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PREDICATE LOGIC IN WITTGENSTEIN'S *TRACTATUS**

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1. *The Tractatus on Logical Truth*

Wittgenstein's *Tractatus logico-philosophicus* is about logic. But, *which* logic is it about? Examples of logical truths (e.g. 6.1201) are from both propositional and (first-order) predicative logic. Wittgenstein appears to countenance higher order logic as well (see e.g. 5.5261), although it would be hard to find room in the *Tractatus* for meaningful talk of second-order properties. On the other hand, we know that he had no use for the theory of classes (6.031), and that he wanted to expunge the theory of identity not just from logic but from "correct conceptual notation" (5.534). He took predicative language as primary:

> "I write elementary propositions as functions of names, so that they have the form 'fx', '$\phi(x, y)$' etc." (4.24b; see 3.318)

apparently viewing propositional language as a kind of shorthand, which could be safely employed whenever the internal structure of an elementary proposition is logically immaterial. Given all this, it looks sensible to assume that *Tractatus* logic is *at least* elementary logic (without identity).

This being so, it would seem that the account of logic contained in the book is radically flawed. For the book's philosophy of logic is totally insensitive to the distinction between propositional logic, or the theory of truth functions, and predicate logic. Indeed, Wittgenstein extends the notion of tautology to cover all logical truths (6.1), explicitly including such predicative truths as '$(\forall x)Px \supset Pa$' (6.1201). Moreover, he appears to believe that elementary logic as a whole is decidable, for he holds that

> "Proof in logic is merely a mechanical expedient to facilitate the recognition of tautologies in complicated cases." (6.1262)

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On contemporary lights, both claims are misguided. First, logical truth in general cannot be reduced to tautologousness: for example, '$(\forall x)Px \supset Pa$' (Wittgenstein's example) is not a tautology, for the antecedent is not a truth function of simpler formulas, and if it is taken as a simple sentence, nothing forbids that it be true while the consequent, 'Pa', is false. Of course -as we shall presently see- this was not Wittgenstein's analysis; it is, however, the standard analysis that is given today.

Secondly, Wittgenstein's claim that proof in logic is merely a *mechanical* expedient to exhibit tautologousness is puzzling to say the least. For the truths of first-order logic are indeed provable; however, in the general case there is nothing mechanical about their proof. If Wittgenstein meant (as his use of the word 'mechanical' would lead us to believe) that there is a mechanical method -an algorithm- which is capable of deciding the truths of first-order logic in general, i.e. an algorithm that, given any formula of first-order language, tells us whether or not that formula is a logical truth, *that* claim is unsound: as Church showed in 1936,[1] no such algorithm exists for first-order logic in general. As Max Black remarked, "[Church's] result is fatal to Wittgenstein's philosophy of logic" (Black 1964, p.319).

Naturally, Church's result was not there for Wittgenstein to take account of in the *Tractatus*.[2] Indeed, that there are crucial differences between predicate logic in general and its propositional fragment was by no means commonplace in, say, 1918. For example, Whitehead and Russell thought in 1910 that "the theory of deduction for propositions containing apparent variables" -i.e. generalised predicate logic- could be deduced from "the theory of deduction for elementary propositions" (i.e. propositional logic) thanks to "certain primitive propositions" (Whitehead an Russell 1910, I, *9). Such primitive propositions amounted to axioms for quantification theory. It is by no means evident that the authors were aware that such axioms determined a notion of logical truth which was in certain respects irreducible to the one holding for propositional logic. Russell himself believed, in the early Twenties, that Wittgenstein's idea of the reducibility of predicate logic to propositional logic was "very interesting" (Russell 1922, p.14).

That in 1922 the difference between predicate logic in general and its propositional fragment had not been brought to light was, of course, no ground for the assertion that there is no such difference. So, the question

[1] See e.g. Mendelson 1964, pp. 155-156.

[2] In 1915, Löwenheim had shown that the *monadic* fragment of (first-order) predicative logic, i.e. the set of predicative formulas in which only one-place predicates occur, was decidable. There is no evidence that Wittgenstein was aware of Löwenheim's result, or that he made any difference between monadic and polyadic predicative formulas. Among his examples some are monadic (e.g. 5.5321, 6.1201), some polyadic (5.531, 5.532).

arises of how could Wittgenstein substantiate his two claims, that all logical truths are tautologies and that they are decidable by a mechanical method.[3] Actually the second claim reduces to the first, for if all logical truths are tautologies in Wittgenstein's sense then they are truth functions of elementary propositions, and therefore decidable.

2. *Logical Truth and Tautologousness*

It is sometimes flatly stated that Wittgenstein believed (in the *Tractatus*) that universally quantified propositions were just conjunctions, and existentially quantified propositions were disjunctions. For example, according to Max Black "It seems certain that [Wittgenstein] wanted to construe general propositions as conjunctions and disjunctions of elementary propositions" (Black 1964, p.281).[4] Bertrand Russell, in his *Introduction* to the *Tractatus*, spoke of "Mr.Wittgenstein's theory of the derivation of general propositions from conjunctions and disjunctions" (Russell 1922, p.15). And Wittgenstein himself, while rejecting his old doctrine of generality in the early Thirties, described it as the view on which "$(\exists x).\phi x$ is a logical sum", i.e. a disjunction:

> "Though its terms aren't enumerated *here*, they are capable of being enumerated (from the dictionary and the grammar of the language). For if they can't be enumerated we don't have a logical sum". (PG p.268).

As it turns out, all three authors' perception of the *Tractatus* doctrine of generality is much subtler than such statements would reveal. However, let us stop for a moment and consider what would follow from just equating general propositions with conjunctions or disjunctions.

Suppose that a universally quantified proposition is a conjunction; i.e. suppose that, for every formula of the form '$(\forall x)\phi x$', there is a set $\{\phi x\}$ of propositions such that '$(\forall x)\phi x$' abbreviates the conjunction of the

[3] As Glock (1996, p.149) rightly points out, when Wittgenstein introduces his own decision procedure -the truth-tabular method for which the *Tractatus* is best known- he explicitly restricts its domain of application to propositions "in which no sign of generality occurs" (T 6.1203). Nevertheless, 6.1262 entails that some such method must be available even for propositions involving quantifiers.

[4] This is clearly wrong as it stands. Even if universal quantification were just conjunction, a proposition of the form '$(\forall x)(\phi x \supset \psi x)$' could not be reduced *in every case* (i.e., for any ϕ and ψ) to a conjunction of elementary propositions, or to a combination of conjunctions and disjunctions of elementary propositions.

propositions belonging to the set (and dually for existentially quantified propositions). Would it follow that all logical truths are tautologies?

If conjunction is genuine conjunction, and disjunction is genuine disjunction it would clearly follow that quantified propositions are truth functions of elementary propositions. However, this alone would not entail that all logical truths are tautologies in Wittgenstein's (and our own) sense, i.e. propositions that are "true for all the truth-possibilities of the elementary propositions" (4.46). For suppose that the internal structure of an elementary proposition had logical import, or in other words, that there were logical relations among elementary propositions depending on the particular predicates occurring in them; so that, for example, there could be predicates P, Q such that, for any x, Px is incompatible with Qx; or a binary predicate I such that, for any x, y, Ixy and Px implied Py for any predicate P. In that case, some combinations of the truth values of certain elementary propositions would be ruled out by their internal structure (e.g. there would be no line in a truth-table such that $Pa=T$, $Qa=T$, or such that $Iab=T$, $Pa=T$, $Pb=F$). The truth value of any (well-formed) combination of elementary propositions by way of connectives and quantifiers would still depend only on the truth values of the constituent elementary propositions, and we could still take logical truths to be the propositions that come out true for all *admissible* combinations of the constituents' truth values. However, logical truths would not be tautologies in the sense of being propositions that are true for all truth-possibilities of the elementary propositions, if that is taken to mean that tautologies are *total* functions over the set of permutations of the truth values of elementary propositions, which assign T to any such permutation.

Consequently, in order for the logical truths to coincide with the tautologies it is not sufficient to interpret the quantified formulas as conjunctions or disjunctions; we also have to guarantee that the internal structure of an elementary proposition has no logical import. Of course, this does not particularly concern propositions involving quantifiers. Even in a quantifier-free predicative language, if (for example) we have a predicate having the semantic properties of identity we are going to have inadmissible combinations of truth values for certain elementary propositions, so that logical truth -truth for all admissible combinations- will not coincide with tautologousness, i.e. with truth for all combinatorially possible combinations.

Wittgenstein was quite aware of these matters: he insisted that two elementary propositions cannot contradict each other (6.3751), and he went to great lengths to exclude identity from adequate symbolism:

> "Expressions like '$a=a$', and those derived from them, are neither elementary propositions nor is there any other way in which they have sense" (4.243c).

For if such expressions *were* elementary propositions, with '=' expressing identity, there would be logical relations among elementary propositions (depending on the special properties of '=') and logical truth would not coincide with tautologousness. There would be well-formed combinations of elementary propositions -such as '$(a=b \ \& \ b=c)' \supset a=c$- that are logical truths without being tautologies, for they are true for all admissible combinations of the constituents' truth values without being true for all combinatorially possible combinations of truth values.

If we want logical truth to coincide with tautologousness we have to deny logical import to the internal structure of elementary propositions, which is exactly what we do in propositional logic. In propositional logic, we customarily write elementary propositions as p, q, etc. just because their internal structure is logically immaterial. Wittgenstein, on the other hand, saw elementary propositions "as functions of names", so he had to explicitly ensure that no such function had logical import. Later on, he became convinced that such an assumption was belied by semantic facts about ordinary language: simple predicates such as '- is red' or '- is green', for example, obviously had logical import (this he had already recognized in the *Tractatus*, 6.3751), but on the other hand it was hard to regard such propositions as '*a* is red' or '*a* is green' as complex. Thus for a relatively short period of his philosophical career he admitted that "there are rules for the truth functions which also deal with the elementary part of the proposition" (PB §82; see SRLF, pp.168-169), i.e. that there may be logical relations among propositions which depend on the particular predicates occurring in them. This required a revision of the whole doctrine of truth-functionality, which, however, did not come out to Wittgenstein's satisfaction; so that, eventually, he gave up (or anyway minimized the role of) the whole semantic doctrine of the proposition he had put forth in the *Tractatus* (see e.g. PG pp.123, 210-211).

To summarize and conclude this discussion: *if* quantified propositions were truth functions (conjunctions or disjunctions), *and* the internal structure of elementary propositions had no logical import, then all logical truths would be tautologies. The latter assumption is also needed to ensure that all *propositional* (i.e. quantifier-free) logical truths are tautologies; however, in the propositional case the assumption may be regarded as embedded in the very choice of propositional language (propositional language *expresses* the logical irrelevance of internal structure by not revealing it).

3. *The Untenability of Wittgenstein's Proposed Reduction*

However, quantified propositions are not truth functions: for example, '$(\forall x)\phi x$' is not equivalent to any conjunction '$\phi a \,\&\, \phi b \,\&\, \phi c \,\&\ldots$'. It is not equivalent to a finite conjunction, for the domain of quantification may be infinite; and it is not equivalent to an infinite conjunction, for we may not have a name in the language for every object in the (infinite) domain of quantification. So, if Wittgenstein had assumed that quantified propositions are equivalent to conjunctions or disjunctions his account of generality would have been mistaken. However, it is not obvious that he did make that assumption in the *Tractatus*. Indeed, he seems to flatly reject it:

> "I dissociate the concept *all* from truth-functions.
>
> Frege and Russell introduced generality in association with logical product or logical sum. This made it difficult to understand the propositions '$(\exists x)fx$' and '$(x)fx$', in which both ideas are embedded" (5.521).

In the case of universally quantified propositions, the two ideas that Wittgenstein is talking about here are the idea of conjunction and the idea of generality: '$(\forall x)\phi x$' is a conjunction of propositions of the form 'ϕx', and it is the conjunction of *all* such propositions.[5] Whether or not Wittgenstein was right in accusing Frege and Russell of obliterating generality proper in their analysis of quantification, it is clear that he meant his own account to be different.

Wittgenstein's positive doctrine can be gathered from the following rather cryptic remarks:

> "What is peculiar to the generality-sign is first, that it indicates a logical prototype (*Urbild*), and secondly, that it gives prominence to constants" (5.522).
>
> "The generality-sign makes its appearance as an argument" (5.523).

"The generality-sign" is any symbolic element that expresses generality in a proposition. Wittgenstein wants to say that what expresses generality in a proposition must have the nature of a *variable*. Thus the "true form" of a universally quantified proposition such as '$(\forall x)\phi x$' is not '$\phi a \,\&\, \phi b \,\&\ldots$' but rather something like '$\phi(\ )$' ("The generality-sign makes its appearance as an argument").[6] The *logische Urbild* which the generality-sign in-

[5] See Black 1964, p.283, Anscombe 1959, p.142.

dicates is the common form of a class of propositions: propositions belonging to such a class differ from one another at a specified argument-place, while being identical otherwise (the identical part is indicated by 'ϕ': in this sense the generality-sign "gives prominence to constants").[7] Quantification theory is reduced to the theory of truth functions by assuming that the truth functions (such as conjunction and disjunction) can apply to sets of propositions that are given not by enumeration $(\phi a, \phi b, \ldots)$ but through a form. As Russell put it (quite accurately in this case):

> "Wittgenstein's method of dealing with general propositions... differs from previous methods by the fact that the generality comes only in specifying the set of propositions concerned, and when this has been done the building up of truth-functions proceeds exactly as it would in the case of a finite number of enumerated arguments p, q, r..." (Russell 1922, p.14).

Thus, Wittgenstein thought that a quantified proposition such as '$\sim(\exists x)fx$' could be identified with the joint negation of the propositions belonging to a set ξ , provided the elements of ξ were *all* the values of fx (5.52). Quantification is generality plus truth-functionality; the generality is given through a form. "A general prop[osition] is *A* truth-function of *all* PROP[OSITION]S of a certain form" (*To Russell*, 19.8.19, CL p.126).

It could be objected that here, Wittgenstein is guilty of a confusion of linguistic levels: for at the level of language, at which such truth functions as conjunction, negation etc. are applied, there are no forms but only propositions. There is no such thing as the application of a truth function such as joint negation to a *form*: joint negation, like any other truth function, only applies to propositions given one by one. However, the objection would beg the question, for Wittgenstein was not suggesting that truth functions applied to propositional forms. He thought that they applied to propositions; he also believed, however, that the actual specification of the propositions to which they applied could be, so to speak, postponed. In the meanwhile, they could be determined by giving their form. Wittgenstein was trying to eat his cake and have it: he wanted the instances of a general proposition to be there in order for the truth functions to apply to them, but he did not want to be obliged to actually enumerate them.

Until what (logical) moment is the actual specification of such instances to be postponed? The answer is, until logic is *applied*. For "the *application*

[6] Of course, '$\phi(\ )$' must be read in the right way. It should not be identified with the propositional function (open formula) 'ϕx'.

[7] For an explanation of the second half of 5.522 see Anscombe 1959, p.143.

of logic decides what elementary propositions there are" (5.557). What elementary propositions there are depends on what semantically different names there are (5.55), and this is a question that cannot be settled by logic alone: the answer depends on the actual institution of language, with its actual discriminations. Once language is in place, we know what distinct names there are (which shows what *objects* there are) and what elementary propositions there are. At that point, we also know what elementary propositions *of a certain form* there are; we know what are the instances of '*f*( )'. They are exactly those elementary propositions that turn out to be of that form.

But there is an obvious difficulty. Wittgenstein says that "What belongs to its application, logic cannot anticipate" (5.557b). Therefore, logic cannot anticipate the actual specification of the instances of a given form: that is left to the application of logic. How, then, can such instances be *logically* relevant, i.e. play the role of arguments of truth functions? "Logic and its application must not overlap", says Wittgenstein (5.557e). Yet, this is exactly what he is trying to bring about by his account of quantification. He needs the instances of a propositional form to be there in order for his truth-functional analysis of the logical truths of quantification theory to go through: for if they are not there, the truth functions cannot apply to them and the logical truth of -say- '$(\forall x)Px \supset Pa$' is not determined.[8] But that means to have logic overlap with its application, for it is only in the application of logic that the instances of a propositional form are actually specified. Thus, Wittgenstein's account of generality fails on the *Tractatus*'s own grounds: granted, logic "has to be in contact with its application" (5.557d); however, it cannot essentially depend on matters that are only determined by its application.

4. "*Dogmatism*"

As he went back to the issue of generality in the early Thirties, Wittgenstein remarked that the *Tractatus* account was "indefensible", and he added:

> "It went with an incorrect notion of logical analysis in that I thought that some day the logical product for a particular $(x).\phi x$ would be found" (PG p.268).

[8] The only alternative to a truth-functional analysis of such logical truths that Wittgenstein considered consisted in taking "$(\forall x)Px$ entails Pa" as a "primary proposition", i.e. as an axiom: see LM p.90, and cf.AM pp.5-6).

The notion of analysis that is here denounced as incorrect is the same as underlies the idea of an elementary proposition. We reach, by *a priori* arguments, the conclusion that there must be elementary propositions, but we cannot indicate any; still, we believe that it will be possible to specify them in the future. "It is held that, although a result is not known, there is a way of finding it" (WWK p.182). Similarly, in the case of generality we believe that a general proposition *must* be equivalent to some definite conjunction or disjunction (for otherwise, how could the logical truth of '$(\forall x)Px \supset Pa$' be established?), and that although the conjuncts or disjuncts cannot be enumerated here and now, they are capable of being enumerated and they will be, some day.

What is wrong with this notion of analysis is that no actual analysis can be based on it, so that no notion (e.g. of elementary proposition) has really been introduced. We seem to have established that there must be *such things as* elementary propositions, but we cannot really say what they look like.

> "Such a procedure is legitimate only if it is a matter of capturing the features of the physiognomy, as it were, of what is only just discernible - and that is my excuse. I saw something from far away and in a very indefinite manner, and I wanted to elicit from it as much as possible" (WWK p.184).

As we don't really know what an elementary proposition looks like, we are in no position to say what the analysis of a proposition really is: only that there must be something, some procedure (vaguely conceived by Wittgenstein on the analogy of Russell's paraphrase of sentences involving definite descriptions: PG p.211), which procedure we would like to call 'analysis'. Such a notion of analysis comes close to being empty.

Wittgenstein refers to the philosophical attitude motivating his old, incorrect notion of analysis as 'dogmatism' (WWK pp.182-184),[9] which may lead one to believe that he is objecting to *a priori* arguments as such: as if he were saying that it is dogmatic, for example, to claim that there are elementary propositions in the lack of any *a posteriori* evidence for them. This is not Wittgenstein's point, however. *A priori* reasoning is entirely legitimate: indeed, logic is its proper place. The trouble with the introduction of the notion of an elementary proposition is not that it is based on *a priori* arguments, but that it is *not really* based on such arguments. For the

[9] On Wittgenstein's criticism of his own early "dogmatism", and the role of such criticism in enhancing the transition to his later philosophy see Schulte 1989, pp.94-95.

notion's actual specification is made to depend on future discoveries, as if it required some information which we happen to lack at the present moment, although it is capable of becoming available in the future. As one might think to have established by theoretical arguments that there must be protons and neutrons, except that our microscopes are not powerful enough for us to actually *see* them - but one day, thanks to better technology, we will. Therefore, the introduction of elementary propositions (like the reduction of quantification to truth-functional logic) is dogmatic not because it is *a priori*, but rather because it is based on a promissory note: we are asked to *believe* that one day, we shall know what elementary propositions are like, or what are the conjuncts in a universally quantified proposition.

But -as Wittgenstein came to see more clearly in the early Thirties- nothing in logic hinges on missing information. On the contrary, all the information we may need is already in. Nothing is hidden.

> "The truth of the matter is that we have already got everything, and we have got it actually *present*; we need not wait for anything. We make our moves in the realm of the grammar of our ordinary language, and this grammar is already there. Thus we have already got everything and need not wait for the future" (WWK p.183).

In the *Tractatus*, Wittgenstein had written that "There can *never* be surprises in logic" (6.1251); but then, he had been unfaithful to his own maxim. Difficulties with generality and, more generally, with the notion of an elementary proposition taught him an important lesson concerning the proper method in the philosophy of language: no philosophical conclusions -i.e., no conclusions about the grammar of our language- can be made to depend on uncovering "deep", hidden features of language. For example, the only viable notion of elementary proposition is one on which the elementary propositions are entirely open to view:

> "You may call the sentence 'Here there is a red rose' an elementary proposition. That is to say, it doesn't contain a truth-function and it isn't defined by an expression which contains one. But if we're to say that a proposition isn't an elementary proposition unless its complete logical analysis shows that it isn't built out of other propositions by truth-functions, we are presupposing that we have an idea of what such an 'analysis' would be" (PG p.211).

The same lesson applies to the analysis of quantification. Here, we must distinguish two cases (PG p.268). When we are talking about finite sets of elements each of which has a name (such as the primary colours or the notes of the C major scale), the truth-functional account of quantification

does apply: the proposition "All the primary colours occur in this picture" is indeed equivalent to a conjunction. But then, we are perfectly able to exhibit the conjuncts: 'Red occurs in this picture', 'Yellow occurs in this picture', 'Blue occurs in this picture'. Whereas in cases such as "All men die before they are 200 years old", the truth-functional analysis does not apply and we are in need of something else.

Wittgenstein himself never provided such an alternative account - Tarski did. And it may be doubted that, even in the finite case, his account is really accurate.[10] However, it is clear that he had seen through his old mistake.

5. *Conclusion*

The *Tractatus*'s account of predicate logic (and thus of logic in general) is indeed flawed, but not trivially. Wittgenstein's error depends on an untenable theory of generality, whose source is confusion concerning the proper method in the philosophy of logic and language: contrary to his own doctrine, Wittgenstein made logic and its application overlap. Reflection on this mistake and its grounds ("dogmatism") led Wittgenstein to reject the *Tractatus*'s account of generality, and the two connected notion of complete logical analysis and elementary proposition. More generally, it made him better aware of the fact that in philosophy "nothing is hidden": a maxim that he had stated, but not fully honoured in the *Tractatus*.

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REFERENCES

G.E.M.Anscombe, *An Introduction to Wittgenstein's Tractatus*, Hutchinson & Co., London, 1959.

M.Black, *A Companion to Wittgenstein's 'Tractatus'*, Cambridge University Press, Cambridge, 1964.

H.-J.Glock, *A Wittgenstein Dictionary*, Blackwell, Oxford, 1996.

[10] Actually, in order for the conjunction to be logically, and not just functionally equivalent to the quantified proposition it is also needed that red, yellow, and blue are *all* the primary colours. Here Wittgenstein appears to be relying on his old *Tractatus* doctrine (a form of "Closed World Assumption") according to which "If objects are given, then at the same time we are given *all* objects. - If elementary propositions are given, then at the same time *all* elementary propositions are given" (5.524). If we are given the names of the primary colours, we are given *all* their names. Somehow, we are supposed to just *know* that *those* names are *the* names of the primary colours.

E.Mendelson, *Introduction to Mathematical Logic*, Van Nostrand, New York, 1964.

G.E.Moore, *Wittgenstein's Lectures in 1930-33*, "Mind" 63(1954), 1-15, 289-315, 64(1955), 1-27, 264; later in PO, 46-114 [=LM].

B.Russell, "Introduction" to T.

J.Schulte, *Wittgenstein - Eine Einführung*, Reclam, Stuttgart, 1989.

A.N.Whitehead and B.Russell, *Principia Mathematica*, Cambridge University Press, Cambridge, 1910; Second edition, 1927.

L.Wittgenstein, *Tractatus Logico-Philosophicus*, Routledge and Kegan Paul, London, 1922 [=T].

L.Wittgenstein, *Some Remarks on Logical Form*, "Proceedings of the Aristotelian Society" Suppl.Vol.9 (1929), 162-71; now in PO, 29-35 [=SRLF].

L.Wittgenstein, *Philosophische Bemerkungen*, ed. by R.Rhees, Blackwell, Oxford 1964; English translation by R.Hargreaves and R.White, Blackwell, Oxford 1975 [=PB].

L.Wittgenstein, *Philosophische Grammatik* , ed. by R.Rhees, Blackwell, Oxford 1969; English translation by A.Kenny, Blackwell, Oxford, 1974 [=PG].

L.Wittgenstein, *Lectures, Cambridge 1932-1935*, from the notes of A.Ambrose and M.Macdonald, ed.by A.Ambrose, Blackwell, Oxford, 1979 [=AM].

L.Wittgenstein, *Philosophical Occasions 1912-1951*, ed.by J.Klagge and A. Nordmann, Hackett, Indianapolis and Cambridge, 1993 [=PO].

L.Wittgenstein, *Cambridge Letters*, ed.by B.McGuinness and G.H.Von Wright, Blackwell, Oxford 1995 [=CL].

Wittgenstein und der Wiener Kreis, hrsg.von B.McGuinness, Suhrkamp, Frankfurt a.M.1967; English translation by J.Schulte and B.McGuinness, Blackwell, Oxford,1979 [=WWK].