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THE ROLE OF AMBIGUITIES IN THE CONSTRUCTION OF COLLECTIVE THEORIES

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Abstract

The paper presents a formal model for theory development, based on a very intuitive ambiguity-adaptive logic. Apart from its simplicity, the model has some interesting features. (i) It allows for the construction of theories that cannot rely on observational data. (ii) It allows to construct a theory starting from zero, and using a small set of predicates. (iii) The model establishes that there is no real difference between the construction of scientific theories and the development of everyday knowledge.

1. *Introduction*

The success as well as the *epitheton ornans* of the 'exact' sciences may give rise to the opinion that the terms used in these theories have exactly one meaning. This opinion on its turn may give rise to the opinion that scientific theories must fulfill the norm *"one term one meaning"*. People who share these opinions clearly forget about the fact that we cannot look inside each other's head, and that therefore we never know for sure what a given term means for someone else. Hence there is no positive test for *"Term T has exactly one meaning"*. There is however a negative test. We can conclude that a term T does not has exactly one meaning when an inconsistency arises *from the collective use of the term.* I give a fictive example:

> A: "You know what X's are ?"
> B: "Of course, they are P and Q; some of them are R and others are R'."
> A: "Indeed. Today I have seen a X that is S."
> B: "That seems unbelievable to me. I would think that no X's are S. Could you show me one ?"

*As so many times, I am extremely indebted to Kristof De Clercq, for his accurate remarks on drafts of this paper.

Throughout their communication, A and B meet an inconsistency. If A was right to have seen some X that is S, then B meets an inconsistency between her 'accepted theory about X's', and the new data. Let us have a look how the story continues:

> A: "Here you are: an X that is S !"
> B: "Dear B, I am afraid to say that this looks very much like an X but it is not an X: you see these little T's here on the bottom ? This is not an X, it is a Y !"

As long as A and B were talking about X's in terms of P, Q, R and R', it was impossible for them to find out that the term X had a larger extension in A's opinion than in B's opinion. At the mean time we see that there are two ways in which the collective inconsistency can be solved: (i) B adapts her theory about X's. "No X's are S" must be replaced by "Some X's are S". (ii) For some members of the group the term X must be replaced by two 'new' terms, *viz.* the term "X" with a more precise meaning and the term Y. It is important to see that this replacement of terms does not necessarily apply to "X". The story might as well have continued in this way:

> A: "Here you are: an X that is S !"
> B: "Dear B, I am afraid to say that this X has a property that looks very much like S, but it is not quite S, it is Z. Look at this other thing, *this* thing is S."
> A: "Oh, I see, I have never made a distinction between S and Z.

In this paper I focus on the use of predicates. When a group of scholars create a new theory about some domain, they need a common language in which they can talk about the domain, and common actions within the domain. *They do not need predicates with an a priori common, transparently clear meaning.* Throughout the scholars' interactions in words and deeds, the meaning of the used predicates can become more precise, and the statements can become more exact, *within the group of scholars*. Maybe it is more correct to say that *the use* of the predicates can become more *common*. In this process inconsistencies play an important role. The derived inconsistencies indicate which predicates are not precise enough, and which statements are not exact enough, or, if you want, the derived inconsistencies indicate which predicates and which statements are not used in a common way.

Where does a group of scholars find new predicates to describe the objects of their domain? A first observation is that the set of *available* predicates is not infinite. Every actual collective vocabulary is finite. If a is the name of an element of the domain, and $P_1, ..., P_n$ are the only predicates of rank

1 available in the common language, and if a group of scholars does want to describe a, then $P_1 a \vee ... \vee P_n a$ *must* be accepted within the group. If some P_i is selected as a candidate for the collective description of a, the meaning of P_i may, and probably will change throughout the use of it. *E.g.* When modern physicists wanted to construct a theory about 'light', some of them decided to use the predicate "wave". At that time this predicate referred to a specific maritime phenomenon, but throughout the use of the term in the field of physics, the term got a domain-specific interpretation. Still, any two physicists cannot know for sure whether they *interpret* the term "wave" in the same way, even if they *use* the term in a common, consistent way.[1] As long as the use of the term does not lead to the derivation of an inconsistency or any other logical abnormality, physicists suppose that, or act as if all physicists interpret the terms they use in the same way.

When scholars create a new theory, they can go and gather data and formulate hypotheses *at random*. It may however be hard to formulate hypotheses and it may even be harder to get and describe new data, if the scholars do not have an appropriate language at their disposal. Scholars often need to be creative and often need to use their imagination. With respect to creativity and imagination we can say that the inspiring sources never are abundant. Creative scholars can, *e.g.* borrow general statements from other domains. When a predicate "P" is borrowed from another domain, in which "$(\forall x)(Px \supset Qx)$" is an accepted statement, the scholars may check whether they can borrow this statement too. The borrowed statement "$(\forall x)(Px \supset Qx)$" may belong to the core of a new theory for quite a long time, even if the meaning of the term "Q" changes completely throughout the development of the theory.

In this paper it is shown that an ambiguity-adaptive logic is an excellent tool for the creation of new collective theories. At an early stage of the construction of a theory, the domain-specific terms are unavoidably vague or ambiguous. Still, the creation of the theory will never take off, if the scholars (who belong to one group) do not assume that all of them use the terms they use in a common way. Relying on the strength of the ambiguity-adaptive logic, the scholars can credulously assume that all of them give the same meaning to a specific term, *unless and until this assumption leads*

[1] Even when logicians write down an axiom scheme in which the *use* of some logical term is exactly defined, they still need a common praxis in order to know about the use of the axiom scheme. If an axiom scheme would be self-evident, every first year student who receives a simple printout of the scheme, would be able to derive all theorems of any logic. I know from my experience with first years students, that many of them need lots of explanation before they can even apply the rule *Modus Ponens*. If they are not told what to do *in their natural language*, they never find out what to do, nor what all these symbols could mean.

to the derivation of an inconsistency. Such a derived inconsistency is an indication of one of the following situations: (i) The use of some term P is 'not common enough', and therefore it should be replaced by (at least) two new terms P^1 and P^2.[2] (ii) Some term P must be subject to conceptual change. An example of the latter situation: the scientists who developed the theory of thermodynamics had to re-interpret the concept "heat" after the derivation of some inconsistency.[3] The former situation is typical for theories that are in full development. The reason is obvious: scientists who work for the first time on a theory about some domain cannot avoid to use terms the meaning of which is only known implicitly or terms that are used in some metaphorical way. For instance, when early Greek philosophers constructed a theory about everything beneath the moon, a group of them took for granted that everything was made of water, wind, earth and fire, although "fire" did not exactly refer to the well-known flames, and "wind" did not exactly refer to the lukewarm breeze moving their beards and stoles. When physicists formulated for the first time a definition of speed, they had to borrow the expressions "distance" and "difference in time" from their everyday language. I bet that these expressions did not have their abstract and theoretical meaning from the beginning.

In section 3 I briefly present the ambiguity-adaptive logic AAL. In section 4 I present a very intuitive model, based on this ambiguity adaptive logic, which may be used to reconstruct old and construct new collective theories. By means of this model I think it can be shown that there is no fundamental difference between the way a young child learns to talk about its world and the way a new theory is constructed. I hope to present the reconstruction of some historical examples of conceptual change by means of the here presented model in another paper. My main interest however is to show that the model I present can be used to construct theories about 'things' that cannot be observed, but are really important to us. I am especially thinking of 'experiences' and 'values'. I hope to establish in this paper that neither the fact that we only have the disposal of vague or metaphorical predicates, nor the fact that it is hard to make any universally valid statement about all values, are obstacles for people who want to construct a *collective* theory about values.

In order to show that all scientific theories are "collective theories", I will start this paper with some philosophical observations.

[2] Of course, either P^1 or P^2 can be the old term P.

[3] This is sharply described in [6] by Joke Meheus.

2. *All about our experiences*

Although Kant already mentioned that we know nothing about 'the object *an sich*', many people seem to forget that our statements 'about objects' are always statements about the objects' appearance *für uns*. We can even say that the expression *"für uns"* is not correct enough: every statement about objects is always a statement about *some individual's* experience of the object. Even when two persons act and communicate within one and the same context, and talk about one and the same object — an object that one of them holds in her hands, while she is showing it to the other person, for example — and even if they use a language they have been communicating with for ages, both persons talk about their own experience of the object. Even if they adhere to a very realistic world-view. These persons may talk about the size of the one and only sheet of paper lying on the table, they may use a very precise measuring tool, and they may agree that this sheet is 29,7 cm by 21,0 cm, they are still talking about their own experience 'of the sheet of paper'. Indeed, none of us can step outside of our own experiences. Still, we often intend to say things about the object itself, and not about our own experience of the object.

The apparent continuities and similarities that emerge within our empirical experiences, and show up between our empirical experiences and our communicative experiences, can be explained in three ways. (i) *Solipsism.* The one and only solipsist may say that these continuities and similarities are the product of her thoughts. (ii) *Scepticism and the chaotic world view.* Very sceptical people may say that the apparent similarities and continuities are *only apparent*, and people who believe that the universe is nothing but chaos may say that the similarities and continuities are all accidental. (iii) Most of us however will agree that these continuities and similarities can be explained by the fact that we have experiences of one and the same world, and the fact that we have learned to use pieces of language in an analogous way. People who adhere to (i) or (ii), and who want to buy some sandwiches at the food store, will have to act as if they adhere to (iii), anyway. Nevertheless, the fact that I believe that the sandwiches I eat are real objects, does not change the fact that *my* life exists of my experiences, nor the fact that whatever I say about a sandwich is mediated by my experiences. We may believe in the existence of an objective world, and I believe most of us do, and we may succeed in indicating (apparently) the same objects with the same pieces of language, but the fact that we agree about the correctness of one and the same statement *about the world*, does not imply that we agree for the same reasons. For my present purpose, the point I want to stress is that we never know for sure that we actually do mean the same when we use the same words. All we can do is communicate and interact and experiment as much as possible, in order to find out if our credulous assumption that we do mean the same when we use the same word, is not contradicted.

Given the fact that all our knowledge of the world, is mediated by individual experiences, and taking into account that we need to interact in order to obtain some common statements about the world, whereas the number of people we interact with is rather small in comparison with the number of all people that ever lived or will live, it is easily seen that we can never reach a universal interpretation of a term but at most a collective interpretation of a term. *A fortiori* all theories about the world are theories belonging to some specific community *i.e* (literally) *collective* theories.

3. *The ambiguity-adaptive logic* AAL

The underlying idea of ambiguity-adaptive logics is double: (i) we have to aim at a common and concise language; (ii) we are often too credulous in assuming that every two occurrences of one and the same non-logical constants (henceforth NLC) always mean the same. NLC may be ambiguous or vague, and even if they are not vague or ambiguous, different people may interpret them in a different way. Even one single person may interpret some words in a different way when she is using the words in different contexts. If, for instance, we derive $(\forall x)(Px \supset Rx)$ from $(\forall x)(Px \supset Qx)$ and $(\forall x)(Qx \supset Rx)$, we have to assume that both occurrences of the predicate Q can be identified with one another.

The technical approach is very easy. We give a different superscript to every NLC occurring in some set of premises. Let $\Gamma = \{(\forall x)(Px \supset Qx), (\exists x){\sim}Qx\}$. The maximally ambiguous interpretation Γ^I of Γ will be $\{(\forall x)(P^1x \supset Q^1x), (\exists x){\sim}Q^2x\}$. Applying Classical Logic (henceforth CL) to Γ^I we can derive $(\exists x){\sim}P^1x \vee (\exists x){\sim}(Q^1x \equiv Q^2x)$. The ambiguity-adaptive logic AAL assumes that $(\forall x)(Q^1x \equiv Q^2x)$ is the case, unless and until $(\exists x){\sim}(Q^1x \equiv Q^2x)$ is derived from the premises. Hence, for the premises at hand, we have $\Gamma \vdash_{\mathsf{AAL}} (\exists x){\sim}Px$. If Γ would be extended with the premise $(\forall x)Px$, $(\exists x){\sim}Px$ would not be an AAL-consequence anymore. $(\exists x){\sim}(P^1x \equiv P^2x) \vee (\exists x){\sim}(Q^1x \equiv Q^2x)$ would be a CL-consequence of the set $\Gamma^I \cup \{(\forall x)P^2x\}$. The latter result can be interpreted in the following way: either the first occurrence of P cannot be identified with the second occurrence of P, or the first occurrence of Q cannot be identified with the second occurrence of Q. *I.e.* at least one of the predicates is not precise enough. The decision whether we should replace P or Q cannot be made on logical grounds, but in specific situations there may be good contextual reasons for a choice, as we shall see in section 4.

The mechanism of the ambiguity-adaptive logic can also be demonstrated as follows. Within one proof from a given set of premises, the logic AAL oscillates between an upper limit logic and a lower limit logic. The upper limit logic at hand is CL plus the assumption that for all NLC C^i, C^j : $\sim C^i \neq C^j$.[4] The lower limit logic at hand is CL plus the assumption that for all NLC $C^i, C^j : C^i \neq C^j$ (where $i \neq j$).

In this paper I use the name AAL to refer to the logic AAL^2 that was presented in [10]. The first ambiguity-adaptive logic was presented in [9].[5] For the present purposes (the construction of theories) we can restrict the NLC in the language to predicates of rank n.

3.1. *The maximally ambiguous interpretation*

Let $\mathcal{L}$ be the language of CL, containing $\supset$, $\sim$, &, $\vee$, $\equiv$, $\forall$, $\exists$, $=$ and the members of $\mathcal{P}^r$, $\mathcal{C}$ and $\mathcal{V}$. $\mathcal{P}^r$ is the set of letters for predicates of rank r ($r \geq 1$). $\mathcal{C}$ is the set of letters for individual constants. $\mathcal{V}$ is the set of letters for individual variables.[6]

Let $\mathcal{L}^{\mathcal{I}}$ be obtained from $\mathcal{L}$, by replacing $\mathcal{P}^r$ by $\mathcal{P}^{r\mathcal{I}}$. For $i = 1, 2, ...,$ $\pi^i \in \mathcal{P}^{r^{\mathcal{I}}}$ iff $\pi \in \mathcal{P}^r$.

Where $C \in \mathcal{P}^r$, we define $\mathcal{I}(C) = \{C^i \mid C^i \in \mathcal{P}^{r\mathcal{I}}\}$. $C^i \in \mathcal{I}(C)$ is called an *indexed* predicate (in general: an *indexed* NLC). Let the normal set $\mathcal{W}$ of well-formed CL-formulas (henceforth wffs) of the language $\mathcal{L}$ be defined as usual and let $\mathcal{W}^{\mathcal{I}}$ be defined in the language $\mathcal{L}^{\mathcal{I}}$ in the same way. In what follows, the language of CL will be $\mathcal{L}^{\mathcal{I}}$.[7]

We need to define an appropriate interpretation of a set of premises $\Gamma \subset \mathcal{W}$. The only requirement is as follows: every predicate P occurring n times in Γ ($n \geq 1$) must get n different superscripts in this interpretation. Obviously, this requirement can be fulfilled in an infinite number of ways. Let us

[4] For the definition of $C^i \neq C^j$, see Definition 1.

[5] The idea of interpreting inconsistencies as ambiguities is also elaborated in [5].

[6] I think it is justified, for the present purposes, to assume that the considered sets of premises do not contain letters for individual constants, nor propositional constants. Still, the language scheme allows for members of $\mathcal{C}$, be it for being able to apply the rules Universal Instantiation and Universal Generalization. If some specific domain requires individual constants for the formulation of a theory, we can either suppose that individual constants are never ambiguous or treat them in the same way as the predicates. In the latter case, ambiguities concerning individual constants are of the form $\sim\alpha^i = \alpha^j$.

[7] For short: $\mathcal{L}$ contains only NLC without indices, $\mathcal{L}^{\mathcal{I}}$ contains only NLC with indices.

call the set of all possible appropriate interpretations of Γ the set $\mathcal{I}(\Gamma)$. We define $\mathcal{I}(\Gamma)$ as follows: let $\mathcal{I}(\Gamma)$ be such that $\Gamma^I \in \mathcal{I}(\Gamma)$ iff
(i) $\Gamma^I \subset \mathcal{W}^{\mathcal{I}}$,
(ii) each element of $\mathcal{P}^{r\mathcal{I}}$ occurs at most once in Γ^I, and
(iii) deleting the superscripts from the elements of $\mathcal{P}^{r\mathcal{I}}$ that occur in Γ^I, results in Γ.

It can be shown that all $\Gamma^I \in \mathcal{I}(\Gamma)$ give an equivalent result, and hence it is justified to restrict our attention to one paradigmatic $\Gamma^I \in \mathcal{I}(\Gamma)$. The simplest convention for a set of premises in an actual proof, is to replace the i-th occurrence of an NLC C in Γ by C^i. If, for instance, P has seven occurrences in Γ, the interpreted set of premises $\Gamma^I \in \mathcal{I}(\Gamma)$ will contain $P^1, ..., P^7$, in that order. In what follows the name Γ^I will always refer to this specific member of $\mathcal{I}(\Gamma)$. Where $A \in \mathcal{W}$, let $\mathcal{I}(A)$ be such that $A^I \in \mathcal{I}(A)$ iff
(i) $A^I \in \mathcal{W}^{\mathcal{I}}$.
(ii) deleting the superscripts from the elements of $\mathcal{P}^{r\mathcal{I}}$ that occur in A^I, results in A.[8]

Definition 1: *For* $C \in \mathcal{P}^r$, $C^i{\neq}C^j =_{\mathsf{df}} (\exists\alpha_1)...(\exists\alpha_n){\sim}(C^i\alpha_1...\alpha_n \equiv C^j\alpha_1...\alpha_n)$

Maybe it is interesting to notice that, *e.g*, $(\exists x){\sim}(P^ix \equiv P^jx)$ and $(\exists y){\sim}(P^iy \equiv P^jy)$ are considered as one and the same ambiguity. Also notice that $C^i{\neq}C^j$ and $C^j{\neq}C^i$ are considered as one and the same ambiguity. In order to get an accurate grip on these equivalent formulas, I define an ambiguity as an equivalence class of formulas.

Definition 2: *An ambiguity is an equivalence class* $[C^i{\neq}C^j] = \{A \mid A \in \mathcal{W}^{\mathcal{I}}$ *and* $\vdash_{\mathsf{CL}} A \equiv C^i \neq C^j\}$. $\mathcal{A}$ *is the set of all ambiguities.*

It is important to have an accurate definition, but when we talk about it, it might be more elegant to consider the formula $C^i{\neq}C^j$ as an ambiguity (instead of the equivalence class $[C^i{\neq}C^j]$).

[8] Some indexed NLC may occur more than once in A^I.

Definition 3: *A DA-formula is a formula of the form* $C_1^{i_1} \neq C_1^{j_1} \vee \ldots \vee C_n^{i_n} \neq C_n^{j_n}$ $(n \geq 1)$, *abbreviated as* $DA(\Delta)$, *in which* $\Delta = \{C_1^{i_1} \neq C_1^{j_1}, \ldots, C_n^{i_n} \neq C_n^{j_n}\}$.[9]

3.2. *Semantics of* AAL

"$\Gamma \models_{\mathsf{AAL}} A$" is defined by means of a selection of the CL-models of Γ^I, the (standard) maximally ambiguous interpretation of Γ.[10] The CL-models of Γ^I that do not verify more ambiguities than required in order to verify the members of Γ^I, are the AAL-models of Γ^I.[11]

Definition 4: *Where* M *is a* CL-*model,* $\mathsf{A}(\mathsf{M}) = \{C^i \neq C^j \mid \mathsf{v}_{\mathsf{M}}(C^i \neq C^j) = 1\}$.

Definition 5: *A* CL-*model* M *is minimally ambiguous with respect to* Γ^I, *iff* M *is a* CL-*model of* Γ^I, *and there is no* CL-*model* M′ *of* Γ^I, *such that* $\mathsf{A}(\mathsf{M}') \subset \mathsf{A}(\mathsf{M})$.

Definition 6: M *is an* AAL-*model of* Γ^I *iff* M *is minimally ambiguous with respect to* Γ^I.

Definition 7: $\Gamma \models_{\mathsf{AAL}} A$ *iff some* $A^I \in \mathcal{I}(A)$ *is true in all* AAL-*models of* Γ^I.

3.3. *Proof-theory*

The semantics is formulated from the point of view of someone who is able to oversee all possible consequences from any set of premises. Although it is possible to define the proof-theoretical consequence relation in an analogous

[9] In order to be maximally accurate, I should define a DA-formula as an equivalence class of all formulas A such that $\vdash_{\mathsf{CL}} A \equiv C_1^{i_1} \neq C_1^{j_1} \vee \ldots \vee C_n^{i_n} \neq C_n^{j_n}$. Hence, Δ can accurately be considered as a set of ambiguities. I believe this would make the notation too ugly.

[10] There are two well-developed strategies to select the adaptive models from the models of the underlying logic: the reliability-strategy and the minimal-abnormality strategy. I refer the reader to papers of Diderik Batens, such as [2]. The logic AAL is based on the minimal-abnormality strategy.

[11] Although it would be more appropriate for my present purposes to define AAL from the interpreted set Γ^I — *i.e.* to define $\Gamma^I \models_{\mathsf{AAL}} A^I$ instead of $\Gamma \models_{\mathsf{AAL}} A$ —, I choose to define it from Γ, because this is more elegant from a general theoretical point of view.

way, we define the proof-theory by means of derivations in concrete dynamic proofs.

We make a distinction between 'derived at a stage of a proof' and 'finally derived'.[12] The latter notion is used to define the consequence relation "$\vdash_{\mathsf{AAL}}$".

In concrete proofs every line gets a fifth element. This fifth element contains a (possibly empty) set of ambiguities. If some of the members of this set of ambiguities turns out to be derivable from the premises under a minimal abnormal interpretation of the premises, this line has to be marked "OUT" and does not belong to the proof anymore. Hence, a formula that was (conditionally) derived at an early stage of the proof, may not be derivable anymore at a later stage of the proof. It is also possible that a line that had to be marked "OUT" at an early stage, becomes unmarked at a later stage. Still it is possible to define the consequence relation $\Gamma \vdash_{\mathsf{AAL}}$, and to proof soundness and completeness.[13]

The notion 'derived at a stage of a AAL-proof' is defined by means of the rules RC, RU, and RM. The reader should not be frightened by the definitions of these rules: when you use them, these rules turn out to be very easy. A first observation is that the rule RU is nothing but a name for all rules valid in CL. RC is really very simple, and RM is nothing but the proof-theoretical counterpart of the semantical selection of minimally ambiguous models.

Definition 8: *The rule RC: from a line (i) to derive a line (j).*

(i)	$A^I \vee DA(\Delta)$	*(linenumbers); Rule*	Δ'
(j)	A^I	(i)*; RC*	$\Delta \cup \Delta'$

Definition 9: *The rule RU: from lines $(i_1), ..., (i_n)$, with resp. $A_1^I, ..., A_n^I$ as second element and $\Delta_1, ..., \Delta_n$ as fifth element $(n \geq 0)$, to derive a line (j) with B^I as second element, $(i_1, ..., i_n)$ as third element, RU as fourth element, and $\Delta_1 \cup ... \cup \Delta_n$ as fifth element, given that $A_1^I, ..., A_n^I \vdash_{\mathsf{CL}} B^I$.*

Definition 10: *$DA(\Delta)$ is a minimal DA-consequence of Γ^I at stage (i) of a proof from Γ^I, iff $DA(\Delta)$ is the second element of a line (j) $(1 \leq j \leq i)$ the fifth element of which is empty, and there is no $\Delta' \subset \Delta$ such that $DA(\Delta')$ is the second element of a line (k) $(1 \leq k \leq i)$ the fifth element of which is empty.*

[12] It is possible to obtain analogous dynamics on the semantical level, by using 'block-semantics'. I refer the reader to Diderik Batens'[1] and [3].

[13] I refer the reader to my [9] for the soundness and completeness proof of the first ambiguity-adaptive logic.

Obviously, at least one member of every minimal DA-consequence of Γ^I is true. What we are interested in are all minimal sets φ of ambiguities, such that, all minimal DA-consequences (at a given stage) are true if these ambiguities are true. These minimal sets are obtained as follows. First you take all sets that contain one element of each minimal DA-consequence. (Remember that equivalent ambiguities are considered as one and the same ambiguity.) From these sets some sets may be proper supersets of other sets, obviously they are not minimal sets.

Definition 11: $\Phi_{(i)}$ *is the set of all sets* φ *that contain exactly one element of each minimal* DA*-consequence of* Γ^I *at line* (i) *of that proof from* Γ^I *and that are no proper supersets of such a set.*

Definition 12: *Line* (j) *with* A^I *as second and* Δ *as fifth element, fulfills the integrity criterion at stage* (i) *of a proof from* Γ^I $(1 \leq j \leq i)$*, iff* $\varphi \cap \Delta = \emptyset$ *for some* $\varphi \in \Phi_{(i)}$*, and for each* $\varphi \in \Phi_{(i)}$ *there is a line* (k) $(1 \leq k \leq i)$ *such that, where* Δ_k *is the fifth element of line* (k)*,* $\varphi \cap \Delta_k = \emptyset$*.*

Definition 13: *The rule RM: If a line does not fulfill the integrity criterion at stage* (i) *of a proof, then the line is market* OUT. *The application of RM is obligatory at every stage of a proof.*

In view of the fact that the application of the rule RM is obligatory at every stage of a proof, lines that are marked at an early stage, may become unmarked at a later stage, and vice versa.

Definition 14: A^I *is derived at stage* (i) *of a proof, iff* A^I *is the second element of a line that is not marked* OUT.

Definition 15: A^I *is finally derived on a line of an* AAL*-proof from* Γ^I*, iff it is the second element of that line and any (possibly infinite) extension of the proof can be further extended in such way that the line is unmarked.*

Definition 16: $\Gamma \vdash_{\mathsf{AAL}} A$ *iff some* $A^I \in \mathcal{I}(A)$ *is finally derived at some line of an* AAL*-proof from* Γ^I*.*

Let Φ_Γ be defined from *all* minimal DA-consequences of Γ^I in the same way as $\Phi_{(i)}$ is defined from the minimal DA-consequences of Γ^I *at stage* (i) of a proof from Γ^I.

Theorem 1: $\Gamma \vdash_{\mathsf{AAL}} A$ *iff there is a* $A^I \in \mathcal{I}(A)$ *such that there are one or more (possibly empty) finite sets* $\Delta_1, \Delta_2, .. \subset \mathcal{A}$, *such that* $\Gamma^I \vdash_{\mathsf{CL}} A^I \vee DA(\Delta_1)$, $\Gamma^I \vdash_{\mathsf{CL}} A^I \vee DA(\Delta_2)$, *..., and for any* $\varphi \in \Phi_\Gamma$, *one of the* Δ_i *is such that* $\Delta_i \cap \varphi = \emptyset$.[14]

Theorem 2: *If* $\Gamma \vdash_{\mathsf{AAL}} A$, *then, for some* $A^I \in \mathcal{I}(A)$, *it is possible to extend any proof from* Γ^I *into a proof in which* A^I *is finally derived.*

3.4. *Some interesting properties of* AAL

Theorem 3: *If* Γ *is* CL*-consistent, then for all* $A \in \mathcal{W}$, $\Gamma \vdash_{\mathsf{AAL}} A$ *iff* $\Gamma \vdash_{\mathsf{CL}} A$.

Theorem 4: Γ *is* CL*-consistent iff for every* AAL*-model* M *of* Γ^I, $\mathsf{A}(\mathsf{M}) = \emptyset$.

Theorem 5: *If* (i) *for all* $D \in \mathcal{W}$, $\Gamma \vdash_{\mathsf{CL}} D$, *and* (ii) *there is some* $E^I \in \mathcal{W}^{\mathcal{I}}$ *such that* $\Gamma^I \nvdash_{\mathsf{CL}} E^I$ *(which is always the case, except for border cases), then the* AAL*-consequence set of* Γ *is not trivial, and there are some NLC* $C_1, ..., C_n$ *occurring in* Γ *such that* $\Gamma^I \vdash_{\mathsf{CL}} C_1^{i1} \neq C_1^{j1} \vee ... \vee C_n^{in} \neq C_n^{jn}$

In the latter formula, the ambiguities $C_1^{i1} \neq C_1^{j1}, ... C_n^{in} \neq C_n^{jn}$ are very interesting with respect to theory-development. If we have, *e.g.* $\Gamma^I \vdash_{\mathsf{CL}} C^i \neq C^j$, then we know that the i-th and the j-th occurrence of C in Γ have a different meaning. At least one of them must be replaced by a new NLC. Hence, we can apply AAL in order to detect ambiguities, vagueness, conceptual change. Specific applications allow for the use of meaningful indexes, *e.g.* time-indexes[15] and source-indexes.

4. *A model for the construction of collective theories*

For specific applications of the ambiguity-adaptive logic AAL, we can introduce restrictions on the use of indices. For our present purpose the restriction is as follows: the indices refer to the scholar or the group of scholars who formulated the premise. For instance, if the premise $(\forall x)(Px \supset Qx)$ belongs to the collective theory of some group Σ, we will write this premise as $(\forall x)(P^\Sigma x \supset Q^\Sigma x)$. If some scholar S^1 introduces the premise Pa, we

[14] For proofs of this and the following theorems, I refer to reader to [10] and [9].

[15] For examples of applications of ambiguity-adaptive logics with time-indexes, see [11].

will write P^1a. It is easily seen that we can derive from these premises the AAL-consequence Qa on condition that $P^\Sigma \neq P^1$ is not CL-derivable from the indexed premises.

4.1. *Ludwig's theory of toys*

As an introduction to my model, I tell the story of the development of little Ludwig's theory of toys.[16] Little Ludwig was very sceptical. He believed what his mother taught him, and he believed what he saw with his own eyes, and he did not believe anything else. He did not even believe his friend Bertrand when he said that he had got a green ball. Ludwig himself had got a yellow ball, and Ludwig's mother has taught him about balls nothing but the fact that balls are round. Hence none of his reliable sources told him that balls can be green. "Bertrand may not know the meaning of the words "green" and "ball" " Ludwig thought, "Maybe he says "green" instead of "yellow", or maybe he thinks that apples are balls."

From his two reliable sources (his own experience and his mother's 'lessons'), Ludwig developed a simple theory about toys. Let Θ be the name of this theory. Θ contains the premises "Balls are round" ($(\forall x)(Bx \supset Rx)$), and "Some ball is yellow" ($(\exists x)(Bx \& Yx)$). Let us write these premises as $M : (\forall x)(Bx \supset Rx)$ or $(\forall x)(B^M x \supset R^M x)$, and $L : (\exists x)(Bx \& Yx)$ or $(\exists x)(B^L x \& Y^L x)$. As he considered all other sources as unreliable, Ludwig explicitly used a *sceptical default* (henceforth LSD):

LSD: | If $\Theta \nvdash (\exists x)A$, then Ludwig takes for granted that $(\forall x){\sim}A$ is the case. | (1)

Notice that Ludwig does not mention any logic (he writes "$\vdash$" and *e.g.* not "$\vdash_{\mathsf{CL}}$"). As so many sceptical people, Ludwig believed in the existence of one true logic — which turned out to be Classical Logic.

The reader may also notice that Ludwig's sceptical default may lead to inconsistencies very easily. For example, suppose that Ludwig's mother teaches him that all things are either solid, fluid or a gas. Ludwig does not know what these words mean, but as his mother tells the truth, we have: $\Theta \vdash (\forall x)(Sx \vee Fx \vee Gx)$. As neither $(\exists x)(Bx \& Sx)$, $(\exists x)(Bx \& Fx)$, nor $(\exists x)(Bx \& Gx)$ is derivable from Θ, it is easily seen that Ludwig — applying his LSD — takes for granted that the following formulas are the case:

$$(\forall x)(Bx \supset {\sim}Sx) \tag{2}$$

[16] This paper is indirectly inspired by the *Tractatus Logico-Philosophicus* by Ludwig Wittgenstein.

$$(\forall x)(Bx \supset \sim Fx) \tag{3}$$

$$(\forall x)(Bx \supset \sim Gx) \tag{4}$$

$$(\forall x)(Bx \supset (Sx \vee Fx \vee Gx)) \tag{5}$$

If Ludwig was a very conservative, classical logician, he would be in big trouble. Indeed, from (2)–(5), one can derive (by means of classical logic)

$$(\forall x)(Bx \supset ((Sx \& \sim Sx) \vee (Fx \& \sim Fx) \vee (Gx \& \sim Gx))) \tag{6}$$

From (6) it follows that there are no balls ($\sim(\exists x)Bx$), which clearly contradicts the reliable fact that he has got a yellow ball. Young Ludwig was not a very conservative classical logician. For him inconsistencies were interesting indications where and how this theory could be improved.

Ludwig realized that his LSD can be overruled. As (5) and $(\exists x)Bx$ can also be derived from Θ without LSD, he considered these formulas as more reliable than (2), (3) and (4). His conclusion was that there is something wrong with the S, F and/or G occurring in resp. (2), (3) and (4). This conclusion was the trigger for new investigations. Young Ludwig went to his mother and asked her whether balls are solid, fluid or gaseous. His mother took the time to show him ice, that is solid, water that is fluid, and steam that is gaseous. She knocked on the solid table, she poured some oil in a little bowl, and said that the air we breath is gaseous. "So, balls are solid !" Ludwig concluded very wisely. Thus, the derivation of the inconsistency resulted in the development of his theory of toys: Θ is extended with the premises $(\forall x)(Bx \supset Sx)$, $(\forall x)(Bx \supset \sim Fx)$ and $(\forall x)(Bx \supset \sim Gx)$, and hence (2) is no longer accepted. Moreover the inconsistencies leaded to a better understanding of the terms "solid", "fluid" and "gaseous". It may be interesting to notice that the fact that Ludwig's mother learned him to distinguish solid from fluid, did not guarantee that Ludwig and his mother make this distinction always in the same way. One day they were eating ice cream. Ludwig thought it was fluid, but his mother said that he had to eat it before it became fluid.

This process can very easily be reconstructed by means of AAL. Our set of premises are the following formulas; Θ and LSD are the respective sources:

$$\Theta : (\forall x)(Bx \supset (Sx \vee Fx \vee Gx)) \tag{7}$$

$$\text{LSD} : (\forall x)(Bx \supset \sim Sx) \tag{8}$$

$$\text{LSD} : (\forall x)(Bx \supset {\sim}Fx) \tag{9}$$

$$\text{LSD} : (\forall x)(Bx \supset {\sim}Gx) \tag{10}$$

If Γ is the set of formulas occurring in (7)–(10), then $\Gamma^I = \{(\forall x)(B^\Theta x \supset (S^\Theta x \vee F^\Theta x \vee G^\Theta x)), (\forall x)(B^D x \supset {\sim}S^D x), (\forall x)(B^D x \supset {\sim}F^D x), (\forall x)(B^D x \supset {\sim}G^D x)\}$, in which the superscript D refers to LSD. It is easily seen that from Γ^I the following formula is CL-derivable:

$$B^\Theta \neq B^D \vee S^\Theta \neq S^D \vee F^\Theta \neq F^D \vee G^\Theta \neq G^D \tag{11}$$

From a strictly logical point of view this disjunction of ambiguities cannot be shortened, but within the context of his investigations, Ludwig quickly concluded that $B^\Theta \neq B^D$ is not the case, for he knew very well what balls are, while both B^Θ and B^D were used by him: these two occurrences had the same meaning. In order to find out if $S^\Theta \neq S^D$ is the case, or $F^\Theta \neq F^D$, or $G^\Theta \neq G^D$, he organized new experiments. In this situation these new experiments were easily done by asking his mother. For those people who want to use one and the same logic in one and the same situation, it may be interesting to remember that the CL-consequences and the AAL-consequences of Θ are exactly the same as long as Θ is consistent, whereas triviality is avoided when some extension of Θ is inconsistent and one chooses to use AAL. If some set Γ is inconsistent, the minimal DA-consequences indicate which terms should be subject to new investigations.

The acceptance of LSD also resulted in another type of inconsistencies. Let us consider Bertrand's statement "I have got a green ball", a statement that was not derivable from Ludwig's theory:

$$\Theta \nvdash (\exists x)(Bx \& Gx) \tag{12}$$

From (12) and LSD, it follows that Ludwig takes the following formula for granted

$$(\forall x)(Bx \supset {\sim}Gx) \tag{13}$$

From

$$B : (\exists x)(Bx \& Gx) \tag{14}$$

together with (13), we can derive

$$B^D \neq B^B \vee G^D \neq G^B \tag{15}$$

In this case Ludwig had no quick solution. In fact, there were four possible solutions:

(1) $B^D \neq B^B$ and $G^D \neq G^B$
(2) $B^D \neq B^B$ and $G^D = G^B$
(3) $B^D = B^B$ and $G^D \neq G^B$
(4) $B^D = B^B$ and $G^D = G^B$

There were four possible solutions, but there was only one way to act: Ludwig had to find out whether he and Bertrand meant the same when they used the same word. Bertrand could not show his green ball; he had borrowed it to his little brother. So, Ludwig showed him an apple and asked whether Bertrand would call it a ball. "Of course not", replied Bertrand, "I know what balls are. Balls are round." Ludwig showed a wheel, and Bertrand said that it was not a ball. Ludwig showed a little yellow ball and Bertrand said it was a little ball. Finally, after 17 'correct answers' Ludwig concluded that he could accept, for the time being, that Bertrand meant "ball" when he said "ball". From the four possible solutions, only the third and the fourth remained. Now, Ludwig showed a banana, and Bertrand had to say it was yellow. Ludwig pointed at the sky and Bertrand had to say it was blue. Ludwig pointed at the grass, Kermit the Frog, a green sweater, ..., and Bertrand said all of it was green. Again, not a single contradiction surfaced, and so Ludwig accepted that Bertrand told the truth when he said that he had a green ball. In other words: Ludwig had to extend his theory with the premise: $(\exists x)(Bx\&Gx)$. This was a quite amazing fact: Ludwig relied on Bertrand's observation and on their collective interactions: Ludwig's theory became a collective theory.

A final part of Ludwig's story concerns Ludwig's flat ball. The dog had been playing with Ludwig's ball, and now it was flat. Ludwig's reliable theory learned him that his ball was round, whereas his reliable observation learned him that his ball was not round. In this case, he had to conclude that (where the superscript O, refers to Ludwig's observation):

$$B^{\Theta} \neq B^O \vee R^{\Theta} \neq R^O \tag{16}$$

Ludwig reasoned as follows: "Round is round, and not round is not round, and I really do know what "round" means. So the observed ball is not the kind of ball that is meant in my theory. I have to rethink the term "ball". In my theory the term "ball" refers to normal balls, to balls that are not broken." Here we see that a new observation does not lead to a reformulation of the theory but to a change with respect to the meaning of some term.

4.2. *The model*

Let Θ be a theory, *i.e.* a set Γ of statements (laws, definitions, facts, ...) and a logic. For our present purposes, we choose the logic AAL. Suppose $(\forall x)A(x)$ is such that $\Gamma \vdash_{\mathsf{AAL}} (\forall x)A(x)$, and for every $B \in \Gamma$: $(\forall x)A(x) \vdash_{\mathsf{AAL}} B$. Θ can be considered as the couple $\langle(\forall x)A(x), \mathsf{AAL}\rangle$. Again we have a sceptical default:

SD: If $(\forall x)A(x) \nvdash_{\mathsf{AAL}} (\exists x)S(x)$, then accept that $(\forall x)(A(x) \supset \sim S(x))$ (17)

In what follows, the formula "$(\forall x)(A(x) \supset \sim S(x))$" will be abbreviated as "$\sim(S \circ A)$". "$S \circ A$" can be read as S is compatible with A. The idea is that SD blocks any extension of Θ, by denying everything that does not follow from the theory. Still we now that scholars working on a theory often bring in new information. In general new information can be of two forms: $(\exists x)S(x)$ or $(\forall x)S(x)$. If some new information is brought in, we stipulate that we always mention the source. If some formula B contains the predicates $P_1, ..., P_n$, $B^i \neq B^j$ is the abbreviation for $P_1^i \neq P_1^j \vee ... \vee P_n^i \neq P_n^j$.

4.2.1. *Case 1:* $K : (\exists x)S(x)$

Suppose some group Σ of scientists accept a theory $\Theta = \langle(\forall x)A(x), \mathsf{AAL}\rangle$. Suppose $(\forall x)A(x) \nvdash_{\mathsf{AAL}} (\exists x)S(x)$. Hence $\sim(S \circ A)$ is accepted by the members of Σ. Suppose some members of Σ organize an experiment K and observe that $(\exists x)S(x)$. Hence we obtain the following set of premises: $(\forall x)A^{\Sigma}(x)$, $\sim(S^D \circ A^D)$, $(\exists x)S^K(x)$. Let us apply CL to it (the lower limit logic of AAL). The main goal of the proof is to derive minimal DA-formulas. If this goal is reached, we have interesting information, *viz.* we see which occurrences of which NLC can not consistently be identified with one another. Unless and until some DA-formula is derivable, we also have an interesting result, *viz.* in this case $(\exists x)S^K(x)$ and $(\exists x)S^{\Sigma}(x)$ can be identified with one another, and hence, the members of Σ can conclude that $(\exists x)S(x)$ is compatible with $(\forall x)A$, even if they suppose that all of them give the same meaning to the (old and new) terms in S. In general, the following heuristic rule will be very fruitful, especially for the new information: try to replace every indexed NLC C^i $(i \neq \Sigma)$ with C^{Σ}. It is indeed the purpose of the members of Σ to derive a collective theory in *their* common language.

1.	$(\forall x)A^{\Sigma}(x)$	— PREM	$\emptyset$
2.	$\sim(S^D \circ A^D)$	— PREM	$\emptyset$

3.	$(\exists x)S^K(x)$	—	PREM	$\emptyset$
4.	$(\exists x)S^\Sigma(x) \vee S^K \neq S^\Sigma$	3	RU	$\emptyset$
5.	$(\exists x)S^\Sigma(x)$	4	RC	$\{S^K \neq S^\Sigma\}$
6.	$\sim(S^\Sigma \circ A^\Sigma) \vee S^D \neq S^\Sigma \vee A^D \neq A^\Sigma$	2	RU	$\emptyset$
7.	$\sim(S^\Sigma \circ A^\Sigma)$	6	RC	$\{S^D \neq S^\Sigma, A^D \neq A^\Sigma\}$
8.	$\sim(\exists x)S^\Sigma(x)$	1,7	RU	$\{S^D \neq S^\Sigma, A^D \neq A^\Sigma\}$
9.	$S^K \neq S^\Sigma \vee S^D \neq S^\Sigma \vee A^D \neq A^\Sigma$	5,8	REC[17]	$\emptyset$

At stage 9, lines 5, 7 and 8 have to be marked OUT. The formulas in the second element of these lines are no longer considered to be derivable. At this stage the scholars must take some non-logical steps: they have to organize experiments in order to localize the abnormalities or ambiguities. There are three possible ways to localize the difference in meaning: (i) $[S^K \neq S^\Sigma]$: some term occurring in the new information from source K does not have a common meaning, (ii) $[S^D \neq S^\Sigma]$: some term occurring in $S(x)$ as it stands in the sceptical default does not have a common meaning, (iii) $[A^D \neq A^\Sigma]$: some other term occurring in the core of the theory does not have a common meaning.

I believe that most groups of scientists will use a local constraint,[18] in that they will easily accept that (iii) $[A^D \neq A^\Sigma]$ is not the case. $A(x)$ is the core of their theory, and does contain nothing but well-known terms. Moreover they have formulated SD themselves. I think there are indeed good reasons to examine the meaning of the NLC occurring in $S(x)$ in the first place, and more exactly: to examine whether all NLC occurring in $S^K(x)$ can be identified with the collective use of the terms.

The first thing to do is to examine for every NLC C in $S(x)$ whether $C^K = C^\Sigma$. I believe this examination can be organized in such a way that the question whether $C^\Sigma = C^D$ can be answered simultaneously. From the previous sections we remember that there is no positive test for $C^K = C^\Sigma$. There is a negative test: the scholars who organized experiment K together with the (other) members of Σ should, through communication, try to 'prove' that $C^K \neq C^\Sigma$. I think it is normal that this communication will be accompanied by actions and observations — almost in the same way as young Ludwig and Bertrand did. For instance, the scholars can try to prove for some object a of their studied domain, that $C^K a$ is the case whereas $C^\Sigma a$ is not the case, or *vice versa*. The scholars who organized K can also

[17] The rule REC (Elimination of conditions) is a derivable rule: if A is derived on condition Δ and $\sim A$ is derived on condition Δ', then $DA(\Delta \cup \Delta')$ can be derived unconditionally.

[18] The term "constraint" is taken from [8].

try to prove that $C^{\Sigma} \neq C^{D}$, which means that the members of Σ were too credulous in believing that they used the term C in a common way. If their interactions are organized rationally, the lack of such 'proves' inductively leads to the assumption that the term C is used in a common way ($C^{\Sigma} = C^{D} = C^{K}$). The reader may object that the scholars who organized K can organize their experiment again, or describe it in such a way that it can be re-organized by other members of the group, but it is easily seen that this is not a positive test: it is nothing but one of the 'lacks of a negative test'. Moreover, experiment K *an sich* does not explain why the members have been assuming that $(\exists x)S(x)$ is not the case, and it does not reveal the weak parts of that assumption.

I give an overview of the possible results:

(1) For some NLC C occurring in $S(x)$, $C^{K} \neq C^{\Sigma}$, and for all NLC C occurring in $S(x)$, $C^{D} = C^{\Sigma}$. In this case, the DA-formula in line 9 of the proof above can be replaced by $C^{K} \neq C^{\Sigma}$, which becomes a new minimal DA-formula. Hence the condition on which the formula in line 5 is derived, remains overruled, whereas the formulas in lines 7 and 8 become derivable. Hence it is the case that $\sim(S \circ A)$: the old common interpretation of S is such that $(\exists x)Sx$ is not compatible with the core of the theory. Still there is an interesting result deriving from the fact that $C^{K} \neq C^{\Sigma}$: if the members of Σ have got good reasons not to identify C^{K} with C^{Σ}, they also have good reasons to introduce a new term for C^{K}, *e.g* D. So let $(\exists x)S'x$ be obtained from $(\exists x)S'x$ by replacing C with D. After the introduction of this new term, the members of Σ have good reasons to replace Θ with $\Theta' = \langle(\forall x)(\exists y)(S'(y)\&A(x)), \mathsf{AAL}\rangle$. For aesthetic reasons we can write $\Theta' = \langle(\forall x)A'(x), \mathsf{AAL}\rangle$. The theory has changed in view of the new information, whereas from a formal point of view we are in the same situation as before.

(2) For some NLC C occurring in $S(x)$, $C^{D} \neq C^{\Sigma}$, and for all NLC C occurring in $S(x)$, $C^{K} = C^{\Sigma}$. This is a logical possibility that may not happen every now and then in real life. Still, as $S(x)$ does not belong to the core of the theory as it is, the members of Σ may not have paid too much attention to the meaning of the NLC occurring in $S(x)$. Anyway, we meet a new situation: the formula in the second element of line is no longer a minimal DA-consequence of the premises, for $C^{D} \neq C^{\Sigma}$ becomes minimal. Still, the conditions on which the formulas in lines 7 and 8 are derived remain overruled ($C^{D} \neq C^{\Sigma}$ is a part of $S^{D} \neq S^{\Sigma}$), whereas the formula in line 5 becomes derivable. This means that the interpretation S^{K} of S becomes the collective interpretation. The scholars will accept

that $S \circ A$, or in other words, that $(\exists x)Sx$ becomes a part of the theory. At the same time, the members of Σ obtain better reasons to accept that their use of the NLC occurring in $S(x)$ is collective. Hence the theory $\Theta = \langle(\forall x)A(x), \mathsf{AAL}\rangle$ can be replaced by the theory $\Theta' = \langle(\forall x)(\exists y)(S(y)\&A(x)), \mathsf{AAL}\rangle$. For aesthetic reasons, we can write: $\Theta' = \langle(\forall x)A'(x), \mathsf{AAL}\rangle$. The theory has changed, but from a formal point of view we are in the same situation as before.

If it was the case that the NLC C also occurs in the original $A(x)$, the statements in which C occur, must be revised. For an example, see the 'white crow' example below.

(3) For all NLC C occurring in $S(x)$, $C^K = C^\Sigma = C^D$. In this case, the DA-formula in line 9 of the proof above can be replaced by $A^D \neq A^\Sigma$.[19] Hence the condition on which the formula in line 5 is derived, is no longer overruled, and hence $(\exists x)S^\Sigma(x)$ becomes derivable, which means that $S \circ A$ is the case: $(\exists x)S^\Sigma(x)$ is compatible with the core of the theory. This means that the sceptical default is overruled. Applying their 'local constraint' the members of Σ may not believe that they use the NLC occurring in $A(x)$ in an ambiguous way and extend their theory $\Theta = \langle(\forall x)A(x), \mathsf{AAL}\rangle$, or in other words, replace it by the theory $\Theta' = \langle(\forall x)(\exists y)(S(y)\&A(x)), \mathsf{AAL}\rangle$. For aesthetic reasons, we can write: $\Theta' = \langle(\forall x)A'(x), \mathsf{AAL}\rangle$. The theory has changed, but from a formal point of view we are in the same situation as before. The new SD does not contain $\sim(S^D \circ A^D)$ anymore. Still, there is a logical possibility that this new theory is inconsistent, for $A^D \neq A^\Sigma$ was derivable from the original premises. This would simply mean that $A(x)$ was ambiguous from the beginning. If the scholars had applied AAL to the premises of their theory from the beginning, they would have discovered that some disjunctions of ambiguities were derivable. The disjuncts of the minimal DA-consequences of the premises would have indicated which NLC may be ambiguous. Hence, the scholars need to communicate about these NLC in the Ludwig-Bertrand-style, in order to solve these ambiguities. The derivation of ambiguities may also be an indication of the fact that some law, rule or definition occurring in the original theory was simply wrong.

A fictive example: suppose some group of biologists accept a theory Θ in which "crows are black" is true. Clearly, the statement "some crow is white"

[19] The main difference with case 2 is that case 3 concerns terms that belong to the core of the theory, whereas the terms considered in case 2, occur in the sceptical default.

does not follow from the theory: if the biologists adhere to SD, they accept that crows are not white. Suppose that one day one of these biologists (viz. K) succeeds to catch a picture of a white crow. Let $(\forall x)A(x)$ be the core of their theory. Clearly we have that (where "C" stands for "crows", "W" for "white"):

$$\Sigma : (\forall x)A(x) \nvdash_{\mathsf{AAL}} (\exists x)(Cx \& Wx) \tag{18}$$

$$\text{SD} : (\forall x)(A(x) \supset {\sim}(Cx \& Wx)) \tag{19}$$

$$K : (\exists x)(Cx \& Wx) \tag{20}$$

From these formulas we can derive:

$$C^K \neq C^\Sigma \vee W^K \neq W^\Sigma \vee C^D \neq C^\Sigma \vee W^D \neq W^\Sigma \vee A^D \neq A^\Sigma \tag{21}$$

together with

$$(\exists x)(C^\Sigma \& W^\Sigma) \text{ on condition that } C^K = C^\Sigma \text{ and } W^K = W^\Sigma \tag{22}$$

$${\sim}(\exists x)(C^\Sigma \& W^\Sigma) \text{ o.c.t. } C^D = C^\Sigma, W^D = W^\Sigma \text{ and } A^D = A^\Sigma \tag{23}$$

If the biologists find no proof that ' "white" is not "white" ' or that ' "crow is not a crow" ', either $(\exists x)(Cx \& Wx)$ is compatible with and becomes part of their theory,[20] or their theory was wrong.[21] It is however more likely, that K did not use the words "crow" in a collective way (and in that case the collective theory can be extended with the discovery of a new kind of bird — unless K was stupid enough to take a seagull for a crow). Another possibility is that $C^D \neq C^\Sigma$: the members of Σ did not use the predicate "crow" in a way that was accurate enough. In that case they can replace the predicate "crow" *e.g.* with "typical crow". Hence the statement "crows are black" is replaced with "typical crows are black and there is some untypical white crow". Another solution is that the statement "crows are black" is replaced with "crows are black or white". A more accurate substitute for

[20] In this case we obtain the strange situation in which a crow can be, at the same time, white by observation and theoretically black.

[21] The theory would have been wrong in that it is not the case that both "all crows are black" and "what is white is not black" can be true.

$(\forall x)(Cx \supset Bx)$ may be:[22]

$$(\exists x)((Cx\&Wx)\&(\forall y)(Cy \supset (By \vee (Wy\&x = y)))) \tag{24}$$

4.2.2. *Case 2:* $M : (\forall x)S(x)$

Suppose some group Σ of scientists accept a theory $\Theta = \langle(\forall x)A(x), \mathsf{AAL}\rangle$. Suppose $(\forall x)A(x) \nvdash_{\mathsf{AAL}} (\exists x)S(x)$. From adding SD to their theory, it follows that they do accept that $\sim(S \circ A)$ is the case. Suppose some members of Σ have good reasons to formulate a new hypothesis M, *viz.* $(\forall x)S(x)$. Hence we obtain the following set of premises: $(\forall x)A^{\Sigma}(x)$, $\sim(S^D \circ A^D)$, $(\forall x)S^M(x)$.

1.	$(\forall x)A^{\Sigma}(x)$	—	PREM	$\emptyset$
2.	$\sim(S^D \circ A^D)$	—	PREM	$\emptyset$
3.	$(\forall x)S^M(x)$	—	PREM	$\emptyset$
4.	$(\forall x)S^{\Sigma}(x) \vee S^M \neq S^{\Sigma}$	3	RU	$\emptyset$
5.	$(\forall x)S^{\Sigma}(x)$	4	RC	$\{S^M \neq S^{\Sigma}\}$
6.	$\sim(S^{\Sigma} \circ A^{\Sigma}) \vee S^D \neq S^{\Sigma} \vee A^D \neq A^{\Sigma}$	2	RU	$\emptyset$
7.	$\sim(S^{\Sigma} \circ A^{\Sigma})$	6	RC	$\{S^D \neq S^{\Sigma}, A^D \neq A^{\Sigma}\}$
8.	$(\forall x)\sim S^{\Sigma}(x)$	1,7	RU	$\{S^D \neq S^{\Sigma}, A^D \neq A^{\Sigma}\}$
9.	$S^M \neq S^{\Sigma} \vee S^D \neq S^{\Sigma} \vee A^D \neq A^{\Sigma}$	5,8	ERC	$\emptyset$

Again, lines 5, 7 and 8 have to be marked OUT at stage 9. The formulas in the second element of these lines are no longer considered to be derivable. From a strictly logical point of view there is no difference between case 2 and case 1, except for the fact that the inconsistency in case 1 concerns an existentially quantified formula, whereas the inconsistency in case 2 occurs within the scope of a universally quantified formula. Hence we do not meet a mere exception on a rule belonging to the theory, or on an exception on a SD-assumption, but we meet a general statement that contradicts an old rule or assumption. I believe that this difference will influence the 'local constraints'. The new information is so flagrantly contradicting the old beliefs of the members or Σ that no scholar will accept $(\forall x)S(x)$ if there is any possibility for another solution. This constraint results in the fact that $S^M \neq S^{\Sigma}$ will be considered as minimal DA-formula. I assume that this decision will be taken in general, for the just mentioned non-logical reason: $(\forall x)S^{\Sigma}x$ should not be derivable. Hence, the thing to do is to examine for

[22] For a logic for general statements that might have exceptions, I refer the reader to my [12].

every NLC C in $S(x)$ whether $C^M = C^\Sigma$. I give an overview of the possible results:

(1) For some NLC C occurring in $S(x)$, $C^M \neq C^\Sigma$. In this case, $C^M \neq C^\Sigma$ becomes a new minimal DA-formula. The condition on which the formula in the second element of line 5 is derived, remains overruled, whereas the formulas in lines 7 and 8 become derivable. Hence it is the case that $\sim(S \circ A)$: the old common interpretation of S is such that $(\forall x)Sx$ is not compatible with the core of the theory. Still there is an interesting result deriving from the fact that $C^M \neq C^\Sigma$: if the members of Σ have got good reasons not to identify C^M with C^Σ, they also have good reasons to introduce a new term for C^M, *e.g* D. So let $(\exists x)S'x$ be obtained from $(\exists x)Sx$ by replacing C with D. After the introduction of this new term, the members of Σ have good reasons to replace Θ with $\Theta' = \langle(\forall x)(S(x)\&A(x)), \mathsf{AAL}\rangle$. For aesthetic reasons we can write $\Theta' = \langle(\forall x)A'(x), \mathsf{AAL}\rangle$. Again, the theory has changed in view of the new information, whereas from a formal point of view we are in the same situation as before.

(2) For all NLC C occurring in $S(x)$, $C^M = C^\Sigma$. This result is classically inconsistent with the local constraint. If the members of Σ give up their local constraint, there are two interesting ways out:

a. $(\forall x)(A(x)\&S(x))$ is consistent. Hence Θ can simply be replaced with $\langle(\forall x)(A(x)\&S(x)), \mathsf{AAL}\rangle$. I think this can only happen at an early stage of the development of a new theory, when the theory still exhibits large lacunae. At this stage, the scholars really should examine the applicability of terms and statements that do not belong to the core of the theory yet.

b. $(\forall x)(A(x)\&S(x))$ is inconsistent. There is a theoretical possibility that for some NLC C occurring in S, $C^D \neq C^\Sigma$ (whereas $C^M = C^\Sigma$), but is seems more likely that $(\forall x)A(x)$ was wrong. Applying AAL to $(\forall x)(A(x)\&S(x))$ will result in the derivation of minimal DA-formulas. The NLC occurring in these DA-formulas should be subject to new investigations. It may also be the case that some (general) statements in which these NLC occur must be rejected. Finally, this will result in a replacement of Θ with $\Theta' = \langle(\forall x)(A'(x)\&S(x)), \mathsf{AAL}\rangle$, or for aesthetic reasons $\Theta'=\langle(\forall x)A''(x), \mathsf{AAL}\rangle$. The theory has changed, but from a formal point of view we are in the same situation as before.

4.3. *A new theory*

I suppose it does not need to be explained any further that this model can be applied to the reconstruction of scientific theories, as well as to the development of everyday knowledge. Still I want to pay some extra attention to some aspects of the model. When some group Σ of people start the creation of a new theory about a new domain, the theory will at first contain nothing but AAL-theorems. Indeed, before any information is harvested, $\Theta = \langle \emptyset, \mathsf{AAL} \rangle$. At this point, all non-theorems will be rejected by the sceptical default. Still, the members will have a vague idea of the elements of their domain, and there must be a way to indicate this elements. Suppose we turn back the clock to God knows when, and we meet some scholars Σ who want to construct a theory about all 'objects' that have no spatial extensiveness but do have a mass. One thing is that these 'objects' clearly cannot be empirically observed. Another thing is that the expressions "to have no spatial extensiveness" and "to have a mass" are not explicitly defined at the moment on which the scholars start to work on their theory. Given the model presented here, these two things do not cause any problem. If "Mx" stands for "x has a mass", and "Ox" stands for "x has no spatial dimension", then the members of Σ are able to define their domain by accepting the following statement:

$$(\forall x)(Mx \& Ox) \tag{25}$$

If they know something about mathematics, they can quantify their expressions. If $\mathcal{R}$ is the set of all real numbers, they can write:[23]

$$(\forall x)(Mx \supset (\exists y)(Ry \& Wxy)) \tag{26}$$

At first this will be contradicted by their sceptical default, but after some communication the members of Σ may agree that it might be useful to allow for a unique quantified mass for every element of the domain. Next they can introduce new expressions, which they borrow from their everyday communications, such as "x moves over a distance of y" (notated as "Sxy") and "x moves during a period of z" (notated as "Txz"). They do not need explicit and sharp definitions of these terms. It is sufficient that they know how to use these terms in a collective way. They can just think of the distance from Ghent to Brussels, and of periods that last some hours or days. Using these vague predicates, they can sharply define new predicates, such as "y is the average speed of an object x" (notated as "Vxy"). As definitions are nothing

[23] For all elements x of the domain, there is some real number y such that the mass of x is y.

but abbreviations, they do not need to confront these definitions with their sceptical default.

$$(\forall x)(\forall y)(\forall z)(\exists z')(z' = \frac{y}{z} \,\&\, ((Sxy\&Txz) \supset Vxz')) \tag{27}$$

They can define a lot this way. One more example: "y is the kinetic energy of x" (notated as "Kxy"):

$$(\forall x)(\forall y)(\forall z)(\exists z')(z' = y \times z^2\&((Mxy\&Vxz) \supset Kxz')) \tag{28}$$

I think this shows sufficiently that it is possible to construct a collective theory by means of vague predicates and the here presented model. The sceptical default will become very useful when these scholars decide to identify every physical object with one of the objects of their domain.

I hope to find some historicists of science who want to reconstruct some historical examples of theory development, construction of new theories and conceptual change. My final remark however concerns those things that are really important to us: our experiences and our values. Some people say that it is impossible to construct scientific theories because we cannot observe experiences or values, and because we do have at our disposal nothing but vague or metaphorically used predicates if we want to describe our experiences and values, and, finally because it seems to be apparent that we cannot formulate any universally valid statement about experiences or values. I hope to have established that we do not need a domain of observable elements, nor predicates that have a clear meaning, and I hope that it is clear that, as all theories are collective theories, we may as well start and create collective theories about experiences and values in small, local groups.

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REFERENCES

[1] Batens, Diderik: "Blocks. The Clue to Dynamic Aspects of Logic." *Logique et Analyse* 150-151-152, 1995, pp. 285–328 (appeared 1997).
[2] Batens, Diderik: "Inconsistency-adaptive Logics." in Ewa Orlowska (ed.) *Logic at Work. Essays Dedicated to the Memory of Helena Rasiowa.* Heidelberg, New-York, Physica Verlag, Springer, 1998, pp. 445–472.
[3] Batens, Diderik: "A Dynamic Semantics for Inconsistency-Adaptive Logics. *Bulletin of the Section of Logic* 27, 1998, pp. 15–18.

[4] Batens, Diderik: "On the remarkable correspondence between paraconsistent logics, modal logics and ambiguity logics." (to appear in the proceedings of WCP2000 Brazil).
[5] Brown, Bryson: "Yes, Virginia, there really are paraconsistent logics." *Journal of Philosophical Logic*, 28: 489–500, 1999.
[6] Meheus, Joke: "Adaptive Logic in Scientific Discovery: the Case of Clausius." *Logique et Analyse* 143–144, 1993 (appeared 1996) pp. 359–389.
[7] Priest, Graham: "The logic of paradox" *Journal of Philosophical logic*, 8, may 1979; pp. 219–241.
[8] Nickles, Thomas: *Scientific Discovery, Logic, and Rationality.* Dordrecht, Reidel, 1980.
[9] Vanackere, Guido: "Ambiguity-adaptive Logic", *Logique & Analyse*, 159, sept. 1997, (appeared in 1999); pp. 261–280.
[10] Vanackere, Guido: "Minimizing ambiguity and paraconsistency", *Logique & Analyse*, to appear.
[11] Vanackere, Guido: "Ontological Causes of Inconsistency, and a Change-Adaptive, Logical Solution." *Proceedings WCP2000*, Campinas, São Paulo. In print.
[12] Vanackere, Guido: "HL2. An Inconsistency-Adaptive and Inconsistency-Resolving Logic for General Statements that Might Have Exceptions."*Journal Of Applied Non-Classical Logics.* 10, 2000, pp. 317–338.
[13] Wittgenstein, Ludwig: *Tractatus logico-philosophicus*. Vertaald en van een nawoord en aantekeningen voorzien door W.F. Hermans. Ataenaeumm-Polak & Van Gennep Amsterdam 1975.