BURIDAN ON INTERVAL SEMANTICS FOR TEMPORAL LOGIC

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In a recent paper(1) I. L. Humberstone has presented a tense logic based on temporal intervals. In this paper he points out that it is important to distinguish between two different kinds of negation in order to formulate an adequate temporal logic based on durations.

In their papers(2),(3) Peter Röper and John P. Burgess have developed these ideas further.

However, the idea of a temporal logic based on durations is not new. It is very interesting that Buridan in his work "Sophismata"(4) formulated a concept of the truth of a proposition during an interval in time. In fact "Sophismata" includes a special chapter in which Buridan deals with the semantics of durational logic. In this paper I intend to outline the basic ideas of Buridan regarding these logical ideas.

1. The definition of truth

There is one fundamental difference between the approach of the modern authors mentioned above and that of Buridan. The modern authors consider a proposition to be true over an interval in time of and only if it is true over all subintervals. Buridan, however, defined the truth of a proposition in the following way:

(1) I. L. HUMBERSTONE, Interval Semantics for Tense Logic: Some Remarks, Jour. of Phil. Logic 8 (1979) p. 171-96.

(2) Peter ROPER, Intervals and Tenses, Jour. of Phil. Logic 9 (1980) p. 451-69.

(3) John P. BURGESS, Axioms for Tense Logic, II: Time Periods, Notre Dame Jour. of Formal Logic, Vol 23, Number 4 (1982).

(4) Johannes BURIDANUS, Sophismata (critical edition with an introduction by T. K. Scott) Frommann-Holzboog 1977.

> "Thus, if in one part of the present time, Socrates stands(5) or is white or is dead, it is simply true to say that he stands or is white or is dead."(6)

According to Buridan "the present" is not a point in time, but it is a duration. It is not determined a priori which duration we ought to use as "the present", but we are allowed to use this hour, this day, this month, this year, etc... Buridan's concept of truth is obviously relative to a choice of "the present". That is: it only makes sense to talk about the truth of a proposition in general if the present is specified.

Let p be the proposition "Socrates stands" and let x be the present time. According to the above definition p is true during x if and only if there exists at least one part of x during which p is true i.e. during which Socrates is in fact standing.

2. *Two kinds of negation*

If in one part of the present time Aristotle is alive and if in another part of the present time he is dead, Buridan is obliged to accept the truth of the conjunction "Aristotle is alive and he is dead". This seems to be a violation of the principle of contradiction.

In order to solve the problem it is very important for Buridan to distinguish between affirmative and negative propositions. He confirms that an affirmative proposition is true if and only if the corresponding negative proposition is false. That is: "S is P" is true if and only if "S is not P" is false. For this reason the conjunctions "S is P and S is not P" and "Aristotle is alive and he is not alive" can never be true. But if this is so, how can there be a duration for which the conjunction "Aristotle is alive and he is dead" is true? Buridan solves the problem by pointing out that while "Aristotle is dead" is an

(5) Corresponding to the critical edition I read "stat" instead of "sedet", which is used in Scott's translation.

(6) John BURIDAN, Sophisms On Meaning and Truth, New York 1966 (translated by T..K. Scott).

affirmative proposition, "Aristotle is not alive" is a negative proposition. Obviously there are two kinds of negation involved in the temporal logic of Buridan:

1) Negation of predicates e.g. "non-alive" (= "dead") is the negation of "alive".
2) Negation of propositions e.g. "Aristotle is not alive" is the negation of the proposition "Aristotle is alive".

In order to symbolize this distinction we need an operator I corresponding to the verb in the proposition. Since 1) is some kind of "inner negation" it seems natural to symbolize it as $I \sim p$, where as 2) is the usual negation: $\sim Ip$.

Let Ip be the proposition "Aristotle is alive". From this proposition we can obviously form three others:

$\sim Ip$ symbolizing "Aristotle is not alive",
$I \sim p$ symbolizing "Aristotle is non-alive (dead)",
$\sim I \sim p$ symbolizing "Aristotle is not non-alive".

It follows from the definition of truth and the principle of contradiction that a negative proposition, $\sim Iq$, is true for a duration, x, if and only if there is no part of x for which Iq is true.

3. *The calculus of I*

It is easy to verify that $\sim I \sim p$ is true for a duration x if and only if Ip is true for all parts of x. Therefore if the truth of $\sim I \sim p$ is given for some duration, it follows that Ip is true for the same duration. That is, the implication

$$\sim I \sim p \supset Ip$$

is a valid thesis in Buridan's temporal logic. But since $Ip \wedge I \sim p$ can be true for some duration, the opposite implication is not valid in general.

According to the definition of truth the proposition $I(p \wedge q)$ is true for some duration x if and only if there is some part of x for which Ip and Iq are both true. For this reason the implication

$$I(p \wedge q) \supset (Ip \wedge Iq)$$

is a valid thesis in Buridan's temporal logic, whereas the opposite implication is not valid.

Regarding repeated use of the operator I it seems natural to interpret IIp as "there is some part of the present time of which there is some part for which p is true". The interpretations of $I\sim Ip$ and $I\sim I\sim p$ are similar. It is easy to verify that the following equivalence is valid if time is dense:

$$IIp \equiv Ip$$

According to the definition of truth a proposition Ip is true for an interval in time, x, if and only if there exists a subinterval, y, of x, so that the proposition is true during y i.e. Ip is true for all parts of y. For this reason it is obvious that the following equivalence is a thesis:

$$Ip \equiv I\sim I\sim p$$

4. The logic of tenses

It is obvious that Buridan takes it for granted that the tense-distinctions (past, present, future) are important to logical reflection. But he is also aware of the fact that a logic of tenses which has to pay due regard to a logic of durations is very complicated. For this reason, probably, he is content to sketch his ideas of tense logic.

Buridan suggests two alternative ideas for the construction of the logic of tenses. The first of these leads to a very natural semantics. The tenses, past and future, are taken absolutely, in the sense that no part of the present time is said to be past or future. If p is an arbitrary proposition and if F and P are the usual tense operators corresponding to "it will be that..." and "it has been that...", the absolute definitions are as follows:

Fp is true for a duration x if and only if there is some duration, y, entirely after x, so that Ip is true for y.
Pp is true for a duration x if and only if there is some duration, y, entirely before x, so that Ip is true for y.

Buridan makes no attempt to formulate the semantics for these tense operators, but he maintains that if tenses are taken in an absolute sense, then the Aristotelian proposition "All which is moved was moved previously" cannot be true. That is, the implication

$$Iq \supset Pq$$

is not a valid thesis in Buridan's temporal logic. This is obviously correct. It is also clear that $Iq \supset Fq$ is invalid in general.

On the other hand, the Aristotelian proposition is a valid thesis if the tenses are taken in the relative sense. In that case we have to use the following definitions:

$F_{rel}p$ is true for a duration x if and only if there is some part, y, of x for which Fp is true.
$P_{rel}p$ is true for a duration x if and only if there is some part, y, of x for which Pp is true.

It is remarkable that these relative tenses can be expressed by means of the operator I:

$$F_{rel}q \equiv IFq \qquad P_{rel}q \equiv IPq$$

Utilizing the equivalence for P_{rel} the Aristotelian proposition with relative tenses can be formulated as follows:

$$Iq \supset IPq$$

If time is dense this implication is obviously a valid thesis. This is also true for the implication:

$$Iq \supset IFq$$

5. *Conclusion*

It should be noted that the theses $Iq \supset IPq$ and $Iq \supset IFq$ correspond to some very important theses in the modern axiomatic systems of durational logic. The basic ideas in these modern systems are, however, different from Buridan's ideas.

I am inclined to believe that the search for an adequate durational logic should take Buridan's ideas into consideration.

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