species ontological hierarchy can explain only those moods containing universal premisses. Indeed, the genus-species ontology is rather favorable to the point of view that the subject is considered as a whole and that the predicate is predicated of it as an individual and not as distributed. But this same idea of the distributed predicate lies at the bottom of the distinction between universal and particular propositions and of syllogistics in general. According to the genus-species ontology, a genus is either predicated of a species or not predicated of it, and in both cases it is predicated or not of the species as a whole, that is, of all the individuals of this species. It is, within the genus-species ontology, meaningless to differentiate between universal propositions such as "all men are animals" and particular propositions such as "some men are animals."<sup>74</sup>

<sup>73</sup> See also Lukanin 1984, p. 104–105.

<sup>74</sup> Another striking example of the incompatibility between the formal — extensional — treatment of propositions and Aristotelian metaphysics is the following. Aristotle sees the ground for a correct syllogism in the existence of a strict ontological subordination between its terms. Let us consider a standard syllogism of the first figure: "all A are B," "all B are C," therefore "all A are C." In this syllogism, the major term C can be a genus ("animal") or an accident ("white"). The middle term B must be either a genus or a species ("man," "ox"). It can also be a property derived from a substance of the genus ("grammarian" for "man"). The minor term A can be a species (genus) or an individual ("Socrates"). The most important consequence of the ontological restrictions on a correct syllogism is that the subject in both premisses cannot be represented by an accident, such as "whiteness," which can appear only in the role of the predicate in the major premise (Lukanin 1984, p. 110). The reason for the rejection of accidents as candidates for the middle term involves the fact that the substance of the middle term, which enters as a subject in the major premise, constitutes in a demonstrative syllogism the very essence of a thing, the principle of its generation. "Therefore, as in syllogisms, substance is the starting-point of everything." (Aristotle, *Metaphysics*, VII, 9, 1034a30–32). It is the middle term, which reveals the cause of the thing and therefore of the syllogism which is based on it, which generates knowledge. Aristotle says about the middle term that it "must be consequentially connected with the minor, and the major with the middle." (Aristotle, *Anal. Post.* I, 6, 75a36–37). If an accident is allowed to take place of the subject, the following parody of a syllogism (known well before Aristotle) would be possible:

White (whiteness) is a color,  
Socrates is white, therefore  
Socrates is a color.

The invalidity of this syllogism is a result of "the part-whole relation" (which is at the foundation of the inference in a syllogism) which holds, in Aristotle's view for substantial relations of the type "a thing — its species or genus" only. "Whiteness" is not a genus which "man" could participate in as its essence. "Although the class of "white things," from the point of view of its extension, is larger than, say, the concept of "an individual white man" (Socrates), it does not constitute its genus, and in this case it is possible only to speak about

The only possible way of constructing particular propositions within the genus-species ontology is by conversion with restriction to propositions like “All A are B” which express the subordination of the species A to the genus B. As a result of conversion, a form of speech would originate which represents an inverse relation of the genus B to its species A, and this relation can be interpreted as the particular proposition “some B are A.” However, the meaning of the latter is different from the meaning which is usually associated with the concept of particular propositions. Indeed, according to Aristotle, a genus must always be larger than any one of its species. Consequently, the meaning of the converse proposition in question cannot be reduced to the standard “some (maybe all) B are A,” which lies at the basis of syllogistic inference, but to a different, more precise formula “some (not all) B are A.” Notice that it is just this form which Vasil’ev will assess as the authentic meaning of particular propositions in contrast to the standard version. But it is impossible to develop the standard Aristotelian syllogistics based on such a restricted version of particular propositions.

Concluding our discussion on the universal/particular distinction in Aristotle, let us note that the main motivation for its introduction was the wish to preserve the validity of the law of excluded middle with respect to propositions (affirmations and negations) with a distributed predicate predicated of a universal subject. Leaving aside the difficult question of how it happened that Aristotle entered into confrontation with his own principles, we will now turn to Vasil’ev’s logical ideas which, as we shall see, are directly related to the problematic coordination between Aristotelian logic and ontology.<sup>75</sup>

A two thousand year old tradition, going back to Aristotle, teaches that logic should be grounded in a set of unconditional principles. Traditional logic indicates four such principles: the law of identity, the law of contradiction, the law of excluded middle, and the law of sufficient reason. The first three go back to Aristotle, the last is attributed to Leibniz. These principles or laws can be understood in two different ways: either in a purely formal

the relation between terms according to their content and not their quantities.” (Lukanin 1989, p. 108).

<sup>75</sup> Note here, Paul Gohlke’s position on the origin of the difference between universal and particular propositions in Aristotle: Paul Gohlke, in *Entstehung der aristotelischen Logik*, Berlin, 1936, believes that the early logic of Aristotle (as that of Plato) neither acknowledged nor terminologically distinguished particular propositions as a special class of propositions, since it was essentially based upon the theory of ideas as separate substances considered as something whole and indivisible. “In such a theory, particular propositions have no metaphysical value” (p. 24). But later, according to Gohlke, Aristotle became more attached to empiricism and “his logic started to free itself from its platonic tenets and acquired a more formal character.” (p. 58). Moving in this direction, Aristotle later introduced particular propositions as a separate species.

way, as postulates of logic from which other logical laws can be inferred, (thus having a status similar to that of the postulates of geometry). Alternatively, this formal treatment can be seen to have a more substantial, meta-physical foundation in which these principles are considered to be logical laws because a) they are ontological principles and b) there is a correspondence between logic and being itself which allows us to transfer the laws of being into the sphere of logic. The latter interpretation was Aristotle's. He considered the traditional logical axioms — the law of contradiction and the law of excluded middle — as general laws (axioms) of being and evaluated them as laws of logic only afterwards. However, the formal treatment of logical laws also has a venerable tradition. The followers of Aristotle from the peripatetic school already separated logic from ontology and understood the former in a much more ‘formal’ way than their teacher. Thus, e.g., Alexander of Afrodisi considered the law of contradiction and the law of excluded middle as laws of thought only.<sup>76</sup>

The similarity between logic and geometry provoked Vasil'ev to pose the following question: is it possible to treat logic in the way Lobachevski and Bolyai treated geometry? Is it possible to change some logical postulates and preserve others? There is, on the one hand, such a thing as absolute geometry — geometry without the fifth postulate. But there are also several other geometries formed by adding to absolute geometry a fifth postulate or one of its negations. Thus, in geometry one can discern two parts: a part which comprises the first four postulates and is fixed, and another part which is variable. If logic is also based on axioms and in this respect is analogous to geometry, could we not try to reject some of its laws? Vasil'ev explores this possibility:

“The axioms of logic are as numerous as the axioms of geometry. What is there, then, to guarantee that one of the logical principles could not be rejected and replaced by another one? A geometer rejects the axiom of parallels, replaces it with another postulate and obtains a scientific system of imaginary (non-Euclidean) geometry. What guarantees that an imaginary non-Aristotelian logic, which is constructed by replacing one of our axioms while preserving the other, is impossible?”<sup>77</sup>

If such a transformation of logic were possible, and Vasil'ev is confident that it is, it becomes possible to discern two parts in logic. A part which comprises laws that are true everywhere and under all circumstances and another part which comprises laws that depend upon the specifics of the universe of discourse. Thus, in a very formal way, the idea emerges to divide

<sup>76</sup> Lukanin 1984, p. 59.

<sup>77</sup> Vasil'ev 1912–1913, p. 97–98.

logic into what Vasil’ev calls metalogic (the stable, fixed part) and empirical logic (the variable part).

As was mentioned before, in his papers Vasil’ev not only states that this possibility exists, but he also proposes a concrete method for the discernment in question. In his view, every logical law should undergo thorough scrutiny. If there is a universe of discourse in which a logical law does not hold, this law belongs to the level of empirical logic. To this category of laws belong, according to Vasil’ev, the Aristotelian axioms of the law of contradiction and the law of excluded middle.

Vasil’ev then divides the realm of ‘Being’ into two sets of “objects”: the first set comprises concepts, the second set comprises sense-impressions and perceptions that are caused by a contact with empirical reality. In accordance with this division, logic is likewise divided into two parts. The one part — metalogic — comprises laws that are valid both for propositions concerning concepts and propositions concerning facts of perception. The other — empirical logic — comprises laws that are valid only in one of the two spheres: either for propositions concerning concepts, or for propositions concerning facts. As was already indicated, the law of excluded middle holds with respect to propositions concerning concepts and therefore belongs to empirical logic (i.e. it can be rejected or replaced). The law of contradiction holds only when negation is taken within a fixed genus. The fixation of a genus has a non-formal, substantial character, so the law of contradiction relies upon non-logical hypotheses which depend upon the specifics of the universe of discourse. Therefore, it also belongs to empirical logic. Negation is an essentially non-formal operation, and consequently the logical minimum which metalogic consists of, does not comprise propositions formed using the notion of negation. Let us now turn to the details.

Vasil’ev’s already mentioned first paper, “On particular Propositions, the Triangle of Oppositions, and the law of Excluded Fourth,” issued in October 1910<sup>78</sup>, is mostly devoted to the demonstration of the thesis that the law of excluded middle holds only for propositions concerning facts of perceptions (that is, facts limited to a certain moment in time and place), but does not hold for propositions concerning concepts; the latter fall under the law of excluded fourth. In the course of his analysis, Vasil’ev also formulates some novel ideas concerning the understanding of logical negation.<sup>79</sup>

<sup>78</sup> Vasil’ev 1910.

<sup>79</sup> In that he follows Sigwart who also criticized the law of excluded middle and thought that “the law of excluded middle, the law of contradiction, and the law of double negation are destined to develop the essence and meaning of negation.” (quote from Sigwart, in Vasil’ev 1910, p. 47).

According to Vasil'ev the objects of logic are of two kinds: realities (of perception) and concepts. “As realities are the matter of the logical world, so concepts are its spirit, its cognizing souls... Logic deals with nothing other than realities and concepts.”<sup>80</sup> And it “operates with them” by means of propositions (or ‘judgments’), which Vasil'ev understands in a traditional way as predications of a subject (the role of which may be played either by a fact of perception or a concept) by a predicate.

Vasil'ev notices that the law of excluded middle can be formulated in two different ways: as a law concerning propositions (as it was understood by Aristotle), or as a law concerning predicates (as it is understood mainly in traditional scholastic logic). As a law concerning propositions, it is formulated in Aristotle in the following way: “But, on the other hand, there cannot be an intermediate between contradictories, but of one subject we must either affirm or deny any one predicate.”<sup>81,82</sup> In the form of a law concerning predicates, the law of excluded middle states the truth of one of the two propositions “A is B” and “A is non-B,” whereby non-B is understood as a predicate that is contradictory to B. In other words, the law of excluded middle states that of a thing (subject) A one of two contradictory predicates is necessarily predicated.<sup>83</sup>

Since the objects of logic are concepts and facts of perception, it is necessary to investigate whether the law of excluded middle holds for propositions concerning concepts, facts, or both.

Vasil'ev begins his analysis with the question of the applicability of the law of excluded middle to propositions concerning concepts.<sup>84</sup> Here he encounters problems, similar to those we have indicated above in the course of our review of oppositions in Aristotelian logic. These problems are closely related to the ambiguity of the notion of negation.

Earlier, we saw that Vasil'ev began his criticism of the law of excluded middle by indicating that in traditional logic particular affirmative propositions such as “some A are B” and particular negative propositions such as “some A are not B” are treated respectively as “some (maybe all) A are B”

<sup>80</sup> Vasil'ev 1910, p. 48.

<sup>81</sup> Vasil'ev also cites the formulations of the law of the excluded middle from the Port-Royal logic “truth and falsity of contradictory propositions are incompatible” and Christian Wolff “propositionum contradictariorum altera necessario vera.” (Vasil'ev 1910, p. 40).

<sup>82</sup> Aristotle, *Metaphysics*, IV, 7, 1011b23–25.

<sup>83</sup> On the difficulty of defining the predicate non-B, see above, and with respect to Vasil'ev, below.

<sup>84</sup> Vasil'ev 1910, p. 33ff.

and "some (maybe all) A are not B." Such a treatment, Vasil'ev remarks, does not correspond to the use of particular propositions in real scientific research. Particular propositions of the form "some (maybe all) A are B" can appear only at the very outset of a scientific investigation, when a scientist notices that some individuals of the species A have a property B. In a complete scientific theory any propositions of this form should turn either into universal propositions "all A are B", or into specific particular propositions such as "some (but not all) A are B." This consideration serves as the ground for distinguishing between two kinds of particular propositions "some (maybe all) A are B" and "some (but not all) A are B." The former may be interpreted as propositions about facts of perception or, rather, as the expression of a psychological attitude, viz. a vacillation between the universal proposition "all A are B" and a (genuine) particular proposition "some (but not all) A are B." Only the latter are propositions concerning concepts.

In this reasoning, Vasil'ev has rightly pointed to the Achilles' heel of the Aristotelian distinction between universal and particular propositions. As we have shown above, this distinction is incompatible with the Aristotelian quasi-biological genus-species hierarchy which reigns in the sublunary world. But, neither is it compatible with Vasil'ev's universe of concepts, which he understands as separate substances similar to platonic ideas. This is the very meaning of Vasil'ev's phrase that particular propositions "some (maybe all) A are B" are not really useful in the practice of scientific research. According to Vasil'ev, the concepts that are used in research are fixed. Equally fixed are the relations between them.

Building on the thesis that the genuine form of particular propositions is "some (but not all) A are B," Vasil'ev launches his criticism of the law of excluded middle. First of all, he notices that the truth of the proposition "some (but not all) A are B" presupposes that a negation of a particular form "some (but not all) A are not B" is also true, and vice versa.

Vasil'ev interprets this logical equivalence as evidence that every particular proposition is in fact the synthesis of a particular affirmative and particular negative proposition and can thus be expressed as a special kind of universal proposition "all A are B or are not B." This new form Vasil'ev designates as  $M(I,O)$ . He writes: "there is no opposition between particular affirmative and particular negative propositions; these are two expressions of one and the same proposition."<sup>85</sup> "Particular propositions are not at all propositions concerning a part of the subject, but rather propositions with a compound predicate."<sup>86</sup> Vasil'ev opposes the proposition  $M(I,O)$  to each of

<sup>85</sup> Vasil'ev 1910, p. 37.

<sup>86</sup> Vasil'ev 1910, p. 28.

the universal propositions A and E. Three kinds of propositions, then, form the so called triangle of oppositions in which “three pairs of oppositions are expressed by the same formula: both propositions cannot be true (at the same time), but both can be false.”<sup>87</sup>

Each of the three propositions concerning concepts — A, E, M(I,O) — are universal propositions (as a matter of fact there are no particular propositions concerning concepts), in which a property is predicated of the whole ‘volume’ (extension) of a universal concept adopting the position of the subject of the proposition. It follows from this classification of propositions that every pair of opposites formed between the propositions A, E and M(I,O) is contrary, but not contradictory, that is, the law of excluded middle does not hold for them.

Vasil’ev corroborates this thesis with the following explanation. For a subject (a concept) and a predicate there are three possibilities of predication<sup>88</sup>:

- the subject always has the given predicate (that is, the predicate expresses a substantial property of the subject, is its *proprium*);
- the subject is always lacking this predicate;
- the subject sometimes has and sometimes does not have this predicate (the predicate expresses an accidental property, an *accidens*).

He then illustrates this argument with the following examples:

- Triangles are closed (geometrical) figures;
- Triangles are virtuous;
- Triangles are equilateral.

The proposition “triangles are closed (geometrical) figures” belongs to type A, and it is true. It is obvious, that both of its opposites — E “triangles are not closed (geometrical) figures” and M(I,O), “triangles are closed (geometrical) figures and are not closed (geometrical) figures” — are false. Consequently the law of excluded middle does not hold for the pair E and M(I,O): from the falsehood of the one the truth of the other does not follow. The proposition “triangles are not virtuous” has the form E, and it is true; but both opposite propositions — A, “triangles are virtuous”, and M(I,O), “triangles are virtuous and are not virtuous”, — are false. The law of excluded middle does not hold for the pair A and M(I,O). The proposition “triangles are equilateral” (i.e., they may be equilateral) has in fact the form “triangles are equilateral and are not equilateral,” belonging to the type M(I,O), and is true (in Vasil’ev’s interpretation). In contrast, both its opposites — A, “(All) triangles are equilateral” and E, “(No) triangles are equilateral” — are false. This means that the law of excluded middle does not hold for the pair A

<sup>87</sup> Vasil’ev 1910, p. 37.

<sup>88</sup> Vasil’ev 1910, p. 38.

and E. On the other hand, for all three triplets of propositions, the law of excluded fourth holds, i.e. if two of the three propositions are false, the third one is true.

Vasil’ev then argues that the law of excluded middle, as it is formulated by Kant, does not hold either. For any subject (concept) and a predicate, there are not two, but three different opposite propositions, only one of which is true:

- the predicate is an attribute of the subject;
- the contradictory predicate is an attribute of the subject;
- both predicates are attributes of the subject (not each of them separately).

As an example, Vasil’ev gives the predicate “to be blond” and as its contradictory — “to be non-blond,” neither of which is separately predicated of the concept “man,” but may be predicated of it together.<sup>89</sup>

Our analysis of the kinds of oppositions in Aristotle has shown that a necessary (but not sufficient) condition of the validity of the law of excluded middle consists in the correct fixation of the genus within which one constructs the negation of a predicate. It is easy to see, that just because of the lack of precision in such a fixation, the law of excluded middle is violated in Vasil’ev’s examples.

Let us in this respect consider the example of a proposition, in which the term “triangle” is ascribed the properties “closedness,” “virtuousness” and “equilaterality.” Each of these predicates belongs to different genera: “closedness” to “geometrical figure,” “virtuousness” to “human quality” and “equilaterality” to “triangle.” If a genus, say “geometrical figure”, is fixed, and within this genus we raise the question whether a figure is closed or non-closed, then the opposites “closed” — “non-closed” will be contradictory. The pair of opposites “closed” and “virtuous” (within this genus) is not contradictory: from the fact that a figure is not closed it does not follow that it is virtuous, and vice versa: from the fact that a figure is not virtuous it does not follow that it is closed (triangles are neither closed nor virtuous). If we fix the genus “triangle,” then within this genus the opposites “equilateral” and “non-equilateral” will constitute a pair of contradictory predicates. But within the genus “geometrical figure” they are not contradictory, since a circle, for example, is neither equilateral nor non-equilateral.

Since the subject of propositions about concepts is always considered as a whole (the predicate is not distributed), the conclusion involving the violation of the law of excluded middle also holds for propositions concerning individual subjects. According to Vasil’ev, an individual term (e.g. “Caesar”) does not necessarily denote a concrete man at a certain place and time,

<sup>89</sup> Vasil’ev 1910, p. 49.

but it may, rather, denote the whole "trajectory" of his life, and thus propositions concerning such an individual term do not differ from propositions about (universal) concepts. Therefore, the law of excluded middle with respect to singular propositions does not hold either.

The situation changes radically when one considers propositions about facts of perception. Vasil'ev writes:

"The subject of singular propositions such as Caesar, Goethe and so on, can be a concept. Such a proposition, then, symbolizes the whole of Caesar's or Goethe's life... [It] subordinates the plurality of the separate moments in the life of Caesar, Goethe to the unity of the concept. But the subject of particular propositions can also be a perception, a reality, a separate moment of time. In the former case, the singular proposition is a proposition about a concept [considered as] a rule, and therefore the triangle of oppositions and the law of Excluded Fourth are applicable. So it is a rule that Caesar was a Roman, a genius, and so on. It is, furthermore, a rule that he was not a Gaul, and so on. It is also rule that he was sometimes ill and that he was in Gaul, in Britannia. In the latter case, the singular proposition is a proposition about a fact, e.g. "Ivan Ivanovich is now drunk", "Yesterday morning at 5 o'clock NN died." In such propositions, the copula always contains a precise indication of a moment of time, since the subjects of such singular propositions — perceptions and representations — are related to a definite moment in time. The difference between singular propositions concerning concepts and [singular propositions about] facts is clearly seen in the following example: "NN is an ill man" is a rule; "NN is now ill" is a proposition concerning a fact."<sup>90</sup> Only for the latter kind of propositions, the law of excluded middle holds.

Vasil'ev concludes his paper with the following remark: "the law of excluded middle is not at all a logical law on a par with the logical laws of identity, contradiction and sufficient reason, which embrace the spheres of logic and thought."<sup>91</sup> From this conclusion it is evident that, when criticizing the law of excluded middle in 1910, Vasil'ev was not expressing doubts about the law of contradiction. But the logic of his argument concerning the law of excluded middle, in particular his treatment of particular propositions by turning them into the form "all A are B and are not B," inevitably had to lead him to have doubts concerning the validity of the law of contradiction.

Aristotle understands the law of contradiction first and foremost as an ontological law. Its main metaphysical formulation runs as follows: "the same attribute cannot at the same time belong and not belong to the same subject

<sup>90</sup> Vasil'ev 1910, p. 51.

<sup>91</sup> Vasil'ev 1910, p. 53.

and in the same respect.”<sup>92</sup> It is transformed into a logical law on the basis of an analogy between being and logic. In its logical form, the law of contradiction appears as an assertion about the incompatibility of an affirmation and a negation: “If then “he is not able to walk” means the same as “he is able not to walk”, capacity to walk and incapacity to walk will belong at the same time to the same person (for the same man can both walk and not-walk, and is possessed of knowledge of what is good and of what is not-good), but an affirmation and a denial which are opposed to one another do not belong at the same time to the same thing.”<sup>93</sup>

In the beginning of his paper On Imaginary Logic Vasil’ev remarks that in the logical formulation of the law of contradiction, which expresses the incompatibility of an affirmation and a negation, there is a vicious circle. The concept of negation already implies the idea of incompatibility, since negation is that which is incompatible with affirmation.<sup>94</sup> From this he draws the conclusion that the law of contradiction is already included in the definition of negation. Therefore, “constructing a logic without the law of contradiction amounts to constructing a logic without our negation which is reducible to incompatibility.”<sup>95</sup> Let us come back to Vasil’ev’s argument.

According to Vasil’ev, there is an essential difference between the affirmation “S is P” and the negation “S is not P.” The affirmation “S is P” is simple and may be obtained as a result of direct experience. Negation has a different, a more complex structure. Consider the negation “S is not P.” According to Vasil’ev, this proposition is not simple, but always the result of a specific syllogism (which coincides with the syllogistic form *Celarent* of the first figure):

<i>N</i> excludes <i>P</i> , [that is, <i>N</i> ] is incompatible with <i>P</i> (incompatibility clause)	
<i>S</i> is <i>N</i> (minor premiss)	
<i>S</i> is not <i>P</i> (inferred negative proposition) <sup>96</sup> .	

This syllogism makes negative propositions dependent on propositions about incompatibility such as “*N* excludes *P*,” or “*N* is incompatible with

<sup>92</sup> Aristotle, *Metaphysics*, IV, 3, 1005b19–20.

<sup>93</sup> Aristotle, *Anal. Pr.* I, 46, 51b17–23.

<sup>94</sup> Vasil’ev 1912a, p. 61.

<sup>95</sup> Vasil’ev 1912a, p. 62.

<sup>96</sup> Vasil’ev 1912a, p. 61.

*P.* Notice that such a treatment of negation is just the reverse of the Aristotelian approach, according to which the opposition of affirmation and negation is more primary than the contrary opposition (to which incompatibility belongs). In particular, Aristotle states that the validity of the law of contradiction with respect to contrary opposites is a derivative of its applicability to the contradictory opposites of affirmation and negation: "Now, since it is impossible that contradictories should be at the same time true of the same thing, obviously contraries also cannot belong at the same time to the same thing."<sup>97</sup>

This syllogism leads Vasil'ev to distinguish between two aspects of negation, a formal one and a material one (as he calls them). The formal aspect has to do with the negative form of the categorical proposition, "(all) S is (are) not P." The material aspect consists in the reduction of negation to an assertion of incompatibility, and constitutes the foundation of the formal aspect. In this two-fold structure of negation, Vasil'ev sees the possibility for a radical reform of logic. The essence of this reform does not consist in a return to the Aristotelian idea of the primacy of negation over incompatibility, but in a total rupture of the logical link between negation and incompatibility. As a substitute, Vasil'ev proposes to conceive a situation (possibly in an imaginary world totally different from ours), in which negation would not be a consequence of an incompatibility clause, but would, rather, appear as the result of direct perception. In such a world, experiences of different kinds are possible: one, e.g., confirming a given proposition, another confirming its negation. As a result of such a synthetic experience the so called "indifferent propositions" originate. They are of the form "S is P and is not P simultaneously," and in them the law of contradiction is implicitly violated. Vasil'ev notices that there are two different formulations of the law of contradiction. One of them goes back to Aristotle and forbids contradictions with respect to propositions. Vasil'ev cites Sigwart: "the propositions "A is B" and "A is not B" cannot both be true at the same time."<sup>98</sup> However, Vasil'ev himself does not consider this to be a formulation of the law of contradiction but, rather, a formulation of the so called *Law of Absolute Difference between Truth and Falsehood*, which, in contrast to the law of contradiction, cannot be violated. It is worth remembering that this was Aristotle's position too.

Another formulation of the law of contradiction is Kant's: "Nothing has a predicate which contradicts it." This is our law of contradiction and it is, according to Vasil'ev, false. In the course of the argumentation in support of

<sup>97</sup> Aristotle, *Metaphysics*, IV, 6, 1011b15–17.

<sup>98</sup> Vasil'ev 1912a, p. 64.

this thesis, Vasil'ev makes some perspicacious remarks concerning negation which show his (partial) allegiance to the Aristotelian view.

What does the proposition “a predicate which contradicts a thing” mean? It means that there is nothing of which a property can be truly predicated which contradicts the essential properties of this thing, its *propria* which constitute its definition. But what does it mean that a predicate C contradicts a predicate B, which is a *proprium* of the thing in question?

Let us consider, following Vasil'ev<sup>99</sup>, the proposition “Columbus was the first European who reached the shores of America” and its negation, “Columbus was not the first European who reached the shores of America.” In the negation, according to Vasil'ev, we are not denying that Columbus was a European nor that he reached the shores of America, but only that of all the Europeans who did so, he was the first. In the negative proposition about Columbus, only one of the three predicates — “first,” “European,” “reaching the shores of America” — (forming the compound predicate “to-be-the-first-European-who-reached-the-shores-of-America”) is negated, i.e. the predicate “first.” We can interpret Vasil'ev's idea in the following way.

Let us transform the negation into the form which is used in the Kantian formulation of the law of contradiction “A is non-B” or “Columbus was the non-first European, who reached the shores of America.” Vasil'ev's argument takes for granted that a genus, namely the genus “European who reached the shores of America,” is implicitly fixed, a necessary condition for a correct negation. Within this genus we take the *differentia specifica* “first” and consider the species “the first European, who reached the shores of America” (it is of no importance that the species in question is constructed to denote only one individual). The negation of this predicate is constructed when, within the same fixed genus “European who reached the shores of America,” we define by means of the *differentia specifica* “non-first” a species, viz. “to be a (the) non-first European, who reached the shores of America.” In the practice of scientific research, such a fixation is usually done implicitly. Within this genus we consider the opposites “first” and “non-first.” It is only due to the fixation of such a genus that the law of contradiction is valid. This is what Vasil'ev actually means.

We can expand a little bit upon this idea of Vasil'ev's. If the genus “European, who reached the shores of America” is not fixed, then both predicates “first” and “non-first” can be predicated of Columbus (with respect to different genera), and consequently the law of contradiction is not valid here. So, Columbus could be “the first European who reached the shores of America,” but also “the non-first-born of the children in his family”. The cause of the violation of the law of contradiction consists in that the predicate “first” and

<sup>99</sup> Vasil'ev 1912a, p. 87.

its negation "non-first" introduce *differentia specifica* within different genera. We could restrict our consideration to the genus "men, who reached the shores of America," in order to exclude genera like "children of Columbus' parents." But in this case too, propositions such as "Columbus was the first European who reached the shores of America" and "Columbus was not the first who reached the shores of America" will both be true, since long before Columbus, inhabitants of the Pacific may have reached the shores of America (at least, that is what is claimed by some anthropologists). The cause of the violation of the law of contradiction consists in that the predicate "first" in the first case belongs to the genus "European, who reached the shores of America" and in the second to another, broader one, viz. "men, who reached the shores of America."

Vasil'ev's ideas concerning conditional negation (with respect to a genus) can be interpreted as an additional argument against the equivalence of external and internal negation and the law of excluded middle. Consider the proposition "Columbus was the first Chinese who reached the shores of America" and its negation "Columbus was the non-first Chinese, who reached the shores of America." Formally, we are talking about propositions of the form "A is B" (1) and "A is non-B" (2). If we assume the equivalence between "A is non-B" and "A is not B", the opposites (1) and (2) should be contradictory, which means that one of them is true and the other is false. However, it is obvious that both propositions (1) and (2) are false (within Aristotelian logic). Therefore the law of excluded middle is violated with respect to these opposites.

Another example<sup>100</sup>, also from Vasil'ev, concerns a universal subject and deals with the proposition "dogs are not human beings." Vasil'ev remarks that in this negation, we do not negate the entire content of the concept "man." Thus, in this proposition it is e.g. not denied that a dog is a mammal. As in the previous example, the introduction of contradictory predicates takes for granted that a genus "mammal" has been previously fixed. Within such a genus, we can oppose to the predicate "man" and the predicate "non-man." The property "to be (a) non-man" can be predicated of a dog. Under these conditions the law of contradiction holds. But if the genus is not fixed at all, or is fixed more broadly, say, as "animal," then a problem arises as to the connection between the subject "dog" and the predicate "mammal." On the one hand, a dog is a mammal (if, by the proposition "dogs are not human beings" we stay within the genus "animal", and only negate the dog's possession of the *differentia specifica* of a man). On the other hand, a dog is not a mammal (if, together with the *differentia specifica* of man, we also negate its "genus proximus" (closest genus) "mammal"). We can describe man as

<sup>100</sup> Vasil'ev 1912a, p. 87.

“rational mammal” or “rational animal.” But by defining the predicate “non-man” we have a problem, since the meaning of this predicate depends upon our choice of the predicate which the particle “non” should be related to, viz., “rational,” “mammal,” or “animal.”

Both examples are important for Vasil’ev, since they provide him with the opportunity to again expose the relative character of our predicative negation and the possibility of positing his own kind of negation (conceivable only in imaginary logic), which he calls “absolute negation.” He defines this negation in the following way.<sup>101</sup> Let us take the proposition “S is A,” where A is a predicate which consists of the properties p, q, r, s, . . . One can, as in our (Aristotelian) logic, consider different predicates that are constructed out of the predicate A by negating some of the properties, while preserving the others: e.g., by negating a species but preserving the genus to which this species belongs, such that the negation consists in a negation within this genus. One can, however, also introduce a predicate non-A, consisting of all the properties non-p, non-q, non-s, non-r, . . ., which is called absolute negation. If we accept such a definition of negation we do not need negative propositions of the form “S is not A” at all. They can be replaced by equivalent affirmative propositions of the form “S is non-A.” Such a “negation” is not variable in the way our ‘normal’ negation whose definition depends upon a fixed genus is. It is an absolute negation.

The following lines, offered in conclusion of the above argument, reveal Vasil’ev’s deep understanding of the problem of the relation between external and internal negations. He writes: “In general, we can either 1) affirm all the properties of A, or 2) negate all the properties of A, or 3) affirm some of the properties and negate some [of the properties of A]. The first case results in an affirmative proposition, the second case proposition is the absolute negation, and the third case results in a proposition with our [regular] negation. In all there are, then, three subdivisions of propositions according to their quality.”<sup>102</sup> In fact, what Vasil’ev is speaking about is the necessity to distinguish between an external negation (which he tries to represent in the form of the absolute negation “S is non-A”) and an internal negation. As Aristotle indicated before him, internal negation is, in fact, a kind of affirmation. In “Logic and Metalogic” (1912–1913), Vasil’ev writes: “If we analyze our negative proposition, we will see that it can have two meanings: 1) S is non-P or 2) S is not P. It is clear that the first type is reduced to an affirmation, it is affirmative *ex via forma*.”<sup>103</sup> Then Vasil’ev draws attention to the fact that the second type, the proposition which has the form of an external

<sup>101</sup> Vasil’ev 1912a, p. 87–88.

<sup>102</sup> Vasil’ev 1912a, p. 88.

<sup>103</sup> Vasil’ev 1912–1913, p. 117.

negation, presupposes the possibility of a mistake which may be corrected by this very same proposition. In a hypothetical situation, when propositions are made by an 'Infallible Spirit', propositions with the form of an external negation are useless because, as we have indicated, metalogic is a 'positive logic' expressing 'infallible knowledge' and therefore contains only affirmative propositions. Here Vasil'ev sees a possibility for an imaginary logic which would be the logic of an 'Infallible Spirit'.

Recent research, to which we will come back in the conclusion, has shown that it is difficult to transform external negations into internal ones by means of formal logic alone. It is impossible to construct for a predicate B its negation non-B in such a way that the two propositions "A is not B" and "A is non-B" would be equivalent. The reason for this lies in the fact that in 'terrestrial' logic, as Vasil'ev called Aristotelian logic (i.e. in the real practice of scientific investigations), we are always dealing with fixed genera, within which we make internal negative propositions. And such a fixation of the universe of discourse is always based on non-formal considerations. This distinction can, however, be potentially very important in logic. To this, we will also come back in the conclusion, but we will now first go into the problem of Vasil'ev's alleged Psychologism and his relation to modern 'non-classical' logic.

##### 5. *Is It True What They say about Vasil'ev?*

On a more general level, and with respect to Vasil'ev's place in the history of logic and of his importance, several questions may be raised for current research in logic. They pertain mainly to the problem of Vasil'ev's alleged Psychologism and the question of his being a 'forerunner' or an 'initiator' of non-classical logics, be they 'many valued' 'paraconsistent' or 'dialetheic ones'.<sup>104</sup> We want here to briefly go into these questions before formulating our concluding remarks where we try to demonstrate and illustrate the actuality of Vasil'ev's ideas.

It is widely accepted that Vasil'ev held a psychologistic view on logic, and this is sometimes even positively valued. So, e.g. Bazhanov states: "Thus, Vasil'ev was evidently a proponent of a psychologistic approach to logic. Most of the supporters of psychologism were opposed to the mathematization of logic. Vasil'ev, however, thought that this process opened up new horizons in the development of logic. Although he was acquainted with them

<sup>104</sup> See e.g. in this respect Kline 1965, Arruda 1980, Priest and Routley 1989.

at least in general terms, Vasil'ev himself did not use the methods of mathematical logic. Yet his psychologism helped to propose radically new, non-classical systems of logic which he sought to formalize by using mathematical logic.”<sup>105</sup> Stelzner comments: “In Vasil'ev's psychologistic treatment, logic in general is related to the psychical organization of the subjects of knowledge.”<sup>106</sup> However, Stelzner also maintains that it is precisely Psychologism that is eminently represented in Vasil'ev and is even a main source of his ideas in the development of his ‘non-classical’ logic: “Jahrhunderts übten psychologistische und teilweise damit verbundene formalistische Konzeptionen der Logik einen eher förderlichen Einfluß aus. Trotz der nicht nur von Frege vertretenen Kritik am Psychologismus war während der Periode des Übergangs von der traditionellen zur modernen Logik das Paradigma des Psychologismus stark genug, um die Entwicklung von interessanten pragmatisch beeinflussten nichtklassischen logischen Ideen zu erlauben und zu fördern. Eines der eindruckvollsten Beispiele dafür liefert das Werk des russischen Logikers Nikolaj Aleksandrovich Vasil'ev (1880–1940), dessen wichtigste Arbeiten zwischen 1910 und 1913 erschienen sind.”<sup>107</sup> Priest, on the basis of what Vasil'ev writes on metalogic in his ‘Logic and Metalogic’ to the effect that since metalogic contains only laws of pure thought and of inference and judgment in general, it reflects only the nature of the cognizing subject, concludes that Vasil'ev's psychologism is evident here<sup>108</sup> and goes on to criticize him in this respect. So, both in the Russian literature on Vasil'ev and in non-Russian sources the idea of Vasil'ev's psychologism is widespread. But the question is whether all of this is true and whether Vasil'ev is justifiedly called a proponent of psychologism.

There are several reasons why we think this is not so. Of course, superficially, one may find certain indications which are in favour of a psychologistic tendency in Vasil'ev's approach to logic in general. So e.g. Stelzner quotes Vasil'ev's paper “Imaginary (non-Aristotelian) Logic” to underpin his argument. Vasil'ev writes e.g.: “However unusual the idea of a different logic may, there is nothing implausible in it. That which is obvious for us, [i.e.] in our world with our structure of mind and our faculty of perception, may be not only not evident, but also completely wrong in a different world,

<sup>105</sup> Bazhanov 2001, p. 212.

<sup>106</sup> Stelzner 2000, p. 135, cfr. also Stelzner 2001, p. 267.

<sup>107</sup> Stelzner 2001, p. 250.

<sup>108</sup> Priest 2000, p. 143.

for beings with a different kind of mental structure."<sup>109</sup> However, the Russian text of Vasil'ev continues as follows: "Is it really true that God should necessarily think according to Aristotelian logic, following the canons of the syllogism and Mill's rules of induction? From very early onwards religious thought has conceived of the idea of a God whose mind was infinitely superior to the human mind. Consequently, there is nothing implausible or absurd in the idea that divine logic would differ from human logic."<sup>110</sup> It is, to say the least, highly remarkable that to illustrate his presumed psychologism Vasil'ev would make an appeal to God as an example of a being with a different mental structure. Moreover, when Priest invokes Vasil'ev's paper 'Logic and Metalogic' where he claims that metalogic reflects only the nature of the cognizing subject<sup>111</sup> a few pages further Vasil'ev, in describing his metalogic, again makes reference to 'God' or to some kind of 'perfect mind'. The logic of such a mind, Vasil'ev says is "Divine logic". (It) must be the logic of perfection and perfect cognition, and that is why it must be a logic only of affirmative propositions and no negative ones. . . This divine logic, the logic of perfect cognition is also metalogic.<sup>112</sup> Now, clearly, Vasil'ev is here referring to some 'idealized (divine) situation' (be it a mental one) which, if certain conditions are fulfilled, would and does give rise to 'metalogical knowledge'. This knowledge, however, is also a kind of knowledge that human beings can (attempt to) attain and which we can come to discover certain features of. So e.g., the law of contradiction may well be a law of empirical logic, but the law of Absolute Difference between Truth and Falsehood is a law of metalogic.<sup>113</sup> And Vasil'ev adds, speaking about the general way this metalogic is constructed, that its construction is quite analogous to what could be done in geometry. "The same metalogic underlies both imaginary and empirical logic, and this explains the possibility of the construction of an imaginary logic. Metalogic is analogous to those most general propositions of geometry that are common to all geometries: Euclidean and imaginary ones; it corresponds to what Bolyai called absolute geometry."<sup>114</sup> Clearly now, every reference to a psychologistic approach

<sup>109</sup> Vasil'ev 1912a, p. 55, quoted in Stelzner 2001, p. 266.

<sup>110</sup> Vasil'ev 1912a, p. 55.

<sup>111</sup> Vasil'ev 1912–1913, p. 115.

<sup>112</sup> Vasil'ev 1912–1913, p. 118.

<sup>113</sup> Vasil'ev 1912a, p. 91.

<sup>114</sup> Vasil'ev 1912a, p. 91.

here has disappeared. To Vasil'ev, the construction of metalogic is some kind of 'an empirical procedure' which is in fact a search for the a priori conditions of any logic whatsoever.

This puts Vasil'ev within a tradition which, within the Russian context, can be called neo-Kantian and to which we will immediately return, but there is also another reason not to 'suspect' Vasil'ev of psychologistic tendencies. In 'Imaginary (non-Aristotelian) Logic' Vasil'ev discusses several views on the nature of logical laws. They can be seen as descriptive psychological laws, normative laws, or as laws expressing ideal truths or generalizations from experience. Vasil'ev says that in these three cases (only the first one of which may be considered to give rise to some psychologistic approach) it is possible to look at logical laws in such a way that each time alternative laws are possible and alternative logics can be constructed. It is noteworthy how Vasil'ev in discussing the several views on logical laws, especially the view of logical laws 'as ideal truths' — a view that can easily be retraced to Frege's idea of logical laws as objective truths<sup>115</sup> —, interprets this view in formalist terms and is then able here to make a case for alternative logical systems. In doing so, he turns Frege's logistic view on the relation between logic and mathematics upside down. In Vasil'ev it is not logic that provides a basis for mathematics but, rather, logic becomes part of mathematics, which allows logic to be treated as a discipline of mathematics. "At each step (in its development) mathematics involves a generalization of its operations and the extension of the field of its objects. In this way it moves, for example, from the real numbers to the imaginary numbers. It is impossible to extract the square root of a negative number, since every squared real number is positive; but mathematicians, by introducing imaginary numbers, can extract the square root of a negative number and [thus] give a more general characterization to the operation of 'taking the root of'. Exactly in the same way as mathematical operations can be generalized, logical operations can be generalized too, and in both cases this generalization may lead to the creation of imaginary objects."<sup>116</sup> Be this as it may, the conclusion seems to be that on the problem of Psychologism Vasil'ev is neutral, as may appear from his discussion of even a fourth view (attributed to Mill) on the nature

<sup>115</sup> Stelzner 2001, p. 260–261.

<sup>116</sup> Vasil'ev 1912a, p. 56–57. Stelzner (2001, p. 262) claims that among the possible views on the nature of logic and logical laws, Vasil'ev did take into account such view as Mill's, Husserl's, Erdmann's, Sigwart's and others but did not take into account Frege's criticisms on (especially Erdmann's) psychologism. This may be right, but we are not claiming that Vasil'ev should be considered a psychologist and he therefore escapes the aforementioned criticism. Vasil'ev himself (1912a, pp. 92–93) explicitly puts his investigations in line with the axiomatic approach introduced by Hilbert.

of logical laws as generalizations from experience. Notice here that he does not want to reinstate this ‘old view’ which he calls ‘old, nearly abandoned’, and which, indeed, may (also) give rise to some kind of psychologism if endorsed. “Thus, whichever of the three major points of view on logical laws we accept, from each one of them there follows (in its own way) the possibility of the existence of a logic different from ours. This possibility also follows from the fourth, old, and now nearly abandoned point of view of Mill, according to which the laws of logic are generalizations from experience. If this is true, then, even more, we can imagine a world in which the generalizations taken from experience, and consequently logic, will be different from ours. *Therefore, the controversy about the nature of the laws of thought does not have much importance for our aim; whatever solution of this controversy is accepted, we have to conclude to the possibility of a logic different from ours.* (italics ours R.V. — E.Z.). So, we will not deal now with this very complex and difficult matter.”<sup>117</sup>

As was said before, our view is that Vasil’ev’s presumed ‘Psychologism’ is in fact to be considered within the neo-Kantian tradition in Russia at that time. The most important representatives of Neo-Kantianism in Russia at the end of the nineteenth and the beginning of the twentieth centuries were Alexander Ivanovich Vvedenski (1856–1925) and Ivan Ivanovich Lapshin (1870–1952),<sup>118</sup> both of which are, incidentally mentioned in Vasil’ev ‘Imaginary (non-Aristotelian) Logic’<sup>119</sup>, so it is reasonable to assume that Vasil’ev was familiar with their work. The problem which fascinated Russian Neo-Kantians was the nature and status of the Kantian ‘Thing-in-Itself’.<sup>120</sup> In his work, *The Laws of Thought and the Forms of Knowledge* (1906) Lapshin, for one, presents himself as a proponent and interpreter of Kant’s critical philosophy. Lapshin argues that Space and Time are first and foremost concepts or categories. In contradistinction to Kant, he argues that

<sup>117</sup> Vasil’ev 1912a, p. 57.

<sup>118</sup> Lossky 1952, p. 167–170. In this book, Lossky also devotes a couple of pages to Vasil’ev’s logical ideas. See also Biryukov 2001, p. 225–227. Cfr. also Zenkovsky 1953, vol. 2, pp. 677–705.

<sup>119</sup> Vasil’ev 1912a, p. 65, note 6.

<sup>120</sup> Actually, this problem has been a constant theme in Russian philosophy (and, of course, not only there) during the whole of the nineteenth century. One may here, e.g., think of A. Herzen who, in response to the ‘Unknowability Thesis of the Thing-in-Itself’ developed a whole philosophy (inspired by Hegel) of what he called ‘The Philosophy of Action’ (Filosofia dela’) in order to restore epistemic access to the reality or ‘Things-in-Themselves.’ (Cf. e.g. Walicki 1979, pp. 127–135 and Copleston 1986, pp. 77–99).

all data of experience, including the ones of our inner sense, have a spatial form. Logical knowledge depends upon the applicability of the law of contradiction to things that can be known. This law is essentially related to the form of time and space and is therefore, one may add, empirical. Objects may have contradictory properties at different moments in time but not at the same time, i.e. not ‘in coexistence’. The present — as coexistence — is nothing but the synthesis of two expressions of the same content, one from the temporal and the other from the spatial point of view. Therefore, the law of contradiction is only applicable to a ‘spatial synthesis’ or ‘spatial coexistence’. It follows that only objects with spatio-temporal content can be known, i.e. only phenomena in experience are knowable. As for Things-in-Themselves, things as they exist independently of any experience, we do not even know whether they are spatial or temporal or whether the law of contradiction is applicable there (it may not be), and therefore we do not know anything about them. We do not even know whether they exist. Also in his paper on ‘Logic and Metalogic’, Vasil’ev writes approvingly on Lapshin, whom he considers to be a ‘fellow traveller’ to a certain degree. “The incompatibility of properties is empirical, hence the law of contradiction is empirical too. The incompatibility of properties is related to spatiality and therefore also the law of contradiction is. It is, by the way, this connection between logic and forms of knowledge which I.I. Lapshin has discovered.”<sup>121</sup> Now, though it is clear that Vasil’ev takes from Lapshin the general (neo-Kantian) idea that there is a connection between logic and (forms of) cognition, he goes one step further. Since he makes a distinction between the law of contradiction and the law of Absolute Difference between Truth and Falsehood (the law of non-self-contradiction), he is able to maintain that Things-in-Themselves may have contradictory properties. Indeed, it is Vasil’ev’s aim to show that a logic of such properties is possible, but the law of Absolute Difference between Truth and Falsehood is absolute (it is, actually, a law of Metalogic). It applies to propositions only<sup>122</sup>, and is valid even for the intelligible world. Therefore Lapshin’s ideas on the applicability of the law of contradiction, understood in the sense of the law of non-self-contradiction, are, according to Vasil’ev, unjustified when he states that the law of contradiction may not be applicable to Things-in-Themselves. It would even be bad philosophy, Vasil’ev adds, to think otherwise: “However, all depends on which of the two laws we mean. If we mean the law of contradiction, then, of course, this law loses its validity in the intelligible world, since it derives its force from experience, from the experience of

<sup>121</sup> Vasil’ev 1912–1913, p. 114 and note 10 on that page.

<sup>122</sup> Vasil’ev 1912a, p. 68.

the ascertained existence of incompatible predicates and it is, in fact, unquestionable only in the world of experience. But to the Thing-in-Itself or God can be considered as the union of contradictory predicates, as a *coincidentia oppositorum*, a view which has been held more than once in the history of philosophy. But the law of Absolute Difference between Truth and Falsehood, the law of non-self-contradiction, preserves its validity also in the case when we argue about the intelligible world. Within God, contradictory predicates may be realized, but there can be no self-contradictions in our affirmations about God. (...) A bad philosopher would be he who would justify the self-contradictions in his/her metaphysics by referring to the fact that it deals with the intelligible world.<sup>123</sup> The other neo-Kantian mentioned is Alexander Ivanovich Vvedenski (1856–1925). Vvedenski, who was in fact Lapshin’s teacher and the ‘founding father’ of Neo-Kantianism in Russia, is invoked by Vasil’ev as a philosopher whose views are fundamental to his own Imaginary Logic and more specifically, his view on negation.<sup>124</sup> Vvedenski called his own system (somewhat surprisingly) ‘logicism’. In one of his main works *Logic as a part of the Theory of Knowledge*, he wanted to found epistemology on logic by means of a theory of inference and methods of proving general synthetic propositions which are arrived at by means of inferences that prove them. This is problematic for Vvedenski because the data of our experience can only justify particular propositions. Such (general) inferences are only possible because they are based upon certain generally accepted synthetic a priori propositions which, in combination with definitions (analytical) and the data of experience, allow us to make inferences which in their conclusions result in new synthetic propositions. The objects of knowledge which are obtained through inferences are not the Things-in-Themselves, for the following reason: Since the validity of an argument is based on the law of contradiction (in the sense that if an argument is valid it cannot be the case that if the premisses are true the conclusion would be false), inferences can only be constructed for things or objects for which the law of contradiction holds. According to Vvedenski, these objects must be (our) representations. Thought itself is not submitted to the law of contradiction, since we can apparently ‘think of’ e.g. a round square though we are not able to ‘imagine’ it. Therefore, inferences are only possible concerning representations or ‘apparent being’, which is being as it appears to us. Things-in-Themselves are

<sup>123</sup> Vasil’ev 1912a, p. 65 note 6.

<sup>124</sup> Vasil’ev 1912a, p. 69.

not known to be submitted to the law of contradiction and therefore no inferences concerning them are possible. Vvedenski, furthermore, takes it that the a-priori principles needed, since they are not proved by experience, are the thoughts of the knowing subject itself which can only represent representations and not Things-in-Themselves. Again, here Vasil'ev agrees with Vvedenski especially as far as the relation between the law of contradiction and thinking is concerned. It is thinking that allows us to construct contradictions ('indifferent propositions', in Vasil'ev) without us, however, being able to 'imagine' them. In thinking, the law that holds there is not the (empirical) law of contradiction but the purely rational, metalogical law of Absolute Difference between Truth and Falsehood. The law of contradiction is 'real' since it applies not to thought but to 'reality', to 'objects', and it states that 'in reality' contradictions cannot exist. But since it is an empirical law one might as well also reason without it and that is precisely what happens in Imaginary Logic. "In corroboration of our analysis of the law of contradiction, we can refer to Prof. Vvedenski. This author teaches that the law of contradiction is a natural law of our representation, since a contradiction cannot be conceived of. Thinking itself is not subject to this law however, because we are able to think a contradiction. Thus we are able to think of, though not able to imagine, a round square or God's Trinity. Thinking is subject to this law as a norm when we aim at adjusting our thoughts to our representations for which the law of contradiction is a natural law. From Prof. Vvedenski's theory, which seems to us to be correct, the idea of an imaginary logic must follow as unavoidable corollary. It also follows from this that if we do not adjust our thought to our representations, but instead think of an imaginary world, a world of different representations, we can think without the law of contradiction, and think a contradiction. Every real thought is always manifested in a proposition. Therefore, to think a contradiction actually means to form a special proposition of contradiction viz. an indifferent one, alongside with the affirmative and negative ones."<sup>125</sup>

However, next to (empirical) logic, Vasil'ev also considers metalogic. Logic as we know it is always a mixture of empirical logic and metalogic (as has already been pointed out, a term coined by Vasil'ev by analogy with Metaphysics) which is described in several ways. Roughly, metalogic is 'empirical logic' (which includes 'imaginary logic') minus the 'empirical elements' (the laws that are empirical), and it constitutes knowledge of thinking regardless of experience. Here is how Vasil'ev himself describes his metalogic: "Therefore, in metalogic we get to know nothing besides thought. In empirical logic we get to know the main properties of our world as well. Empirical logic is a combination of the metalogical and the experiential, of the rational

<sup>125</sup> Vasil'ev 1912a, p. 68–69.

and the empirical. Therefore, empirical logic lies at the foundation of any ‘terrestrial’ reasoning, since scientists or representatives of any specific science will make mistakes if they are reasoning contrary to the main properties of our world. So, scientists have to reason and, in fact, do reason according to both the principles of metalogic and empirical logic. But philosophers can, and in certain cases have to, reason according to the principles of metalogic only. Metalogic in itself constitutes a certain logical minimum; it is that, which is contained in all possible logics, real and imaginary ones. It is that, which makes logic be logic. Empirical logic (but also any of the imaginary logics) is richer in content; it is more concrete, more definite; it contains all of the metalogical and something specific to empirical logic only. Therefore, a person who is reasoning in accordance with empirical logic (or any imaginary logic), is also reasoning in accordance with metalogic, but not at all vice versa.”<sup>126</sup> Vasil’ev claims that Metalogic contains logical truths which follow from the very definition of the logical, and which are of absolute validity for logic. Here he also indicates that he does not agree with Husserl’s idea on logical laws, since Husserl insists on the unchangeability of ‘all basic logical postulates’, and that is what Vasil’ev cannot accept. But the principles and laws formulated in metalogic are of absolute validity for every logic.<sup>127</sup> What then, are the principles and laws that hold here (and not in empirical logic)? We have already mentioned that in Vasil’ev’s view metalogic contains only ‘positive propositions’, but there are also ‘laws’ that apply not to reality but to thoughts or propositions that belong there. From our earlier discussion we know that the law of Absolute Difference between Truth and Falsehood is one of them (it is the ‘analogon’ in metalogic of the law of contradiction in empirical logic, but it is absolute). Vasil’ev also mentions the law of identity and the law of sufficient reason — each of which is linked to a ‘real’ law or a ‘law of reality’ — as belonging to metalogic: “*Contrariwise, the formal laws of thought apply to thought only, and not to reality; they apply to propositions, and not to objects.* They are laws about propositions and about propositions only. In contrast to the empirical changeability of things, the law of identity, e.g., establishes the logical constancy of concepts, i.e. of parts of propositions. It is a law about propositions and [it] tells absolutely nothing about objects. Exactly in the same way the law of absolute difference between truth and falsehood and the law of sufficient reason tell us about propositions only, and not about objects. Consequently, one should strictly distinguish between them and those real laws (about objects) they

<sup>126</sup> Vasil’ev 1912a, p. 90. In *Logic and Metalogic* p. 115ff. Vasil’ev tries to circumscribe this metalogic even further. We have already in an earlier part of this paper hinted at what he says there.

<sup>127</sup> Vasil’ev 1912–1913, p. 97.

can easily be confused with. A formal law of propositions — e.g. the law of sufficient reason — “every proposition has to be justified” — should be distinguished from the “real” law of causality which says that “every phenomenon must have a cause.” A formal law such as “propositions should not contradict each other” should be distinguished from a real law such as “there is no contradiction within objects.” *Thus, the real law of contradiction stands to the formal law of absolute difference between truth and falsehood just as the real law of causality stands to the formal law of sufficient reason.* We have seen that one can deny the law of contradiction without denying the law of absolute difference between truth and falsehood. Exactly in the same way, the denial of the law of causality does not imply the denial of the law of sufficient reason. Indeterminism is the negation of the universal validity of the law of causality, but it too should be justified and demonstrated according to the law of sufficient reason.”<sup>128</sup> We have now the strange situation that from the ‘traditional logical laws’ mentioned by Vasil’ev<sup>129</sup> viz. the laws of identity, of contradiction, of excluded middle and of sufficient reason, the laws of identity and sufficient reason are moved to the level of metalogic, which contains laws of pure thought and “reflects only the nature of the cognizing subject”. One does not immediately see a reason for this and Priest, e.g., castigates Vasil’ev for not providing one since in any case from the point of view of modern non-classical logics even the laws of Vasil’ev’s metalogic are not any longer recognized as absolute<sup>130</sup>. The reasons Vasil’ev gives, to the extent that these laws or principles are the invariant part of logic or genuine laws of thought, e.g. in the case of the law of Absolute Difference between Truth and Falsehood that it is a law that forbids self-contradiction and is therefore a law without which no logic would be possible, are, according to Priest, all unconvincing. “The law of identity is harder, but there are certain relevance logics in which it fails. The principle of sufficient reason is not a part of modern logic at all, and in any case has been cast into doubt by quantum mechanics. And the law of absolute difference fails in the semantics of say, first degree entailment where sentences may have more than one truth value. . . . Logics where some things may be both true and false are just as possible — and coherent — as logics where some things may be neither.

<sup>128</sup> Vasil’ev 1912a, p. 68.

<sup>129</sup> Vasil’ev 1912–1913, p. 97.

<sup>130</sup> Priest 2000, p. 142–143. This also brings up the question of and in what sense Vasil’ev may be called a forerunner of (certain) modern non-classical logics. We will come back to this a little bit later.

Certainly, if one violates this law, one may end up endorsing contradictions. But so what?”<sup>131</sup>

Without going into this criticism of Priest’s which is of course constructed with hindsight and which, if true, might mean that maybe all of logic is then somehow empirical thus challenging in a very serious way any (be it even a small) universality-claim for logic, it seems to us that there may indeed have been reasons for Vasil’ev to set up things the way he did and it is precisely within a broadly (neo)-Kantian framework that it makes sense. Our claim is not that Vasil’ev held Kant’s view on logic<sup>132</sup> but that certain distinctions he makes can be traced back to Kant. In order to see what this may mean, let us first go back to what Vasil’ev says about the relation between logic and metalogic. Metalogic, according to Vasil’ev, is in itself not an instrument of knowledge. It must be supplemented by ‘empirical logic’ which introduces material principles. It does not provide knowledge of any specific world since it only expresses the nature of the knowing subject. It therefore is a purely theoretical science and has no relation to reality. However, once ‘mediated’ (Vasil’ev uses the term ‘Logos’ here) through material or empirical elements, it gets a practical meaning. Empirical logic originates in life, in the interaction between man and his environment, and in that sense logic serves knowledge and knowledge serves life which implies that logic serves life.<sup>133</sup>

In his ‘Critique of Pure Reason’ Kant formally makes a distinction between general (pure) logic and what he calls transcendental logic. Pure logic, according to Kant<sup>134</sup>, is a body of demonstrated doctrine in which everything is a-priori. It has nothing to do with empirical principles and it abstracts from all content. It is concerned altogether with the form of thought. General (pure) logic exhibits the absolutely necessary rules of thought without which there can be no employment whatsoever of understanding. The rules of general logic, then, are not concerned with some kind of an objective relation between understanding on the one hand and objects on the other, and therefore its rules cannot be viewed as sufficient for any metaphysical knowledge of reality. Transcendental logic, which is not a rival or alternative to formal

<sup>131</sup> Priest 2000, p. 142–143.

<sup>132</sup> As a matter of fact, in ‘Logic and Metalogic’ (p. 121) Vasil’ev states that indeed Kant’s view on logic was refuted adding that Kant himself helped to free logic from the impasse of Aristotelian scholastic tradition. Whether that is at all true as far as Kant is concerned is unclear, unless one understands this in the sense that Vasil’ev himself did, which implies that it was from within a Kantian paradigm that Vasil’ev developed his own criticism of traditional logic.

<sup>133</sup> Vasil’ev 1912–1913, p. 116–117.

<sup>134</sup> Cf. e.g. Aschenbrenner 1983, pp. 72ff. and Grier 2001, pp. 71ff.

(pure) logic is, in Kant's view, given the task of demonstrating certain truths that are indispensable to the pursuit of empirical knowledge, more specifically synthetic a priori knowledge of objects. The term 'transcendental' in transcendental logic in Kant is used in two ways with regard to the relation between thought and its objects. On the one hand, in as far as transcendental logic abstracts from all empirical content of thought, it can be seen as a discipline concerned with the rules for thinking objects which hold independently of our sensibility. On the other hand, in as far as transcendental logic wants to clarify the conditions under which pure thought can be applied to objects, 'transcendental' also indicates the investigation into the necessary conditions under which any knowledge of objects is possible.

The analogy with the relation between Logic and Metalogic in Vasil'ev seems to us to be obvious. *What Vasil'ev seems to have done is to apply the Kantian distinction between General (pure) Logic and Transcendental Logic to formal (or pure or general) logic itself.* Metalogic, in this view, becomes the (Kantian) pure logic since it is not concerned with empirical knowledge at all. Of course, in doing this, Vasil'ev cannot stick to what Kant did, since the 'pure-transcendental' distinction is now an internal property of logic as such. So, e.g. the law of contradiction in Kant belongs to 'pure logic', but in Vasil'ev it belongs to 'empirical logic'. *Indeed, in the distinction Pure-Transcendental, 'empirical logic' in Vasil'ev is comparable to transcendental logic in Kant. Empirical logic is concerned with the relation between thinking and objects (or 'reality'). Furthermore, it constitutes an investigation into how we can think objects independently of our own sensibility.* Vasil'ev's Imaginary Logic as a 'logic of concepts' wants to show how different logics are possible given 'different sensibilities' and it attempts to lay down the principles governing these logics. Moreover, in formulating the laws of empirical logic, imaginary logic also constitutes an investigation into the conditions of knowing objects in general. However, while using the Kantian scheme, Vasil'ev at the same time turns it 'upside down', the big difference being that in Kant logical principles cannot be but a priori while in Vasil'ev (think of the discussion of the law of contradiction) they may be a posteriori (and even synthetic). This, of course, is not Kantian, but we think nevertheless to have shown that Vasil'ev's way of working is comparable to the Kantian one.

Where now, does all of this lead us to with respect to Vasil'ev's presumed psychologism? Priest accuses Vasil'ev of "a psychologism of a Kantish kind"<sup>135</sup>, but is that even justified? In again the 'Critique of Pure Reason' Kant most notoriously distinguishes between the 'question of fact' of the 'physiological derivation' of a priori concepts (their occurrence in the mind

<sup>135</sup> Priest 2000, p. 141, n. 13.

or consciousness of man) and the question of their validity, which requires a transcendental deduction. This is all the more important, since for Kant it means that the truth of empirical knowledge does not depend on the psychological mechanisms but on a priori conditions independent of such mechanisms. Indeed, this was even the reason why Kant was accused of rejecting psychologism by its proponents. Vasil'ev stresses that metalogic (which is a priori) is part and parcel of any empirical logic and seems to us to be following the lead of Kant here. There is, thus, every reason not to consider Vasil'ev a 'psychologist', the more so since in the last decades of the nineteenth century the neo-Kantians argued against the psychologistic presentation of philosophy, but admittedly this last fact presents only circumstantial evidence. Just as does the fact that we know from Vasil'ev's biography that somewhere around 1908 he realized that his research in psychology, which he had been into for several years, could only be some kind of prolegomenon to more 'serious things', to wit the study of logic and philosophy, and as a consequence of which he abandoned his psychology studies.<sup>136</sup>

The other problem we want to go briefly into is the question of the place of Vasil'ev in the history of logic. And here the answer will have to be more variegated. We would like to make a distinction between the question whether Vasil'ev can be considered to a forerunner of (certain) non-classical logics and whether in general his method of imaginary logic can still be useful today. As might be imagined, the answer to the question whether Vasil'ev can be called a forerunner of non-classical logic has received divergent answers.<sup>137</sup> On the one hand some, like A.I. Arruda and N. Da Costa, seem to have not only been convinced that Vasil'ev can be called a forerunner of Paraconsistent logic but they actually tried to reconstruct Vasil'ev's Imaginary Logic as a paraconsistent system which could be useful for constructing other and more sophisticated systems.<sup>138</sup> N. Rescher considers him a forerunner of many-valued logic.<sup>139</sup> Moreover, since the 'rediscovery' of Vasil'ev in the former Soviet Union especially under the influence

<sup>136</sup> Stelzner (Stelzner 2001, p. 250) quotes Vasil'ev as writing in 1908 (Vasil'ev 1908, p. 141) that "psychology should serve as a firm foundation for logic" and calls this a classical confession of psychologism. From what we said before, it should be clear that in the important logical works Vasil'ev produced afterwards (from 1910 on), he has definitely changed his mind. Moreover, in the text quoted Vasil'ev, continues to write that psychology and the history of philosophy should also come before the study of ethics. Would it, then, also follow that Vasil'ev is a Psychologist and Historicist in ethics?

<sup>137</sup> Poli 1993 for details.

<sup>138</sup> See e.g. Arruda 1977, 1980, 1984, 1989, Puga (a.o.) 1988.

<sup>139</sup> Rescher 1969. See in this respect also Kline 1965.

of V.A. Smirnov and later also V.A. Bazhanov,<sup>140</sup> there is a whole lot of papers that have been coming out and are still coming out (mostly in Russian) that try to use Vasil’ev’s work in the context of modern formal logic, and the bibliography of papers and books devoted to Vasil’ev that we have added attests to this. Originally, this interest in the Soviet Union could also be explained by the fact that Vasil’ev’s ideas supposedly could be used to explain the thesis of Hegelianism and Marxism that contradictions may be fundamental for understanding reality and that the claim that reality exhibits contradictions is a claim about how the world is. Nevertheless, even in recent decades<sup>141</sup> Vasil’ev’s ideas are considered to be important, and rightly so. On the other hand, there are those holding a position ranging from ‘cautious’ to ‘outright negative’. The cautious approach to Vasil’ev is represented by W. Stelzner who, on the one hand, admits that Vasil’ev never constructed a fully worked-out system of non-classical logic but who also, on the other hand, considers him as a true reformer of logic (albeit from within traditional logic) who nevertheless might have missed the ‘other reforms’ proposed by Frege, Russell a.o. “Vasil’ev wird heute als Begründer oder zumindest Vorläufer einer kaum zu überblickenden Vielfalt nichtklassischer logischer Theorien in Anspruch genommen. Und obwohl auch Vasil’ev, wie Vladislavlev, kein entwickeltes System einer nichtklassischen Logik aufgebaut hat und bestimmte seiner Ideen, die Ansprüche auf Überwindung klassischer logischer Positionen erheben (vor allem sein erster Ansatz zu den partikulären Urteilen), in ihrer Kritik an klassisch gültigen Prinzipien der traditionellen Logik tatsächlich eher auf einem Mißverständnis der klassischen Positionen beruhen, hat Vasil’ev seine logischen Arbeiten bewußt als Gegenentwürfe zur aristotelischen klassischen Logik verfaßt und als solche Gegenentwürfe propagiert. Vor allem aber hat er die Intuitionen, auf denen diese Gegenentwürfe beruhen, in unvergleichlich stärkeren Maße thematisiert, entwickelt und begründet als dies bei Vladislavlev der Fall ist, bei dem von einer solchen Begründung keine Rede ist (...) Interessant für das Verhältnis von moderner und traditioneller Logik ist, daß auch Vasil’evs Ansätze zur Reform der Logik vollständig auf Basis der traditionellen Logik entwickelt wurden, die zu Vasil’evs Zeit durch die Arbeiten von Frege und Russell (nach heute herrschender wissenschaftshistorischer Auffassung) theoretisch bereits überlebt war.”<sup>142</sup> The negative approach gives less or no credit at

<sup>140</sup> Cfr. the bibliography on Vasil’ev for works by Smirnov and Bazhanov.

<sup>141</sup> See Priest and Routley for an historical overview of paraconsistent dialethic approaches in logic. According to Arruda (a.o., eds, 1980) (who is following Łukasiewicz here), the paraconsistent approach can even be traced back to Aristotle.

<sup>142</sup> Stelzner 2001, pp. 250–251.

all to Vasil'ev's work. A good example here is G. Priest's evaluation of the thought of Vasil'ev, an evaluation which seems to have radicalized over the years. In 1989<sup>143</sup> he still gives some credit to Vasil'ev's thought if it comes to the question whether Vasil'ev can be called a forerunner of paraconsistent systems in view of the fact that his ideas either merely come down to predicate negation or that the Law of Contradiction (LC) is somehow still retained: "However, in treating Vasil'ev as a forerunner of paraconsistent logic one must tread with great care, first in what his rejection of LC amounts to. For although one might in a logical reconstruction take "S is P and not P" ("S is P $\uparrow$ ") to be equivalent to "S is P and S is not P", Vasil'ev maintains that there are cases where "S is P $\uparrow$ " is true but both "S is P" and "S is not P" are false. This suggests that the negation in "S is P $\uparrow$ " is merely predicate negation, and this would make his position quite compatible with classical sentential logic as we have seen in the case of Meinong. But even if it is something more like sentential negation that is involved (and hence the conjunction is non-standard) there are problems. . . . But the underpinning theory must lead to predicate paraconsistency in the shape of the rejection of S is P and not P  $\vdash$  B, and so *should* lead, given that indifferent judgments retain the intended LC refuting features, to an underlying paraconsistent logic. In this tenuous sense Vasil'ev can be accounted a forerunner of paraconsistent logic."<sup>144</sup> Not much credit is, then, given to the theory, and it is concluded that Arruda's claim that Vasil'ev is a forerunner of paraconsistent logic is not much more substantial.<sup>145</sup> However, in a more recent paper, in which Priest discusses Vasil'ev's place in the history of logic he is even less positive and the only credit he is now willing to give him is that (as Quine) he realized that logic was revisable — but even that is nothing special since at the same time logic was indeed being thoroughly revised by such logicians as Frege, Russell and others — and that there is need for a certain logical pluralism where different kinds of objects require different logics that may be dependent on the empirical properties of the objects in question. Finally, Priest claims that Vasil'ev's Psychologism is of a kind that is not longer thinkable after the work of Frege and Husserl. If at all on the technical side something positive has to be mentioned Priest suggests that Vasil'ev pioneered some kind of '(im)possible worlds' logic, but there too the real breakthrough was to come only several decades later. In this sense Vasil'ev's work was too early. He

<sup>143</sup> Priest and Routley 1989.

<sup>144</sup> Priest and Routley 1989, p. 33–34.

<sup>145</sup> Priest and Routley 1989, p. 30. There it is also stated even to claim that Vasil'ev's logic is 'many-valued' is only true in 'a misleading sense' in which intensional logics are accounted many-valued.

was, in other words, a visionary who had nevertheless no real idea of what was going to come. “Now that we have looked at Vasil’ev’s ideas, both logical and philosophical, it is time to assess his significance in the history of logic. I think that this can hardly reside in the content of his work. The logic itself is now little more than a curiosity, and the philosophical ideas, I have argued, do not stand up to inspection. . . . Characterizing Vasil’ev’s radicality is much harder. As a logician, it has been claimed, Vasil’ev was a forerunner of many-valued logic. This is hardly the case. Imaginary logic is not a many-valued logic in anything close to the modern sense. It is not a modern logic at all. Sometimes, he is claimed to be a forerunner of paraconsistent logic. This is closer to the truth, but still incorrect. Paraconsistent logics challenge the claim that everything follows from a contradiction. Vasil’ev did not even have the logical machinery to formulate this claim. Perhaps closer still, he is a forerunner of dialethic logic, which endorses some contradictions. But even this is not right, since, after all, he endorsed the law of non-contradiction. . . . Still, however one locates Vasil’ev’s originality — be it in anticipating paraconsistent logic, impossible worlds, revisability or pluralism — the developments that would have to happen to make it possible to appreciate this were some time into the future. In this sense, his work was too early.”<sup>146</sup>

We are not convinced of this evaluation. On the one hand we have shown earlier that Vasil’ev can hardly be accused of Psychologism, on the other hand we will here not try to show that Vasil’ev was in fact a forerunner of paraconsistent or many-valued logic or even an intuitionist for that matter. This discussion we will reserve for another paper. What we will now, by way of conclusion, try to show (pace Priest) is how Vasil’ev’s ideas may still be important in logic by analyzing critically, along the lines of Vasil’ev, an important procedure in mathematics (or logic), viz. Cantor’s Diagonal Procedure. In doing this we will start from Vasil’ev’s distinction between ‘external’ and ‘internal’ negation which was already discussed earlier in this paper.

#### 6. *Conclusion: On the Difference Between External and Internal Negation: A Vasil’evian Critique of Cantor’s Diagonal Procedure*

As was already pointed at, in Vasil’ev’s work there are three main ideas on the essence of negation.<sup>147</sup> The first relates to the fact that Vasil’ev, following

<sup>146</sup> Priest 2000, p. 143–144.

<sup>147</sup> E.g. Vasil’ev 1912a, pp. 85–88.

Aristotle, systematically discerns between two kinds of negation: external negation of the form " $A$  is not  $B$ ," which is a 'genuine' negation, and internal negation of the form " $A$  is non- $B$ ," which is in fact an affirmation about the thing  $A$  having the predicate non- $B$ . Another insight consists of the acknowledgement of the conditional character of internal negation, that is, of the dependency of the predication of  $B$  and non- $B$  upon a previously fixed genus. The third idea concerns Vasil'ev's view that in our "terrestrial" logic, that is, in the logic of actual scientific research, we pass over the problem of the transformation of external to internal negation, by implicitly setting up an appropriate genus, which is not transgressed during the course of reasoning, and within which we define negation. Since the definition of an appropriate genus is essentially 'informal' (it is not the logician, but the specialist in a specific scientific field who fixes the universe of discourse), Vasil'ev considers our "terrestrial" negation and all that relies on it as belonging not to the logical nucleus (the metalogic), but to empirical logic, which is the variable part of logic. As belonging to the variable part of logic, such a negation may be replaced by another one, which Vasil'ev calls "absolute negation." Absolute negation is introduced in such a way, that internal negation becomes always (without the fixation of an appropriate genus) equivalent to external negation.

But in our "terrestrial" scientific investigations (and not in the sphere of the "Spirit who never makes mistakes", as Vasil'ev says), in order to correctly convert an external negation into an internal one, we always have to fix an appropriate genus. We will give a simple example of scientific reasoning in which a correct fixation of the genus allows us to correctly transform an external negation into an internal one. Subsequently, we will give another (slightly different) example, in which such a transformation becomes incorrect owing to an erroneous way of defining the genus. Let us, firstly, consider the true proposition, "7 is not an even number," which has the form of an external negation. We can transform it into a further correct statement "7 is an un-even (odd) number," which has the form of an internal negation. The correctness of this transformation is guaranteed by the fact that we are — which is what mathematicians implicitly assume — dealing with the genus "natural number," which consists of two opposite species, viz. "even numbers" and "odd numbers." Such a transformation of an external negation into an internal one is valid, and thus systematically used in conditional proofs (*reductio ad absurdum*). The situation changes when we consider another example of a true proposition, which has the form of external negation, such as " $1/2$  is not an even number." Routinely transforming this judgment into internal negation we get " $1/2$  is an un-even (odd) number," results in a proposition which is obviously false. Thus, the transformation of an external negation into an internal one will in this case be incorrect. The two propositions " $1/2$  is

an even number” and “ $\frac{1}{2}$  is not an even number” are contradictory, and the former is false while the latter is true (this would, in Vasil’ev’s view, be an example of “the absolute distinction between truth and falsehood”). By contrast, both propositions “ $\frac{1}{2}$  is an even number” and “ $\frac{1}{2}$  is an odd number” are false. The reason for their falsehood lies in the fact that the number  $\frac{1}{2}$  is not a natural number, and consequently the predicates “even” and “non-even (odd)” cannot be applied to it at all. Therefore, mathematicians will never infer from the falsity of the proposition “ $\frac{1}{2}$  is an even number” that the proposition “ $\frac{1}{2}$  is an odd number” is true.

However, there are situations in which mathematicians seem to forget about the fact that a correct transformation of an external negation into the corresponding internal variety presupposes the fixing of an appropriate genus. This is the case in Cantor’s Diagonal Procedure which is used in, e.g., proving that the cardinal number of a set of subsets of a given set is larger than the cardinal number of the set itself.<sup>148</sup>

Let us begin our analysis of the use of the diagonal procedure in this demonstration by posing a preliminary question as to what kind of object a “set” is. There are several places in G. Cantor’s work where he is trying to give content to the concept of a “set”. In a paper on transfinite set-theory dating from 1895 he says the following: “By a ‘set’, we understand every collection into a Whole  $M$  of definite and well-differentiated objects  $m$  of our intuition or our thought (which are called the ‘elements’ of  $M$ )”.<sup>149</sup> According to e.g. Max Black, the reference to definite and well-differentiated objects means that Cantor wants to recognize as elements of a set only sharply demarcated objects and not ‘fuzzy ones’ (so e.g. numbers but not clouds), that are indeed subject to sharp criteria of identity and difference (e.g. ‘men’ vs. ‘electrons’).<sup>150</sup> In a letter to Dedekind from 1899, Cantor goes into more detail, making a distinction between ‘absolutely infinite or inconsistent Multiplicities’ and ‘consistent Multiplicities’ which he calls ‘sets’: “If we start from the notion of a definite multiplicity (a system, a totality) of things, it became clear to me that we must necessarily distinguish between

<sup>148</sup> The incorrectness of Cantor’s Diagonal Procedure has been first indicated by Sergei Bychkov, who, in doing so, was not influenced by Vasil’ev but, rather, by Aristotle. Using the results of Daniil Vinner (in Vinner 1997) on the non-equivalence of the external and the internal negation, Bychkov (a.o.) developed a criticism of Cantor’s Diagonal Procedure in a number of papers: Bychkov 1997, 1999, and 2000.

<sup>149</sup> Cantor 1895 §1, p. 481: “Unter einer ‘Menge’ verstehen wir jede Zusammenfassung  $M$  von bestimmten wohlunterschiedenen Objekten  $m$  unsrer Anschauung oder unseres Denkens (welche die ‘Elemente’ von  $M$  genannt werden) zu einem Ganzen.”

<sup>150</sup> Black 1971, pp. 618–619. There is a whole literature on the concept of a set in Cantor. Hallett’s book (Hallett, 1984) provides a lot of details and is very informative in this respect.

two kinds of multiplicities (by this I always mean *definite* multiplicities). For, on the one hand a multiplicity can be such that the assumption that *all* of its elements ‘are together’ leads to a contradiction, so that it is impossible to conceive of the multiplicity as a unity, as ‘one finished thing’. Such multiplicities I call *absolutely infinite or inconsistent multiplicities*. As one can easily see, the ‘totality of everything thinkable’, for example, is such a multiplicity; later still other examples will present themselves. When, on the other hand, the totality of elements of a multiplicity can be thought without contradiction as ‘being together’, so that their collection into ‘one thing’ is possible I call it a *consistent multiplicity* or a *set*.<sup>151</sup> Sets are then, multiplicities that are ‘well-formed’ i.e. that can be thought of as a whole without contradiction. Apart from the relation that Cantor here makes between ‘absolutely infinite’ and ‘inconsistent multiplicities’<sup>152</sup>, it is striking that he now explicitly mentions the idea of non-contradiction (or ‘consistency’) in relation to a ‘real set’. This gives more content to his 1895 definition of set which only maintained that sets consist of definite well-differentiated things in our intention or thought. So, the condition of ‘definiteness’ and ‘well-differentiatedness’ is here specified in terms of what can actually be called the law of non-contradiction, of which we know that Vasil’ev treated it as an ‘empirical law’.

In order to see what this implies for Cantor’s diagonal procedure in view of Vasil’ev’s analysis, we will consider Cantor’s idea of a multiplicity (‘Vielheit’) as a genus and use the term ‘class’ here. If we now consider a genus such as ‘Class’ (taken as ‘primitive’), by what definition can we then discern sets from classes or, in terms of traditional logic, what is the *differentia specifica*, which permits of the definition of the species, “Set,” within the genus “Class”? One answer, which is usually given in most handbooks (that are based on naïve, non axiomatized, set-theory), is the following: a “Class”  $X$

<sup>151</sup> Cantor, letter to Dedekind. 3.8.1899, in Cantor 1991, p. 407: “Gehen wir von dem Begriff einer bestimmten Vielheit (eines Systems, eines Inbegriffs) von Dingen aus, so hat sich mir die Notwendigkeit herausgestellt, zweierlei Vielheiten (ich meine immer *bestimmte* Vielheiten) zu unterscheiden. Eine Vielheit kann nämlich so beschaffen sein, daß die Annahme eines ‘Zusammenseins’ *aller* ihrer Elemente auf einem Widerspruch führt, so daß es unmöglich ist, die Vielheit als eine Einheit, als ‘ein fertiges Ding’ aufzufassen. Solche Vielheiten nenne ich *absolut unendliche* oder *inconsistente Vielheiten*. Wie man sich leicht überzeugt, is z.B. der ‘Inbegriff alles Denkbaren’ eine solchen Vielheit; später werden sich noch andere Beispiele darbieten. Wenn hingegen die Gesamtheit der Elemente einer Vielheit ohne Widerspruch als ‘zusammenseiend’ gedacht werden kann, so daß ihr Zusammengefaßt werden zu ‘*einem* Ding’ möglich ist, nenne ich sie eine *consistente Vielheit* oder eine ‘Menge’”.

<sup>152</sup> See Hallett 1984, p. 186ff. for details.

is called a “Set,” when for any arbitrarily chosen object  $y$  one of the opposites is true — viz. whether this object belongs or does not belong to this class. In symbolical form, whether  $y \in X$  or  $y \notin X$ . The possibility of indicating a correct answer (choosing between two variants  $y \in X$  or  $y \notin X$ ) is the only condition for a class to be a set. The definition of a set also permits of the definition of classes that are not sets. In order to define classes that are not sets (let us call these “Proper Classes”) we should take the negation of the definition of set. Thus we may call a class  $X$  a “Proper Class” (non-set) if there exists an object  $y$  such that either a) both  $y \in X$  and  $y \notin X$  are true, or b)  $y \in X$  and  $y \notin X$  are false.

Consider, now, the proof of Cantor’s theorem. Let us take an arbitrary set  $X$  and the class of its subsets  $P(X)$ . Cantor’s theorem states that the cardinal number of  $P(X)$  is larger than the cardinal number of  $X$ . The proof of the theorem comprises three steps.

Step 1. Consider an arbitrary mapping  $f$  from  $X$  into  $P(X)$ , that is the function  $f$ , which makes  $x$ , for every  $x \in X$  correspond to a subset  $f(x) \in P(X)$ . Then, let us, after Cantor, define a subclass  $Z$  of the set  $X$  in the following way. An element  $x \in X$  belongs to the class  $Z$ , if and only if  $x \notin f(x)$ .

Step 2 (the diagonal procedure itself). Suppose that there exists an element  $t \in X$ , such that  $f(t) = Z$ . There are two possibilities: whether  $t \in Z$ , or  $t \notin Z$ . If  $t \in Z$ , then it follows from the definition of  $Z$ , that  $t \notin f(t) = Z$ . If  $t \notin Z$ , then to the contrary,  $t$  should belong to  $Z$ , because  $t \notin f(t)$ . This contradiction shows that the hypothesis about the existence of an element  $t$ , such that  $f(t) = Z$  is false.

Step 3. Let us assume that there exists a mapping  $f : X \rightarrow P(X)$  which effectuates a one-to-one correspondence between  $X$  and  $P(X)$ . If we construct a subclass  $Z$  for the given function  $f$  as in step 2, this subclass will not have an inverse image (there will be no  $t \in X$ , such that  $f(t) = Z$ ). Since there exists a correspondence between the set  $X$  and a proper subclass of the class  $P(X)$ , the cardinal number of the class  $P(X)$  is larger than the cardinal number of the set  $X$ .

This proof, however, is an illusion. Let us focus our attention upon the second step of the proof, that is upon the diagonal procedure itself. Or, to be more precise, on that part of it in which we consider just two possibilities for the element  $t$ : either  $t \in Z$ , or  $t \notin Z$  (each of these possibilities leading to a contradiction). Two questions arise in respect with this reasoning. (i) Why do we think that both variants —  $t \in Z$  and  $t \notin Z$  — cannot be realized at once? (ii) Why do we think that the two variants —  $t \in Z$  and  $t \notin Z$  — exhaust all the possibilities? The answer to both questions seems to be at hand: we accept both hypotheses because we tacitly assume that *the class  $Z$  is a set* (a subset of the class  $X$ ). Indeed, it follows from the definition of a set that if we deal with a set  $Z$ , then both the law of contradiction and

the law of the excluded middle hold with respect to the opposites  $t \in Z$  and  $t \notin Z$ .

But the method of definition of the class  $Z$  in Cantor’s diagonal procedure does not, in fact, guarantee that  $Z$  is a set. If this is not the case, that is, if  $Z$  is not a set, but a proper class, there is an object  $x \in X$ , such that a) both  $x \in Z$  and  $x \notin Z$  are true, or b) both  $x \in Z$  and  $x \notin Z$  are false. Furthermore, nothing precludes that the element  $t$ , which we supposed to be the inverse image of  $Z$  by the mapping  $f$ , is the object in question.

Therefore, the reasoning by means of the diagonal procedure does not actually imply that  $Z$  does not belong to the image of the function  $f$ . It only means that the class  $Z$  is not a set at all.

In Bychkov (Bychkov, 1999) it is shown that Cantor himself, in order to guarantee that  $Z$  is a set, used two *ad hoc* hypotheses treated by him as postulates (without offering any sound arguments in support of them). The first of these postulates states that for every set  $X$  the class of its subsets  $P(X)$  is also a set. The second states that if a class  $Y$  is a subclass of a set  $X$ , then the class  $Y$  is a set. Both postulates are usually accepted in axiomatized set theories.

The difficulty in the argument (the diagonal procedure) arises because the principle of the correct transfer from external to internal negation is neglected. Indeed, consider a proposition of the form “ $y$  belongs to  $X$ ,” or symbolically  $y \in X$ . This proposition has in Aristotelian (or Vasil’ev’s terms) the form of the affirmation “ $y$  is (an element) of  $X$ .” The (external) negation of this proposition is “ $y$  is not (an element) of  $X$ ,” or in symbolic form  $\neg(y \in X)$ , the opposites  $y \in X$  and  $\neg(y \in X)$  being contradictory as affirmation and negation. But, in the Diagonal Procedure, a conditional proof is employed, in which the external negation of the form  $\neg(y \in X)$  is routinely transformed into internal negation of the form  $y \notin X$ , or “ $y$  is (an element) not belonging to  $X$ .” Such a transformation is, however, only correct if and only if an appropriate genus of the species “set” is fixed. There is only one genus which encompasses the species “set,” and that is the genus “class.” But, as we have noted above, when formulating the definition of a set and a proper class respectively, a class is something indefinite as regards the membership (or the lack of thereof) of any arbitrarily chosen object. If we take an arbitrary object and a given class, we cannot decide whether this object belongs to this class or not. (This would only be possible if the class in question was a set). Thus, we must conclude that the reasoning used in the diagonal procedure is in fact incorrect. As a consequence, we can infer that the so-called hierarchy of sets, introduced by Cantor, is in limited to finite and denumerably infinite sets only.<sup>153</sup>

<sup>153</sup> In Bychkov 1999, the (incorrect) argument used in the Cantor’s diagonal procedure is compared to three examples of proofs — in arithmetic, geometry and analysis — in which

Thus, what is a routine mathematical procedure — the transformation of an external negation into an internal one — becomes problematic when it is applied in conditional proofs to the highest genera of mathematical (set-theoretical) concepts, viz. sets and classes. But this fact is usually neglected by mathematicians, who do not seem to acknowledge that Cantor’s application of set theory goes far beyond the usual mathematical ways of reasoning (in which the fixation of an appropriate genus determines the correctness of the transformation of external negation into internal negation). In this respect, we would like to emphasize the importance of Vasil’ev’s logical ideas, which, along with the Aristotelian concept of negation, may be useful for a critical reappraisal of the set theoretical foundations of modern mathematics.

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the fixation of an appropriate genus assumes the correctness of the transfer from external to internal negation (the first one is the theorem about the irrationality of the ratio of the diagonal to the side of a square, Euclid’s proof by contradiction). In this paper a theological rationale behind Cantor’s uncritical attitude towards the diagonal procedure is also pointed out. The construction of the infinite hierarchy of cardinal numbers, which never attains the genuine infinite Absolute, was a nucleus of Cantor’s mathematical theology.

\* The bibliography consists of two parts. The first part (A) consists of Vasil’ev’s published work. Prof. V.A. Bazhanov (Ulyanovsk State University, Russian Federation) has an archive on Vasil’ev containing more (as yet unpublished) work. The second part (B) is a bibliography on Vasil’ev with publications in Russian and other languages next to the references of the literature used in this paper. Though the bibliography is not intended to be exhaustive, its aim is to nevertheless provide a good overview of recent and less recent work on Vasil’ev’s ideas (*R.V. – E.Z.*).

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