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ADDING A FEW NAMES

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Substitutional semantics can be done in one of two ways. One can forget about objects altogether, and deal with names (that is, individual constants) only: call this substitutional semantics proper (SSP). Or one can keep objects in the background, but not let them play any direct role (via sequences, assignments, or what have you) and use names to stand for objects: call this substitutional *style* semantics (SSS). From a philosophical point of view, SSS is quite interesting, since it allows one to inquire upon how far, and at what conditions, linguistic resources can go in matching ontological demands. But, most often, this opportunity gets lost in the context of semantical projects concerned with proving completeness and other global adequacy results, and unable to appreciate the significance of some subtle matters of *in*adequacy. In the present note, I want to alert the reader to one such matter.

A minimal requirement for the plausibility of SSS is the following:

(R) Every object must have at least one name.

Without imposing (R), we would have no guarantee that reference to names can effectively replace reference to objects: for example, that $(x)A$ is in fact true when $A(a/x)$ is true for every name a (which would make no sense as a problem for SSP, but is a serious issue for SSS). So let us impose this requirement and consider a model M in which every object has *exactly one* name. It is common to refer to the model M' which coincides with a model M, except for the fact that some name a denotes a given object o. In the case of our M here, however, such an M' will in general not exist, because if o is distinct from the denotation o' of a then, were one to assign o the name a, o' would be left with no name, against (R).

Consideration of a model M' of the kind discussed above is often important in proving a crucial substitution lemma, on the way to the soundness theorem, that is: in proving that A is verifiable if $A(b/a)$ is. If the substitution lemma is all we care about, the solution is easy: it is enough, for example, to rotate all the names not directly involved in (the statement and proof of) the lemma in such a way that every object is still covered. But this easy solution should not make us forget that a problem remains: a problem that has nothing to do with the substitution lemma or the soundness theorem, but rather with a serious limitation (R) has (somewhat surprisingly) generated in our formal construe of logical possibility. For models, at this level of logical analysis, are the formal counterparts of possible worlds, and hence the conclusion reached at the end of the last paragraph amounts to the following: in presence of (R), it is often the case that *it is impossible* to (simply) assign a name a different denotation. Given a possible state of affairs, the attempt at simply relocating a name in it results in something impossible.

The problem is a perfectly general one, and one that would not be resolved by strengthening (R) to, say,

(R') Every object must have at least two names.

For with (R') at our disposal, it would certainly be the case that, were we to assign the name *a* a different denotation, the object originally denoted by *a* would not be left nameless; however, it would often be left with less than *two* names, and hence (R') — though not (R) — would not be satisfied (and the outcome, once again, would not be a "possible" world). The same is true of any strengthening

(Rn) Every object must have at least n names

of (R), where n is finite.

Where the problem ceases to arise is at the first infinite ordinal. For consider now

(Rω) Every object must have at least ω names.

Given any model M which satisfies (Rω), one sees immediately that the structure M' which coincides with M except that the name a denotes the object o is always one satisfying (Rω).

In conclusion, we can expect names to make do for objects only if with each object we correlate an infinite stock of names. Which gives some formal content to the commonplace philosophical claim that objects cannot be exhausted by (finite) conceptual (and hence also linguistic) means.

Substantive discussion of the claim above goes beyond the scope of the present note. But even the elementary considerations brought up here are an illustration of how quickly one can get into deep, significant issues when semantical analysis is used as a sensitive tool of philosophical probe rather than as a piece of machinery designed to establish "technical results."

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