A PRAGMATIC REQUIREMENT FOR CLASSICALLY VALID ARGUMENTS

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One context in which argument occurs is when one person, say Bob, asks for reasons to accept a statement, say 'Q', and somebody else, Ann, gives him a reason, say 'P'. Logic can adjudicate some questions about an exchange of this kind, but not all. We may require that the argument 'P, so Q' be valid; but we cannot by logic alone ensure that 'P' is true. There are, however, aspects of this interchange susceptible to logical analysis which go well beyond mere validity, even though they do not extend to the truth of 'P'; and these aspects are rarely discussed in any detail by logicians. It is my purpose in the present paper to consider one of these aspects. Neglect of it has, I believe, contributed to the suspicion of classical logic prevalent among some logicians recently.

The classical definition of 'implication' is that 'P' implies 'Q' just in case it is logically impossible that both 'P' is true and 'Q' is false. But if 'P' is self-contradictory, then it is logically impossible for 'P' to be true; and hence the definition has the consequence, which some logicians find to be objectionable, that a self-contradiction implies any statement whatsoever. Indeed, in a fourteenth century work formerly attributed to John Duns Scotus, but now thought to be by one of his students (known therefore as the Pseudo-Scot), a formal proof was given that a statement of the form '$P \& \sim P$' implies an arbitrary statement 'Q'. (references in Kneale /1/ *pp.* 281*ff*.). Nonetheless, the classical definition of 'implication' does not entail that if Bob asks for an argument for 'Q' Ann's reply will be logically satisfactory if it is self-contradictory. There is demonstrably more to giving a logical reply to such a request than merely saying something which implies 'Q', or even something which is possibly true and which implies 'Q'. To show this, we must examine the logical structure of the context: its pragmatics.

We suppose Bob to have accepted, at the stage n at which he presents his request for an argument for 'Q', some finite, possibly empty, consistent set B_n of sentences. The set $\{s : B_n \vdash s\}$ may be regarded as Bob's theory at n of the field he is investigating. If B_n is not consistent, Ann should be pointing this out to Bob, rather than adding to his confusion. He asks Ann for reasons to accept 'Q'. Here 'Q' may but need not be among the things he has hitherto accepted. Immediately after he asks this, *i. e.* at stage $n+1$, 'Q' is not among the sentences Bob is now accepting; if it was in B_n it should be removed to get B_{n+1}. Remembering that B_n is a set of sentences (and not, for example, of propositions), we can achieve this effect by relative complementation:

$$(1)\quad B_{n+1} = B_n - Q$$

(In set-theoretical contexts, braces and quotes around schematic letters holding place for sentences have been suppressed.)

Let us suppose Ann to respond positively by giving an argument for 'Q'. It is, we may suppose, some sentence (or conjunction of sentences) 'P'. By offering 'P' in this context, she is implicitly affirming that if P then Q, in other words she is committing herself to the truth of '$P \supset Q$'. We abbreviate this conditional as 'R', and construe her argument as 'P, R, so Q'. Inferential validity is thus assured. Of course if Bob doubts her explicit premiss 'P', he may go on to question it; and if he thinks her conclusion 'Q' does not follow from 'P', he may question 'R'. Until he does one or the other, and in particular immediately after her response, we construe him as accepting them. Thus at that stage,

$$(2)\quad B_{n+2} = B_{n+1} \cup P, R = (B_n - Q) \cup P, R$$

There is one further condition on this exchange. If there were in B_n sentences other than 'Q' from which 'Q' followed, Bob's challenge of 'Q' would have been inconsistent, and the appropriate response for Ann would have been to point out this inconsistency ("But you already know why Q") rather than to give an argument for 'Q'. Since she did not do this, we can conclude that he was not being inconsistent; that is, that

$$(3)\quad B_n - Q \nvdash Q$$

It is clear that $B_{n+2} \vdash Q$, given (2). But we must also require of Ann's reply that it should leave Bob in a consistent position; that

$B_{n+2} \nvdash \sim Q$. Otherwise her argument for 'Q' would not have given Bob reason to accept 'Q' rather than '$\sim Q$'.

A simple argument is sufficient to show that this result is achieved by requiring that Bob's store before he challenged 'Q', together with 'Q' itself, does not imply the negation of Ann's response 'P'; in other words

(4) $B_{n+2} \nvdash \sim Q \equiv B_n \cup Q \nvdash \sim P$

To show that if $B_n \cup Q \nvdash \sim P$ then $B_{n+2} \nvdash \sim Q$, we assume (a) $B_n \cup Q \nvdash \sim P$ and (b) $B_{n+2} \vdash \sim Q$. From (b) and (2) we get $B_{n+1} \cup P, R \vdash \sim Q$. By the deduction theorem this gives $B_{n+1} \cup P \vdash R \supset \sim Q$, which by truth tables gives $B_{n+1} \cup P \vdash \sim Q$. By the deduction theorem again this gives $B_{n+1} \vdash P \supset \sim Q$, which is $B_{n+1} \vdash Q \supset \sim P$, which by the deduction theorem again gives $B_{n+1} \cup Q \vdash \sim P$, contra assumption (a).

To show that if $B_{n+2} \nvdash \sim Q$ then $B_n \cup Q \nvdash \sim P$, we assume that (c) $B_{n+2} \nvdash \sim Q$ and (d) $B_n \cup Q \vdash \sim P$, and consider two cases. Case (*i*) $Q \notin B_n$. From (d) and the deduction theorem $B_n \vdash Q \supset \sim P$. By (*i*), $B_n = B_{n+1}$, so $B_{n+1} \vdash Q \supset \sim P$, which can be weakened to $B_{n+1} \cup P, R \vdash Q \supset \sim P$. Of course $P, R \vdash Q$, so $B_{n+1} \cup P, R \vdash \sim P$, which by (2) is $B_{n+2} \vdash \sim P$. But by (2) again, $P \in B_{n+2}$, so $B_{n+2} \vdash P \,\&\, \sim P$, which by the Pseudo-Scot's argument yields $B_{n+2} \vdash \sim Q$, contra assumption (c).

Case (*ii*) $Q \in B_n$. From this $B_n \cup Q = B_n$, so by (d) $B_n \vdash \sim P$. By the deduction theorem $B_n - Q \vdash Q \supset \sim P$, which by (1) is $B_{n+1} \vdash Q \supset \sim P$, which can be weakened to $B_{n+2} \vdash Q \supset \sim P$. Now $P, R \vdash Q$, which can be weakened to give $B_{n+2} \vdash Q$; so $B_{n+2} \vdash \sim P$. But $P \in B_{n+2}$, so $B_{n+2} \vdash P \,\&\, \sim P$, which by the Pseudo-Scot's argument again gives $B_{n+2} \vdash \sim Q$, contra assumption (c).

Thus when Bob, committed to statements B_n, asks for reasons to accept 'Q', Ann's response 'Because P' will not be logically satisfactory unless Bob's commitments B_n together with 'Q' do not imply the negation of her response. In particular, if 'P' is a self-contradiction, so that its negation is classically implied by any set of sentences, then her reply will not be acceptable. This is the pragmatic result which preserves classical logic from the absurd consequences alleged against it by the relevantists.

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REFERENCE

William and Martha Kneale, *The Development of Logic*, Oxford Clarendon 1962.