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RESPONSE TO "WHAT IS THE GOAL OF PROOF?"

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Mathematicians will only use deductive proofs to establish the truth of mathematical results. For example, they will not use probabilistic proofs, or other forms of empirical confirmation, to establish the truth of mathematical results. In "What Do Mathematicians Want?," I try to use the framework of means/ends reasoning to explain this methodological choice. However, I conclude that there may be no epistemic objective of mathematicians that provides a satisfying explanation of the rejection of probabilistic proofs.

Of course, scientists as well as mathematicians make use of mathematical results. In "What is the Goal of Proof?," Aaron Lercher suggests that the epistemic objectives of *scientists* may explain the rejection of probabilistic proofs. In order to try out this suggestion, Lercher considers three examples from the history of science.

In each of Lercher's examples, a scientist needs to make use of a particular geometrical result and prefers to have a proof of this result rather than an empirical confirmation. For example, in order to confirm a hypothesis about acceleration, Galileo needed to make use of certain results about the areas of triangles; Galileo proved these geometrical results instead of empirically confirming them by measuring several actual triangles. Similarly, in order to explain how the lens of the eye refracts light, Descartes needed to make use of certain results about the tangents of curves; Descartes proved these geometrical results instead of designing and constructing a mechanical device to empirically confirm them.

In each of Lercher's examples, proofs seem to have two important advantages over empirical confirmation. First, the proofs are easier to obtain. Empirical confirmation of these results would require much more effort. For example, as Lercher notes, "Descartes uses non-empirical methods to solve this problem in natural science, when strictly empirical methods would have been too difficult for him." Second, the proofs are more reliable. For example, Descartes would certainly be much more likely to make a mistake in the design and/or construction of such a complicated mechanical device. These two advantages can explain why the scientists prefer proof to empirical confirmation in these examples.

Of course, I do not deny that scientists often (indeed almost always) have good epistemic reasons to prefer proof to empirical confirmation. In fact, mathematicians as well as scientists recognize the advantages of ease-of-use and reliability. Thus, many of Lercher's points could probably have been made with examples from the history of mathematics as well as the history of science. The open question, however, is whether scientists and mathematicians *always* have good epistemic reasons to prefer proof to empirical confirmation. My contention is still that the answer to this question may be no.

There are other examples where proofs do not have these two important advantages over empirical confirmation. For example, suppose that a scientist is developing an encryption scheme for secure communication and needs to know whether particular numbers are prime. In this case, empirical confirmation (e.g., using the Rabin test) seems to be easier and more reliable than proving primality. It is not clear from what Lercher says why scientists should still prefer proof to empirical confirmation when empirical confirmation is easier and more reliable. In fact, it is not even clear that scientists *would* prefer proof to empirical confirmation in such a case. As a result, Lercher has not provided a complete explanation of the rejection of probabilistic proofs and other forms of empirical confirmation.

While I think that Lercher has yet to explain this methodological choice of mathematicians, considering the epistemic objectives of scientists as well as mathematicians is an interesting and potentially fruitful strategy. However, as Lercher himself points out, his strategy only helps the scientific naturalist (e.g., Quine), and not the mathematical naturalist (e.g., Maddy), explain this methodological choice. According to the mathematician Carl Pomerance though, there is "a qualitative difference between probabilistic verification and mathematical proof that is important to *mathematicians*" (emphasis added). Thus, we might have hoped that this methodological choice could be explained by appealing only to the epistemic objectives of mathematicians themselves. Consequently, an explanation that appeals to the epistemic objectives of non-mathematicians would be somewhat unsatisfying.