ON COPI'S MISAPPLICATION OF A DECISION PROCEDURE

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In his widely-used text,(1) Professor Irving Copi provides an automatic procedure for determining the invalidity (and hence also the validity) of arguments formulated within the monadic predicate calculus. However, the procedure described will often lead to the decision that an argument is valid when it in fact is not. I will show that Professor Copi either must restrict the class of arguments to which his algorithm is applicable so that it applies only to those arguments which contain no occurrences of individual constants, or else he must amend his algorithm along the lines suggested in Part II of this paper.

I

Professor Copi initially considers arguments formulated within a language containing the usual logical symbols, individual constants, individual variables and monadic predicate letters, but not containing any function symbols. Each of these classes of symbols is to be understood in the usual way; we may also assume, along with Professor Copi, that no symbolic sentence contains free occurrences of a variable.

The procedure which is given consists essentially in eliminating quantifiers with respect to a domain of individuals, and then applying a truth-table test for invalidity. More specifically, the procedure involves interpreting a sentence of the form

$$(x)\,\Phi x$$

in a finite domain of objects

$$\{a, b, \ldots k\}$$

as a conjunction of the form

$$(\Phi a \cdot \Phi b \cdot \ldots \cdot \Phi k),$$

and interpreting a sentence of the form

$(\exists x)\, \Phi x$

in the same domain as a disjunction of the form

$(\Phi a \vee \Phi b \vee \ldots \vee \Phi k)$.

Let us call these the *Copi-expansions* of a universally quantified sentence and of an existentially quantified sentence, respectively.

Given a Copi-expansion of the premises and conclusion of an argument, a truth-table test will show whether or not the premises tautologically imply the conclusion. And an argument involving quantifiers is valid just in case every Copi-expansion of the argument is such that the premises tautologically imply the conclusion.

Professor Copi notes that different Copi-expansions of a given argument may lead to different results concerning the given arguments' validity, depending on the cardinality of the set of objects chosen as domain. In answer to the question of how large a domain must be considered before being assured that an argument involving quantifiers is either valid or invalid, he remarks:

> A theoretically satisfactory answer to this question has been found. If an argument contains n different predicate letters then if it is valid for a model containing 2^n individuals then it is valid for every model, or universally valid. This result holds only for propositional functions of one variable, and is not true of ... relational predicates ... (2)

Yet Professor Copi fails to make the additional restriction that such arguments must not contain occurrences of individual constants. Without this restriction, the following argument, containing only one predicate letter, is classified as universally valid by the Copi-expansion test in the theoretically crucial domain of two objects;

Fa
Fb
$\therefore (x)\, Fx$

For in the domain $\{a, b\}$ the Copi-expansion of this argument is

Fa
Fb
∴ (Fa · Fb),

in which the premises tautologically imply the conclusion. Yet the original argument is certainly *not* universally valid.

One reason that Copi's procedure gives incorrect results in connection with such arguments in this: in the result to which Professor Copi refers the word «valid» in the second antecedent must be understood in the model-theoretic sense of being true under every interpretation in a domain of 2^n individuals; it must not be understood as tautological implication of a Copi-expansion in a domain of 2^n individuals. For Copi-expansions, unlike model-theoretic interpretations, provide no rules for dealing with expressions containing occurrences of individual constants. Thus, while the *wff* '(Fa · Fb) ⊃ (x) Fx' is not 2-valid in the model-theoretic sense of being true under every interpretation in a domain of exactly two objects (since the terms 'a' and 'b' may be assigned the same denotation), its Copi-expansion must lead to tautological implication in such a domain, as we have just seen.

Therefore, as the procedure is stated by Professor Copi, it must be restricted in its application to arguments which do not contain occurrences of any terms other than (bound) variables. In order to remove this restriction, Copi will have to amend his algorithm along the lines suggested in the following section.

II

In order to amend his algorithm, there are a couple of alternatives open to Professor Copi. (3)

The alternative I favor is one which corresponds most closely to the model-theoretic result stated by Ackermann. The method, to be perfectly perspicuous, would require more meta-theoretic apparatus than Copi has at his disposal in the early part of his text, but is nonetheless simple in its application.

Given an argument in monadic predicate calculus which contains occurrences of individual constants, first choose a set of individual constants $\{c_1, c_2, \ldots, c_k\}$ which do not occur in the argument. Quantifiers may then be eliminated as before. Next, assign each constant symbol of the argument one of the c_i, perhaps the same c_i for different constants. (This third step introduces additional Copi-expansions to be considered). Finally, a truth-table test for tautological implication can be carried out. On this approach it remains true that, where n is the number of distinct predicate letters in an argument, if tautological implication is obtained with respect to every Copi-expansion for a set $\{c_1, \ldots, c_k\}$, where $k = 2^n$, then the argument is universally valid.

Thus, while the argument considered in the previous section had only one Copi-expansion (indeed, one in which 'a' and 'b' denote different elements of the domain), with the addition of step 3 above there are three additional expansions to be considered for tautological implication. And consideration of the expansion with respect to {c, d} such that

$$a: \rightarrow c$$

and

$$b: \rightarrow c$$

(and completing the other required steps) is sufficient to establish the invalidity of that argument.

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NOTES

(1) I.M. Copi, *Symbolic Logic* (4th Ed.). Macmillan Publishing Co., New York, 1973. The section under criticism is found on pp. 78-83.

(2) Copi, p. 81. Copi's reference for this result is to W. Ackermann, *Solvable Cases of the Decision Problem*, North-Holland Publishing Co., Amsterdam, 1954, Chap. IV.

(3) Another alternative, suggested by this Journal's anonymous referee, is to test the Copi-expansion in a set of $2^n + m$ individual constants, where n is the number of distinct predicate letters and m the number of distinct individual constants which occur in the given argument.