

## DIVIDING BY ZERO

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Among the many queer claims made by physicists, there are the following two.

- 1) If a material object moves at the velocity of light, then its mass is infinite.
- 2) If a virtual particle of a certain kind is emitted and reabsorbed, then its energy is infinite.

For only a few examples of such claims, the reader is referred to p. 132 of [1], p. 71 of [2], and p. 100 of [4]. It should be observed that there is no way to understand these examples as assertions of finite quantities increasing since they explicitly assert the existence of infinities. It is the purpose of the present note to call attention to this way of thinking, to make clear what it presupposes and thereby why it is absurd, and finally to suggest some alternatives to it. Of these, only one is due to the author. That all of this is needed is clear from the examples referred to above.

Both 1) and 2) can be explicated, or made precise, as conjunctions of two equations, one only curious and the other somewhat spectacular. First, some measure  $m$  of an object  $x$  is being expressed as a fraction  $r/0$  with  $r$  a positive real number. Formally, this must be translated into an equation like the following one.

3)  $m(x) = r/0$ .

Second,  $r/0$  is for some reason informally assumed to be infinitely large. This intuition is perhaps based on the notion that 0 goes infinitely many times into  $r$ . The formal assertion must be the following one.

4)  $r/0 = \infty$  ( $r$  a positive rest number).

It is also a definition of arithmetic that

- 5)  $x/y$  is indiscernible from the unique  $z$  such that  $y \cdot z = x$  for any real numbers  $x$  and  $y$ .

It should also be assumed that

- 6) If  $o$  is a binary operation on real numbers and  $z$  is not a real number, then  $yo z$  is not a real number, or at best 0.

In particular,  $0 \cdot \infty = 0$  and  $r \cdot \infty = \infty$  if  $r$  is a positive real number. The reader who recalls that the scalar product of two vectors and the product of an imaginary complex number with its conjugate are real numbers should note that these products are *not* operations on real numbers. Multiplication by  $\infty$  is of course not so either, but close enough since it satisfies 6). That is, it can be understood as an operation on real numbers applied elsewhere according to 6).

The claims 1) and 2) are strong evidence for that 3) through 5) and implicitly 6) are together considered to be quite proper by physicists and perhaps others. However, 5) and 6) imply that both 3) and 4) and thereby 1) and 2) are absurd. The reason is quite simple. By 5) and 6), there is *no* object  $z$ , numerical or of some other kind, which is the unique  $z$  such that  $r/0 = z$ . This is because no object  $z$  whatever satisfies  $0 \cdot z = r$  if  $r$  is a positive real number. Even if  $r$  were 0, there would be no unique  $z$  satisfying the identity since all real numbers do.

The logical way out is of course to swallow the indubitable consequences of 5) and 6) and to drop 3) and 4) for the following.

- 3') Neither  $m(x)$  nor  $r/0$  exists.

This, however, requires a modification of standard logics with their unnatural overly strong existence assumptions. For the needed logics, see [3]. Those who prefer standard logics must use  $=$  in place of indiscernibility (which is  $=$  or double non-existence) and add the clause  $y \neq 0$  in 5). 3) then becomes the following.

- 3'') Both  $m(x)$  and  $r/0$  are undefined or meaningless.

A more complex and artificial alternative is to identify  $m(x)$  and  $r/0$  with some arbitrarily selected null entity after making the changes in 5). The mysterious  $\infty$  is best avoided here because of its fallacious flavor of infinity. The null entities or entity employed should preferably also be real numbers and identical for the sake of elegance and

simplicity. The natural choice is then 0 and 3) becomes the following.

$$3''') \quad m(x) = 0 = r/0.$$

This choice is also of interest since 0 satisfies the two multiplication rules for  $\infty$  given above. Nevertheless, 3''') is misleading since it suggests that some magnitude has been measured or calculated to be 0. This is not what has happened. It has simply been *selected* to be 0. It is just this kind of confusion which all use of null entities generates. It is therefore best to use either 3'') or 3'). 1) and 2) are then converted into the modest statements that the mass and energy in question are undefined or non-existent.

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#### REFERENCES

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