Logique & Analyse **153-154** (1996), 201-204

EFFECTIVE IDENTIFICATION IN THE LIMIT OF FIRST ORDER STRUCTURES AND CREATIVE SETS

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Abstract
An alternative criterion of effective identification of first order structures will be proposed. This criterion uses the fact that all creative sets are recursively isomorphic and allows non decidable structures to be effectivelly identifiable.

1. *The original conception of effective identification*

Osherson and Weinstein (1986) define the concepts of identification and effective identification in the limit of first order structures in the following manner:

Let L be a countable, first order language with equality.

(i) A basic formula of L is an atomic formula of L or the negation of an atomic formula of L.
(ii) An L-structure is a countable structure that interprets L.
(iii) An L-environment is an ω-sequence of basic formulas of L.
(iv) $Rng(e)$ is the set of basic formulas occuring in an L-enviroment e.
(v) An L-environment e is said to be *for* an L-structure S if and only if there is an assignment function h from the set of variables of L onto the domain of S such that $Rng(e) = \{B \mid B$ is a basic formula of L and $B[h]$ is true in $S\}$.
(vi) An *L-theory* is a subset of the set of all sentences of L.
(vii) SEQ_L is the set of all finite sequences of basic formulas of L.
(viii) $Rng(\sigma)$ is the set of basic formulas occuring in a member σ of SEQ_L.
(ix) An *L-learning function* is any total function from SEQ_L to the set of all L-theories.
(x) An L-learning function φ is said to converge on an L-environment e to an L-theory T if and only if $\varphi(\ddot{e}_n) = T$ for all but finitely many n belonging to ω (here $\ddot{e}_n$ is the finite initial segment of lenght n belonging to ω in e).
(xi) $Th(S)$ is the set of all sentences of L that are true in a given L-structure S.

(xii) An L-learning function φ *identifies* an L-structure S if and only if for all L-environments e for S, φ converges on e to $Th(S)$. Given a collection H of L-structures, an L-learning function φ *identifies* H if and only if φ identifies every L-structure S belonging to H.

Before proceeding I present the motivation of Osherson and Weinstein for their original definition of effective identification.

> "We now turn our atention from the informational to the computational component of identification problems by focussing on effective learning functions. If mental processes are mechanical then scientists implement computable learning functions. Furthermore, the theories that mechanical scientists emit must be finitely characterizable". (Osherson and Weinstein[1986], p. 68).

(xii) Let $\varphi_0, \varphi_1, \varphi_2, \ldots$ be an effective enumeration of the partial recursive functions from ω to ω.

(xiii) Let SEQ be the set of all finite sequences of natural numbers.

(xiv) For i belonging to ω and σ belonging to SEQ or SEQ_L, let σ_i be the ith member of σ.

(xv) Let F be the set of all total functions from SEQ to ω.

(xvi) Let gn be a fixed Gödel numbering of all formulas of L. For σ belonging to SEQ_L, let $gn(\sigma)$ belonging to SEQ denote the n-tuple $(gn(\sigma_0), gn(\sigma_1),\ldots gn(s_{lenght(\sigma)}))$. For an L-theory T, let $gn(T)$ denote the set $\{gn(A) : A$ belongs to $T\}$.

(xvii) Given i belonging to ω, let $C_i = \{x$ belonging to $\omega : \varphi_i(x) = \text{o}\}$ (a simple dovetailing argument shows that each C_i is recursively enumerable).

(xviii) An L-learning function φ is *computable* if and only if there is a computable ψ belonging to F such that for all σ belonging to SEQ_L, $gn(\varphi(\sigma)) = C_{\psi(gn(\sigma))}$.

(xix) An L structure S is *effectively identifiable* if and only if some computable L-learning function φ identifies S. A collection H of L-structures is effectively identifiable if and only if some computable L-learning function φ identifies each member of H.

(xx) An L-structure S is decidable if and only if $gn(Th(S))$ is recursive.

Osherson and Weinstein note that using the above definition of effective identification, only decidable L-structures are effectively identifiable. This is the case due to the following series of standard results:

(a) $Th(S)$ is a logically closed and complete theory,

(b) Every recursively enumerable and logically closed theory is axiomatizable,

(c) Any axiomatizable complete and logically closed theory is decidable.

Together with the fact that:

(d) effective identification establishes that every theory in the range of a computable learning function is recursively enumerable.

Having noted this limitation, Osherson and Weinstein offered two possible alternatives in order to overcome it. The first eliminates condition (d) above and thus, in my opinion, conflicts with the original motivation previously quoted. The second could be used to relax the requirement that learning functions converge to *Th*(*S*). I propose a third alternative that keeps condition (d) and abandons the convergence of learning functions to *Th*(*S*) in a way different from that suggested by Osherson and Weinstein.

2. *Effective Identification with possible stabilization in creative sets*

Definition: An *L*-structure *S* is effectively identifiable with possible stabilization in creative sets if and only if *S* is effectively identifiable (in the sense of Osherson and Weinstein[1986]) or there is a computable *L*-learning function φ such that for every *L*-environment *e* for *S* there is a natural number n_0 such that for $n > n_0$ there is an *L*-theory T_n satisfying the following conditions:

a) $\varphi(\ddot{e}_n) = T_n$
b) $gn(T_n)$ is a creative set
c) T_n is a subset of $Th(S)$

The idea behind the above definition is that by being all creative sets recursively isomorphic (see, for instance, Rogers [1967], p.183) they are also, in a certain sense, equivalent, for any mechanical learner. Thus we can still have stabilization despite the fact that the learning function can oscilate among different theories.

The choice of the isomorphism type of the creative sets was made in order to give the result below.

Result: The standard structure for the language of arithmetic is effectively identifiable with possible stabilization in creative sets.

Of course, this structure is not decidable (in fact, by Tarski's indefinability theorem, the set of Gödel numbers of sentences true in the standard structure for the language of arithmetic is not even arithmetical) but using the fact that the Gödel numbers of the theorems of Peano Arithmetic form a creative set (see Rogers [1967], p. 98) we can see that the standard structure for the language of arithmetic is, indeed, effectively identifiable with pos sible stabilization in creative sets. (we made the assumption that all theorems of Peano Arithmetic are true in the standard structure).

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