THE STRANGE MODAL LOGIC OF INDETERMINACY

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Think of "indeterminately" as a modal operator ∇.

Imagine that the vagueness of objects requires at least one proposition A to be indeterminate in truth-value. Imagine that this fact may be captured by the true and hence determinate proposition ∇A.

Then, thinking of "determinately" as the dual operator Δ, the modal logic I of indeterminacy will have as a thesis

(a) $\nabla A \rightarrow \Delta\nabla A$

which is the characteristic axiom of S5. ∇ and Δ will indeed be duals in I because every proposition will be determinate iff not indeterminate, and determinate (indeterminate) iff its negation is also.

Gareth Evans [Analysis *38* (1979) 208] (1) seems to suggest that I is at least as strong as S5. But this it cannot be. For though I can be expected to have the thesis

(b) $\Delta(A \rightarrow B) \rightarrow (\Delta A \rightarrow \Delta B)$

and the rule (where PC is the classical predicate calculus with identity)

(1) Recent responses to Evans' article fall into two classes. There are those that accept argument and conclusion. Thus Nathan Salmon in his *Reference and Essence*, Blackwell, Oxford (1982) pp. 243-4 provides the reader with an informal versions of them. HW Noonan, in his 'Vague Objects', Analysis (1982) *42* pp. 3-6 repeats Evans' argument, though with some expanion.
Then there are those that accept neither argument nor conclusion. Thus Richmond H Thomason in his 'Identity and Vagueness' Phil. Studies. (1982) *42* pp. 329-332 offers an alternative account of vague identity in which Evans' argument is exposed as invalid. The spirit of my paper is closer to those of the first class. But my worry is methodological. Should one set about the deep problem of vagueness in this (admittedly fashionable) way?

(c) $\vdash_{\overline{PC}}$ A implies $\vdash_{\overline{I}}$ ΔA,

nevertheless I must lack the *axiom of determinacy*

(d) $\Delta A \rightarrow A$

and its dual

(e) $A \rightarrow \nabla A$.

For some propositions may be determinately false and some true without being indeterminate.

Evans' aim is to refute the claim that objects may be vague by showing that indeterminate identity-statements like

(1) $\nabla(a = b)$

are self-contradictory. His argument is more than reminiscent of Kripke's for the necessity of identity-statements. However, Evans' appeal to the determinacy of self-identity

(3) $\sim \nabla(a = a)$

and to Leibniz' Law enables him to derive directly only

(5) $\sim(a = b)$.

But if ∇ behaves like "possibly", it is no more obvious that (5) contradicts (1) than that "$\sim$A" contradicts "possibly A" for arbitrary A.

Evans therefore *seems* to need the assumption that I is at least as strong as S5 in order to derive his

(5′) $\Delta \sim(a = b)$

which is "straightforwardly inconsistent with (1)" in view of the duality of ∇ and Δ. For, given this assumption, (1) implies

(1′) $\Delta\nabla(a = b)$

using (a), and (3) implies

(3′) $\Delta \sim \nabla(a = a)$

using $\Delta \sim(a = a)$, the equivalent of (3) via duality, and $\Delta A \rightarrow \Delta\Delta A$, the characteristic axiom of S4 contained in S5. Prefixing Leibniz' Law with Δ then allows Evans to infer his (5′). The argument is unduly roundabout. I is not as strong as S5. But, as it happens, this does not matter. For notice that (5) is true, therefore using the expected I-thesis $A \rightarrow \Delta A$ we can infer

$$(5'') \quad \Delta \sim (a = b)$$

which, by duality, straightforwardly contradicts (1).

What then is the modal logic of indeterminacy if not a modal logic as strong as S5? How can $\nabla A \rightarrow \Delta \nabla A$, $\Delta A \rightarrow \Delta\Delta A$ and $A \rightarrow \Delta A$ be among its theses? There is a simple answer to these questions, though it is one which trivialises Evans' argument. It is this.

Let I be determined by the class C of *standard models* (following the terminology of Chellas, *Modal Logic*, Cambridge, 1980) whose accessibility relation is *empty*. Then one has for every A, $\models_{\overline{C}} \Delta A$ and $\models_{\overline{C}} \Delta \sim A$, and also $\models_{\overline{C}} \sim \nabla A$ and $\models_{\overline{C}} \sim \nabla \sim A$. (For ΔA (∇A) is true in a world ω in a standard model iff A is true in every (at least one) world in that model accessible to ω.) The expected theses (rules) of I are all valid (validity-preserving) in C. I is a *normal modal logic* whose characteristic axiom is the strange

$$(h) \quad \Delta A$$

The proofs of soundness and completeness of I with respect to C are routine and trivial.

Now one sees immediately that the indeterminate identity-statement (1) is an I-contradiction. The appeal to Leibniz' Law is redundant but, I think, of interest. For in I we have the derived rule

$$(DR) \quad \frac{\nabla(a = b)}{\nabla\phi a \leftrightarrow \nabla\phi b}$$

There is perhaps an ambiguity in formulae like $\nabla\phi a$ which is obscured by the notation convention. Strictly, $\nabla\phi a$ should be read as $\nabla(\phi a)$ since ∇ is a sentential operator. But the appeal to Leibniz' Law assumes that $(\nabla\phi x)$ is an indeterminate predicate denoting, presumably, an indeterminate property. But may properties be indeterminate, particularly if objects may not be?

Construing the extension in the world ω of the indeterminate predicate (∇ϕ)x as the set of objects in ω of which ϕ is true in some world accessible to ω, we justify the notation convention. But we do so only at the cost of making every indeterminate predicate have empty extension. If there are indeterminate properties, they inhere in nothing.

In our picture of indeterminacy (that is, on Evans' and mine, which I take to be an articulation of the former), there can indeed be no true indeterminate identity-statements. But indeterminacy generates a strange modal logic. The semantical business of there being classes of indeterminate worlds accessible to no worlds not even to themselves is strange and not intuitively attractive. The actual world is determinate because indeterminate but not accessible to itself. All of which suggests to me not that there can be no vague objects, but rather that the modal logic of indeterminacy, constructed as an extension of maximally determinate classical logic, affords a poor model for the deep idea of vagueness *de re*.

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