Logique & Analyse **125-126** (1989), 113-130

MODAL MEINONGIAN LOGIC

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Abstract
A formalization of four distinct quantificational modal Meinongian logics is given, with nonstandard model set thwrctical semantics. The derivation of Barcan and converse Barcan theorems prompts revision of two systems of modal Meinongian logic to restore congruence of semantic and inferential structures of Meinongian systems with identical uniform domain distributions of existent and nonexistent objects. The problem of transworld identity of incomplete and impossible objects is examined, and world-indexing of nuclear properties is recommended as a solution.

I. *Meinongian Modalities De Dicto and De Re*

Meinong's object theory provides the basis for an informal modal logic. The semantics of Meinongian objects describe properties of actual, merely possible, and metaphysically impossible intentional objects. This is a ***de re*** theory of Meinongian modalities, involving the modal status of objects of thought (though not every Meinongian ***'res' cogitabilis*** exists, subsists, or has any mode of being in any logically possible world). Meinong developed an elaborate informal theory of possibility and probability that is different in many ways from contemporary model set theoretical semantics. To formally express ***de dicto*** and ***de re*** modalities in a possible worlds context it is necessary to extend Meinong's object theory to construct a Meinongian counterpart of standard non-object-theoretical modal logic.

There have been several attempts to develop modal Meinongian logics. But these do not always take sufficient account of the ontic peculiarities of Meinongian systems, and thereby fail to demonstrate semantic connections between the Meinongian domain and model set theoretical operations on domains, worlds, and models. Without this philosophical groundwork modal Meinongian logic remains an empty formalism that cannot contribute to a more thorough understanding of Meinong's object theory.

II. Differences in Standard and *Meinongian* Modal Logics

The semantic structures of standard systems of alethic modal logic standardly involve a Henkin-type recursive procedure for assembling maximally consistent sets of propositions, each of which constitutes or at least completely describes what for heuristic purposes is sometimes referred to as a logically possible world.(1) A model for standard modal logic is an ordered quadruplet $\langle \Sigma, \Gamma, R, V \rangle$, consisting of the set Σ of all maximally consistent sets of propositions, $\Sigma = \{W_1, W_2, W_3, \ldots\}$; an element or member $\Gamma \in$ C, sometimes distinguished as or as representing the actual world; a (usually minimally reflexive) relation R on C; and a valuation function V, which for $V(p, W_i)$ assigns truth value T or F to each proposition p of each maximally consistent set of propositions or world $W_i \in \Sigma$ $(i \geq 1)$. For truth functionally complex propositions, $V({\sim}p, W_i) = T$ if and only if $V(p, W_i) = F$; $V((p \supset q), W_i) = T$ if and only if $V(p, W_i) = F$ or $V(q, W_i) = T$. Logical necessity is defined on the Leibnizian conception as truth in every logically possible world, $V(\Box p, W_i) = T$ if and only if $V(p, W_j) = T$ for every W_j such that $R(W_i, W_j)$. Logical possibility is reducibly defined in terms of necessity, $\Diamond p \equiv {\sim}\Box{\sim}p$ (or conversely by duality; $\Box p \equiv {\sim}\Diamond{\sim}p$, where $V(\Diamond p, W_i) = T$ if and only if $V(p, W_j) = T$ in at least some W_j such that $R(W_i, W_j)$). Relation R is often interpreted as world-accessibility. When a world W_j is accessible from world W_i, then any proposition true in W_j is logically possible in accessible world W_i. Distinct systems of modal logic are semantically determined by distinct models with distinct accessibility relations, such as combinations of reflexivity with symmetry, transitivity, and other more exotic variants.

Quantificational or predicate modal logic is interpreted by means of an expanded semantic model, an ordered quintuplet $\langle \Sigma, \Gamma, R, D, V \rangle$, in which Σ, Γ, and R are as before, and where D is a function which for $D(W_i)$ assigns a domain of existent objects to each world $W_i \in \Sigma$, and valuation function V assigns a set of n-tuples of members of $D(W_i)$ to n-ary predicate 'P^n' if $n > 0$, and otherwise if $n = 0$ assigns T or F to P^n in $V(P^n, W_i)$.

Truth functional valuations are defined as before for propositional connectives. If $p = P^n x_1 \ldots x_n$, then $V(p, W_i) = T$, relative to an assignment of $a_1, \ldots, a_n$ to the x_i if and only if the n-tuple $a_1, \ldots, a_n \in V(P^n, W_i)$; if $p = (\forall x) q(x, y_1, \ldots, y_n)$, then $V(p, W_i) = T$ relative to an assignment of $b_1, \ldots, b_n$ to the y_i if and only if $V(q(x, y_1, \ldots, y_n), W_i) = T$ for every assignment of a member $d \in D(W_i)$ to x.(2) Axiom schemata are devised to assure convergence of semantic models and deductive inference methods for logically valid propositions in each modal system.(3)

Modal Meinongian logics can now be similarly defined. Nonquantificational versions have precisely the same kind of semantic model, but are different because of differences in the propositions included in and excluded from their worlds. In Meinongian semantics the proposition that the round square is round is true, and so belongs to maximally consistent sets of true propositions in modal Meinongian semantic models, but not to the models of standard non-Meinongian modal logics.(4) The independence thesis in Meinongian, unlike standard propositional semantics, permits the true predication of properties to nonexistent objects. A further distinction that complicates modal Meinongian logics occurs because of the three-valued interpretation of Meinongian propositional logic for some nuclear predications to indeterminate nonexistent objects.(5) The proposition that an incomplete Meinongian object has (or does not have) a (nuclear) property for which the object is indeterminate (such as 'Macbeth spoke Italian', 'Macbeth did not speak Italian') is most naturally classified as neither true nor false but undetermined in truth value. Standard propositional semantics in model set theoretical interpretations of standard modal logics on the contrary are classically bivalent. A maximally consistent set of propositions in a modal Meinongian model might include the proposition that not every proposition is true or false, which no standard model would contain. There are also true propositions of the models of standard modal logic that are not true and therefore not part of the models of modal Meinongian logic, such as the proposition that every object exists, or that every proposition is true or false. Quantificational modal Meinongian logic also parallels to some extent the formalization of standard quantificational modal logic. But here important differences emerge. The domain function D^m of a modal Meinongian quantificational model assigns the same domain consisting of both existent and nonexistent Meinongian objects to each Meinongian world $W^m_i \in \Sigma^m$. This guarantees uniform population or homogenous distribution of objects across every logically possible world in every model for each distinct system of modal Meinongian logic. The numerical identity of Meinongian domains and nonselective occurrence of Meinongianobjects in every world of every model has important formal and philosophical consequences. The result of these distinctions is that logical necessity, possibility, and impossibility, do not coincide in standard and Meinongian modal logics. Standard modal logics cannot embed and are not embeddahle in modal Meinongian logics.

III. *Alternative Systems of Modal Meinongian Logic*

There is a plethora of systems of modal logic, just as there is a continuum of inductive methods, and of standard and nonstandard deductive logics.(6) It would not be appropriate to undertake the formalization of each and every system of modal Meinongian logic, since there are indefinitely many. For most philosophical, scientific, and mathematical purposes, only a few modal logics are needed. It will therefore suffice to outline Meinongian counterparts of the four most common and useful systems of modal logic, and to provide Meinongian semantic models for their interpretation. Modal Meinongian logic in its formal treatment of extranuclear necessity, possibility, and impossibility, is part of the classically bivalent extranuclear subtheory of the otherwise nonstandardly three-valued Meinongian logic. The modal propositions of modal Meinongian logic are exclusively either true or false, even though they are about or involve modal operations on at least some nuclear predications that are neither true nor false but undetermined in truth value. The nonmodal logical truths of object theory are also truths of every modal Meinongian logic.

The four basic systems of alethic modal logic are T (Feys-Godel), S_4 (Lewis), S_5 (Lewis), and *B* ('Brouwersche'). Listed here are characteristic definitions and inference principles for the four corresponding systems of modal Meinongian logic.

Axioms and Necessitation Rule

If p, q are object theory wffs:

T^m (Meinongian variant of Feys-Godel T)

(M1) $\Box p \equiv \sim\Diamond\sim p$
(M2) $\Diamond p \equiv \sim\Box\sim p$

(M3) $(p \mathrel{-\!\!3} q) \equiv \Box(p \supset q)$
(M4) $\Box p \supset p$
(M5) $\Box(p \supset q) \supset (\Box p \supset \Box q)$
(NR) $\vdash p \supset \vdash \Box p$

S_4^m (Meinongian variant of Lewis S_4)
(M1)-(M5)-(NR)
(M6) $\Box p \supset \Box\Box p$

S_5^m (Meinongian variant of Lewis S_5)
(M1)-(M6)-(NR)
(M7) $\Diamond p \supset \Box \Diamond p$

B^m (Meinongian variant of Brouwersche system B)
(M1)-(M5)-(NR)
(M8) $p \supset \Box \Diamond p$

The inference structures of these systems are the same as those of their corresponding standard non-Meinongian modal logics. Differences between the two kinds of systems are hidden away in the formal semantics. The models of modal Meinongian logic constitute a DeMorgan lattice. For any Meinongian worlds $W^m{}_i$, $W^m{}_j$, $W^m{}_k \in E^m$, the following conditions are obviously satisfied.(7)

$$W^m{}_i \cap W^m{}_i = W^m{}_i$$
$$W^m{}_i \cup W^m{}_i = W^m{}_i$$
$$W^m{}_i \cap W^m{}_j = W^m{}_j \cap W^m{}_i$$
$$W^m{}_i \cup W^m{}_j = W^m{}_j \cup W^m{}_i$$
$$W^m{}_i \cap (W^m{}_j \cap W^m{}_k) = (W^m{}_i \cap W^m{}_j) \cap W^m{}_k$$
$$W^m{}_i \cup (W^m{}_j \cup W^m{}_k) = (W^m{}_i \cup W^m{}_j) \cup W^m{}_k$$
$$W^m{}_i \cap (W^m{}_i \cup W^m{}_j) = W^m{}_i \cup (W^m{}_i \cap W^m{}_j) = W^m{}_i$$

This makes it possible to define Boolean set theoretical relations on the lattice of all Meinongian worlds or maximally consistent sets of Meinongian propositions.(8)

Truth valuation $V^m(p, W^m{}_i) = T$ (F or U) if and only if propositionp has Meinongian truth valuation $V^m(p) = T$ (F or U) in world $W^m{}_i$. The modal Meinongian semantic models for T, $S_4{}^m$, $S_5{}^m$, and B^m can be formally defined. Combinations of accessibility relations holding between worlds and propositions true in worlds within a model are indicated by '+'.

T^m $\langle \Sigma^m, \Gamma^m, \textit{Reflexivity}, V^m \rangle$
$S_4{}^m$ $\langle \Sigma^m, \Gamma^m, \textit{Reflexivity} + \textit{Transitivity}, V^m \rangle$
$S_5{}^m$ $\langle \Sigma^m, \Gamma^m, \textit{Reflexivity} + \textit{Transitivity} + \textit{Symmetry}, V^m \rangle$
B^m $\langle \Sigma^m, \Gamma^m, \textit{Reflexivity} + \textit{Symmetry}, V^m \rangle$

Nonstandard truth valuations for primitive propositional connectives negation and the conditional are defined. $V^m(\sim p, W^m{}_i) = T$ if and only if

$V^m(p,W^m_i) = F$; $P(-p,W^n_i) = U$ if and only if $V^m(p,W^m_i) = U$; $V^m((p \supset q),W^m_i) = T$ if and only if $V^m(p,W^m_i) = F$, or $V^m(p,W^m_i) = T$ and $V^m(q,W^m_i) = T$ or $V^m(p,W^m_i) = U$ and $V^m(q,W^m_i) = U$; $V^m((p \supset q),W^m_i) = F$ if and only if $V^m(p,W^m_i) = T$ and $V^m(q,W^m_i) = F$; $V^m((p \supset q),W^m_i) = U$ if and only if $V^m(p,W^m_i) = T$, $V^m(q,W^m_i) = U$, or $V^m(p,W^m_i) = U$ and $P(q,W^m_i) = F$.

Modal truth conditions under V^m can be described in a completely general way for any system of modal Meinongian logic. Let Σ^m represent the set of all maximally consistent sets of propositions in modal Meinongian theory m, and let T^m, F^m, U^m represent the truth values of propositions in m. Modal Meinongian theory m can be defined by its corresponding model. These conventions save rewriting the truth value conditions for each system of modal Meinongian logic when only the accessibility relations of a particular model differ.

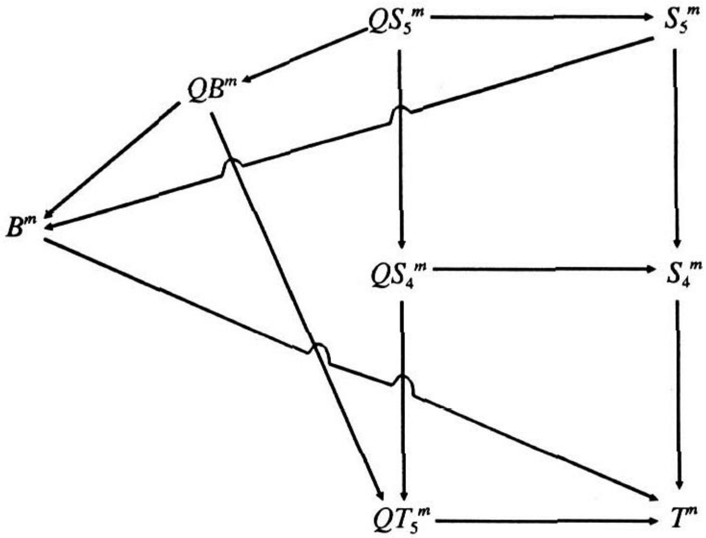
The following simplified principles for alethic modal Meinongian operators may be advanced. Truth conditions of the semantic model serve the same purpose as Kripke's 'models', intercalating a truth value function into a so-called normal model. Quantification in $(\forall W^m)(\ldots W^m \ldots)$ ranges over the Meinongian worlds of a particular Meinongian model. Relation R is any specialized (complex of) accessibility relation(s). Modal expressions in which a necessity operator applies to a proposition are classically bivalent, logically equivalent to corresponding extranuclear necessity predications.

$$V^m(\Box p) = T^m \equiv (\forall W^m)[(W^m \in \Sigma^m \,\&\, R(\Gamma^m,W^m)) \equiv V^m(p,W^m) = T]$$
$$V^m(\Box p) = F^m \equiv (\exists W^m)[(W^m \in \Sigma^m \,\&\, R(\Gamma^m,W^m)) \,\&\, V^m(p,W^m) \neq T]$$

Quantificational versions of modal Meinongian logic are obtained by adding the characteristic axioms of nonquantificational modal systems to quantificational Meinongian object theory. It is important to recall that 'existential' quantification in Meinongian logic has no real existential or ontic import, but merely indicates membership in the Meinongian domain of existent and nonexistent objects.(9)

A quantificational model for quantiticational modal Meinongian logic is an ordered quintuplet $\langle \Sigma^m,\Gamma^m,R,D^m,V^m \rangle$, in which Σ^m, Γ^m, and R are as before in semantic models for nonquantiticational modal Meinongian logic. Meinongian domain function D^m in $D^m(W^m_i)$ uniformly assigns the same domain of existent and nonexistent Meinongian objects to each and every world $W^m_i \in \Sigma^m$. Meinongian valuation function V^m combines the previously described effects of V in $\langle D,I,V \rangle$ for nonmodal quantificational Meinongian logic, and of V^m in $\langle \Sigma^m, \Gamma^m, R, V^m \rangle$ for nonquantificational modal Meinongian logic.

Inference relations between quantificational and nonquantificational systems of modal Meinongian logic are represented diagrammatically. The arrow indicates a transitive theoremhood containment relation, where L $\rightarrow$ *L'* means that system L contains all the theorems of system L' (and may contain more). Here at a glance are the formal interconnections among alternative modal Meinongian logics.



IV. Barcan and Converse Barcan Theorems in Quantificational Modal Meinongian Logic

The formal systems T^m, S_4^m, S_5^m, and B^m are distinct nonquantificational modal Meinongian logics. But the most straightforward unqualified quantificational versions of these systems are distinct only because of their inferentially distinct nonquantificational fragments. The nonvacuously quantified theorems of quantificational modal Meinongian logics QT^m, QS_4^m, QS_5^m, and QB^m are identical. Even such standardly distinguishing propositions as $(\forall x)(\Diamond Px \supset \Box \Diamond Px)$, true in ordinary S_5, but not in ordinary T or S_4, are true in every quantified modal Meinongian system. The semantic collapse of quantificational modal Meinongian logics is determined by quantification over an identical domain of existent and nonexistent Meinongian objects in the models of unqualified quantificational systems. This dissolves the semipermeable accessibility membranes of accessibility relations that otherwise hold between worlds in standard modal semantics.

Standard quantificational modal logics are inferentially distinguished on the basis among other things of whether or not they contain as theorems the Barcan or converse Barcan formulas.[10] The converse Barcan formula $\Box(\forall x)Px \supset (\forall x)\Box Px$ is a theorem of standard modal systems QT, QS_4, QS_5, and QB. But the Barcan formula $(\forall x)\Box Px \supset \Box(\forall x)Px$ is a theorem only of QS_5 and QB, and not of QT or QS_4.

These inferential asymmetries do not provide a satisfactory method of distinguishing any of the quantificational systems of modal Meinongian logic. There is a difficulty in the construction of quantificational modal Meinongian logic which must now be resolved. The reason why the converse Barcan but not the Barcan formula is a theorem of most standard quantificational modal logics is more easily seen in the modal-quantificational duals of these propositions. The converse Barcan formula is logically equivalent to its dual, $(\exists x)\Diamond Px \supset \Diamond(\exists x)Px$. This standardly states that if there actually exists an entity that in some logically possible world has property P, then it is possible or there is another accessible logically possible world in which there exists an entity that has property P. But the Barcan formula $\Diamond(\exists x)Px \supset (\exists x)\Diamond Px$ under standard interpretation states that if in some logically possible world there exists an entity that has property P, then there actually exists an entity that possibly or in some other accessible logically possible world has property P. The truth of this proposition depends on the accessibility relations in the semantic models of particular systems of standard modal logic, since it need not follow that an entity that possibly exists or exists in some other world also exists in the actual world where possibly it has property P. Accessibility gradients and uneven distribution of existent entities across logically possible worlds in standard modal semantics determine whether or not standard modal logics contain the Barcan or converse Barcan formulas as theorems. If there were a uniform distribution of existent entities in every world of every modal semantic model, if any entity existing in any logically possible world existed in every logically possible world, then the Barcan and converse Barcan formulas would be theorems of every standard system of quantified modal

In modal Meinongian logic there is an entirely uniform distribution of existent and nonexistent Meinongian objects in every Meinongian domain of every world of every modal Meinongian model, so that every modal

Meinongian semantic model has precisely the same Meinongiandomain. The combined biconditional $(\forall x)\Box Px \equiv \Box(\forall x)Px$ ($\Diamond(\exists x)Px \equiv (\exists x)\Diamond Px$) under modal Meinongian interpretation says no more than that the domain of a logically possible world contains an existent or nonexistent Meinongian object with property P if and only if the domain of the actual world contains an existent or nonexistent Meinongian object which in some accessible logically possible world has property P. The truth of the proposition is trivially guaranteed by the construction of modal Meinongian semantic models. The domains of the actual world and all other logically possible worlds are identical, and '$(\exists x)\Diamond Px$' carries no real existential or ontological import in Meinongian quantificational semantics.[13]

As things stand, it is not possible validly to deduce the Barcan formula in quantificational modal Meinongian systems QT^m and QS_4^m. The Barcan formula is true in QT^m and QS_4^m as determined by their modal quantificational semantic models, but the inference schemata of the logics are not powerful enough to derive the Barcan formula as a theorem. (The axioms of QT^m and QS_4^m are the same as those of QT and QS_4, from which the Barcan formula standardly cannot be derived.)

The situation must be corrected to regain convergence of semantic and inference structures for QT^m and QS_4^m. The Barcan formula can be added as a nonlogical axiom to QT^m and QS_4^m to produce QT^{m+} and QS_4^{m+}, without strengthening these systems to QS_5^m.[14] The validly deducible theorems of QT^{m+} and QS_4^{m+} will then have perfect congruity with their semantic models. QT^{m+} and QS_4^{m+} accordingly must replace QT^m and QS_4^m *as* the legitimate modal Meinongian counterparts of QT and QS_4. This undermines the inferential isomorphism between quantificational standard and modal Meinongian logics, but in a sense provides the most direct solution to the problem. Another method is to modify the semantic models for QT^m and QS_4^m so that only the converse Barcan and not the Barcan formula remains true. This can be done by placing restrictions on the domain function D^m, limiting it to D^{m-}, which in $D^{m-}(W^m_i)$ assigns to W^m_i a domain of *existent* objects only, rather than a full Meinongian domain of existent and nonexistent objects. This proposal also restores convergence of inferential and semantic, structures to QT^{m-} and QS_4^{m-}, preserving intact the inferential isomorphism between quantificational standard and Meinongian modal logics. (None of these solutions to the semantic and inferential incongruities of QT^m and QS_4^m over the truth and derivability of the unmodified Barcan formula require adjustment to the theoremhood containment relations among quantificational and nonquantificational modal Meinongian logics.)

The existential intent of the Barcan formula cannot be expressed in unqualified quantificational modal Meinongian logic except by an extranuclear existence predication:

$$O(\exists x)(E!x \ \& \ Px) \supset (\exists x)(E!x \ \& \ O Px)$$

This complex expression cannot be derived in modal Meinongian logic without supplementary addition of further specific nonlogical axioms about the nature of the extranuclear existence property E!. It remains to be seen whether and in which systems of quantificational modal Meinongian logic this version of the Barcan formula or its converse are theorems.

The unmodified converse Barcan formula can be proved in QT^m (and hence in QS_4^m, QS_5^m, and QB^m). The implementation of necessitation rule (NR) in a natural deduction environment is that whenever there is a proof of p (not subordinate to any other proof), then there is a proof of $\Box p$.([15])

(MT1) $(\exists x)\Diamond Px \supset \Diamond(\exists x)Px$

1.	$\Box(\forall x){\sim}Px$	
2.	$\Box(\forall x){\sim}Px \supset (\forall x){\sim}Px$	(M4)
3.	$(\forall x){\sim}Px$	(1,2)
4.	${\sim}Po_i$	(3)
5.	$\Box{\sim}Po_i$	(1-4,NR)
6.	$(\forall x)\Box{\sim}Px$	(5)
7.	$\Box(\forall x){\sim}Px \supset (\forall x)\Box{\sim}Px$	(6)
8.	${\sim}(\forall x)\Box{\sim}Px \supset {\sim}\Box(\forall x){\sim}Px$	(7)
9.	$(\exists x)\Diamond Px \supset \Diamond(\exists x)Px$	(8,M1,M2)

The unmodified Barcan formula is derivable in unqualified quantificational modal Meinongian logic QS_5^m. The QS_5^m proof is unavailable in QT^m and QS_4^m, as indicated by appeal to characteristic QS_5^m axiom (M7) in steps (13) and (33). This completes discussion of the problem of quantifying into modal contexts in modal Meinongian logic.([16])

(MT2) $\Diamond(\exists x)Px \supset (\exists x)\Diamond Px$

1.	$(\forall x)\Box{\sim}Px$	
2.	$\Box{\sim}Po_i$	(1)
3.	$(\forall x)\Box{\sim}Px \supset \Box{\sim}Po_i$	(2)

4.	$\sim\Box\sim Po_i \supset \sim(\forall x)\Box\sim Px$	(3)
5.	$\Box(\sim\Box\sim Po_i \supset \sim(\forall x)\Box\sim Px)$	(1-4,NR)
6.	$\Box(\sim\Box\sim Po_i \supset \sim(\forall x)\Box\sim Px) \supset$ $(\Box\sim\Box\sim Po_i \supset \Box\sim(\forall x)\Box\sim Px)$	(M5)
7.	$\Box\sim\Box\sim Po_i \supset \Box\sim(\forall x)\Box\sim Px$	(5,6)
8.	$\sim\Box\sim(\forall x)\Box\sim Px \supset \sim\Box\sim\Box\sim Po_i$	(7)
9.	$\Diamond(\forall x)\Box\sim Px \supset \sim\Box\sim\Box\sim Po_i$	(8,M2)
10.	$\Box\sim Po_i \supset \sim Po_i$	(M4)
11.	$\sim\sim Po_i \supset \sim\Box\sim Po_i$	(10)
12.	$Po_i \supset \Diamond Po_i$	(11,M2)
13.	$\Diamond Po_i \supset \Box\Diamond Po_i$	(M7)
14.	$Po_i \supset \Box\Diamond Po_i$	(12,13)
15.	$\Box\Diamond Po_i \supset \sim\Diamond\Box\sim Po_i$	(M1,M2)
16.	$Po_i \supset \sim\Diamond\Box\sim Po_i$	(14,15)
17.	$\sim\sim\Diamond\Box\sim Po_i \supset \sim Po_i$	(16)
18.	$\Box\Diamond Po_i \supset \Diamond Po_i$	(M4)
19.	$\sim\Diamond Po_i \supset \sim\Box\Diamond Po_i$	(18)
20.	$\Box\sim Po_i \supset \Diamond\Box\sim Po_i$	(19,M1,M2)
21.	$\Diamond\Box\sim Po_i$	(2,20)
22.	$\sim\sim\Diamond\Box\sim Po_i$	(21)
23.	$\sim Po_i$	(16,22)
24.	$(\forall x)\sim Px$	(23)
25.	$\Diamond(\forall x)\Box\sim Px \supset (\forall x)\sim Px$	(24)
26.	$\Box(\Diamond(\forall x)\Box\sim Px \supset (\forall x)\sim Px)$	(6-25,NR)
27.	$\Box(\Diamond(\forall x)\Box\sim Px \supset (\forall x)\sim Px) \supset$ $(\Box\Diamond(\forall x)\Box\sim Px \supset \Box(\forall x)\sim Px)$	(M5)
28.	$\Box\Diamond(\forall x)\Box\sim Px \supset \Box(\forall x)\sim Px$	(26,27)
29.	$\Box\sim(\forall x)\Box\sim Px \supset \sim(\forall x)\Box\sim Px$	(M4)
30.	$\sim\sim(\forall x)\Box\sim Px \supset \sim\Box\sim(\forall x)\Box\sim Px$	(29)
31.	$(\forall x)\Box\sim Px \supset \Diamond(\forall x)\Box\sim Px$	(30,M2)
32.	$\Diamond(\forall x)\Box\sim Px$	(1,31)
33.	$\Diamond(\forall x)\Box\sim Px \supset \Box\Diamond(\forall x)\Box\sim Px$	(M7)
34.	$\Box\Diamond(\forall x)\Box\sim Px$	(32,33)
35.	$\Box(\forall x)\sim Px$	(28,34)
36.	$(\forall x)\Box\sim Px \supset \Box(\forall x)\sim Px$	(35)
37.	$\sim\Box(\forall x)\sim Px \supset \sim(\forall x)\Box\sim Px$	(36)
38.	$\Diamond(\exists x)Px \supset (\exists x)\Diamond Px$	(37,M2)

V. Transworld Identity of Incomplete and Impossible Meinongian Objects

The possible existence of incomplete Meinongian objects presents a special problem for modal object theory semantics. Consider the proposition that the golden mountain is possible or possibly exists. The semantics for modal Meinongian logic interprets possible existence as existence in some world or worlds accessible to the actual world. In the actual world the golden mountain is incomplete, lacking many nuclear properties and their complements in its uniquely identifying ***Sosein.*** But in a world containing the golden mountain as actual and not a mere Meinongian object, the golden mountain exists and is complete, with a full selection of nuclear properties including exclusively every nuclear property or its complement, featuring especially the nuclear properties of being golden and a mountain.

This suggests that some worlds may contain complete existent objects that are incomplete in other logically possible worlds. There seems nothing paradoxical or metaphysically unacceptable about this. It is natural to suppose that if a square table had also been round, then instead of existing it would be an impossible round square table. If the round square table had not been square, it might exist. Again there appears to be no limit (beyond essential property or natural kind restrictions) to any combination of nuclear properties among possible, actually existent, or nonexistent Meinongian objects in different worlds. But it might be objected that this latitudinarian approach to transworld identity for incomplete and impossible Meinongian objects in the domains of alternative accessible Meinongian worlds implies that an impossible object like the round square is possible after all, in the sense that there are worlds in which the round square is not round or not square. If this were true, it might preclude the intelligible categorization of any Meinongian objects as impossible.

Routley has challenged the intuitive picture of transworld identity among existent, incomplete, and impossible Meinongian objects, by arguing that an object incomplete in a given world is essentially incompleteor incomplete in every logically possible world. He writes:

> Consider, for instance, the round squash: as a pure deductively (unclosed) object this is round and a squash and has no other properties. Thus it is incomplete, e.g. it is neither blue nor not blue. Hence it does not exist. Nor can *it* exist: to exist it would have to be completed, but any such completion is a different object.([17])

The modal Meinongian counterpart theory developed by Routley in accord with this criticism is like the standard counterpart modal logics described by Leibniz and David Lewis.[18] But Routley's version of counterpart modal Meinongian logic is different in that he seems to permit transworld identity of existent and nonexistent objects, provided that no existent object in a given world is nonexistent in another accessible logically possible world, or conversely. This posits an ordinary counterpart theory for Meinongian objects restricted to contingently existent or nonexistent objects.[19]

Routley's proposal contradicts well-entrenched beliefs about the possible existence of contingently nonexistent objects. When someone says that Pym in Edgar Allen Poe's *The* Narrative of Arthur Gordon *Pym* is possible, that he is a person who might have lived and had the adventures attributed to him in Poe's story, it is undoubtedly meant that the very same object described by Poe and not merely another relevantly like him is possible, even though Pym in the actual world is incomplete and indeterminate with respect to many nuclear properties and their complements. If it were true that actually incomplete objects are incomplete in every logically possible world, then modal object theory would be exceedingly uninteresting. It would then be possible only for actually existent or nonexistent objects to have different complete or incomplete sets of nuclear properties than they happen to have in the actual world (and even this might be prohihited hy strict adherence to Routley's criterion). But if objects are identified and distinguished by the unique unordered sets of constitutive nuclear properties in their Sosein, then it remains at least a technical problem to explain how Meinongian objects could be incomplete or impossible in some logically possible worlds, but complete and existent in others.

The difficulty is removed by indexing an object's nuclear properties to particular worlds.[20] An analogous problem arises for the indiscernibility of identicals over time. The objection is sometimes made that a man cannot be identical to his youthful self if the man is bald and the youth is not. But this is a superficial criticism of the identity principle overcome by requiring that properties are incompletely and incorrectly specified unless indexed to time. The man does not have the property of being bald simpliciter, but the property of being bald at time t. The youth does not have the complement of the property of being bald simpliciter, but has the complement of the property at time t' ($\neq t$). The indiscernibility of identicals is not contradicted by the example on this reformulation because both the old man and the youth have the properties of being bald at t and not bald at t'.

The same idea enables modal Meinongian logic to include objects in the domains of its semantic models that are complete in some worlds, but incomplete or even impossible in others.(21) According to the world-indexing proposal, Arthur Gordon Pym does not simply have the nuclear property of being a shipwrecked cannibal, he has the nuclear property of being a shipwrecked cannibal in Meinongian world W^m_{Poe} (and other worlds of the modal Meinongian semantic model). Pym does not simply lack the nuclear property of speaking Italian, he lacks both this property and its complement in the actual world, and in some but not all alternative logically possible worlds. The world-indexing solution to the transworld identity problem for nonexistent Meinongian objects does not entail that the round square is not impossible, but only that the Meinongian object which in or relative to some logically possible worlds is an impossible round square is a possible round object in or relative to other logically possible worlds in which it is not square, and in other worlds a possible square object that is not round.(22)

In this way, the very same object, the man described in Poe's tales, can correctly be said to be possible, or such that he might have existed in the actual world. He is an incomplete object in or relative to some worlds, but in others be exists and is fully determinate. Pym has both the incomplete set of properties completely characterized by Poe in the actual world, and the complete set of properties partially characterized by Poe in some of the fictional logically possible worlds in which Pym exists. By similar token, Edgar Allen Poe, though complete and existent in the actual world, is in some worlds a fictional, incomplete, and nonexistent but logically possible Meinongian object — in some worlds he is the literary inventionof Pym!(23)

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Notes

(1) It is convenient but unnecessary to refer to maximally consistent sets of propositions as 'worlds'. The problem of whether nonactual logically possible worlds exist has motivated attempts to eliminate reference to worlds in standard modal semantics. See Leblanc [1973], pp. 241-59. In Meinongian semantics, logically possible worlds need not exist or subsist in order meaningfully to enter into interpretations of modal logic.

(2) The model set theoretical semantics are derived from Kripke, [1959], pp. 1-14; [1963a], pp. 67-96; [1963b], pp. 83-94.

([3]) Axioms for standard modal systems are found in Feys, [1937], pp. 517-53; [1938], pp. 217-52. Gödel, [1933], pp. 34-40. Sobocinski, [1953], pp. 171-78. Lewis and Langford, [1932]. See Hughes and Cresswell, [1980], pp. 31, 46, 49, 58. Lemmon (with Scott), [1977], pp. 20-78.
([4]) Meinong, [1960], p. 82.
([5]) Ibid., pp. 83-6. Lambert, [1983].
([6]) Snyder, [1971], pp. 166-89.
([7]) Birkoff, [1967], pp. 244-45.
([8]) Rescher and Brandom, [1979], pp. 92-8, 158-59.
([9]) That the ∃ 'existential' quantifier has no real existential or ontological import in Meinongian semantics is also aftirmed by Parsons, [1980], pp. 69-70, and Routley, [1981], p. 174.
([10]) Barcan Marcus, [1946], pp. 1-16.
([12]) Hughes and Cresswell, [1968], p. 142. Lemmon, [1960], pp. 391-92.
([13]) Snyder, [1971], pp. 143-51. The modal semantic theories of some versions of logical atomism also posit a uniform distribution of existents across every logically possible world (usually in different terminology). See Wittgenstein, *Tractatus Logico-Philosophicus*, 2.014-2.0231. Wittgenstein's theory does not provide transworld uniform populations of complex existents.
([14]) The predominance of modal Meinongian versions of S_5 is suggested by Parsons, [1980], pp. 100-3. Zalta, [1988], pp. 61-7. Routley favors a quantificational version of Lewis' S_2 in [1981], pp. 207-21.
([15]) Adding the Barcan formula to standard quantificational versions of T and S_4 without strengthening them to quantificational S_5 is proposed by Hughes and Cresswell, [1968], p. 144.
([16]) A simpler 19-step proof of the Barcan formula in QB^m, using (M8) instead of (M4), can be transcribed from a similar proof in non-Meinongian quantificational Brouwersche system QB in Hughes and Cresswell [1968], p. 145. I am grateful to a consultant of *Logique et Analyse* for bringing this to my attention.
([17]) Hughes and Cresswell, [1968], Appendix I, 'Natural Deduction and Modal Systems', p. 333.
([18]) Routley, [1981], p. 247.
([19]) Leibniz, [1685]; [1846]. Lewis, [1973].
([20]) Routley, [1981], pp. 247-53.
([21]) World-indexing is proposed as a solution to problems of transworld identity for standard modal logics by Plantinga, [1974], pp. 92-7.

([22]) Meinong has a different approach to the possible existence of actually nonexistent objects that avoids the need for transworld identity of incomplete objects. Meinong argues that incomplete but possible objects have implexive being or are implected [*implektiert*] in existent or possible complete objects. The possibility of the incomplete golden mountain is explained on this proposal by the claim that all nuclear properties of the golden mountain are shared by another possible complete existent object, subsumed in its larger complete set of properties. The incomplete object is not literally a part of the possible complete object in which it is implected, but its possibility is accounted for by the claim that the possible complete object absorbs the incomplete object's smaller complement of nuclear properties as a subset. Meinong, [1915], pp. 211-24. Findlay, [1963], pp. 168-70, 181-82,209-15. Although Meinong's thesis is in some sense an alternative to transworld identity and counterpart modal semantics, it resembles counterpart theory in that the (incomplete) golden mountain is not literally identical to any possible complete object nor to any complete object in any logically possible world. The same arguments raised against counterpart semantics therefore also apply to Meinong's theory of implexive being.

([23]) It might be objected that the world-indexing solution to transworld identity of actually nonexistent objects invites a certain kind of confusion. Consider three worlds, W, W, *W''*. World W contains the round square table, T_1. By stipulation in W it might lack the property of being square while gaining other compatible nuclear properties, so that in W it exists as an actual complete round table, or at least as an incomplete but possible round table. In W', the table might lack the property being round while gaining other compatible nuclear properties, so that in *W''* it exists as an actual complete square table, or at least as an incomplete but possible square table. There presumably is also a nonexistent incomplete object T_2 that has just the nuclear properties of being round, square, and a table, in all three worlds. The round square table T_1 is arguably referentially identical to the round square table T_2 in W where they share all nuclear non-converse-intentional properties, but nonidentical to T_2 in W and *W''*, where they do not. This would violate intuitive conceptions of identity, especially if 'T_1' and 'T_2' are supposed to be rigid designators. If we take the world-indexing approach seriously, then there is an easy solution to the apparent problem. The world-indexed properties of the two objects keep them distinct, where $T_1 = RST_W\text{-}RT_{W'}\text{-}ST_{W''}$ and $T_2 = RST_W\text{-}RST_{W'}\text{-}RST_{W''}$. These complex rigidly designative terms preserve transworld distinctions between T_1 and T_2, while accounting for their exact coincidence of properties in some logically possible worlds.

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